TWO DIMENSIONAL TRANSIENT HEAT TRANSFER IN HE II

J. M. Pfotenhauer and X. Huang
Applied Superconductivity Center
University of Wisconsin
1500 Johnson Drive
Madison, WI 53706

Abstract

A comparison is presented of transient heat flow in He II as measured experimentally and as predicted by analysis based on the Gorter-Mellink equation. The geometry is that envisioned for the He II cooling of a SMES system, viz, an annular layer of He II in direct contact with one layer of a solenoid and extending the full height of the coil. A normal zone over a fraction of a turn provides for two dimensional heat flow in the annular layer of He II. The comparison is given for both adiabatic and isothermal boundary conditions at the ends of the channel. We find good agreement between the analysis and the experimental data, and thereby verify the usefulness of the analysis for large scale systems. In addition, discrepancies between the analysis and data provide insight into the stability process of the He II cooled superconductor.

Introduction

Heat transfer in He II has been analyzed and researched extensively since the discovery of superfluid helium in 1940. However, whereas the aspects of heat transfer in one dimension have received abundant attention from both the physics and engineering communities, aspects of heat transfer in two or three dimensional geometries have been relatively untouched. The large effective thermal conductivity and rapid thermal diffusion of He II result in transient heat flow which is quite sensitive to the geometrical boundary conditions, yet the non-linear nature of the Gorter Mellink heat conduction equation prohibits analytical descriptions of that heat flow for all but the one dimensional linear geometry. On the other hand, the use of He II in engineering applications - notably, cooling of superconducting magnets - commonly involves geometries which easily diverge from simple one dimensional channels. One solution to this problem is to develop finite difference computational techniques to address the problem.

In this report we address the problem of transient heat flow in an annular layer of He II which is used to cool a superconducting solenoid. The geometry is the same as that envisioned for a Superconducting Magnetic Energy Storage (SMES) coil - but on a smaller scale - that of a laboratory size magnet. In particular, the annulus is thin when compared with its radius and extends vertically over the full height of the magnet. In this investigation heat is generated in the magnet in a horizontal strip at the mid height of the annulus and along a length which is some fraction of the circumference. Thus the heat flow is two dimensional, flowing from the heater towards the vertical ends of the annulus, and is symmetric vertically about the mid height and horizontally about the mid point of the heater.

Predictions of finite difference computations are compared with temperature measurements in the lab scale experiment. The obvious purpose of this comparison is to verify the accuracy of the computations, in order that the same computational method may be confidently used for large scale magnets. In addition, it is hoped that parametric investigations using the computations can provide general information about geometry dependent heat flow in He II which can be communicated without the use of computer codes.

Manuscript received August 22, 1988.

Features of Model

A detailed description of the computer model has been given elsewhere[1]; however, a few of the important features are worth repeating here. First, heat transfer in He II is governed by the Gorter Mellink relation

$$\vec{q} = [f^{-1}(T)]^{1/3} \frac{\vec{\nabla}T}{|\nabla T|^{2/3}} \tag{1}$$

where $\vec{q}$ is the heat flux, $\vec{\nabla}T$ is the temperature gradient in the direction of a streamline, and $f^{-1}(T)$ is an effective thermal conductivity function as given by Van Sciver.[2] Further it is assumed that $\vec{q}$ is always parallel to the temperature gradient $\vec{\nabla}T$. Beyond this the model assumes that the temperature at the heated surface is clamped at T_λ (= 2.163 K), there are no temperature gradients across the width of the annulus (i.e. this is a 2-D heat flow model rather than 3-D), and that heat flow in the horizontal direction at exactly one half turn away from the heater's center is zero. Boundary conditions at the vertical ends of the annulus can be set for adiabatic or isothermal conditions.

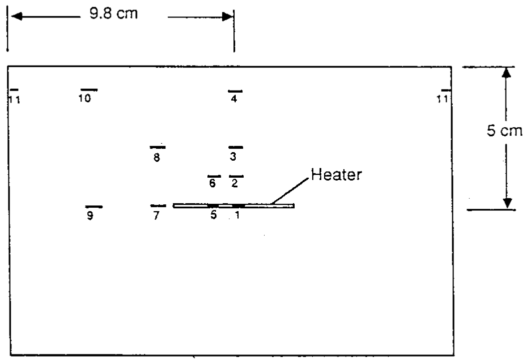
Finally it should be mentioned that the numerical instability inherent to an integration of the Gorter Mellink equation (see Dresner[3]) was avoided by keeping the incremental time step small enough to avoid oscillations in the temperature calculations greater than 5×10^{-4} K.

Experimental Parameters

Temperature measurements have been gathered during a superconductor stability experiment[4,5,6] in which a single layer solenoid is cooled on its inside diameter by an annulus of subcooled He II at 1.8 K and 0.1 MPa. The helium annulus has dimensions of 31.1 mm mean radius, 0.7 mm thickness and a height of 100 mm (adiabatic condition) or 114 mm (isothermal condition). Calibrated Allen Bradley carbon resistors, in contact with the inner diameter surface of the annulus, gather the temperature data with a response time of 1-3 msec and afford a temperature precision of a few millikelvin. In the experiment, a heater located at the vertical midpoint of the solenoid (also the vertical midpoint of the annulus) dissipates a 10 msec pulse of energy into the superconductor which subsequently generates its own heat over the length of conductor which is driven normal.

The location of the thermometers and the heater are shown in Figure 1. Here the annulus of He II is

0018-9464/89/0300-1508$01.00©1989 IEEE



Thermometer Grid

Figure 1: Thermometer and heater locations. All thermometers are mounted on the inner-radius side of the helium annulus and are in direct contact with the helium.

"unwrapped" and presented as a two dimensional surface. The horizontal dimension is equivalent to one full turn and as can be seen, the heater used in this investigation is approximately a quarter turn in length. The exact length is 5.2 cm. Temperatures are measured in only one quadrant of the channel since we expect the temperature profiles in the channel to be symmetric vertically about the mid height of the channel and horizontally about the midpoint of the heater. The physical parameters of the experiment are used as input to the computer program. One dimension which is not precisely known from the experimental data and which is included in the input to the computer program is the length of the normal zone subsequent to the thermal disturbance. Voltage tap measurements indicate that it extends beyond the length of the heater, but the exact length is both difficult to determine and may in fact be changing during the stability process.

The magnitude of the power dissipated in the heater is between 70 W/cm^2 and 170 W/cm^2. This same power, communicated to the conductor-helium interface results in the rapid formation of a helium vapor layer. The time (Δt^*) associated with the formation of the vapor layer is given by

$$\Delta t^* = 110\ q^{-4} \tag{2}$$

(see Van Sciver[7]), from which we find that Δt^* is less than 5 x 10^{-6} sec. Between the vapor layer and the He II there exists a thin layer of He I and therefore also a phase boundary with its temperature locked at T_λ. For the data included in this report, the duration of the stability process and the time during which there exists a normal zone (as measured with voltage taps) is greater than 200 msec. As is shown below, the existence of the T_λ phase boundary lasts an equivalent amount of time. In all of the data shown below, the comparison is limited in time to avoid temperature profiles realized during the collapse of the normal zone.

The adiabatic and isothermal conditions are realized experimentally as follows: In the adiabatic case the micarta cylinder defining the inner radius of the annulus includes lips at the vertical ends which are 7.0 mm tall by approximately 0.7 mm thick. The remaining cracks between these lips and the solenoid are filled with RTV silicone sealant. It is important to note here that this seal is not a perfect thermal barrier as the RTV cracks when cooled to helium temperatures. In addition three small holes (total cross sectional area ~ 0.06 cm^2) penetrate to the top of the annulus in order to allow helium to fill the annulus. Thus the adiabatic conditions are not ideal. In the isothermal case the lips are removed leaving the 0.7 mm annulus open at the vertical ends to a surrounding bath of He II.

Results and Discussion

The results of this comparison for a sample of the thermometers are presented in Figures 2 through 6. In general the agreement is good, and disagreements become obvious where the computer code is unable to include the complexities of the physical processes occurring. In this respect a number of comments are worthwhile.

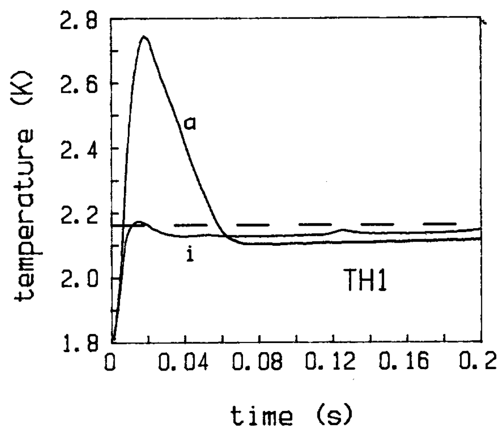


Figure 2: Temperature response of thermometer 1. Distance from center of heater: horizontal, 0 cm; vertical, 0 cm.

Figure 2 demonstrates that for the adiabatic (or closed channel) case the temperature directly across the annulus from the heater's center rises well above T_λ, and subsequently falls to a steady value somewhat below T_λ. This behavior reflects that the phase boundary separating the He II from the He I initially moves across the annulus and beyond this thermometer, but returns past it to some steady position between the heater and this thermometer. In the isothermal (or open channel) data, the intial temperature rise recorded by TH1 is not as great as in the adiabatic case. It is believed that this is due to the greater freedom with which the heat can move in the vertical direction with the isothermal boundary conditions. It is also assumed therefore that the warm He I region spreads further in the two dimensional plane of the annulus with the isothermal condition than with the adiabatic one. Indeed, a significant improvement in the isothermal computations for TH2, TH4, TH6, TH8, TH10 and TH11 is obtained by assuming that the T_λ boundary is vertically positioned 2 mm further from the midheight of the channel than in the adiabatic case.

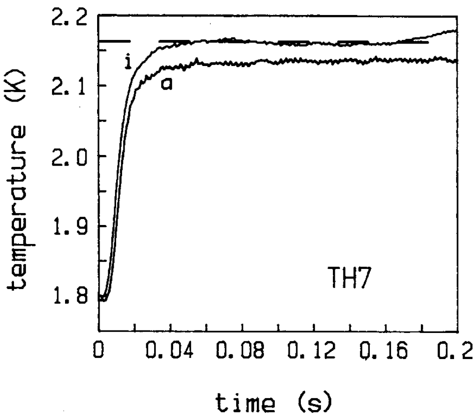


Figure 3: Temperature response of thermometer 7. Distance from center of heater: horizontal, 3 cm; vertical, 0 cm.

In both data traces of TH1, the initial transient gives way to a steady temperature as assumed in the computational model. The fact that a steady temperature approximately equal to T_λ is recorded by TH1 (and TH5 and TH7) indicates the existence of the fixed temperature phase boundary in the nearby vicinity. Further, the fact that the steady temperature as recorded by TH1 (and TH5 and TH7) is below T_λ suggests the existence of a temperature gradient across the 0.7 mm annulus. This suggestion is supported by observations of smaller thermal disturbances resulting in slightly lower "steady" temperatures as recorded by TH1 (and TH5 and TH7). Thus, a 3-D detail of the heat flow, not modelled by the computer, appears in the experimental data.

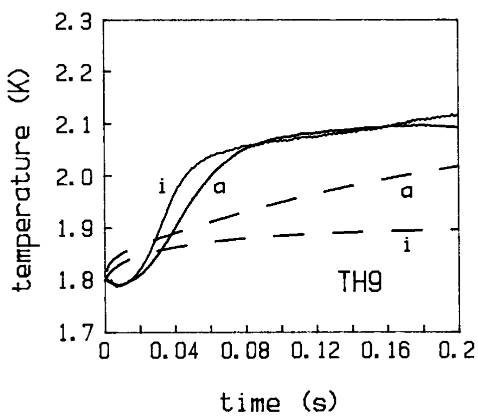


Figure 4: Temperature response of thermometer 9. Distance from center of heater: horizontal, 6.51 cm; vertical, 0 cm. Demonstrates growth of normal zone past this thermometer.

The second obvious discrepancy in the comparison between the data and the computational model appears in Figure 4. In this case our computer input set the half-length of the normal zone shorter than the distance between TH1 and TH9; however, the experimental data portray a normal zone which grows beyond that length. This scenario is similar to that observed in our stability data[6], which is that the normal zone appears to be growing in length while it is cooling. The time dependence of this growth is difficult to model. The increase in heat flux resultant from this growing normal zone or more significantly, the extension of the T_λ boundary in the horizontal direction, also produces discrepancies between the model and data for TH8, TH10 and TH11. This is especially visible in the isothermal comparison of TH11 in Figure 5. Here the computational results denoted by line i are derived with a T_λ boundary half length fixed at 6 cm (as are the isothermal and adiabatic results shown for all other thermometers) while the line i' represents the computations with a T_λ boundary half length fixed at 9 cm. The actual half length of the T_λ boundary grows with time between these two values. The adiabatic comparison shown in Figure 5 also reveals a discrepancy, but in this case the data consistently falls below the computations. This is due to the non ideal adiabatic boundary conditions mentioned above.

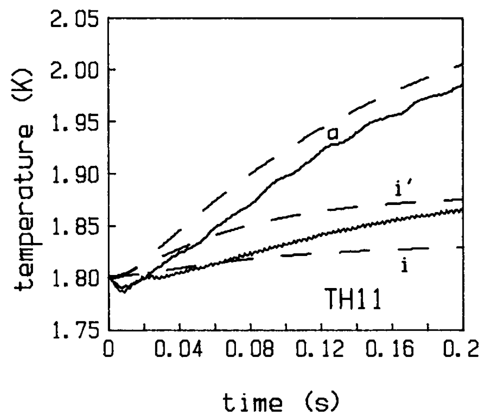


Figure 5: Temperature response of thermometer 11. Line i represents isothermal calculation with fixed T_λ half length of 6 cm; line i' represents isothermal calculation with fixed T_λ half length of 9 cm.

One further physical complexity observed in the data is evident in the first 20 milliseconds of Figures 4, 5, and 6. Here the channel temperatures appear to decrease before growing. We believe this behavior indicates a localized pressure increase due to a short lived helium vapor bubble at the heated surface. Electrically or magnetically induced transients are not large enough to account for the magnitude of this dip. However, liquid helium does have negative values of dT/dP in this temperature range. Indeed, the Joule Thompson coefficient $\mu_j = \left(\frac{dT}{dP}\right)_H$ at saturated vapor pressure are reported[8] to be negative. In addition, calculating μ_j for the subcooled He II from

$$\mu_j = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] \quad (3)$$

where C_p and V are the specific heat and specific volume respectively at 1 atm, reveals that $\mu_j = -2.27 \times 10^{-6}$ KPa^{-1}. The temperature changes observed are

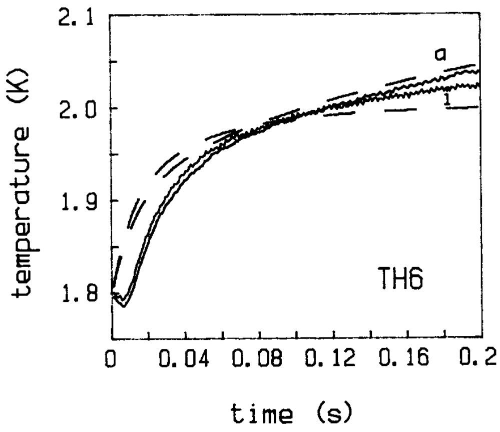


Figure 6.: Temperature response of thermometer 6. Distance from center of heater: horizontal, 1 cm; vertical, 1 cm.

approximately 12.5 mk implying a pressure rise of 5506 Pa, or a 5.5% increase. Note that the speed of first sound in He II is 200 m/sec, thus the pressure rise appears at all the thermometers at essentially the same time. The temperature recorded then by each thermometer is the sum of the thermally induced temperature rise and the pressure induced temperature decrease. Those thermometers close to the heater show no net temperature decrease because of the large and immediate thermal signal, while those thermometers far from the heater display the immediate pressure induced temperature decrease followed by the slower thermally induced temperature increase. Analysis of 1-D heat flow in He II predicts a "thermal diffusion" time τ_D which is roughly given by $\sim \frac{1}{D_T} L^{4/3} \Delta T^{2/3}$ where L is a length and D_T is an effective diffusivity. Calculations based on the exact form of the 1-D analysis reveal that for L = 5 cm, τ_D = 10 msec for a temperature rise equivalent to 1% of $T_\lambda - T_b$ and τ_D = 54 msec for a temperature rise equivalent to 10% of $T_\lambda - T_b$.

Conclusion

The computational model of Eyssa et al provides an accurate tool for predicting temperature profiles realized in two dimensional heat flow in He II. The temperature data gathered during a stability experiment in a small laboratory scale magnet agrees well with the predictions of the model in most respects. Discrepancies become obvious when the computer model is unable to include the complexities of the stability process. Two notable complexities are the growth of a normal zone, and the pressure increase resultant from a transient bubble of helium vapor. With these considerations taken into account, we are quite pleased with the accuracy of the computer model and conclude that it is a worthwhile tool for predicting temperature profiles and heat flow in the two dimensional geometry of large scale SMES systems. Further refinements of the model to include aspects of superconductor stability are in progress.

Acknowledgement

This work was completed under EPRI Contract No. RP22572-2.

References

[1] Y. M. Eyssa, X. Huang and J. Waynert, "Heat Transfer in Helium II for Two-Layer Energy Storage Magnets," IEEE Trans. Mag., vol. 23, p. 561, March 1987.

[2] S. W. Van Sciver, Helium Cryogenics. New York: Plenum Press, 1986, ch. 5, p. 144.

[3] L. Dresner, Nonlinear Differential Equations. Oak Ridge National Laboratory report ORNL/TM-10655, 1988, ch. 7, pp. 115-117.

[4] J. M. Pfotenhauer and S. W. Van Sciver, "Stability Measurements of a Superconductor Cooled by a Two-Dimensional Channel of He II," Adv. Cryo. Eng., vol. 31, pp. 391-398, 1986.

[5] J. M. Pfotenhauer, "Experimental Investigations on the Stability of a He II Cooled Superconducting Solenoid," IEEE Trans. Mag., vol. 23, pp. 926-929, March 1987.

[6] R. W. Boom, princ. investigator, "Superconducting Magnetic Energy Storage: Basic R&D" - Final Report to EPRI covering the period Feb. 1, 1987 to Jan. 31, 1988, ch. 3, pp. 3.10-3.13.

[7] S. W. Van Sciver, "Transient Heat Transport in He II," Cryogenics, vol. 19, pp. 385-392, July 1979.

[8] B. J. Huang, "Joule-Thomson Effect in Liquid He II," Cryogenics, vol. 26, pp. 475-477, August 1986.