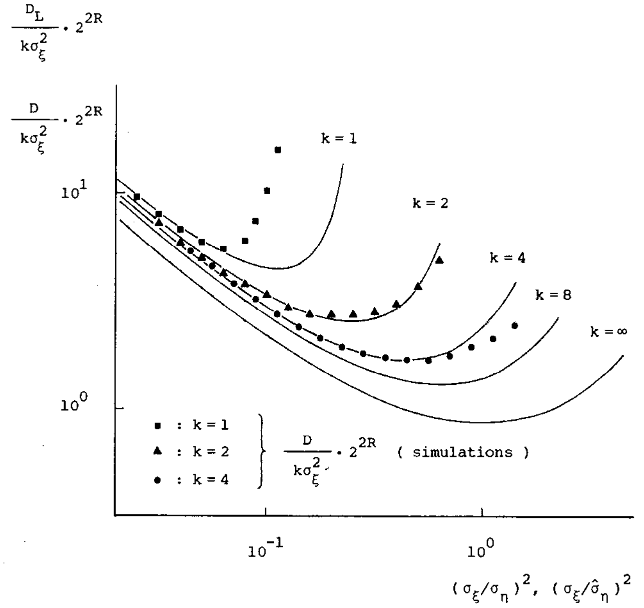

Fig. 2. Average distortions resulting from variance-mismatched vector quantizers.

measure and can be easily obtained, for example, from (4.3.1) of [8].

The relationship between $D_L/k\sigma_\xi^2$ and $(\sigma_\xi/\sigma_\eta)^2$ is calculated from (18) with $k = 1, 2, 4, 8$, and ∞, and shown in Fig. 2. The minima of the distortions become broader as k increases. This means that well-designed vector quantizers are inherently invulnerable to variance mismatch compared with conventional scalar quantizers. It is interesting to note that the value of σ_ξ/σ_η that gives the minimum depends on the block length k, and approaches unity as k increases. That is, the reproduction alphabet with variance σ_η^2 optimally matches with the source with the variance

$$\sigma_\xi^2 = \left\{\frac{k}{k+r}\right\}^2 \sigma_\eta^2.$$

In the asymptotic case where k is sufficiently large, the reproduction alphabet whose variance is equal to that of the source output (quantizer input) gives a nearly optimal vector quantizer.

Computer simulations were performed using the locally optimal quantizers with block lengths $k = 1, 2$, and 4. The iterative optimization methods [9], [10], [11] are used for obtaining the quantizers assuming that the variance of the source output is unity. The code book rate per block, kR, of all quantizers equals 8 [bits], and therefore $R = 8$, 4, and 2 bits per sample for $k = 1$-, 2-, and 4-dimensional quantizers, respectively. The results, $D/k\sigma_\xi^2$ versus $(\sigma_\xi/\hat{\sigma}_\eta)^2$, are also shown in Fig. 2, where

$$\hat{\sigma}_\eta^2 = \left\{\frac{k+r}{k}\right\}^2$$

is the variance of the optimal reproduction alphabet for the source with unit variance. It seems that approximately $kR = 6$ to 8 bits per block are required to ensure the tightness of the lower bound. From Fig. 2, we can conclude that the asymptotic equations (13) and (18) for sufficiently large $N = 2^{kR}$ give good bounds of the performance of variance-mismatched vector quantizers provided that the overload is negligible.

VI. Conclusion

The asymptotic performance of variance-mismatched vector quantizers was derived for distortion measures that are rth powers of a norm of the error vector. It was shown that the performance degradation due to the variance mismatch can be expressed by the simple degradation factor. This asymptotic formula was then applied to the memoryless Laplacian source with the squared-error distortion measure. As a result, through both asymptotic analysis and computer simulations, we have found the interesting fact that well-designed vector quantizers are more invulnerable to the mismatch than are conventional scalar quantizers.

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Multidimensional Digitization of Data Followed by a Mapping

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***Abstract*—In many applications it is necessary to digitize data, knowing only that later on some random function of the digitized data will be of interest. This problem is investigated when the data digitizers are allowed to be multidimensional, i.e., they map a K-dimensional data vector into one of a set of K-dimensional output vectors. It is shown that very complex distortion measures arise naturally. Results are given for the error measure defined as the squared value of the difference between the function of the digitized data and the function of the undigitized data.**

I. Introduction

Let X be a K-dimensional random vector taking sample values $x = (x^1, \cdots, x^K)$ with joint probability density $p(x)$. An N level "block" or vector quantizer is a function $Q_N(\cdot)$ which maps $x \in R^K$ into one of N K-dimensional output vectors $y_1, \cdots, y_N$.

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There are two principal quantizer design problems.

Type I: For a fixed N find the best $Q_N(\cdot)$ to minimize some criterion of distortion between X and $Q_N(X)$.

Type II: Define $P_i = P\{Q_N(X) = y_i\}$; then $H_{Q_N} \triangleq -\sum_{i=1}^{N} P_i \log P_i$ is the entropy of the random variable $Q_N(X)$. For a fixed H, find the best $Q_N(\cdot)$ to minimize some criterion of distortion between X and $Q_N(X)$ subject to the constraint $H_{Q_N} \leq H$.

We will denote the optimal quantizers associated with the above two problems as Type I and Type II. Many authors have considered aspects of these design problems for difference distortion measures for which the distortion between X and $Q_N(X)$ is some function of $X - Q_N(X)$. Max [1] gives necessary but not sufficient conditions required for the Type I optimal minimum mean square error one-dimensional quantizer. Fleisher [2] provides a sufficient condition for the same distortion measure that requires certain convexity properties on the density function of the quantizer input random variable. Panter and Dite [3] derive an expression for the expected mean squared error of a minimum mean-square error one-dimensional quantizer, where they assume the number of output levels N to be very large. Algazi [4] generalizes Panter and Dite's equation to an rth power distortion measure. Wood [5] using equations derived by Roe [6], rederives Panter and Dite's result and gives formulas for the asymptotic (large N) quantizer's output levels. Zador [7] generalizes the work of Panter and Dite to several dimensions and to a more general difference distortion measure. Zador's equation for the minimal rth power distortion error per sample for large N is

$$\frac{1}{K} E\{|X - Q_N(X)|^r\} = J_{K,r} N^{-r/K} \|p\|_{K/(K+r)}, \tag{1}$$

where E denotes expectation, $J_{K,r}$ is a constant depending only upon K and r, and $\|p\|_\alpha = [\int p(x)^\alpha dx^1, \cdots, dx^K]^{1/\alpha}$, $|x| = (\sum_{i=1}^{K} (x^i)^2)^{1/2}$.

For the generally used case of a mean-square error fidelity criterion, r is 2. Yamada *et al.* [8] derive lower bounds to vector quantizer performance for more general difference distortion measures. In [9], upper bounds are derived which are shown to be in many cases very close to the lower bounds in [8]. The constant $J_{K,r}$ is known for only a few values of K and r. However, Zador shows $\lim_{K\to\infty} J_{K,2} = 1/(2\pi e)$, and it is known that $J_{1,2} = 1/12$.

If we define $H(P) \triangleq -\int_{R^K} p(x) \log p(x)\, dx$ as the differential entropy of the random vector X, Zador shows that the corresponding equation for the fixed H (or Type II) quantizer is

$$\frac{1}{K} E\{|X - Q_N(X)|^r\} = C_{K,r} e^{-(r/K)(H_{Q_N} - H(P))}, \tag{2}$$

where $C_{K,r}$ is a constant depending only upon K and r.

The above expressions are asymptotic, i.e., N is assumed to be large. As a practical matter, (1) and (2) can be extremely accurate predictors of distortion performance for even moderate values of N and H. For example, in single-dimensional mean-squared error quantization of a Gaussian random variable, (1) gives within 6 percent of the correct distortion for $N = 32$ or 5 bits/sample. As a comparison, the telephone system uses 6–7 bits/sample in speech digitization. Hence for many cases of practical interest, these expressions are useful indicators of source encoder performance.

This theory is useful for the cases when one is only interested in the distortion the data undergoes in the source coding process. It is frequently the case, however, that after the data is decoded at the receiver, we will wish to perform some sort of signal processing operation on it. In general, it is not X we are interested in but some $g_\alpha(X)$, where α is a random variable in some index set I. For example, $g_\alpha(\cdot)$ may be the discrete Fourier transform of X or perhaps the sum of the squares of the components of X. We will suppose that the transmitter has knowledge only of the distribution function of α and the possible functions $\{g_\alpha(\cdot)\}_{\alpha\in I}$ that will operate on the data.

II. Development

Let $g_\alpha(\cdot) = (g_\alpha^1(\cdot), \cdots, g_\alpha^{l_\alpha})$ be a differentiable mapping from R^K into R^{l_α}. Define $l = \max_{\alpha\in I} l_\alpha < \infty$. Let $B_r(x)$ denote a K-dimensional sphere of radius r centered at x. Let $V(S)$ denote the volume of the set S. A "point density function" $\lambda(x)$ for a quantizer describes the density of the output levels of that quantizer in a region about x. For large N we have the relationship

$$\lambda(x) V(B_r(x)) = \frac{\text{number of output levels of } Q_N(\cdot) \text{ contained in } B_r(x)}{N}.$$

Note that $\int_{R^K} \lambda(x)\, dx = 1$.

We further suppose our quantizer will have the "white property"

$$E\Big\{\big(X^i - Q_N(X)^i\big)\big(X^j - Q_N(X)^j\big)\Big\} = 0, \qquad \text{if } i \neq j.$$

All optimal quantizers known to date have this property, as do "random" quantizers [12]. We define our distortion measure D as the mean-squared error between $g_\alpha(x)$ and $g_\alpha(Q_N(X))$,

$$D \triangleq E\left\{ \sum_{m=1}^{l} \left(g_\alpha^m(X) - g_\alpha^m(Q_N(X))\right)^2 \right\}.$$

If N is large (or $X - Q_N(X)$ is small), we have

$$D \cong E\left\{ \sum_{i=1}^{K} \sum_{m=1}^{l} \left(g_{i,\alpha}^m(X)\big(X^i - Q_N(X)^i\big)\right)^2 \right\} \tag{3}$$

by the multidimensional form of Taylor's theorem, where

$$g_{i,\alpha}^m(x) \triangleq \frac{\partial g_\alpha^m(x)}{\partial x^i},$$

the partial derivative of the mth component of $g_\alpha(\cdot)$ in the ith direction. Define $g_i(x)^2 \triangleq \sum_{m=1}^{l} \underset{\alpha}{E}\{(g_{i,\alpha}^m(X))^2 | X = x\}$. To avoid confusion we specify that $g_i(x) \triangleq (g_i(x)^2)^{1/2}$. Then we have that

$$D \cong \sum_{i=1}^{K} E\left\{ g_i(X)^2 \big(X^i - Q_N(X)^i\big)^2 \right\}. \tag{4}$$

We note that we have been lead to a distortion measure between X and $Q_N(X)$ that is *not* a difference distortion measure. Our problem then is to find an expression for D in terms of $\lambda(x)$, the point density function of $Q_N(\cdot)$, and subsequently to minimize this equation over $\lambda(x)$. Denote the N output levels of $Q_N(\cdot)$ by $y_1, y_2, \cdots, y_N$. Let $S_i \triangleq \{x\colon Q_N(x) = y_i\}$, where $i = 1, \cdots, N$. Let $p(x)$ be the probability density (of compact support) of the data. So that

$$P_i = \int_{S_i} p(x)\, dx, \quad \text{and} \quad V(S_i) = \int_{S_i} dx.$$

We may then approximate (4), for large N, by

$$D \cong \sum_{i=1}^{N} \frac{P_i}{V(S_i)} \int_{S_i} \sum_{n=1}^{K} g_n(y_i)^2 (x^n - y_i^n)^2\, dx. \tag{5}$$

We now note that the locus of points x such that

$$\sum_{n=1}^{K} g_n(y_i)^2 (x^n - y_i^n)^2 \leq r > 0$$

forms a hyperelipsoid in K dimensions. By a result shown in [8] we then know that

$$I_i \triangleq \int_{S_i} \sum_{n=1}^{K} g_n(y_i)^2 (x^n - y_i^n)\, dx$$

is minimized for a given $V(S_i)$ by S_i taking the shape of a hyperelipsoid of the correct minor–major axes ratios. We cannot tile Euclidean space in general with hyperelipsoids. Therefore any results we obtain from this point onward must be lower bounds to possible distortion performance. I_i is minimized for a given $V(S_i)$ by

$$I_i = \int_{\{x:(x-y_i)^T M_i (x-y_i) \le c^2\}} \sum_{n=1}^{K} g_n(y_i)^2 (x^n - y_i^n)^2 \, dx$$

where

$$M_i = \begin{bmatrix} g_1(y_i)^2 & 0 & 0 & \cdots & 0 \\ 0 & g_2(y_i)^2 & 0 & \cdots & 0 \\ 0 & 0 & & \cdots & 0 \\ 0 & 0 & 0 & \cdots & g_K(y_i)^2 \end{bmatrix}.$$

Note $|M_i| = \prod_{n=1}^{K} g_n(y_i)^2)$ and c^2 is chosen such that

$$V\left(\left\{x: (x-y_i)^T M_i (x-y_i) \le c^2\right\}\right) = V(S_i),$$

so that

$$c^2 = V(S_i)^{2/k} \Gamma\left(\frac{k+2}{2}\right)^{2/k} |M_i|^{1/k}/\pi.$$

Therefore

$$I_i = \int_{\{x: x^T M_i x \le c^2\}} \sum_{n=1}^{K} g_n(y_i)^2 (x^n)^2 \, dx$$

$$= \frac{\pi^{K/2}}{\sqrt{|M_i|}} \left(\frac{K/2}{K/2+1}\right) \frac{c^{K+2}}{\Gamma\left(\frac{K+2}{2}\right)}.$$

We now place this result in (5) to obtain a lower bound on asymptotic distortion

$$D \ge \sum_{i=1}^{N} \frac{P_i}{\pi} V(S_i)^{2/K} \Gamma\left(\frac{K+2}{2}\right)^{2/K} |M_i|^{1/K} \left(\frac{K/2}{K/2+1}\right).$$

In [10] Gersho argues that $V(S_i) \approx 1/(N\lambda(x))$. Making this substitution and noting that for large N the above sum can be replaced by an integral, we have

$$D \ge \frac{N^{-2/K}}{V_K^{2/K}} \left(\frac{K/2}{K/2+1}\right) \int \frac{p(x)}{\lambda(x)^{2/K}} \left(\prod_{n=1}^{K} g_n(x)^2\right)^{1/K} dx, \quad (6)$$

where $V_k \triangleq$ volume of the k-dimensional unit radius sphere. An exercise with Hölder's inequality gives that the right-hand side of the above inequality is minimized for $\lambda(x)_{\text{opt}}$ proportional to $(\prod_{n=1}^{K} g_n(x)^2) p(x)^{K/K+2}$.

To construct an upper bound all we need do is consider the asymptotic distortion of a particular quantizer. Let $Q_{N'}(x)$ be the optimum minimum mean-square error quantizer for a uniform probability distribution on the unit hypercube $C = X_{i=1}^{K}[0,1]$, i.e., when $K = 1$, C is the unit interval, when $K = 2$, C is the square, etc. It is known that $\lambda_{\text{opt}}(x) = 1$ for $x \in C$ for this quantizer. Let $Y = (y_1, \cdots, y_{N'})$ denote the N' K-dimensional output levels of $Q_{N'}(\cdot)$. Note that $y_i = (y_{i1}, y_{i2}, \cdots, y_{iK})$ for $i = 1, \cdots, N'$. Consider the set of points $y_i' = (a_1 y_{i1}, a_2 y_{i2}, \cdots, a_K y_{iK})$ for $i = 1, \cdots, N'$, where the $a_j \ge 1$ for $j = 1, \cdots, K$. Let Y' equal the set of all y_i' that lie in C. Let N'' equal the number of distinct y_i' in Y', and let N' be large. Since $\lambda(x) = 1$, we have $N'' = N'/\prod_{i=1}^{K} a_i$. Define $Q_{N''}^{S}(x)$ be the closest $y_i' \in Y'$ to x. We now note two basic facts about $Q_{N'}(x)$, proven in [11]: its point density function $\lambda(x) = 1$ for $x \in C$ and $E\{(x^i - Q_{N'}(x)^i)^2\} \cong J_{K,2}(N')^{-2/K}$, independently of i. Compare this expression with (1), noting that $\|p\|_{K/K+r} = 1$. Suppose that $g_i(x)^2 = C_i^2$ and $p(x) = 1$. Our distortion measure may then be written from (4) as

$$D = E\left\{\sum_{i=1}^{K} C_i^2 \left(x^i - Q_{N''}^{S}(x)^i\right)^2\right\}$$

$$= E\left\{\sum_{i=1}^{K} C_i^2 \left(x^i - Q_{N'}(x)^i\right)^2 a_i^2\right\}$$

where the C_i^2 are arbitrary positive constants

$$D = \frac{C_{\min}^2 J_{K,2} (N'')^{-2/K}}{K} \sum_{i=1}^{K} \frac{C_i'^2 a_i^2}{(a_i)^{2/K}},$$

and where $C_{\min}^2 \triangleq \min_i C_i^2 > 0$ and $C_i'^2 = C_i^2 / C_{\min}^2 \ge 1$.

This expression is minimized if we choose the a_j's ≥ 1 such that $a_j^2 C_j'^2 = T$, a constant ≥ 1. Hence

$$D_{\min} = J_{k,2}(N'')^{-2/K} \left(\prod_{j=1}^{K} C_j^2\right)$$

and

$$N'' = N''\left(\prod_{j=1}^{K} C_j / T^K C_{\min}^K\right)^{1/2}.$$

Thus, by adjusting T and choosing N' large enough, we can get any value of N'' desired.

We now describe our quantizer for the general case of arbitrary $g_i(x)^2$ and $p(x)$. Divide the range space of the random vector X into M disjoint hypercubes, $m_1, \cdots, m_M$. Define

$$N_n'' = \left(\int_{m_n} \lambda(x) \, dx\right) N,$$

where $\lambda(x)$ is some probability density. If M is large and the m_i small enough, then $g_i(x)^2 \approx g_{in}^2$ and $p(x) \approx$ constant for $i = 1, \cdots, K$, and $x \in m_n$. So for approximation purposes, distortion measure (4) becomes

$$D = \sum_{i=1}^{K} \sum_{n=1}^{M} g_{in}^2 E\left\{\left(x^i - Q_N(x)^i\right)^2 | X \in m_n\right\} P(X \in m_n).$$

Now, for each n design an N_n'' level quantizer on m_n, $Q_{N_n''}^{S}(\cdot)$, as done previously but for the distortion measure

$$D_n = E\left\{\sum_{i=1}^{K} g_{in}^2 \left(x^i - Q_{N_n''}^{S}(x)^i\right)^2 | X \in m_n\right\},$$

i.e., g_{in}^2 takes the place of C_i^2. Our quantizer will then be $Q_N(X)$ defined as the quantizer whose output levels are the union of the $Q_{N_n''}^{S}(\cdot)$ output levels for $n = 1, \cdots, M$. We can then show as $M \to \infty$ that

$$D \cong N^{-2/K} J_{K,2} \int \frac{p(x)}{\lambda(x)^{2/K}} \left(\prod_{n=1}^{K} g_n(x)^2\right)^{1/K} dx. \quad (7)$$

We note that the form of the upper bound given here is the same as that of the lower bound given in (6) with $J_{K,2}$ replaced by $(K/2)/[(K/2+1)V_K^{2/K}]$. It is shown in [10] that the difference between these two constants approaches zero as K approaches infinity.

We now take up the Type II quantization problem and sketch a short proof following the general derivation given first by Gersho [10]. We know from (6) and (7) (the upper and lower bounds) that the distortion of an N level quantizer may be written as

$$D = N^{-2/K} C' \int p(x) \exp\left[-\frac{2}{K} \log \frac{\lambda(x)}{\prod_{n=1} g_n(x)}\right] dx$$

where

$$\frac{K/2}{(K/2+1)V_K^{2/K}} \le C' \le J_{K,2}.$$

Gersho shows that

$$H_{Q_N} - H(P) = -E\left\{\log \frac{1}{N\lambda(x)}\right\}.$$

Using Jensen's inequality we have that

$$D \ge N^{-2/K} C' \exp\left[-\frac{2}{K}\int p(x)\log\frac{\lambda(x)}{\prod g_n(x)}\,dx\right]$$
$$= N^{-2/K} C' e^{-(2/K)\int p(x)\log\lambda(x)\,dx} e^{(2/K)\int p(x)\log\prod_{n=1}^K g_n(x)\,dx}$$
$$= C' e^{-(2/K)(H_{Q_N}-H(P))} e^{(2/K)\int p(x)\log\prod_{n=1}^K g_n(x)\,dx} \tag{8}$$

with equality if and only if $\lambda(x)$ is proportional to $\prod_{n=1}^K g_n(x)$. Hence $\lambda_{\mathrm{opt}}(x) = \prod_{n=1}^K g_n(x)$ and the minimum distortion is given by (8) where

$$\frac{K/2}{(K/2+1)V_K^{2/K}} \le C' \le J_{K,2}.$$

III. An Application to Detection Theory

In the practical implementation of optimal detectors one receives the data sequence $(X^1, X^2, \cdots, X^K)$. One quantizes this sequence in some fashion to obtain $(\hat{X}^1, \hat{X}^2, \cdots, \hat{X}^K)$. On a digital computer is calculated the quantity

$$\Lambda(\hat{X}) = \frac{p_1(\hat{X}^1, \cdots, \hat{X}^K)}{p_0(\hat{X}^1, \cdots, \hat{X}^K)},$$

the quantized likelihood ratio, where $p_i(X^1, \cdots, X^K)$ is the probability density of $(X^1, X^2, \cdots, X^K)$ given that the hypotheses H_i is true. The decision process is then to check if $\hat{\Lambda}(X)$ is greater than or less than some threshold. A reasonable criterion to design a Type I quantizer for this system would be to minimize $E\{(\Lambda(X) - \Lambda(\hat{X}))^2\}$. See Kassam [13] for another approach to this problem. Let π_i be the *a priori* probability that H_i is true. In this case

$$\left(\frac{\partial \dfrac{p_1(x)}{p_0(x)}}{\partial x^n}\right)^2 \triangleq g_n(x)^2$$

and $\lambda(x)_{\mathrm{opt}}$ is proportional to (see sentence following (6))

$$\prod_{n=1}^{K}\left(\frac{\partial \dfrac{p_1(x)}{p_0(x)}}{\partial x^n}\right)^2 p(x)^{K/K+2}$$

where $p(x) = \pi_0 p_0(x) + \pi_1 p_1(x)$. More specifically, consider the case of testing for the presence of a zero-mean Gaussian signal in the presence of zero-mean Gaussian noise. In this case $\Lambda(X)$ can be reduced to $\Sigma_{n=1}^K (X^n)^2$. Hence $g_n(X)^2 = 4(X^n)^2$ and $\lambda(x)$ is proportional to $(\prod_{n=1}^K (x^n)^2) p(x)^{K/K+2}$. Therefore in this detection problem (for $K = 1$) we should design a quantizer whose output level density function is the "double-sided" Maxwell probability density.

IV. Discussion

We have generalized (in a heuristic fashion) the asymptotic theory of quantization to include the case where some function of the data may be the main quantity of interest and not the data itself. We note that in Section II we could have developed the upper bound to quantizer performance by a "random quantizer argument" in which the quantizer output levels are chosen to be independent samples from a probability density $\lambda(x)$. This type of argument is considered in some detail in [9]. The only difference in (7) is that instead of $J_{K,2}$ we have the so called "Zador upper bound coefficients" which are presented in [10]. We give the argument we did here because the upper bound is tighter and the constructive nature of the proof suggests a possible method to construct such quantizers. In the detection example we see that the optimum detection quantizer is *not* in general the optimum quantizer matched to the data. This could lead to more efficient quantizer designs in these problems. To date many people have justified the use of uniform quantizers based upon the fact that they always are the best quantizer for a fixed quantizer output entropy no matter what the underlying distribution is. In (8) we see that this need no longer hold and the optimal entropic quantizer (Type II) becomes a function of the expected data transformations.

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Poor Error Correction Codes Are Poor Error Detection Codes

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***Abstract*—We show that binary group codes that do not satisfy the asymptotic Varshamov-Gilbert bound have an undesirable characteristic when used as error detection codes for transmission over the binary symmetric channel.**

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