the RF current, or from unintentional interaction with other electronic instruments [8]. If simple capacitive electrodes do ultimately prove to be superior in practice, their performance could be further improved by the same zonal division and current-leveling scheme which we have discussed here.

The conditions which prevail during actual electrosurgery are highly variable from one patient or one surgical procedure to another. It would therefore be convenient if the dispersive electrode were *adaptive* in the sense that it could adjust to variable conditions in order to maintain safe current densities. This could be arranged by replacing the passive series resistors with active current-limiting elements. It is very straightforward, for example, to design small, bidirectional, FET current limiters which would allow any zone of the electrode to accept only a selected fraction of the total current. If this approach were combined with azimuthal division of the zones, the well-known "leading-edge" effect [1], [2] could be alleviated or eliminated.

Finally, it should be pointed out that the added complications which are inherent in our proposed multiply-connected electrodes do not necessarily imply that such electrodes would be difficult or expensive to manufacture or use. The passive or active components which are used for current leveling could be incorporated in the RF generator, in the connecting cable, or in the specially-designed connector which would be required for attachment to the electrode. The electrode, itself, could remain relatively simple, inexpensive, and disposable.

**REFERENCES**


**Distributed Equivalent-Circuit Models for Circular Dispersive Electrodes**

**J. D. WILEY AND J. G. WEBSTER**

Abstract—Analytically solvable distributed equivalent-circuit models have been developed for circular electrosurgical dispersive electrodes which are either resistively or capacitively coupled to the body. Calculations based on these models show that for either electrode type it is possible to define a characteristic length, the magnitude of which governs the current distribution under the electrode. The well-known perimmetrical burn problem occurs when the current transfer length is much smaller than the electrode radius: a problem which may arise

Fig. 1. Cross-sectional view of the structure to be modeled. A circular dispersive electrode of radius *a* on a homogeneous slab of conductive material of thickness *W* and resistivity *ρ*. The layer labeled *A* is a conductive gel in the case of resistivity coupled electrodes or a dielectric film in the case of capacitively coupled electrodes. The RF source drives a total current *I* between the annular electrode and the circular dispersive electrode.

with either capacitive or resistive electrodes. Design guidelines are given for the optimization of simple circular dispersive electrodes, and suggestions for further improvements are discussed.

**I. INTRODUCTION**

In electrosurgery, a strong RF current (typically several hundred milliamperes at 1 MHz) is delivered by the active electrode to the surgical site, and returned to the generator via a large-area dispersive electrode. In current practice, the area of the dispersive electrode is selected in accordance with a simple power-density guideline which calls for 1 cm² of electrode area for each 1.5 W of applied RF power [1]. The intent of this guideline is to assure that the current density at the dispersive electrode site is sufficiently low to avoid patient burns. Recent investigations, however, have shown [2]–[4] that the current densities under dispersive electrodes are often highly nonuniform, and that severe burns may occur, even with electrodes which are conservatively within the "area" guidelines. These current nonuniformities depend on a number of factors including electrode placement, quality and uniformity of the electrode/skin interface, and the effective electrical and thermal conductivities of the tissue immediately beneath the contact. Thus, a complete treatment of the problem is expected to be beyond the scope of analytical calculations. Nevertheless, idealized models have already proved useful in explaining some aspects of the burn problem, and may help point the way toward better electrode designs. Numerical [2] and analytical [3] calculations of the electric field pattern under idealized circular electrodes, for example, have shown that the current density is highest near the perimeter, with about half the current being collected by the outer 15 percent of the contact. The results of these calculations have been used to suggest improved electrode designs as reported elsewhere [5]–[7].

Equivalent-circuit models provide a second useful approach for exploring the behavior of complex systems. In the present paper, simple distributed equivalent circuits are proposed for two types of circular dispersive electrodes: 1) conventional resistive (gel-pad) electrodes and 2) capacitive electrodes. Although the models are highly idealized, they provide considerable insight into the problems and limitations of existing electrodes, and should be useful in the development of improved design criteria.

**II. EQUIVALENT CIRCUITS**

Fig. 1 shows a cross-sectional view of the idealized contact structure to be modeled. We assume a slab of homogeneous conductive material having thickness *W* and electrical resistivity *ρ*. The circular dispersive electrode consists of a planar metal disk of radius *a* separated from the conductive medium by the layer which is labeled *A* in Fig. 1. This layer is assumed to
be a conductive gel in the case of gel-pad electrodes, or a dielectric in the case of capacitive electrodes. The counterelectrode is assumed to be an annular ring in order to preserve azimuthal symmetry. In actual electrosurgery, of course, the counterelectrode would be more nearly a point source which would impart a directionality to the current pattern. Electrolyte-tank measurements [7] have shown that the assumption of azimuthal symmetry does not introduce any serious errors as long as the counterelectrode is several radii away from the dispersive electrode.

A. Resistive (Gel-Pad) Electrodes

Fig. 2 shows a distributed resistive equivalent circuit in which the underlying tissue is modeled by a sheet resistance $R_s = \rho/w$ and the interfacial layer (including skin, contact, and gel resistances) is modeled by a specific contact resistance $R_c$. Although Fig. 2 shows only a few discrete resistors, it is intended to symbolize a continuous distribution of differential elements of resistance given by

$$dR_c = \frac{R_c}{2\pi r} \frac{dr}{d\eta}$$

and

$$dR_s = \frac{\rho}{2\pi r} \frac{dr}{d\eta}$$

in the vertical and horizontal directions, respectively. The question which will be asked of the model is this—-if a total current $i_0$ flows to ground through this contact, what is the distribution $i(r)$ of current in the resistive slab (i.e., in the tissue) under the contact? Even without calculation, it is clear that the qualitative behavior will depend on the ratio $R_c/R_s$. If this ratio is very small, most of the current will be collected near the perimeter of the contact, leading to possible burns in this area. If $R_c/R_s$ is very large, the current will be forced to flow further underneath the contact before being fully transferred to ground. In view of this behavior, and on dimensional grounds, it is reasonable to define a characteristic length for the problem

$$L_t \equiv \left( R_c/R_s \right)^{1/2}.$$  

(3)

The quantity $L_t$ will be called the current-transfer length. ($R_c$, being a specific contact resistance, has units of $\Omega \cdot m^2$, and $R_s$ has units of $\Omega$. Thus, $L_t$ is indeed a length.) By applying Ohm’s law to the differential elements of resistance, it is straightforward to show that the radial current under the electrode obeys the equation

$$\eta^2 \frac{d^2 i(\eta)}{d\eta^2} - \eta \frac{di(\eta)}{d\eta} - \eta^2 i(\eta) = 0$$

(4)

where $\eta$ is a normalized radial coordinate

$$\eta \equiv \frac{r}{L_t}.$$  

(5)

Equation (4) is one form of Bessel’s equation [8]. By symmetry, one of the necessary boundary conditions is $i(0) = 0$. The second is $i(a) = i_0$. These boundary conditions, together with (4), yield the solution

$$i(\eta) = \frac{i_0 L_t \eta}{a} \left[ \frac{I_1(\eta)}{I_1(a/L_t)} \right]$$

(6)

where $I_1(x)$ is the modified Bessel function of the first kind of order 1 [8]. The potential under the electrode is related to the current by

$$v(\eta) = \frac{R_s}{2\pi} \frac{1}{\eta} \frac{di(\eta)}{d\eta}$$

(7)

which, when combined with (6), gives

$$v(\eta) = \frac{i_0 R_s L_t}{2\pi a} \left[ \frac{I_0(\eta)}{I_1(a/L_t)} \right]$$

(8)

where $I_0(x)$ is the modified Bessel function of the first kind of order zero [8]. The terminal resistance of the dispersive electrode is now easily obtained as

$$R_t = \frac{v(a/L_t)}{i_0} = \frac{R_s L_t}{2\pi a} \frac{I_0(a/L_t)}{I_1(a/L_t)}.$$  

(9)

The principal results of this model are contained in (6), (8), and (9), which can be evaluated for arbitrary values of $R_s$, $R_c$, and $a$, using tabulated values of the Bessel functions [8]. It is more illuminating, however, to consider the two limiting cases discussed earlier—small $L_t$ and large $L_t$.

In the limit of small $L_t$ we have

$$0 \ll \frac{r}{L_t} \ll \frac{a}{L_t} \gg 1.$$  

(10)

In this limit, the arguments of the Bessel functions are large, allowing the use of the asymptotic expansions [8]

$$I_0(x) \sim \frac{e^x}{(2\pi x)^{1/2}} \left[ 1 + \frac{1}{8x} + \frac{9}{128x^2} + \cdots \right]$$

(11)

and

$$I_1(x) \sim \frac{e^x}{(2\pi x)^{1/2}} \left[ 1 - \frac{3}{8x} - \frac{15}{128x^2} - \cdots \right].$$

(12)

Using (11) in (6) gives

$$i(\eta) \approx i_0 \frac{r}{a} \sqrt{\frac{a}{r}} e^{-a\sqrt{L_t}}.$$  

(13)

This result confirms the earlier claim that in the small $L_t$ limit virtually all of the current is collected by the perimeter of the electrode. The total resistance of the distributed network becomes

$$R_t \approx \left( \frac{L_t}{2\pi a} \right) R_s.$$  

Thus, in the small $L_t$ limit, the overall resistance of the contact is considerably less than the sheet resistance of the underlying tissue.

In the large $L_t$ limit, we have

$$0 \ll \frac{r}{L_t} \ll \frac{a}{L_t} \ll 1.$$  

(14)
In this limit, the Bessel functions can be expanded in power series [8]

\[ I_0(x) \approx 1 + \frac{1}{4} x^2 + \frac{1}{64} x^4 + \cdots \] (14)

and

\[ I_1(x) \approx \frac{1}{4} x^2 + \frac{1}{16} x^4 + \frac{1}{384} x^6 + \cdots \] (15)

The current distribution now reduces to

\[ i(r) \approx \left( \frac{l}{a} \right)^2 i_0 \] (16)

and the electrode resistance becomes

\[ R_t \approx \frac{1}{\pi} \left( \frac{l}{a} \right)^2 R_s. \] (17)

The \( i(r) \) result for this case is particularly interesting. It must be emphasized, first, that the entire model is quasi-one-dimensional in that the current flowing in the tissue is assumed to be purely radial (horizontal in Fig. 1). A vertical component is present (implicitly) in the sense that, as the horizontal current flows under an annular ring \( dr \) at radius \( r \), it loses current \( dl \) to the electrode. This decrement can be calculated by differentiating \( i(r) \)

\[ di(r) = \frac{2i_0 dr}{a^2}. \] (18)

Thus, the vertical current density at radius \( r \) is given by

\[ J_v(r) = \frac{dr}{2\pi r di} = \frac{i_0}{r a^2}. \] (19)

In the large \( L_t \) limit, the vertical current density is constant (uniform over the entire contact area). Of course, the heating effect of \( J \) is independent of direction and, in fact, is proportional to \( J^2 \)

\[ J^2 = J_H^2 + J_D^2 = \left( \frac{i_0}{\pi a^2} \right)^2 \left[ 1 + \left( \frac{r}{w} \right)^2 \right]. \] (20)

Thus, although the vertical component is uniform, the overall current density and the heating effects are still enhanced at the perimeter. Nevertheless, the enhancement is considerably less than that obtained for the small \( L_t \) case.

**B. Capacitive Electrodes**

It has recently been suggested [9] that the current nonuniformity associated with gel-pad electrodes can be alleviated or eliminated by the use of electrodes which are *capacitively* coupled to the skin. Fig. 3 shows a distributed equivalent circuit for such an electrode. This model is conceptually and geometrically identical to that of Fig. 2 except that the contact resistance \( R_c \) has been replaced with a distributed capacitance per unit area \( C_a = \varepsilon d \) where \( \varepsilon \) and \( d \) are the dielectric permittivity and thickness of an assumed insulating layer between the electrode metal and the skin. Once again, the qualitative behavior of the contact can be deduced from the equivalent circuit, without calculation. If the reactance \( 1/(\omega C_a) \equiv \Omega/(C) \) of the capacitive layer is very small compared to the body resistance \( R_s \) the current will be transferred to the electrode metal within a very short distance, i.e., if \( \omega R_s C \gg 1 \), current concentration at the perimeter is to be expected. If \( \omega R_s C \ll 1 \), a nearly uniform current distribution is to be expected. An immediate conclusion is that capacitive electrodes do not automatically result in improved uniformity of the current distribution.

The distributed \( RC \) equivalent circuit shown in Fig. 3 has been analyzed in detail elsewhere [10]. The analysis will not be repeated here, except to note that, as in the case of the gel-pad electrode, the current flowing in the tissue layer obeys another form of Bessel's equation

\[ \frac{d^2 i(\eta)}{d\eta^2} - \frac{1}{\eta} \frac{di(\eta)}{d\eta} + j(\eta) = 0 \] (21)

where \( j = \sqrt{-1} \) and \( \eta \) is again a normalized radial coordinate

\[ \eta \equiv \frac{r}{\lambda f}. \] (22)

The transfer length \( \lambda_f \) for this problem is defined by

\[ \lambda_f \equiv 1/(\omega R_s C_a)^{1/2}. \] (23)

Note that the \( C_a \) in (23) is the capacitance per unit area, \( C_a = \varepsilon d \), so that \( \lambda_f \) has dimensions of length. The solution to (21) is most conveniently expressed in terms of the Kelvin functions [8] \( ber(x) \) and \( bei(x) \)

\[ i(r) = i_0 r ber'(r/\lambda_f) + j bei'(r/\lambda_f) \] (24)

where the primes indicate differentiation with respect to argument. After obtaining a similar expression for the radial potential distribution [10], it is straightforward to show that the impedance of the contact is given by

\[ Z = \frac{R_s \lambda_f}{2\pi a} \left[ \frac{A_1}{A_1^2 + A_2^2} \right] \] (25)

where

\[ A_1 = \frac{ber(a/\lambda_f) ber'(a/\lambda_f) + bei(a/\lambda_f) bei'(a/\lambda_f)}{ber^2(a/\lambda_f) + bei^2(a/\lambda_f)} \] (26)

and

\[ A_2 = \frac{ber(a/\lambda_f) bei'(a/\lambda_f) - bei(a/\lambda_f) ber'(a/\lambda_f)}{ber^2(a/\lambda_f) + bei^2(a/\lambda_f)}. \] (27)

Equations (24) and (25) may be evaluated by computer, using tabulated values or appropriate series expansions for the Kelvin functions [8], [10]. As might be expected, however, the results for general values of \( R_s, a, \) and \( \lambda_f \) are rather complicated. It is much more informative to consider the limiting cases \( a/\lambda_f \gg 1 \) and \( a/\lambda_f \ll 1 \) (or, equivalently, \( \omega R_s C \gg \pi \) and \( \omega R_s C \ll \pi \), respectively, where \( C = \pi a^2 C_a \)).

In the limit \( a/\lambda_f \gg 1 \), the magnitude of the current drops off exponentially for \( r < a \)

\[ i(r) \approx i_0 \left( \frac{r}{a} \right)^{1/2} e^{(r-a)/(2)^{1/2} \lambda_f}. \] (28)

This expression is only valid for \( a/\lambda_f \gg 1 \) and \( r \approx a \). Computer calculations verify, however, that the magnitude of \( i \) remains small for all \( r < a \) in the limit \( a/\lambda_f \gg 1 \). Thus, this
regime is characterized by a current-density profile which is very strongly peaked at the perimeter of the contact. The impedance of the contact in this limit is given by

\[ Z = \frac{R_s\lambda_t}{2\pi a} \left( \frac{1 - j}{(2)^{1/2}} \right) \]

This impedance has real imaginary parts which are equal (i.e., the voltage lags the current by 45°), and a magnitude which is small compared to \( R_s \). The equivalent circuit in this limit (which might be called the perimeter-dominated limit) can be simplified to a lumped series \( RC \) circuit with

\[ R = R_\infty \equiv \frac{R_s}{\pi(2)^{1/2}} \left( \frac{\lambda_t}{a} \right) \]

and

\[ C = C_\infty \equiv \frac{\pi}{\omega R_s} \left( \frac{a}{\lambda_t} \right)^2 \]

In the other extreme limit, \( r/\lambda_t \ll a/\lambda_t \ll 1 \), the magnitude of the current reduces to

\[ i(r) \approx i_0 \left( \frac{r}{a} \right)^2 \]

This is identical to the result obtained in the large \( L_t \) limit for resistive electrodes, and the discussion given there applies to the present case as well. The impedance reduces to

\[ Z = \frac{R_s}{8\pi} \left[ 1 - j 8 \frac{\lambda_t}{a} \right]^2 \]

Since \( \lambda_t/a \gg 1 \), this impedance is primarily reactive, and much larger than \( R_s \). The simple result given by (33) shows that a lumped series \( RC \) equivalent circuit could be used in this limit also, with

\[ R = R_0 \equiv \frac{R_s}{8\pi} \]

and

\[ C = C_0 \equiv \frac{\pi}{\omega R_s} \left( \frac{a}{\lambda_t} \right)^2 = \frac{\epsilon A}{d} \]

where \( A = \pi a^2 \). From this it is seen that the impedance is essentially equal to the capacitive reactance of the total electrode capacitance, \( \epsilon A/d \) (modified by a much smaller series resistance \( R_s/8\pi \)).

### III. DISCUSSION

The models presented here are quasi-one-dimensional in the sense that the current flowing under the electrode is assumed to flow parallel to the interface. This is a reasonable approximation if the contact radius is comparable to or greater than the effective thickness of the underlying conductive tissue (i.e., if \( a \geq \omega \) in Fig. 1). There is some evidence that subsurface layers of adipose tissue cause the return current to be concentrated near the skin surface [2]. Under these conditions, the assumption \( a \geq \omega \) would be extremely well satisfied, so that the present models may actually be more realistic than models based on the solution of Laplace's equation for thick, homogeneously tissue [2], [3]. It is interesting and important to note, however, that both the three-dimensional field calculations [2], [3] and the present quasi-one-dimensional circuit models predict enhanced current densities at the perimeter of the contact. This phenomenon is thus somewhat model-independent, and appears to be an inherent feature of simply connected, large-area dispersive electrodes. (Changing the shape of the electrode from circular to, say, oval, figure eight, or rectangular would alter the detailed analytical relationships but not the overall behavior.)

The equivalent circuit models show that the current-density enhancement at the perimeter can be reduced by increasing the current transfer length. The implications of this observation will now be discussed separately for resistive and capacitive electrodes.

#### A. Resistive (Gel-Pad) Electrodes

The current-transfer length is given by (3) as

\[ L_t = (R_c/R_s)^{1/2} \]

The quantity \( R_s = \rho/w \) is not available as an adjustable parameter, so any desired increase in \( L_t \) must be accomplished by increasing the contact resistance, \( R_c \). An ironic (and, at first, somewhat counterintuitive) conclusion is that the conductive gel which has traditionally been used to “improve” the contact by reducing \( R_c \), may actually be contributing to the burn problem! It does not automatically follow, though, that \( R_c \) should be made as large as possible. Equation (17) shows that the limit \( L_t \gg a \) is characterized by a very high overall resistance for the contact. This has at least two potentially detrimental effects 1) with a high resistance in the return path to ground, there is a danger that some of the RF current will be shunted to other electrodes (such as ECG monitoring electrodes) or, indeed, to any lower resistance ground paths; and 2) if \( R_c \) is too large, direct power dissipation in \( R_c \) could pose a new burn hazard to the patient.

A numerical example will serve to illustrate the design tradeoff involved in attempting to increase the current uniformity by increasing \( L_t \). Consider a circular dispersive electrode of radius \( a = 5 \text{ cm} \) (area \( A = 79 \text{ cm}^2 \)), and assume \( R_s = 100 \Omega \) (corresponding, perhaps, to \( \rho = 100 \Omega \cdot \text{cm} \) and \( w = 1 \text{ cm} \)). A specific contact resistance of 25 Ω·cm² would give \( L_t = 0.5 \text{ cm} \). Using (6) it is easy to verify that an annular ring within 0.5 cm of the perimeter (i.e., the ring \( 4.5 \text{ cm} \leq r \leq 5 \text{ cm} \) would collect 63.5 percent of the total current. As this ring constitutes only 19 percent of the total contact area, the current density is enhanced at the perimeter by at least a factor of 3.4 over a hypothetical uniform distribution. The heating effect at the perimeter would be enhanced by a factor of about (3.4)² = 11.5. The total resistance of the contact is found from (9) to be \( 1.6 \Omega \).

Suppose now, that the specific contact resistance were increased to \( R_c = 100 \Omega \cdot \text{cm}^2 \). This would give \( L_t = 1 \text{ cm} \), and \( R_t = 3.6 \Omega \). The same perimetal ring would now collect 43.1 percent of the current, giving a current-density enhancement factor of 2.3 and a heating enhancement factor of 5.2. Thus, some improvement has been achieved at the cost of doubling the total resistance.

A further increase in \( R_c \) to 2500 Ω·cm² gives \( L_t = 5 \text{ cm} \), \( R_t = 35 \Omega \), and enhancement factors of 1.09 and 1.18, respectively.

Equations (6) and (9) can be used to perform similar calculations for other assumed values of the parameters. In general, it is found that if \( L_t \approx a \) the current density is satisfactorily uniform and the total contact resistance is \( R_t = 0.36 R_s \).

#### B. Capacitive Electrodes

The current-transfer length for the capacitive electrode model is given by (23) as

\[ L_t = \left( \frac{1}{\omega R_s C_a} \right)^{1/2} = \left( \frac{d}{\omega R_s \epsilon_r \epsilon_a} \right)^{1/2} \]

where \( d \) and \( \epsilon_r \) are the thickness and relative permittivity of the dielectric layer. In this case, \( d \) is likely to be the most conveniently adjustable parameter. For purposes of numerical illustration, consider \( \omega = 2\pi \times 10^9 \text{ Hz} \), \( R_s = 100 \Omega \), \( d = 0.5 \text{ mils} \), and \( \epsilon_r = 2.2 \) (the latter two numbers correspond to a
commercially available polyethylene film). With these values, \( \lambda_e = 3.4 \) cm. For an electrode radius of \( a = 5 \) cm, this value of \( \lambda_e \) is large enough to avoid excessive current concentration at the perimeter and provides a reasonable contact impedance of \( Z = 4.0-14.7 \Omega \) or \( |Z| = 15.2 \Omega \). Increasing the thickness of the dielectric improves the current uniformity, but causes \( Z \) to rise very rapidly. A value of \( d = 10 \) mls, for example, gives \( \lambda_e = 14.4 \) cm, \( Z = 4.8-262 \Omega \), and \( |Z| = 262 \Omega \).

IV. CONCLUSIONS

We have investigated the behavior of distributed equivalent-circuit models for circular dispersive electrodes which are resistively or capacitively coupled to the body. In both cases, it is possible to define a current-transfer length \( L_t \) or \( \lambda_t \) which determines the current distribution and the total electrode impedance as follows:

For \( L_t \) or \( \lambda_t >> a \), where \( a \) is the electrode radius, the entire electrode participates in current collection, and there is only a slight excess current density at the perimeter. The electrode impedance, however, is extremely large in this limit.

For \( L_t \) or \( \lambda_t \approx a \), there is a moderate current concentration at the perimeter, and the magnitude of the impedance of the electrode is \( \approx 0.36 R_g \) where \( R_g \) is the sheet resistance of the underlying tissue. Whether or not the current concentration in this regime is sufficient to cause burns is a question which can only be answered by experimental measurements.

For \( L_t \) or \( \lambda_t \ll a \), only a narrow perimetric segment of the electrode is effective in current collection. This causes extremely high current densities near the perimeter, and clearly poses a burn threat. The magnitude of the electrode impedance is very small in this limit.

In view of these findings, it appears that the design of simple dispersive electrodes presents a classical engineering tradeoff. The electrode must provide a low-impedance return path for the RF current, but must simultaneously collect this current over a sufficiently large area to avoid burns. These requirements are in conflict, and a compromise must be made. Our models suggest that optimum designs will probably be those for which the current-transfer length is approximately equal to the electrode radius.

Finally, we believe that the phenomenon of excess current density at the perimeter and the uniformity/impedance tradeoff will be shared by all simply connected electrodes having totally convex perimeters, independent of shape (although analytical models would be considerably more difficult to formulate for noncircular shapes). This observation leads us to suggest that multiply-connected electrodes (e.g., several small disks in parallel, annular rings, etc.) may provide a fruitful area for further research. One such electrode has already been proposed [6] and further work along these lines is in progress [7].

REFERENCES


Correction to “An Instrument for Self-Measurement of Intraocular Pressure”

On page 180 of the above paper, Fig. 3 appears incorrectly. The correct positioning appears below.

Fig. 3. General view of the self tonometer in operation.