SYSTEM THEORETIC ANALYSIS OF BATTERY EQUALIZATION SYSTEMS

by

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A Thesis Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Master of Science

in Engineering

at

The University of Wisconsin-Milwaukee

May, 2014

ABSTRACT
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The University of Wisconsin-Milwaukee, 2014
Under the Supervision of Liang Zhang

Battery equalizers are widely used in multi-battery systems to maintain balanced charge among individual battery cells. While the research on the hardware realization of battery equalizers has received significant attention, rigorous analysis of battery equalization from the system's point of view remains largely unexplored. In this research, we study three types of battery equalization system structures: series-based, layer-based, and module-based. Specifically, we develop mathematical models that describe the system-level behavior of the battery equalization processes under these equalization structures. Then, based on the mathematical models, analytical methods are derived to evaluate the performance of the equalization processes. We also carry out statistical analysis to compare the performance of the three equalization structures considered. In addition, these systems will be studied with energy loss.

Note to Practitioners—This research work develops computationally efficient tools to evaluate the performance of battery equalization systems under series-based, layer-based, and module-based structures, respectively. Using these tools, engineers and designers can predict the equalization time of a battery system in real-time

with high accuracy. In addition, based on these tools, one can compare the equalization performance under different structures without using time-consuming simulations. Numerical experiments suggest that the layer-based and module-based structures have better average performance than the traditional series-based equalization structure.

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1 Introduction

In recent years, hybrid electric, development electric and plug-in hybrid electric technology have become one of the most important directions of vehicular research [1]. Although there already exist multiple HEV models in the market, researchers and manufacturers never stop to pursue better energy efficiency. In the above electric models, battery is the primary energy storage device for the power supply. However, due to material and manufacturing constraints, individual battery cells are designed that could only provide limited voltage and power. A single battery cell usually is not able to provide enough power say, driven a hybrid electric vehicle. Therefore, in applications which require high voltage and/or high power, multiple battery cells are often connected together in order to meet the power demand. Paper [2] implies that batteries connected in series are able to provide enough power/high voltage and have been employed in many applications. However, due to variations in manufacturing process, temperature, usage, and other sources, the serially connected battery cells may contain different amounts of charge, and the unbalanced charge may lead to lower efficiency, shorter lifetime, and higher risk of fire and explosion. To maintain balanced charge in a battery system, various types of battery equalizers have been developed. Review paper [3] summarized different types of technologies used in the battery equalization process. Technologies used in the hardware realization of battery equalizers include dissipative resistance shunt [4], bidirectional dc-dc converters [5, 6, 7], switch capacitors [8, 9, 10], multi-winding transformers [11], and two-step buck-boost converters [12], etc.

Lately, increasing research efforts have been invested into developing faster and more efficient battery equalization systems. For instance, a fuzzy logic-based controllers is developed in [13] to reduce equalization time for series-connected battery system. The controller is tested for a 3-battery system. In addition, paper [14] proposes a new battery equalizer configuration and analyzes its performance and cost

effectiveness. The proposed equalizer is tested using both computer simulations and physical experiment on a 5-cell prototype system. In paper [15], a layer-based battery equalization system is introduced. The efficacy of the layer structure is compared with the traditional series-based equalization structure, where all equalizers and batteries are connected in a series manner, using MATLAB simulations and physical circuit experiments of a 4-battery system. The result shows that the layer structure could result in a shorter system equalization time. In paper [16], a module-based battery equalization structure is proposed and is further studied in [12, 17, 18], where the developed equalization system is tested on an 88-lithium-ion-cell battery system. The experimental result implies that the modularized equalizer outperforms the conventional serial approach.

Despite these achievements, system-theoretic investigations of the equalization processes have rarely been carried out. Such studies are critical in system-level performance prediction, parameter optimization, and design of control algorithms. For instance, in papers [12, 13, 14, 15, 16, 17, 18], the efficacy of each equalization system is demonstrated by only one or two specific example. This approach, however, may be misleading since the system equalization performance may depend on the initial status of battery charge and it is not clear how well the examples used can represent the general case. One or two examples are not enough to generate conclusions regarding system's general performance in these cases. Therefore, the performance of the equalization systems under general initial conditions must be studied based on sufficiently large sample size and rigorous statistical analysis to provide justified conclusions. While extensively experimenting with the physical systems is often impractical, computer simulations can be used to accomplish this task. In the current literature, computer software, such as MATLAB/Simulink, PSpice, etc., have already been applied to simulate the behavior of the electrical circuit of battery equalization systems. For instance, a simulation framework of battery operation is developed in

[14] based on MATLAB Simulink. However, due to the limitations of the software, the simulations are usually time-consuming, inflexible and difficult to build or manage for systems with a large number of battery cells. Moreover, it is sometimes not necessary to include detailed circuit components to study the system-level behavior of the battery equalization process. The simulations we carried out in this research are coded in C++, which has a comparatively short running time even when the model is large.

The purpose of this research is to study battery equalization by introducing system-theoretic models of equalization processes in the frameworks of series-based equalization, layer-based equalization, and module-based equalization, respectively. To carry out the analysis, mathematical models are constructed for each case considered. Then, analytical procedures are developed to evaluate the system equalization time based on the initial charge of the batteries. In terms of the equalizer type, paper [19] introduces different types of technologies used in the equalizers. There are equalizers which could transfer charge from the most charged cell to the least charged cell such as flying capacitor, or from multiple higher charged to multiple lower charged cells such as energy converter, etc. In this research however, we only focus on one type of equalizer which transfer charge between the two adjacent battery/battery groups connected to it. It should also be emphasized again that this research will focus on the system-level behavior of the systems instead of their hardware realizations. Moreover, the research will further analyze the systems with energy loss and with non-identical equalizers. In reality, a battery system will experience energy loss as the system is running. Paper [20] investigates the battery energy loss due to the electrical contact resistance (ECR). Meanwhile, research effort has been put into reducing the energy loss or optimizing the system's performance and a method of energy optimization in a battery/supercapacitor hybrid energy system is studied in paper [21]. Here, in order to carry out the system analysis, we first assume that all battery equalization

systems are energy lossless then the models with energy loss will be studied.

The remainder of the thesis is organized as follows: Section 2 presents the assumptions for battery systems with series equalization and formulates the mathematical model. Then, based on the mathematical model, we develop analytical methods for evaluation of the system equalization time. Similar analysis for layer-based and module-based equalization systems are described in Sections 3 and 4, respectively. A comparative statistical study of the three equalization structures is carried out in Section 5. System with energy loss and their analysis will be studied in section 6. Finally, conclusions and future work are given in Section 7. All proofs are included in the Appendix.

2 Series-based Equalization Structure

2.1 Model

2.1.1 Descriptive Model

Consider a battery charge equalization system shown in figure 2.1 defined by the following assumptions:

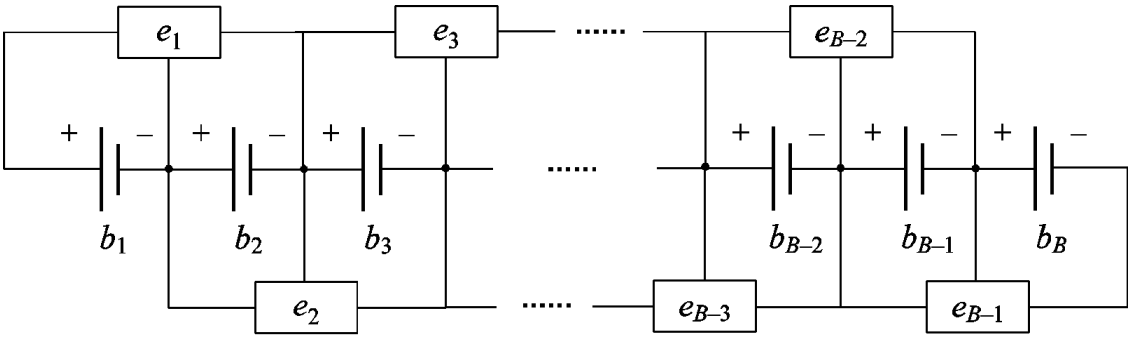


Figure 2.1: Series-based battery equalization system

(I) The system consists of B batteries, b_1,b_2, ...b_B, connected in series, and $B-1$ equalizers, $e_1, e_2, ...e_{B-1}$, connecting each pair of consecutive batteries.

(II) The equalizers have the same working cycle, τ. The time axis is slotted with slot duration τ . Equalizers begin operating at the beginning of each time slot.

(III) Each equalizer is characterized by its equalization rate r_i (unit of charge per time slot), $i = 1, 2, ...B-1$.

(IV) At the beginning of time slot t, if battery b_i has a larger state of charge (SOC) then battery b_{i+1}, then equalizer e_i transfer r_i unit of charge from b_i to b_{i+1} during this time slot or working cycle. If b_{i+1} has a large SOC then b_i, equalizer e_i makes the opposite transfer. If two batteries have the same SOC, no transfer takes place between these two batteries in this time slot.

(V) Assume that all energy within batteries and equalizers are lossless in this model.

In practice, the equalizers under serial structure of Figure 2.1 are designed to have identical parameters. Thus we can assume that

$$r_i = r, \forall i \in \{1, ...B\}. \tag{2.1}$$

Although the model defined by assumptions (I)-(V) contains no electrical components, the transition of energy during equalization is captured at the system-level. Clearly, this model applies to many types of equalizers regardless of the type of technology used. Therefore, this simplified model can focus the study on the system performance of equalization rather than on the hardware realization. Moreover, for simplicity, we assume in (V) that no energy is lost during the equalization process. Extensions of the study to systems with energy losses in batteries and/or equalizers will be considered in section 6.

2.1.2 Mathematical Model

Let $x_i(n) \in [0, 1]$ denote the SOC of battery b_i at the end of time slot (i.e working cycle) n, $i = 1, ..., B$, $n = 0, 1, ...$ The evolution of the system can be given by:

$$x_1(n+1) = x_1(n) + sgn(x_2(n) - x_1(n)) \cdot r, \tag{2.2}$$

$$x_i(n+1) = x_i(n) + sgn(x_{i-1}(n) - x_i(n)) \cdot r + sgn(x_{i+1}(n) - x_i(n)) \cdot r, i = 2, ..., B-1, \tag{2.3}$$

$$x_B(n+1) = x_B(n) + sgn(x_{B-1}(n) - x_B(n)) \cdot r, \tag{2.4}$$

where

$$sgn(u) = \begin{cases} -1, \ if \quad u < 0, \\ 0, \ if \quad u = 0, \\ 1, \ if \quad u > 0. \end{cases} \tag{2.5}$$

Let

$$\bar{x} = \frac{1}{B} \sum_{i=1}^{B} x_i(0). \tag{2.6}$$

Clearly, the equilibrium of the above system is $[\bar{x}, \bar{x}, ..., \bar{x}]$ and it is globally attractive. However, since the system defined by (I)-(V) operates in discrete time, limit cycle convergence around this equilibrium is possible. The equilibrium or the balance state of the actual system will be close to the ideal equilibrium above. Let N_e^{series} denote the smallest number of time slot (working cycles), for the system to reach its equilibrium or the limit cycle. Then the equalization time of the system, denoted as T_e^{series}, is given by

$$T_e^{series} = N_e^{series} \cdot \tau. \tag{2.7}$$

For the practical equalization process, T_e^{series} is one of the most important performance measures of a battery system. The smaller it is, the faster the battery system will balance itself. Therefore, in the following subsections, we will study the evaluation of T_e^{series} and its properties.

2.2 Performance Evaluation

To evaluate the performance of the system defined by (I)-(V), we defined T_e^{series} as its equalization time. One can "simulate" the mathematical model by iteratively calculate $x_i(n)$ using equation (2.2)-(2.5) until convergence is observed. Although

this approach can reduce the evaluation time significantly compared to simulating the actual electrical system, the amount of calculation may increase substantially when the number of battery B increases. The long computation time become the main obstacle to embed the evaluation process into a battery management system (BMS) for active control of the battery operation. Therefore, in this research, the goal is to develop computationally efficient algorithm/procedures to evaluate (or approximate) the equalization time of a battery system. Noted that the equalization rate of a equalizer cannot be too large since it is costly of producing such equalizer. In reality, it is very common that $r \ll 1$. Thus we consider the following continuous time version of the system defined by (2.8)-(2.10):

$$\dot{x}_1(t) = sgn(x_2(t) - x_1(t)) \cdot \widetilde{r}, \tag{2.8}$$

$$\dot{x}_i(t) = sgn(x_{i-1}(t) - x_i(t)) \cdot \widetilde{r} + sgn(x_{i+1}(t) - x_i(t)) \cdot \widetilde{r}, i = 2, ..., B-1, \tag{2.9}$$

$$\dot{x}_B(t) = sgn(x_{B-1}(t) - x_B(t)) \cdot \widetilde{r}, \tag{2.10}$$

where

$$\widetilde{r} = \frac{r}{\tau}. \tag{2.11}$$

For this system, $[\bar{x}, \bar{x}, ..., \bar{x}]$ is the unique equilibrium and it is globally attractive. Moreover, since this system operates in continuous time, limit cycle convergence does not appear. Furthermore, let $y_i(t) = x_i(t) - \overline{x}, i = 1, ...B$. Then, the system evolution can be expressed as:

$$\dot{y}_1(t) = sgn(y_2(t) - y_1(t)) \cdot \widetilde{r}, \tag{2.12}$$

$$\dot{y}_i(t) = sgn(y_{i-1}(t) - y_i(t)) \cdot \widetilde{r} + sgn(y_{i+1}(t) - y_i(t)) \cdot \widetilde{r}, i = 2, ..., B-1, \tag{2.13}$$

$$\dot{y}_B(t) = sgn(y_{B-1}(t) - y_B(t)) \cdot \widetilde{r}, \tag{2.14}$$

where

$$\sum_{i=1}^{B} y_i(0) = 0, \ t \geq 0. \tag{2.15}$$

Under this continuous formulation, $[0, 0, ..., 0]$ becomes the system equilibrium.

Let $\widetilde{T}_e^{series}$ denote the equalization time of the system defined by (2.12)-(2.15), i.e., the smallest t such that $y_i(t) = 0$, for all $i \in \{1, ...B\}$. Then for $r \ll 1$, the equalization time of the original system, i.e., the discrete time system defined by (2.2)-(2.5), and its continuous time counterpart (2.12)-(2.15), should satisfy the following relationship:

$$T_e^{series} \approx \widetilde{T}_e^{series}. \tag{2.16}$$

Theorem 1. *For $B \in \{2, 3\}$, the equalization time of a battery charge system defined by (2.12)-(2.15) can be calculated as follows:*

$$\widetilde{T}_e^{series}(B = 2) = \frac{|y_1(0)|}{\widetilde{r}} = \frac{|y_2(0)|}{\widetilde{r}}, \tag{2.17}$$

$$\widetilde{T}_e^{series}(B = 3) = max \ \{\frac{|y_1(0)|}{\widetilde{r}}, \frac{|y_3(0)|}{\widetilde{r}}\}. \tag{2.18}$$

By using (2.17) and (2.18), the equalization time of a battery system defined by assumption (I)-(V) with $B = 2$ or $B = 3$ can be calculated (approximated) for any given initial condition of battery charge. To evaluate the equalization time for systems with $B \geq 4$ batteries, we introduce the following notation:

Definition 2. Battery group (BG): a group of consecutively connected batteries in a system defined by assumptions (I)-(V).

Let g denote the number of batteries in a battery group (BG). Then for a B-battery string, g takes the value from $\{1, ..., B-1\}$ and, there are a total of $\frac{(B-1)(B+2)}{2}$ different BGs. Let $BG(g, i)$ denote the g-battery BG starting with battery b_i, $g \in \{1, ..., B-1\}$, $i \in \{1, ..., B-g+1\}$. In other words, $BG(g, i)$ represents battery string $b_i - b_{i+1} ... - b_{i+g-1}$. In addition, battery group $BG(g, i)$ is referred to as a *boundary battery group* if $i = 1$ or $i = B - g + 1$; and *internal battery group* otherwise.

During the equalization process, the SOC of each battery, $x_i(t)$, tends to $\overline{x}$, and $y_i(t)$ tends to0. Moreover, for each battery group, summation $\sum_{j=i}^{i+g-1} y_j(t)$ also tends to 0. For boundary battery groups, if consider it as a single battery cell, the rate of equalization process is upper-bounded by the equalization rate of the equalizer at the end, i.e., $\widetilde{r}$; while for internal groups, the upper bound is the total equalization rates of the equalizers on both ends of the BG, i.e., $2\widetilde{r}$. Therefore, we define the *idea equalization time*, $t_{ideal}(g, i)$, of battery group $BG(g, i)$, $g \in \{1, ..., B-1\}$, $i \in \{1, ..., B-g+1\}$, as follows:

- If $BG(g, i)$ is a boundary battery group,

$$t_{ideal}(g, i) = \frac{\left| \sum_{j=i}^{i+g-1} y_j(0) \right|}{\widetilde{r}}, \tag{2.19}$$

- If $BG(g,i)$ is an internal battery group,

$$t_{ideal}(g,i) = \frac{\left|\sum_{j=i}^{i+g-1} y_j(0)\right|}{2\widetilde{r}}, \tag{2.20}$$

It is also possible to combine the two expressions above into one as follows:

$$t_{ideal}(g,i) = \frac{\left|\sum_{j=i}^{i+g-1} y_j(0)\right|}{\widetilde{r}[2 - I_{\{1\}}(i) - I_{\{B-g+1\}}(i)]},\ g \in \{1,...,B-1\},\ i \in \{1,...,B-g+1\}, \tag{2.21}$$

where $I_A(x)$ is the indicator function given by:

$$I_A(x) = \begin{cases} 1,\ if\ x \in A, \\ 0,\ otherwise. \end{cases} \tag{2.22}$$

Definition 3. bottleneck battery group (BNBG): the battery group with the longest idea equalization time.

Let $\widehat{T}_e^{series}$ denote the ideal equalization time of the BNBG. Then based on Definition 3,

$$\widehat{T}_e^{series} = \max_{g\in\{1,...,B-1\},\ i\in\{1,...,B-g+1\}} t_{ideal}(g,i), \tag{2.23}$$

Proposition 4. *The equalization time of a battery charge system defined by (12)-(15) is equal to the ideal equalization time of the BNBG of the system, i.e.,*

$$\widehat{T}_e^{series} = \widetilde{T}_e^{series}, \tag{2.24}$$

and, therefore because of (2.16)

$$T_e^{series} \approx \widehat{T}_e^{series}. \tag{2.25}$$

B	4	8	16
Average ϵ_s	0.0037%	0.0096%	0.0287%
B	32	64	
Average ϵ_s	0.0827%	0.2324%	

Table 1: Accuracy of Proposition 4

Justification: To justify Proposition 4, numerical experiments are carried out. Specifically, as we introduced in the introduction section, a C++ program is developed to calculate the state evolution of the battery equalization system defined by assumption (I)-(V) using equation (2.2)-(2.5). Using this program, the equalization time of the discrete system, N_e^{series}, can be obtained. Then N_e^{series} is compared with the ideal equalization time of the BNBG, $\widehat{T}_e^{series}$. Clearly, if Proposition 1 is true, then the relationship $N_e^{series} \cdot \tau \approx \widehat{T}_e^{series}$ should be observed under the same initial condition of battery SOC. To carry out the experiments, we selected battery number $B \in \{4, 8, ..., 64\}$and $r = 0.00001$. For each B, a total of $50,000$ different formations of initial battery charge are generated with $x_i(0)$'s selected randomly and independently according to uniform distribution $U(0, 1)$. Without loss of generality, let $\tau = 1$. For each initial charge formation, we evaluate its corresponding N_e^{series} using simulation and calculate $\widehat{T}_e^{series}$ based on equation (2.23), and evaluate the relative error based on:

$$\epsilon_s = \frac{\left| N_e^{series} - \widehat{T}_e^{series} \right|}{N_e^{series}} \cdot 100\%. \tag{2.26}$$

The average values of ϵ_s for various B's are summarized in Table . The errors observed are mainly due to the limit cycle convergence observed in the discrete time system. Other than that, we claim that Proposition 4 holds.

Clearly, Proposition 4 provides an efficient approach for analytical evaluation of the equalization time in a battery equalization system defined by assumption (I)-(V) without extensive calculation of system states evolution. Using Proposition 4, one

only needs to calculate the ideal equalization time of all $\frac{(B-1)(B+2)}{2}$ battery groups and identify the BNBG(s). Moreover, it turns out that this procedure can be further simplified thanks to the following properties:

Theorem 5. *Consider a battery equalization system defined by assumptions (I)-(V). Among all BNBGs of the system, there exists at least one boundary battery group with no more that* $\lfloor B/2 \rfloor$ *batteries.*

Therefore, based on Theorem 2, the equalization time of the system can be calculated as:

$$T_e^{series} = \max_{g \in \{1,\ldots,\lfloor B/2 \rfloor\},\, i \in \{1,\ldots,B-g+1\}} t_{ideal}(g,i), \tag{2.27}$$

In other words, the number of battery groups to evaluate is reduced from $\frac{(B-1)(B+2)}{2}$ to B. This reduction may be significant considering that B is usually large in practice. Finally, the amount of calculation required by this method would also allow embedding the procedure into the battery management system for potential active control of the equalization process in the future.

2.3 Initial Battery SOC Permutation

Since a battery equalization system consists of multiple battery cells with different initial SOCs, the permutation of all batteries will affect the equalization time of the system. Here is an example: consider a series-based equalization system with 4 batteries, the initial SOC of these batteries are 0.2, 0.4, 0.6 and 0.8. Assume the equalization rate is 0.0001. The permutations and their corresponding equalization times are given in Table 2. Intuitively, the monotonic permutation has the longest equalization time since it need to transfer charge from one end of the battery string to another.

Below is a theorem and its proof is given in the appendix.

Initial SOC Permutation	Equalization Time
0.4-0.8-0.2-0.6	2000
0.2-0.6-0.4-0.8	3000
0.2-0.4-0.6-0.8	4000

Table 2: Initial SOC permutation and equalization time example

Theorem 6. *If the initial SOC of all batteries are given but the permutation is not fixed and all equalizers are identical, then the monotonic permutation has the longest equalization time.*

3 Layer-based Equalization Structure

3.1 Model

3.1.1 Descriptive Model

The series-based equalization structure is widely used in practice. However, this structure may be inefficient and have long equalization time under certain initial battery charge combinations. To improve the equalization process, a layer-based battery equalization structure is developed in [15]. The block diagram of such equalization system is given in Figure 3.1. To carry out analysis of this equalization structure, we assume that the system operates under assumption (I)-(III) and (V) of subsection 2.1 and the following:

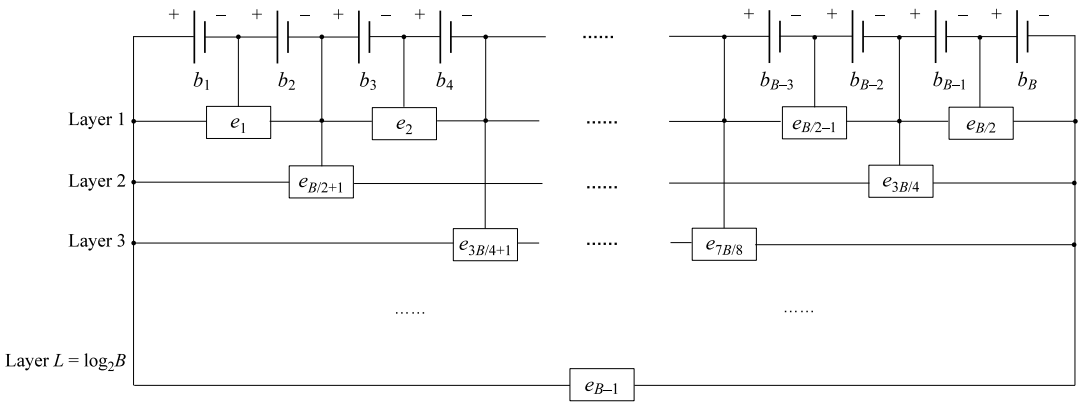


Figure 3.1: Layer-based battery equalization system

(VI) The equalizers are arranged in $L = log_2 B$ layers. In layer$l$, $l \in \{1, 2, ..., L\}$, there are $B/2^l$ equalizers. Assume that all equalizers in layer l have identical equalization rate r_l.

(VII) Under the layer-based structure, the jth equalizer in layer l is used to equalize the charge between battery group $BG(2^{l-1}, 1 + (j-1)2^l)$ and $BG(2^{l-1}, 1 + (2j-1)2^{l-1})$: If battery group $BG(2^{l-1}, 1 + (j-1)2^l)$ has a larger total SOC than $BG(2^{l-1}, 1 + (2j-1)2^{l-1})$ at the beginning of a time slot, then the equalizer takes $r_l/2^{l-1}$ units of charge from each battery in $BG(2^{l-1}, 1 + (j-1)2^l)$, and send $r_l/2^{l-1}$

units of charge to each battery in $BG(2^{l-1}, 1+(2j-1)2^{l-1})$ during the time slot; if $BG(2^{l-1}, 1+(j-1)2^{l})$ has a lower SOC, the equalize makes the opposite transfer; if $BG(2^{l-1}, 1+(j-1)2^{l})$ and $BG(2^{l-1}, 1+(2j-1)2^{l-1})$ have the same SOC, no transfer takes place between the two battery groups during this time period.

In the series-based equalization structure considered in Section 2, boundary battery is affected by one equalizer and internal battery is affected by two equalizers. In the layer-based equalization structure, each battery is affected by a total of L equalizers, that is one equalizer in each layer. In addition, due to assumption (VII) which indicates that the energy transferred by equalizer is evenly distributed among all batteries involved, the equalizers in the layer-based structure operate rather "independently." For instance, consider the 8-battery layer-based equalization system shown in Figure 3.2. In this system, there are $log_2 8 = 3$ layers. Equalizere_1 operates to eliminate the difference in SOC between b_1 and b_2 while e_5 balances the total charge between battery group $BG(2,1)$ and $BG(2,3)$, and e_7 balances the total charge between battery group $BG(4,1)$ and $BG(4,5)$. Under the layer structure, the equalization process of e_1 will not be accelerated or slowed down by other equalizers (e.g., e_5 and e_7) in the system, since those equalizers can only increase or decrease the SOCs of both b_1 and b_2 simultaneously by the same amount.

3.1.2 Mathematical Model

Let $x_i(n)$, again, denote the SOC of batteryb_i at the end of time slot n, $i = 1, ..., B$, $n = 0, 1, 2....$ Then, the evolution of the layer-based system is given by :

$$x_i(n+1) = x_i(n) + \sum_{l=1}^{L} \sum_{j=1}^{B/2^l} (sgn(\Delta_{l,j}(n)) \cdot [I_{S^+_{l,j}}(i) - I_{S^-_{l,j}}(i)] \cdot \frac{r_l}{2^{l-1}}), \ i = 1, ..., B, \quad (3.1)$$

where the signum function $sgn(\cdot)$ and the indicator function $I_A(\cdot)$ are defined in

(2.5) and (2.22) respectively, and

$$\Delta_{l,j}(n) = \sum_{m \in S^-_{l,j}} x_m(n) - \sum_{m \in S^+_{l,j}} x_m(n), \tag{3.2}$$

$$S^+_{l,j} = \{(j-1)2^l + 1,\ (j-1)2^l + 2, ...,\ (j-1)2^l + 2^{l-1}\}, \tag{3.3}$$

$$S^-_{l,j} = \{(2j-1)2^{l-1} + 1,\ (2j-1)2^{l-1} + 2, ...,\ (2j-1)2^{l-1} + 2^{l-1}\}. \tag{3.4}$$

In this formulation, $S^+_{l,j}$ and $S^-_{l,j}$ represent the indices of batteries in the two battery groups balanced by the jth equalizer in the lth layer, while $\Delta_{l,j}(n)$ is the difference of total SOCs between the two BGs at time slot n.

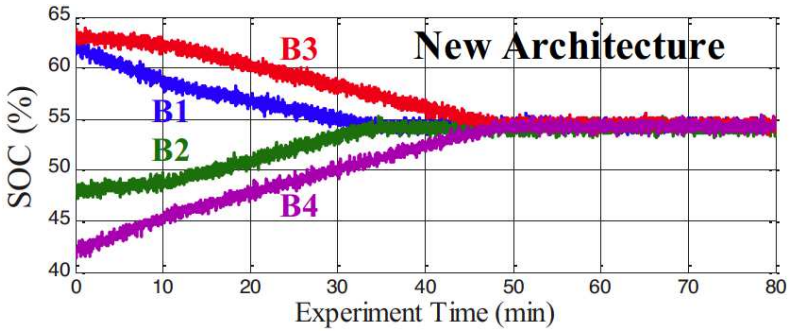
An illustration of the mathematical model for a 4-battery pack is given in Figure 3.2. The initial SOCs of these batteries are $x_1(0) = 0.62$, $x_2(0) = 0.48$, $x_3(0) = 0.63$, and $x_4(0) = 0.42$. Part (a) of the figure shows the actual experimental result obtained in [15] for the system under layer-based structure, while part (b) provides the system evolution obtained based on iterative calculation of equation (3.1).

3.2 Performance Evaluation

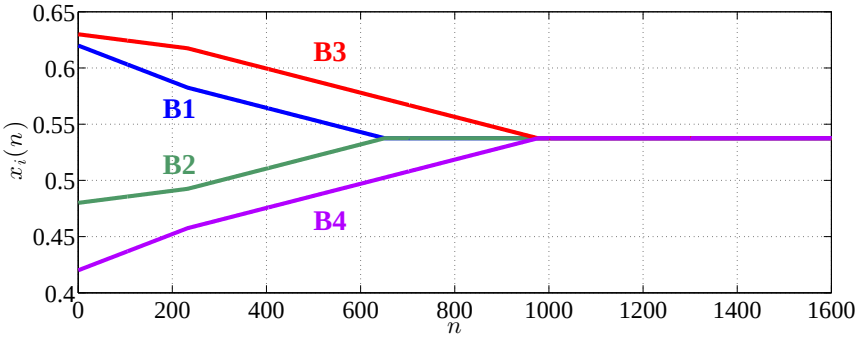
To calculate the equalization time of a layer-based equalization system defined by assumption (I)-(III), (V)-(VII), consider the continuous version of the system model formulation (3.1)-(3.4):

$$\dot{x}_i(t) = \sum_{l=1}^{L} \sum_{j=1}^{B/2^l} (sgn(\Delta_{l,j}(n)) \cdot [I_{S^+_{l,j}}(i) - I_{S^-_{l,j}}(i)] \cdot \frac{\widetilde{r}_l}{2^{l-1}}),\ i = 1, ..., B, \tag{3.5}$$

where



(a) Actual experimental result [15]



(b) Simulation of the mathematical model ($r = 0.000125$)

Figure 3.2: Comparison between the mathematical model and the actual physical system of layer-based battery equalization system

$$\widetilde{r_l} = \frac{r_l}{\tau}. \tag{3.6}$$

For equalizer e_k, $k = 1, ..., B-1$, introduce:

$$l(k) = \left\lceil -log_2(1 - \frac{k}{B}) \right\rceil, \tag{3.7}$$

$$j(k) = k - [1 - 2^{1-l(k)}] \cdot B. \tag{3.8}$$

Then, it can be shown that e_k is the jth equalizer in layer $l(k)$ of the layer-based battery equalization system defined above. In addition, since, as stated above, the equalizers operate "independently" of each other under layer-based structure, the time needed for e_k to balance the charge between the two battery groups involved can be

calculated as:

$$t_{eq}(k) = \frac{\left|\Delta_{l(k),j(k)}(0)\right|}{2\widetilde{r}_{l(k)}}, \tag{3.9}$$

where $\Delta_{l,j}(0)$, $l(k)$ and $j(k)$ are defined in (3.2), (3.7) and (3.8), respectively.

Definition 7. Bottleneck equalizer (BNEQ): the equalizer with the longest equalization time.

Theorem 8. *The equalization time of a battery charge system defined by (3.5) and (3.6) is equal to the equalization time of the BNEQ of the system,*

$$\widetilde{T}_e^{layer} = \max_{k\in\{1,\ldots,B-1\}} t_{eq}(k). \tag{3.10}$$

Therefore, for $r_l \ll 1$, the equalization time of a layer-based equalization system defined by assumption (I)-(III), (V)-(VII) can be approximated using Theorem 7:

$$T_e^{layer} = \widetilde{T}_e^{layer}. \tag{3.11}$$

4 Module-based Equalization Structure

4.1 Model

4.1.1 Descriptive Model

To improve the performance of a battery equalization system, another structure is proposed and studied in [12, 17, 18]. In this structure, the entire battery string is divided into several "modules", each having the same number of battery cells. Within each module, equalizers are deployed based on the conventional series-based equalization structure. These equalizers are referred to as the *cell-level equalizers*. Then, each module is viewed as a "virtual" battery and a group of equalizers are connected to these modules based, again, on the series structure. These equalizers are referred to as the *module-level equalizers*. The block-diagram of such a equalization system is shown in Figure 4.1. Th carry out the analysis of this equalization structure, we assume that the system operates under assumption (I)-(III) and (V) of subsection 2.1.1 and the following:

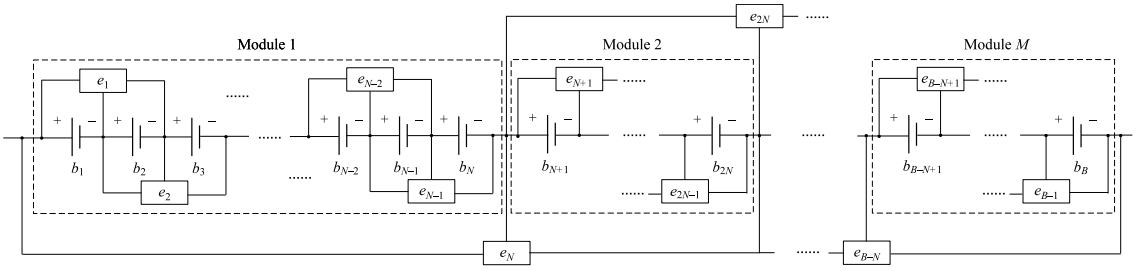


Figure 4.1: Module-based battery equalization system

(VIII) The battery string is divided into M modules, each with N individual batteries. Clearly, $M \cdot N = B$.

(IX) The equalizers are divided into two groups: cell-level equalizers, $S_c = \{jN + k \mid j = 0, ..., M-1;\ k = 1, ..., N-1\}$, and module-level equalizers, $S_M = \{N, 2N, ..., (M-1)N\}$. All cell-level equalizers have identical equalization rate r_c, and all module-level equalizers have identical equalization rate r_m.

(X) For a cell-level equalizer e_i, $i \in S_c$, if battery b_i has a larger SOC than b_{i+1} at the beginning of a time slot, then e_i transfers r_c units of charge from b_i to b_{i+1} during the time slot; if b_i has a lower SOC, then e_i makes the opposite transfer; if b_i and b_{i+1} have the same SOC, no transfer takes place between the two batteries during this time slot.

(XI) For a module-level equalizer e_i, $i \in S_M$, if battery group $BG(N, i-N+1)$ has a larger total SOC than $BG(N, i+1)$ at the beginning of time slot, then the equalizer takes r_m/N unites of charge from each battery in $BG(N, i-N+1)$ and send the same amount of charge to each battery in $BG(N, i+1)$ during the time slot; if $BG(N, i-N+1)$ has a lower total SOC, the equalizer makes the opposite transfer; if $BG(N, i-N+1)$ and $BG(N, i+1)$ have the same SOC, no transfer takes place between the two battery groups during the time slot.

Under this model framework, the equalization within a module is "independent" from all other modules since module-level equalizer modify the state of all batteries in a given module with the same amount.

4.1.2 Mathematical Model

Let $x_i(n)$, again, denote the SOC of battery b_i at the end of time slot n, $i = 1, ..., B$, $n = 0, 1, ...$ Then, the evolution of the system is given by:

$$x_i(n+1) = x_i(n) + \sum_{j=1}^{B-1} (sgn(\Delta_j(n)) \cdot [I_{S_j^+}(i) - I_{S_j^-}(i)] \cdot \frac{r_c I_{S_c}(j) + r_m I_{S_M}(j)}{1 + (N-1) \cdot I_{S_M}(j)}),\ i = 1, ..., B, \tag{4.1}$$

where the signum function $sgn(\cdot)$ and the indicator function $I_A(\cdot)$ are defined in (2.5) and (2.22), respectively, and

$$\Delta_j(n) = \sum_{m \in S_j^-} x_m(n) - \sum_{m \in S_j^+} x_m(n) \tag{4.2}$$

$$S_j^+ = \begin{cases} \{j-N+1, j-N+2, ...j\}, & if\ j \in S_M, \\ \{j\} & if\ j \in S_C, \end{cases} \tag{4.3}$$

$$S_j^- = \begin{cases} \{j+1, j+2, ...j+N\}, & if\ j \in S_M, \\ \{j+1\} & if\ j \in S_C, \end{cases} \tag{4.4}$$

$$S_C = \{jN+k \mid j = 0, ..., M-1; k = 1, ..., N-1\}, \tag{4.5}$$

$$S_M = \{N, 2N, ..., (M-1)N\}. \tag{4.6}$$

4.2 Performance Evaluation

Since the equalization within a module is not affected by other modules, and since the inter-module equalization is independent of the intra-module equalization, the equalization time of the overall system can be calculated using the following algorithm:

Algorithm 4.1

Step 1: Consider each module as an isolated battery system with series-based equalization.

Step 2: Identify the BNBG of each isolated battery system considered in Step 1 and calculated its ideal equalization time. This equalization time is the intra-module equalization of each module denoted as $\widehat{T}_{e,i}^{intra}$, $i = 1, ..., M$.

Step 3: For each module, aggregate all batteries in this module into one single virtual battery and let the initial state of the aggregated battery equal to the sum of the SOCs of all individual batteries in the module. This results in a new battery system with M aggregated batteries and serially connected. Identify the BNBG of this M-battery system and calculate its ideal equalization time. This equalization denoted as $\widehat{T}_e^{inter}$.

Step 4: The equalization time of the original system is the maximum of the inter-module equalization time and the longest intra-module equalization time, i.e.,

$$\widehat{T}_e^{module} = max\{\widehat{T}_e^{inter}, \max_{i=1,...,M} \widehat{T}_{e,i}^{intra}\}. \tag{4.7}$$

Proposition 9. *The equalization time of a battery equalization system defined by (4.1)-(4.6) can be approximated using Algorithm 4.1 as follows:*

$$T_e^{module} \approx \widehat{T}_e^{module} \tag{4.8}$$

Justification: The accuracy of Proposition 8 is justified using a similar numerical approach used in the justification of Proposition 4. Specifically, a C++ program is developed to calculate the state evolution of the module-based battery equalization system using mathematical equation (4.1)-(4.6). Using this program, the equalization time of the system, i.e., N_e^{module} can be obtained. Then N_e^{module} is compared with the equalization time obtained using Algorithm 4.1, i.e., $\widehat{T}_e^{module}$. To carry out the experiments, we selected battery number $B \in \{4, 8, 16, 32, 64\}$ and equalization rate $r = 0.00001$ for all equalizers. For each B, we selected $M \in \{2, 4, ..., B/2\}$. Then, for each combination of B and M, a total of $50,000$ different formations of initial battery charge are generated with $x_i(0)$'s selected randomly and independently according to uniform distribution $U(0, 1)$. Without loss of generality, let $\tau = 1$. For each initial buffer charge formation, we calculate its corresponding N_e^{module} and $\widehat{T}_e^{module}$ and evaluate their relative error based on:

$$\epsilon_m = \frac{\left| N_e^{module} - \widehat{T}_e^{module} \right|}{N_e^{module}} \cdot 100\% \tag{4.9}$$

The average values of ϵ_m for various combinations of B and M are summarized in Table 3. The errors observed are mainly due to the limit cycle convergence observed in the discrete time system. Other than that, we claim that Proposition 8 holds.

$B = 4, M = 2$	$B = 8, M = 2$	$B = 8, M = 4$
0.0010%	0.0027%	0.0022%
$B = 16, M = 2$	$B = 16, M = 4$	$B = 16, M = 8$
0.0079%	0.0037%	0.0074%
$B = 32, M = 2$	$B = 32, M = 4$	$B = 32, M = 8$
0.0215%	0.0077%	0.0075%
$B = 32, M = 16$	$B = 64, M = 2$	$B = 64, M = 4$
0.0211%	0.0583%	0.0174%
$B = 64, M = 8$	$B = 64, M = 16$	$B = 64, M = 32$
0.0106%	0.0177%	0.0581%

Table 3: Accuracy of Proposition 8

4.3 Equalizer Rate Control

As discussed above, there are two types of equalizer in the module-based structure: cell-level equalizer and module-level equalizer. Due to manufacturing constraint and other reasons, in the current stage it is unlikely that each equalizer in the system has different equalization rate. However, it is reasonable to have different equalization rate between cell-level equalizer and module-level equalizer.

Let r_m denote the equalization rate of module-level equalizer and r_c denote the equalization rate of cell-level equalizer. Define α to be the ratio of r_m and r_c, thus $\alpha = \frac{r_m}{r_c}$. By selecting different values on α, the equalization time of the system could be changed or even reduced. Note that in algorithm 4.1, the system equalization time depends on either the inter-module equalization time(equalization time of the aggregated system) or the longest intra-module equalization time(equalization time of a certain battery module). Since the value of α will only affect the transfer rate among different modules, it will not change the total equalization time if the total equalization time is associated with the intra-module equalization time. On the other hand, if the system equalization time is associated with the inter-module equalization time, a larger α would increase the transfer rate among the bottleneck modules. However, the value of α cannot increase infinitely from both manufacturing perspective and system

analysis perspective. When the value of α increases to a certain value, the BNBG no longer exists in the aggregated battery system but transfer into a certain battery modules. As discussed before, in this circumstance, the increase of α will not affect the system equalization time since it is associated with the intra-module equalization time and the system equalization time would converge. In order to demonstrate the above discussion, an example is given to illustrate case 1 in which the BNBG is within a certain battery module and simulation is carried out to identify the "most efficient α."

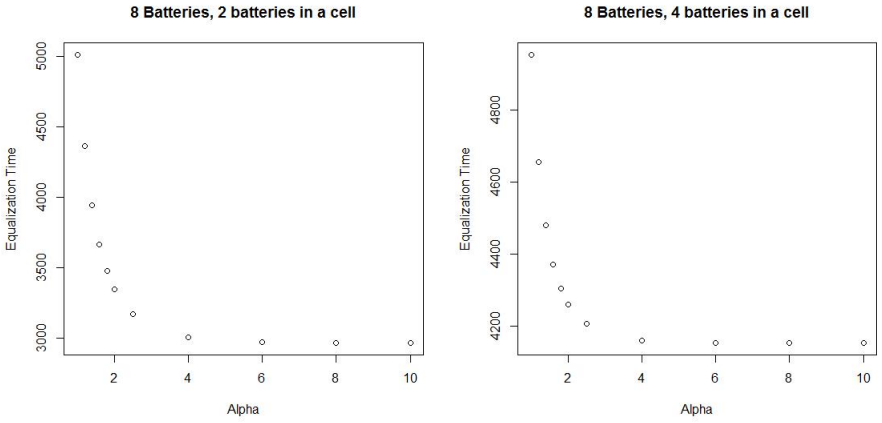
Example 1: Consider a 8-battery module-based system with initial SOCs given in table 4 and $N = 2$ (two batteries in a module). The BNBG is b_3 which is in the second module. The calculated equalization time needed for this module to balance itself is 2832.5 time slots and the simulated equalization time of the whole system is 2833 time slots. The change of α does not affect the equalization process of this BNBG since it only increase or reduce the charge of these batteries at the same time.

Battery	1	2	3	4	5	6	7	8
Initial State	0.6847	0.6596	0.3485	0.915	0.3145	0.8751	0.6209	0.3685

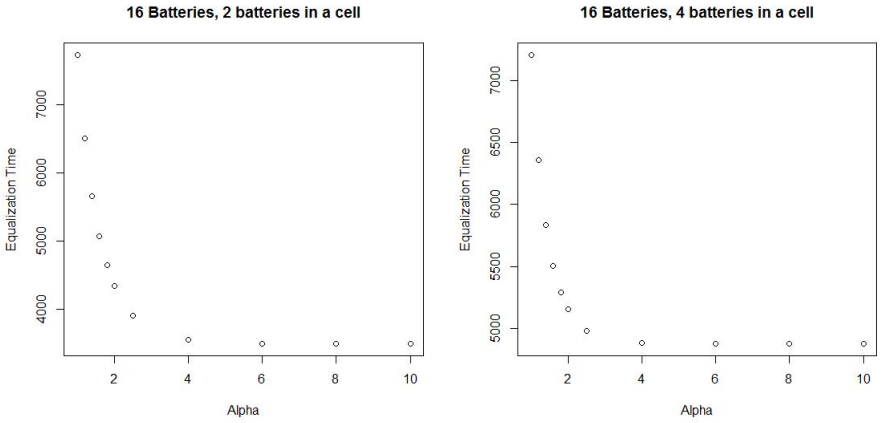
Table 4: Initial battery state of Example 1

Simulation of case 2: In each combination, $30,000$ simulations were carried out for each α value, $\alpha \in \{1, 1.2, 1.4, 1.6, 1.8, 2, 2.5, 4, 6, 8, 10\}$.

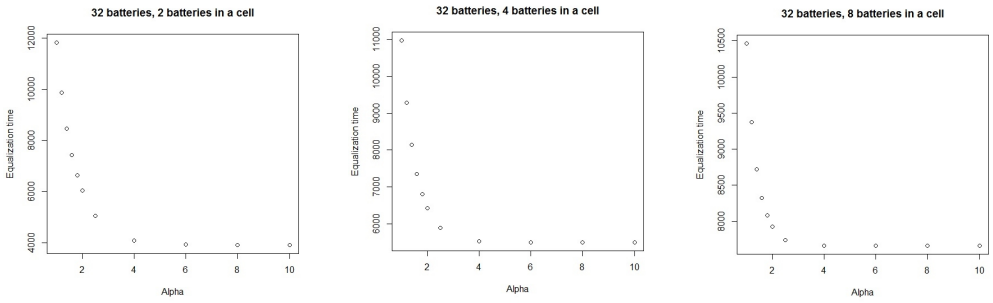
The numerical experiments in figure 4.2 show that the equalization time starts to converge when $\alpha = 2$. Thus we can conclude that the α value should be less than or equal to 2 in order to reduce the system's equalization time most efficiently.



(a) Module-based with $B = 8$



(b) Module-based with $B = 16$



(c) Module-based with $B = 32$

Figure 4.2: Module-based equalization with different α value

5 Comparison

5.1 Statistical comparison

In the current literature (see, for instance, papers [12, 13, 14, 15, 16, 17]), the efficacy of a newly developed equalization system is often tested by only one or two examples. Apparently, this approach ignores the random variation of initial battery states as well as the sensitivity of system performance with respect to initial battery states. Therefore, the resulting conclusions, strictly speaking, cannot be considered justified.

In this subsection, a Monte Carlo statistical study is carried out to compare the three equalization structures discussed above. Specifically, we selected B from $\{4, 8, 16, 32, 64\}$ and randomly generated $50,000$ for each B with $x_i(0)$'s selected randomly and independently according to uniform distribution $U(0, 1)$. For the $250,000$ cases, simulations of mathematical models are carried out for all three equalization structures. For the module-based system, we enumerate all possible M's from $\{2, 4, ..., B/2\}$. In addition, we assume that all equalizers have the same equalization rate $r = 0.00001$. The results are summarized in Table 5 and Table 6 . For the module-based structure, only the results from the best M for each battery number B is given in the table. The sensitivity of the system equalization performance with respect to M will be studied in future work.

As one can see from the table, the layer-based structure provides the best average performance among the three for all B's considered, while the series-based structure appears to have the worst average performance. Specifically, the layer-based structure can reduce the average equalization time by about 25% for $B = 64$ from the series-based structure. The module-based structure, on the other hand, can reduce the average equalization time by about 18% for $B = 64$. It should be noted that, even though the layer-based and module-based structures can improve the average equalization performance over the traditional series-based structure, there are cases in

B	Series-based	Layer-based	Module-based
4	33662	32191	32191($M = 2$)
8	54839	48226	49670($M = 4$)
16	84543	68729	72283($M = 4$)
32	126502	97422	104932($M = 4$)
64	184786	137489	151237($M = 4$)

Table 5: Average equalization times

B	Series-based	Layer-based	Module-based
4	14075	13167	13167($M = 2$)
8	19790	17005	16662($M = 4$)
16	28908	24309	25708($M = 4$)
32	41723	34924	35358($M = 4$)
64	59276	49468	48591($M = 4$)

Table 6: Standard deviation of equalization times

which the series-based structure outperform these two. The frequency of cases, where the layer-based and module-based structures result in shorter equalization time than the series-based structure, is given in Table 7. As one can see, for $B = 4$, although the average equalization times under the layer-based and module-based structures are shorter than those under the series-based structure, improvement only occurs in about 50% of cases. However, as the number of batteries increases, the superiority of the layer-based and module-based equalization structures becomes more and more prominent. For instance, when $B = 64$, the layer-based structure leads to a shorter equalization time than the series-based structure in about 90% of cases, while the module-based structure outperforms the series-based structure in nearly 85% of cases. Since the number of battery cells in real applications (e.g., electrical vehicles) are usually large, the layer-based or module-based equalization structure should be considered.

B	Layer-based	Module-based
4	49.55%	49.55%($M = 2$)
8	62.66%	49.81%($M = 4$)
16	76.98%	68.35%($M = 4$)
32	85.91%	79.47%($M = 4$)
64	90.79%	84.77%($M = 4$)

Table 7: Superiority of the layer-based and module-based equalization structures

5.2 Illustration

In order to further demonstrate the performance of these three structures, the following examples are given. In each example, three equalization structures are compared under the same initial conditions. To be fair, the equalization rate of all equalizers is $r = 0.0001$. In addition, for the module-based equalization system, we assume that each module contains two battery cells. The initial battery states are selected so that each of the three structures has the shortest equalization time in one example.

Example 1: Consider an 8-battery system with initial battery SOCs given in Table 8 and illustrated in Figure 5.1. The equalization process of this battery system using series-based, layer-based and module-based structures are provided in Figure 5.2. As a result, the equalization time of the series-based structure is 3913 time slots, while both layer-based and module-based structure need 4703 time slots for all batteries to balance.

In this example, batteries $b_1$and b_2 form the BNBG, and the series-based structure allows for consistent transfer of SOC to the rest of the battery system. Indeed, at the beginning of the equalization process, the two equalizers on both sides of battery b_2 transfer SOC from b_2 to b_1 and b_3 simultaneously, leading to a fast drop in b_2's SOC. This eventually results in faster equalization compared to the other two structures, which reduces the SOC from b_2 at a lower rate because the transfers are evenly distributed among both $b_1$and b_2.

Example 2: Consider a 8-battery system with initial battery SOCs given in Table

$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
0.0006	0.9412	0.2586	0.1626	0.0561	0.4017	0.3747	0.0054

Table 8: Initial battery state of Example 1

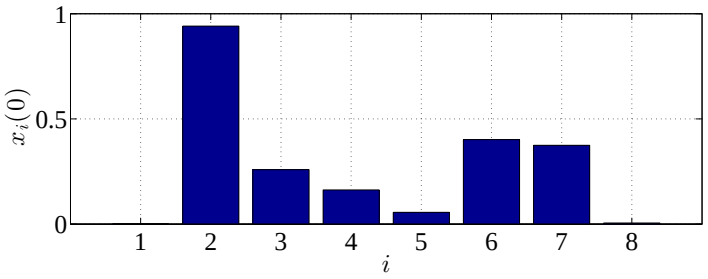


Figure 5.1: Initial battery state of Example 1

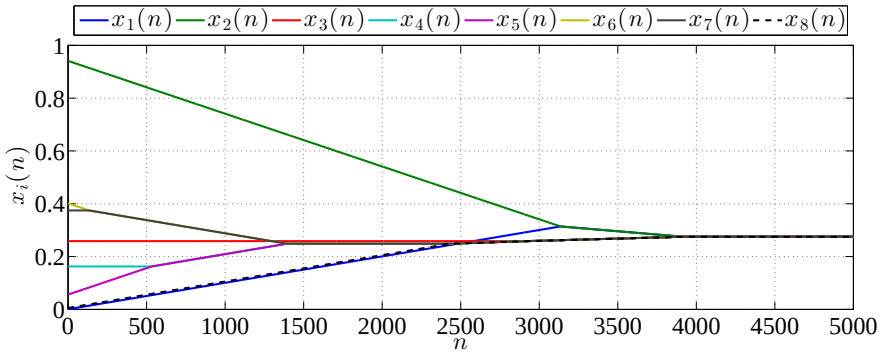
9 and illustrated in Figure 5.3. The equalization process of this battery system using series-based, layer-based and module-based structure are provided in Figure 5.4. As a result, the equalization time of the layer-based structure is 6115 time slots, while both series-based and module-based structure need 7300 time slots fro all batteries to balance.

In this example, batteries b_7 and b_8 form the BNBG in the series-based equalization structure. However, only one equalizer is responsible to transfer charge out of the BNBG in this structure. Similar issue also slows down the equalization process under the module-based structure. On the other hand, under the layer-based structure, equalizers in all three layers can work simultaneously to reduce the charge in b_7 and b_8. As a result, the layer-based structure has the shortest equalization time among all three structures.

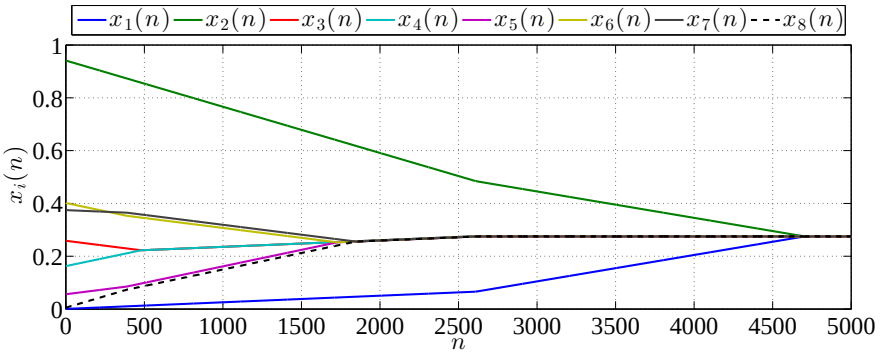
Example 3: Consider an 8-battery system with initial battery SOCs given in Table 10 and illustrated in Figure 5.5. The equalization processes of this battery system using series-based, layer-based and module-based structures are provided in

$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
0.0014	0.3653	0.5324	0.6265	0.08308	0.1193	0.9027	0.8963

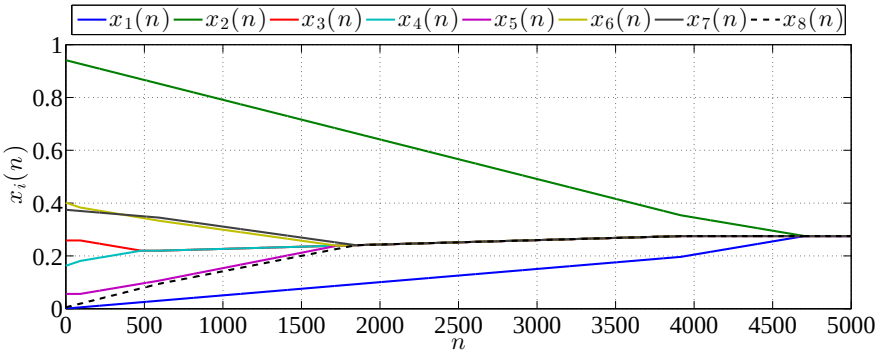
Table 9: Initial battery state of Example 2



(a) Series-based equalization



(b) Layer-based equalization



(c) Module-based equalization

Figure 5.2: Battery equalization process under three structures: Example 1

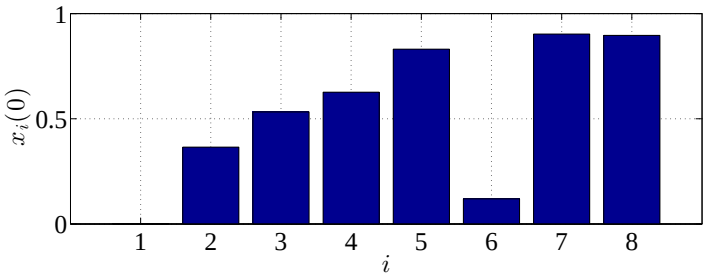


Figure 5.3: Initial battery state of Example 2

$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
0.0009	0.9132	0.8288	0.0317	0.0227	0.0641	0.2329	0.8997

Table 10: Initial battery state of Example 3

Figure 5.6. As a result, the equalization time of the module-based structure is 4562 time slots, while the series-based and layer-based structures need 6200 and 5228 time slots, respectively, for all batteries to balance.

In this example, the BNBG of the series-based equalization structure is $b_1 - b_2 - b_3$, while another boundary battery group formed by $b_7 - b_8$ also requires long equalization time. In the layer-based structure, both battery groups can transfer charge faster to the middle due to the additional channels provided in different layers. In the module-based structure, the reduction in equalization time is even more significant as the module-level equalizers further accelerate the process.

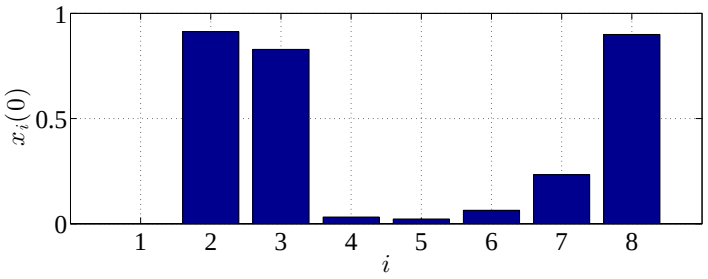
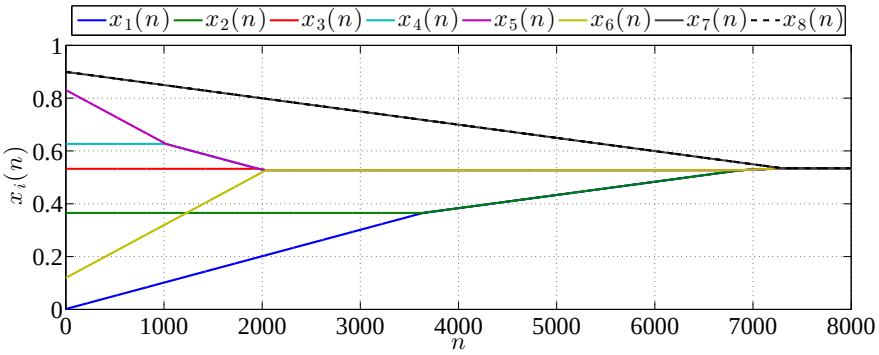
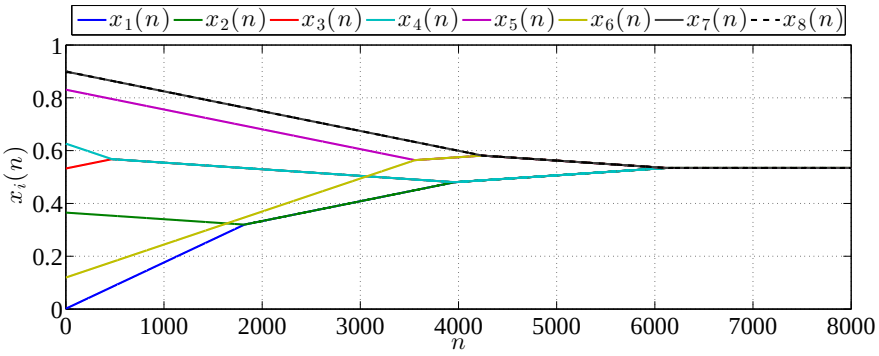


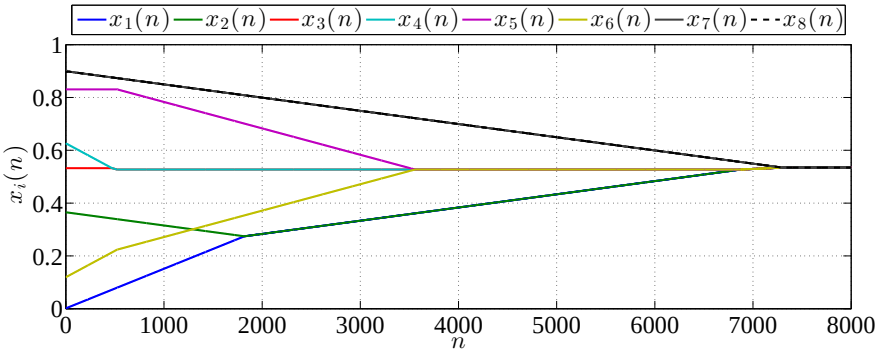
Figure 5.5: Initial battery state of Example 3



(a) Series-based equalization

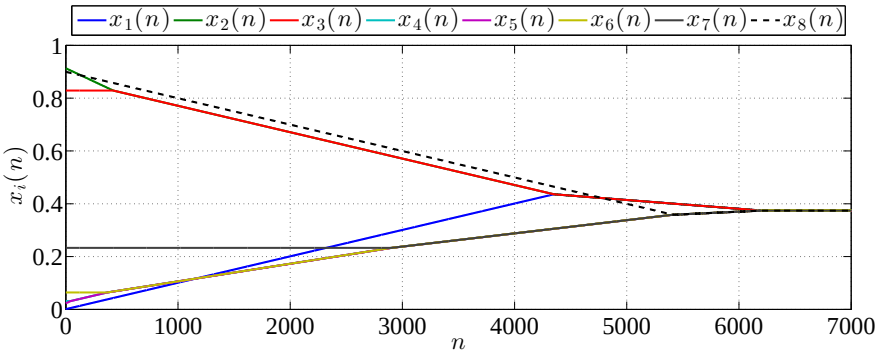


(b) Layer-based equalization

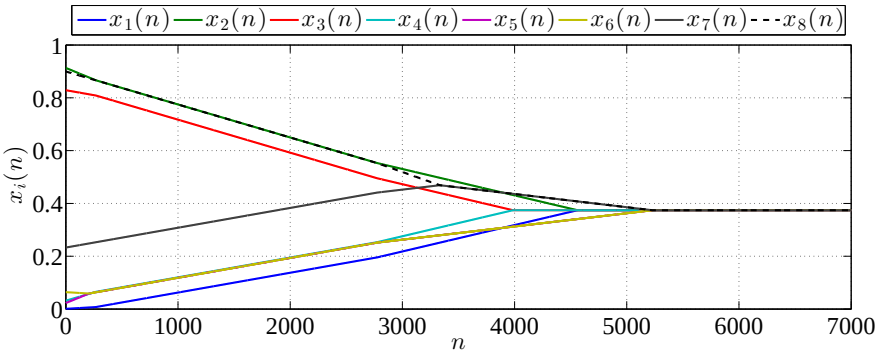


(c) Module-based equalization

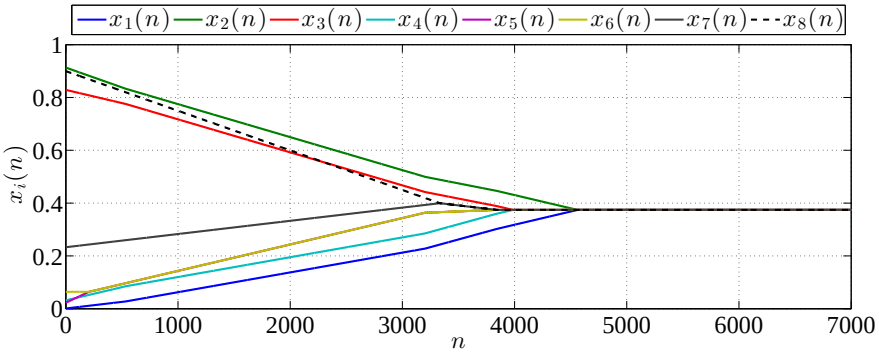
Figure 5.4: Battery equalization process under three structures: Example 2



(a) Series-based equalization



(b) Layer-based equalization



(c) Module-based equalization

Figure 5.6: Battery equalization process under three structures: Example 3

6 Systems With Energy Loss

In practical, energy within a battery cell will loss when the system is running. The battery systems in section 2, 3 and 4 are assumed to be lossless in order to carry out the system analysis. Here in this section we take energy loss of the system into consideration and analyze the system performance for series-based, layer-based and module-based structures.

6.1 Series-based Battery System

Let h_i denote the energy loss rate of equalizer i during one time slot (i.e., working cycle) when the equalizer is operating. Assume a series-based battery equalization system under the assumptions (I)-(III) from section 2 with the following property:

If there is charge transfer in equalizer e_i, it will loss $r_i \cdot h_i$ units of charge during this transfer. That is to say, at time slot t, if battery b_i has a larger SOC than battery b_{i+1}, equalizer e_i will take r_i units of charge from battery b_i but only transfer $r_i(1-h_i)$ units of charge to b_{i+1}. If the adjacent batteries has the same SOC, no loss would occur during this time period. In practice, the equalizers used in Figure 2.1 are designed to have identical parameter. Thus in this section, we also assume that:

$$r_i = r,\ h_i = h,\ \forall i \in \{1, ...B\}. \tag{6.1}$$

6.1.1 Mathematical Model

Let $x_i(n) \in [0,1]$denote the SOC of battery b_i at the end of time slot (i.e., working cycle) n, $i = 1, ...B$, $n = 0, 1, ..., B$, $n = 0, 1, ...$ Then, the evolution of the system is given by:

$$x_1(n+1) = x_1(n) + sgm(x_2(n) - x_1(n), r, h), \tag{6.2}$$

$$x_i(n+1) = x_i(n) + sgm(x_{i-1}(n) - x_i(n), r, h) + sgm(x_{i+1}(n) - x_i(n), r, h), \quad (6.3)$$

$$x_B(n+1) = x_B(n) + sgm(x_{B-1}(n) - x_B(n), r, h) \cdot r, \quad (6.4)$$

where $sgm(\cdot)$ is a modified signum function defined as follows:

$$sgm(u, v, w) = \begin{cases} -v, \ if & u < 0, \\ 0, \ if & u = 0, \\ v(1-w), \ if & u > 0 \end{cases} \quad (6.5)$$

Let N_e^{series} denote the smallest number of time slots, i.e., working cycles, for the system to be equalized. Then, the equalization time of the system, denoted as T_e^{series}, is given by

$$T_e^{series} = N_e^{series}\tau. \quad (6.6)$$

Since it is practically impossible to have two batteries with identical SOCs, we assume that all $B-1$ equalizers operate in every working cycle until equalization is achieved. Thus the terminal SOCs of all batteries, denoted as x_{ter}, can be calculated as follows:

$$B \cdot x_{ter} = \sum_{i=1}^{B} x_i(0) - r \cdot h \cdot (B-1) \cdot N_e^{series}, \quad (6.7)$$

i.e.,

$$x_{ter} = \overline{x}_0 - \frac{rh(B-1)N_e^{series}}{B}, \quad (6.8)$$

where

$$\bar{x}_0 = \frac{1}{B}\sum_{i=1}^{B} x_i(0). \tag{6.9}$$

The equalization efficiency of the system is defined as the ratio of the total SOC at equalization to the total initial SOC, i.e.,

$$\eta_{series} = \frac{x_{ter}}{\bar{x}_0} = 1 - \frac{rh(B-1)N_e^{series}}{B\bar{x}_0}. \tag{6.10}$$

6.1.2 Performance Evaluation

For the case where all equalizers are lossless (i.e., $h_i = 0$), an analytical method has been proposed in to calculate the system equalization time with high accuracy based on system parameters and initial condition. Below, we extend the method to the case where $h_i = h,\ h > 0$.

To facilitate the derivation, we introduce the following notion:

First consider a boundary battery group $BG(g, i)$. Assume this battery group has a higher SOC, then the fastest way to have this battery group reach the equalization status is to have equalizer $e_g$or e_{B-g} continuously discharge this battery group, transfer charge to the outside of this battery group. Let$n_{dis}(g, i)$ denote the number of working cycle for the battery group $BG(g, i)$ to reach equalization, i.e., all battery with SOC equal to x_{ter} assuming the e_g or e_{B-g} only discharge this battery group during this process. Then it can be obtained that

$$\sum_{i=1}^{g} x_i(0) - gx_{ter} = rh(g-1)n_{dis} + rn_{dis} \tag{6.11}$$

$$\Rightarrow \sum_{i=1}^{g} x_i(0) - g\bar{x}_0 = rh(g-1)n_{dis} + rn_{dis} - \frac{rhg(B-1)n_{dis}}{B}. \tag{6.12}$$

Thus,

$$n_{dis}(g,i) = \frac{\sum_{i=1}^{g} x_i(0) - g\bar{x}_0}{[1-(1-\frac{g}{B})h]r}. \tag{6.13}$$

On the other hand, if the battery group has a relatively lower charge, then the fastest way to have it reach the equalization is continuously charge the battery group during the equalization process through e_g or e_{B-g}. Then, the charging time $n_{chg}(g,i)$ can be calculated as follows:

$$gx_{ter} - \sum_{i=1}^{g} x_i(0) = r(1-h)n_{chg} - rh(g-1)n_{chg} \tag{6.14}$$

$$\Rightarrow \sum_{i=1}^{g} x_i(0) - g\bar{x}_0 = r(1-h)n_{chg} + rh(g-1)n_{chg} - \frac{rhg(B-1)n_{chg}}{B}. \tag{6.15}$$

Thus

$$n_{chg}(g,i) = \frac{g\bar{x}_0 - \sum_{i=1}^{g} x_i(0)}{(1-\frac{g}{B}h)r}. \tag{6.16}$$

Clearly, given the initial SOC of the battery system, one of n_{dis} and n_{chg} is positive, while the other is negative or both being zero. Therefore, we defined the estimated equalization time n_{eq} for the battery group $BG(g,i)$ as follows:

$$n_{eq}(g,i) = max\{n_{dis}(g,i), n_{chg}(g,i)\}. \tag{6.17}$$

It should be noted that $n_{eq}(g,i)$ is not necessarily the actual time span for the battery group to balance, but rather the shortest possible time assuming the equalizers on the boundary only operate in "one direction". Therefore, we referred to $n_{eq}(g,i)$ as the ideal equalization time of the battery group.

Next we consider the case of internal battery group $BG(g,i)$, $1 < i < B-g+1$. Clearly, the battery group now has two equalizers connecting it to the outside: e_{i-1}

and e_{i+g-1}. The fastest possible discharging and charging times for the battery group to have it reach balance assuming both e_{i-1} and e_{i+g-1} operate in "one direction" can be obtained as follows:

$$n_{dis}(g,i) = \frac{\sum_{i=1}^{g} x_i(0) - g\bar{x}_0}{[2-(1-\frac{g}{B})h]r}, \tag{6.18}$$

$$n_{chg}(g,i) = \frac{g\bar{x}_0 - \sum_{i=1}^{g} x_i(0)}{[2-(1+\frac{g}{B})h]r}. \tag{6.19}$$

The ideal equalization time is defined in the same manner:

$$n_{eq}(g,i) = max\{n_{dis}(g,i), n_{chg}(g,i)\}. \tag{6.20}$$

Let $\widehat{N}_{eq}$ denote the longest ideal equalization time of all battery groups, i.e.,

$$\widehat{N}_{eq} = \max_{\forall g,i} n_{eq}(g,i). \tag{6.21}$$

Proposition 10. *For a battery equalization system defined by assumptions (I)-(III) and the property introduced above, the system equalization time, equalization efficiency, and the batteries' terminal SOC can be approximated as:*

$$\widehat{x}_{ter} = \bar{x}_0 - \frac{rh(B-1)\widehat{N}_{eq}}{B}, \tag{6.22}$$

$$\widehat{T}_e^{series} = \widehat{N}_{eq}\tau, \tag{6.23}$$

$$\widehat{\eta} = 1 - \frac{rh(B-1)\widehat{N}_{eq}}{B\bar{x}_0}. \tag{6.24}$$

Assume that the equalizers keep running after the calculated equalization time from equation (6.21) and (6.23), since there is charge loss every time when the equal-

izers operating, the total SOC of the system will drop. From a system's point of view, during a certain period of time, the total drop of SOC could be calculated and the average SOC will drop continuously with a certain slope. This concept applies to layer-based and module-based structure as well. Suppose the system keeps running for another n units of time (i.e., working cycle) after the calculated equalization time $\widehat{T}_e^{series}$ and let $\hat{x}_{ter}(n)$ be the average SOC of the system at the end of time slot n. Let $\Delta_{loss}(n)$ be the total loss of SOC during this period and it could be calculated as:

$$\Delta_{loss}(n) = rnh(B-1), \tag{6.25}$$

Let k be the slope of the drop of the average SOC, it could be calculated as:

$$k = -\frac{\widehat{x}_{ter} - \hat{x}_{ter}(n)}{n} = -\frac{rnh(B-1)}{Bn} = -\frac{rh(B-1)}{B} \tag{6.26}$$

6.1.3 Justification

An illustration of the performance estimation method is given in Figure 6.1 for a 32-battery system with randomly generated initial SOCs for all batteries. The colored lined in the figure illustrate the SOCs of the batteries during the equalization process. The black vertical line indicates the equalization time calculated based on equation (6.21). Note that the balanced SOC decreases continuously with the slope calculated by equation (6.26).

6.2 Layer-based battery system

Let a layer-based battery equalization system with assumption (I)-(III) and (VI) with the following property:

Under the layer-based structure, the jth equalizer in layer l is used to equalize the charge between battery group $BG(2^{l-1}, 1+(j-1)2^l)$ and $BG(2^{l-1}, 1+(2j-1)2^{l-1})$:

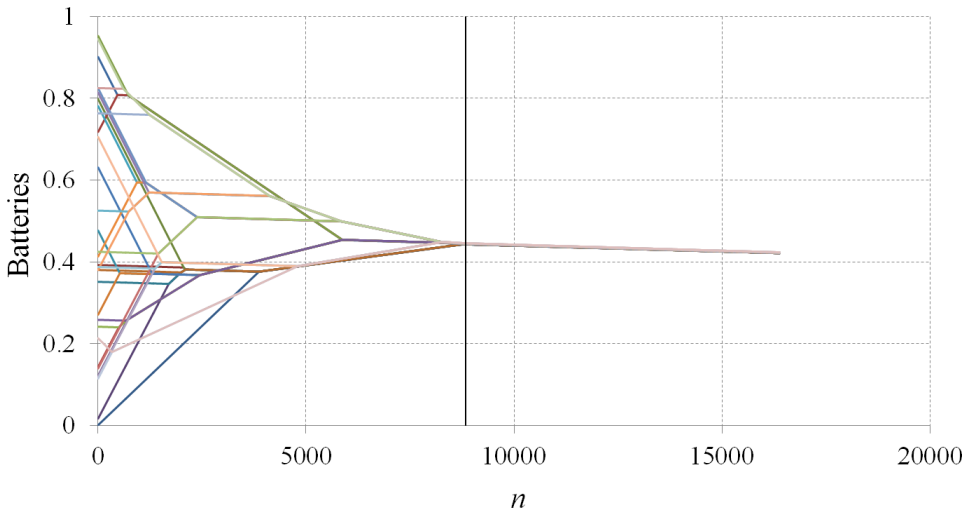


Figure 6.1: Illustration of performance of series-based structure

If battery group $BG(2^{l-1}, 1+(j-1)2^l)$ has a larger total SOC than $BG(2^{l-1}, 1+(2j-1)2^{l-1})$ at the beginning of a time slot, then the equalizer takes $r_l/2^{l-1}$ units of charge from each battery in $BG(2^{l-1}, 1+(j-1)2^l)$, and send $(1-h)\cdot r_l/2^{l-1}$ units of charge to each battery in $BG(2^{l-1}, 1+(2j-1)2^{l-1})$ during the time slot; if $BG(2^{l-1}, 1+(j-1)2^l)$ has a lower SOC, the equalize makes the opposite transfer; if $BG(2^{l-1}, 1+(j-1)2^l)$ and $BG(2^{l-1}, 1+(2j-1)2^{l-1})$ have the same SOC, no transfer takes place between the two battery groups during this time period.

6.2.1 Mathematical Model

Let $x_i(n)$, again, denote the SOC of batteryb_i at the end of time slot n, $i = 1, ..., B$, $n = 0, 1, 2....$ Then, the evolution of the layer-based system is given by :

$$x_i(n+1) = x_i(n) + \sum_{l=1}^{L}\sum_{j=1}^{B/2^l}[sgm((I_{S^+_{l,j}}(i) - I_{S^-_{l,j}}(i))\cdot\Delta_{l,j}(n), \frac{r_l}{2^{l-1}}, h)],\ i = 1, ..., B, \tag{6.27}$$

where the signum function $sgm(\cdot)$ and the indicator function $I_A(\cdot)$ are defined in (6.5) and (2.22) respectively, and

$$\Delta_{l,j}(n) = \sum_{m \in S^-_{l,j}} x_m(n) - \sum_{m \in S^+_{l,j}} x_m(n), \tag{6.28}$$

$$S^+_{l,j} = \{(j-1)2^l + 1,\ (j-1)2^l + 2, ...,\ (j-1)2^l + 2^{l-1}\}, \tag{6.29}$$

$$S^-_{l,j} = \{(2j-1)2^{l-1} + 1,\ (2j-1)2^{l-1} + 2, ...,\ (2j-1)2^{l-1} + 2^{l-1}\}. \tag{6.30}$$

In this formulation, $S^+_{l,j}$ and $S^-_{l,j}$ represent the indices of batteries in the two battery groups balanced by the jth equalizer in the lth layer, while $\Delta_{l,j}(n)$ is the difference of total SOCs between the two BGs at time slot n.

6.2.2 Performance Evaluation

To calculate the equalization time of a layer-based equalization system defined by assumption (I)-(III), (VI) and the above property, consider the continuous version of the system model formulation (6.34)-(6.37):

$$\dot{x}_i(n+1) = \sum_{l=1}^{L} \sum_{j=1}^{B/2^l} [sgm((I_{S^+_{l,j}}(i) - I_{S^-_{l,j}}(i)) \cdot \Delta_{l,j}(n), \frac{\widetilde{r}_l}{2^{l-1}}, h)],\ i = 1, ..., B, \tag{6.31}$$

where

$$\widetilde{r}_l = \frac{r_l}{\tau}. \tag{6.32}$$

For equalizer e_k, $k = 1, ..., B-1$, introduce $l(k)$ and $j(k)$ the same as in section 3 :

$$l(k) = \left\lceil -log_2(1 - \frac{k}{B}) \right\rceil, \tag{6.33}$$

$$j(k) = k - [1 - 2^{1-l(k)}] \cdot B. \tag{6.34}$$

It can be shown that e_k is the jth equalizer in layer $l(k)$ of the layer-based battery equalization system defined above. In addition, as stated in section 3, the equalizers operate "independently" of each other under layer-based structure. Let n_k denote the number of time slots (i.e., working cycle) for e_k to balance the charge between its two adjacent battery groups. After n_k units of time slot, the total terminal SOCs x_{ter}^k of the two adjacent battery groups under the affect of equalizer e_k could be calculated as:

$$x_{ter}^k = \widehat{x}_0 - \frac{n_k hr}{2} \tag{6.35}$$

where

$$\widehat{x}_0 = \frac{\sum_{m \in S^-_{l(k),j(k)}} x_m(0) + \sum_{m \in S^+_{l(k),j(k)}} x_m(n)}{2} \tag{6.36}$$

and $S^-_{l(k),j(k)}$, $S^+_{l(k),j(k)}$ is defined in (6.32) and (6.33) respectively. Let $\hat{\Delta}_k$ denote the the difference of the total SOC between equalizer e_k's left battery group and terminal SOC x_{ter}^k, i.e.,

$$\hat{\Delta}_k = \sum_{m \in S^+_{l(k),j(k)}} x_m(0) - x_{ter}^k. \tag{6.37}$$

If this battery group (left battery group) has a relatively low SOC at the beginning, it will constantly be charged by e_k until it reach the same SOC with the battery group on the right. Thus the time (i.e., working cycle) t_{chg} for equalizer e_k to balance its adjacent battery groups could be calculated as:

$$t_{chg} = \frac{x^k_{ter} - \sum_{m \in S^+_{l(k),j(k)}} x_m(0)}{r(1-h)}, \tag{6.38}$$

Note that t_{chg} is also the ideal (i.e. shortest) time need for equalizer e_k to balance its adjacent battery groups, thus it can be calculated as:

$$t_{chg} = \frac{(\widehat{x}_0 - \frac{t_{chg} hr}{2}) - \sum_{m \in S^+_{l(k),j(k)}} x_m(0)}{r(1-h)}, \tag{6.39}$$

$$\Rightarrow t_{chg}[1 + \frac{hr}{2r(1-h)}] = \frac{\widehat{x}_0 - \sum_{m \in S^+_{l(k),j(k)}} x_m(0)}{r(1-h)}. \tag{6.40}$$

Thus

$$t_{chg} = \frac{\sum_{m \in S^-_{l(k),j(k)}} x_m(0) - \sum_{m \in S^+_{l(k),j(k)}} x_m(0)}{r(2-h)}. \tag{6.41}$$

On the other hand, if this battery group has a relatively higher SOC, it will constantly transfer charge to the battery group on the right. The discharge time t_{dis} for equalizer e_k to balance its adjacent battery groups then could be calculated as:

$$t_{dis} = \frac{\sum_{m \in S^+_{l(k),j(k)}} x_m(0) - x^k_{ter}}{r}. \tag{6.42}$$

$$t_{dis} = \frac{\sum_{m \in S^+_{l(k),j(k)}} x_m(0) - (\widehat{x}_0 - \frac{t_{dis} hr}{2})}{r}, \tag{6.43}$$

$$\Rightarrow t_{dis} r(1 - \frac{h}{2}) = \sum_{m \in S^+_{l(k),j(k)}} x_m(0) - \widehat{x}_0. \tag{6.44}$$

Thus

$$t_{dis} = \frac{\sum_{m \in S^+_{l(k),j(k)}} x_m(0) - \sum_{m \in S^-_{l(k),j(k)}} x_m(0)}{r(2-h)}. \tag{6.45}$$

Notice that

$$t_{chg} = -t_{dis}. \tag{6.46}$$

Thus the ideal (i.e. the shortest) time t_k needed for equalizer e_k to balance its adjacent battery groups is:

$$t_k = |t_{chg}| = |t_{dis}| = \frac{\left|\sum_{m\in S^+_{l(k),j(k)}} x_m(0) - \sum_{m\in S^-_{l(k),j(k)}} x_m(0)\right|}{r(2-h)} = \frac{\left|\Delta_{l(k),j(k)}(0)\right|}{r(2-h)} \tag{6.47}$$

By using the same definition from section 3 for bottleneck equalizer, the equalization time $\widetilde{T}_e^{layer}$ of a layer-based battery systems with energy loss rate h could be calculated as:

Theorem 11. *The equalization time of a battery charge system defined by (6.27)-(6.30) is equal to the maximum equalization time of the equalizer of the system,*

$$\widetilde{T}_e^{layer} = \max_{k\in\{1,\ldots,B-1\}} t_k. \tag{6.48}$$

Therefore, for $r_l \ll 1$, the equalization time of a layer-based equalization system with energy loss can be approximated using Theorem 7:

$$T_e^{layer} = \widetilde{T}_e^{layer}. \tag{6.49}$$

The equalization efficiency $\widehat{\eta}$ could be calculated as:

$$\widehat{\eta} = 1 - \frac{rh(B-1)\widetilde{T}_e^{layer}}{B\bar{x}_0}. \tag{6.50}$$

where $\bar{x}_0$is defined as:

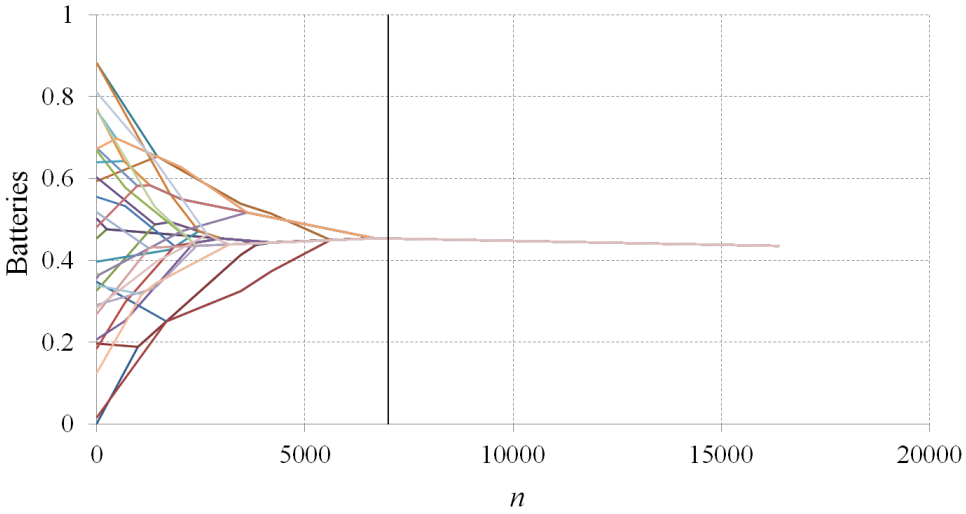


Figure 6.2: Illustration of performance of layer-based structure

$$\bar{x}_0 = \frac{1}{B}\sum_{i=1}^{B} x_i(0). \tag{6.51}$$

6.2.3 Justification

A illustration of the performance estimation method is given in figure 6.2 for a 32-battery layer-based system with random generated initial SOCs. The colored line in the figure illustrate the SOCs of batteries during the equalization process while the black indicates the equalization time calculated according to equation (6.47) and (6.48) .

6.3 Module-based battery system

Let a module-based battery system under the assumption (I)-(III), (VIII) and (IX) with the following properties:

For a cell-level equalizer e_i, $i \in S_c$, if battery b_i has a larger SOC than b_{i+1}at the beginning of a time slot, then e_i transfers $r_c$units out of charge from b_i an d send $h_i r_c$ units of charge to b_{i+1} during the time slot; if b_i has a lower SOC, then e_i makes the

opposite transfer; if b_i and b_{i+1} have the same SOC, no transfer takes place between the two batteries during this time slot.

Moreover, for a module-level equalizer e_i, $i \in S_M$, if battery group $BG(N, i-N+1)$ has a larger total SOC than $BG(N, i+1)$ at the beginning of time slot, then the equalizer takes r_m/N unites of charge from each battery in $BG(N, i-N+1)$ and send $(1-h_i)\frac{r_m}{N}$ amount of charge to each battery in $BG(N, i+1)$ during the time slot; if $BG(N, i-N+1)$ has a lower total SOC, the equalizer makes the opposite transfer; if $BG(N, i-N+1)$ and $BG(N, i+1)$ have the same SOC, no transfer takes place between the two battery groups during the time slot.

As discussed in section 4, under this framework, the equalization within a module is "independent" from all other modules since module-level equalizer modify the state of all batteries in a given module with the same amount.

6.3.1 Mathematical Model

Let $x_i(n)$, again, denote the SOC of battery b_i at the end of time slot n, $i = 1, ..., B$, $n = 0, 1, ...$ Then, the evolution of the system is given by:

$$x_i(n+1) = x_i(n) + \sum_{j=1}^{B-1} (sgm[(I_{S_j^+}(i) - I_{S_j^-}(i)] \cdot \Delta_j(n), \frac{r_c I_{S_c}(j) + r_m I_{S_M}(j)}{1 + (N-1) \cdot I_{S_M}(j)}, h)), \ i = 1, ..., B, \tag{6.52}$$

where the signum function $sgm(\cdot)$ and the indicator function $I_A(\cdot)$ are defined in (6.5) and (2.22), respectively, and

$$\Delta_j(n) = \sum_{m \in S_j^-} x_m(n) - \sum_{m \in S_j^+} x_m(n) \tag{6.53}$$

$$S_j^+ = \begin{cases} \{j-N+1, j-N+2, ...j\}, & if\ j \in S_M, \\ \{j\} & if\ j \in S_C, \end{cases} \tag{6.54}$$

$$S_j^- = \begin{cases} \{j+1, j+2, ...j+N\}, & if\ j \in S_M, \\ \{j+1\} & if\ j \in S_C, \end{cases} \tag{6.55}$$

$$S_C = \{jN + k \mid j = 0, ..., M-1; k = 1, ..., N-1\}, \tag{6.56}$$

$$S_M = \{N, 2N, ..., (M-1)N\}. \tag{6.57}$$

6.3.2 Performance Evaluation

Since the equalization within a module is not affected by other modules, and since the inter-module equalization is independent of the intra-module equalization, the equalization time of the module-based system with energy loss can be calculated using similar idea of algorithm 4.1:

Algorithm 4.2:

Step 1: Consider each module as an isolated series-based battery system with energy loss.

Step 2: Identify the battery group of each isolated battery system considered in Step 1 and calculated its ideal equalization time. This equalization time is the intra-module equalization of each module denoted as $\widehat{T}_{e,i}^{intra}$, $i = 1, ..., M$.

Step 3: For each module, aggregate all batteries in this module into one single virtual battery and let the initial state of the aggregated battery equal to the sum of the SOCs of all individual batteries in the module. This results in a new battery system with M aggregated batteries and serially connected. Identify the BNBG of this M-battery system with energy loss and calculate its ideal equalization time. This equalization denoted as $\widehat{T}_e^{inter}$.

Step 4: The equalization time of the original system is the maximum of the inter-module equalization time and the longest intra-module equalization time, i.e.,

$$\widehat{T}_e^{module} = max\{\widehat{T}_e^{inter}, \max_{i=1,\ldots,M} \widehat{T}_{e,i}^{intra}\}. \tag{6.58}$$

Proposition 12. *The equalization time of a battery equalization system defined by (6.52)-(6.57) can be approximated using Algorithm 4.2 as follows:*

$$T_e^{module} \approx \widehat{T}_e^{module}. \tag{6.59}$$

The equalization efficiency could be calculated as:

$$\widehat{\eta} = 1 - \frac{rh(B-1)\widehat{T}_e^{module}}{B\bar{x}_0}. \tag{6.60}$$

where $\bar{x}_0$ is defined as:

$$\bar{x}_0 = \frac{1}{B}\sum_{i=1}^{B} x_i(0). \tag{6.61}$$

6.3.3 Justification

A illustration of the performance estimation method is given in figure 6.3 for a 32-battery module-based system with random generated initial SOCs. The colored line in the figure illustrate the SOCs of batteries during the equalization process while the black line indicates the equalization time calculated according to algorithm 4.2.

6.4 Statistical Comparison

In this subsection, a statistical study is carried out to compare the three equalization structures with energy loss. Specifically, we selected B from $\{8, 16, 32, 64\}$ and randomly generated $25,000$ for each B with $x_i(0)$'s selected randomly and independently according to uniform distribution $U(0, 1)$. For the module-based system, we select M as 2. In addition, we assume that all equalizers have the same equalization rate

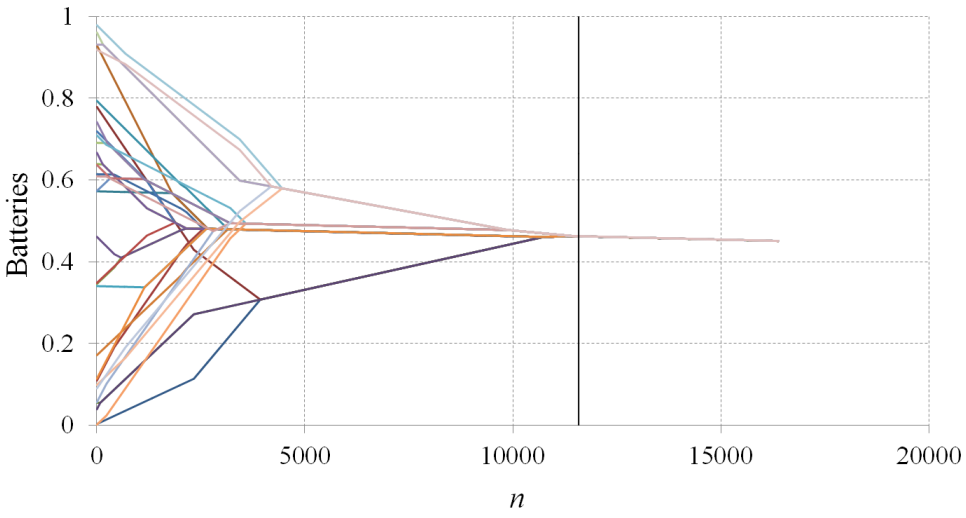


Figure 6.3: Illustration of performance of module-based structure

$r = 0.0001$. The results are summarized in Table 8.

As table 11 shows, the difference of equalization time between the original system and the one with energy loss increases as the energy loss rate increases; it also increases as the battery number increases. On the other hand, the system efficiency decreases as the energy loss rate increases; it also decreases as the number of battery increases. Moreover, layer-based structure provides the best efficiency since it has the shortest equalization time (i.e., equalizer working cycle), which leads to less loss of the SOC within the system.

B($h = 0.5\%$)	Time	Time with loss	Difference	Efficiency
8	5526.71	5546.57	19.86	99.49%
16	8427.75	8456.99	29.24	99.19%
32	12599.34	12642.16	42.82	98.76%
64	18539.97	18602.38	62.41	98.16%

(a) Series-based equalization with $h = 0.5\%$

B($h = 1\%$)	Time	Time with loss	Difference	Efficiency
8	5460.45	5500.09	39.64	99.00%
16	8480.51	8539.96	59.45	98.36%
32	12640.91	12727.66	86.75	97.53%
64	18649.97	18776.76	126.78	96.28%

(b) Series-based equalization with $h = 1\%$

B($h = 0.5\%$)	Time	Time with loss	Difference	Efficiency
8	4836.92	4849.04	12.12	99.56%
16	6854.86	6872.04	17.18	99.34%
32	9703.28	9727.60	24.32	99.05%
64	13799.42	13834.01	34.59	98.63%

(c) Layer-based equalization with $h = 0.5\%$

B($h = 1\%$)	Time	Time with loss	Difference	Efficiency
8	4793.58	4817.67	24.09	99.12%
16	6886.28	6920.88	34.60	98.67%
32	9749.63	9798.63	48.99	98.08%
64	13816.34	13885.77	69.43	97.25%

(d) Layer-based equalization with $h = 1\%$

B($h = 0.5\%$)	Time	Time with loss	Difference	Efficiency
8	5044.88	5065.68	20.80	99.54%
16	7723.45	7751.75	28.29	99.26%
32	11825.75	11866.74	40.99	98.84%
64	17740.75	17801.03	60.27	98.24%

(e) Module-based equalization with $h = 0.5\%$

B($h = 1\%$)	Time	Time with loss	Difference	Efficiency
8	4992.58	5034.30	41.72	99.09%
16	7753.92	7810.88	56.96	98.50%
32	11860.69	11943.27	82.58	97.66%
64	17894.12	18016.82	122.69	96.43%

(f) Module-based equalization with $h = 1\%$

Table 11: Battery equalization system with energy loss

7 Conclusion and Future Work

In this research, we study the series-based, layer-based and module-based battery equalization systems. Specifically, mathematical models are derived to describe the system-level dynamics of the equalization processes under the three equalization structures. To analyze the performance of these systems without energy loss, definitions of bottleneck battery group (BNEQ) and bottleneck equalizer (BNEQ) are proposed. Then, based on these notions. analytical algorithm are developed to calculate the equalization time of the system under consideration. Finally, a Monte Carlo statistical analysis is used to compare the effectiveness of the three equalization structure. As a result, both layer-based and module-based structures result in shorter equalization time on the average. The superiority of the two structures becomes more significant for systems with a large number of batteries. We also demonstrate that the initial SOC permutation would affect the total equalization time of the battery system and the monotonic permutation results in the longest equalization time. To analyze the performance of these systems with energy loss, modification to the models were made and numerical experiments were carried out. The comparison between systems with energy loss and the ones without energy loss are also given in section 6. The efficiency of the system is calculated by its original average SOC and its final average SOC after equalization process. Note that the layer-based structure provides the best average efficiency since it has the shortest equalization time, which result in less loss of energy. Thus we can conclude that, in a systematic perspective, the layer-based equalization structure on average performances better than both series-based and module-based structures under different circumstances. The analytical algorithms and equations developed to calculate the equalization time could be good approximations in order for engineers to predict the system's behavior.

In terms of the real battery systems, there are much more components within the system then what we have studied in this research. Thus a direction of the future

work could be introducing more components to the model and make it more similar to the real circuit. For example, in this research, all equalizers (cell-level equalizer and module-equalizer may have different equalization rate) are assumed to have identical parameters. However, in order to achieve a better efficiency, the equalization rate of each individual equalizer could be controlled by some decision methods based on system's feedback. Moreover, the current system studied in the research operates without recharge. It might have a different performance if there is an external power source connected to it. Under this direction, the future work includes:

- investigation of the system-theoretic properties of the equalization time with respect to system parameters;
- extension of the study to systems with non-identical equalizers;
- extension of the study to systems with feedback control;
- generalization of the study to systems with external power source and/or with load.

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Appendix

Proof of Theorem 1 For $B = 2$, equation (2.17) can be immediately obtained through the mathematical model.

Below, we give the proof for $B = 3$. Without loss of generality, the following three cases are studied:

Case 1: $y_1(0) > y_3(0) > 0 > y_2(0)$;

Case 2: $y_1(0) > y_2(0) > 0 > y_3(0)$;

Case 3: $y_2(0) > y_1(0) > 0 > y_3(0)$.

Since the system is symmetric, it can be shown that any buffer charge formation can be transformed to one of these three cases given above. Next we derive the formula for calculating the equalization time for each case:

Case 1: $y_1(0) > y_3(0) > 0 > y_2(0)$. Based on (2.15), we have $y_2(0) = -[y_1(0) + y_3(0)]$.

The equalization process of the three-battery system under this initial formation can be dived into two phases. During the first phase, both $b_1$and b_3 discharge with rate $\widetilde{r}$, while b_2 is charged with rate $2\widetilde{r}$. This phase ends when b_2 and b_3 contains the same charge for the first time. The duration of this phase is

$T_1 = \dfrac{y_1(0) + 2y_3(0)}{3\widetilde{r}}$.

At the end of this phase, we have:

$y_1(T_1) = \dfrac{2[y_1(0) - y_3(0)]}{3}$, $y_2(T_1) = \dfrac{y_3(0) - y_1(0)}{3}$, $y_3(T_1) = \dfrac{y_3(0) - y_1(0)}{3}$.

Next, battery $b_1$continues to discharge with rate $\widetilde{r}$, while b_2 and b_3 are both being charged but with rate $\widetilde{r}/2$ until $y_1(t) = y_2(t) = y_3(t) = 0$. The duration of this phase is

$T_2 = \dfrac{|y_1(T_1)|}{\widetilde{r}} = \dfrac{2[y_1(0) - y_3(0)]}{3\widetilde{r}}$.

Therefore, the equalization time T_e can be calculated as

$T_e^{series} = T_1 + T_2 = \dfrac{y_1(0)}{\widetilde{r}}$.

Case 2: $y_1(0) > y_2(0) > 0 > y_3(0)$. Based on (2.15), we have $y_3(0) = -[y_1(0) + y_2(0)]$.

Using similar approach as above, we can divide the equalization process under this

initial formation into two phases. During the first phase, $b_1$transfers charge to $b_2$with rate $\widetilde{r}$, while b_2 transfers energy to b_3 with the same rate. As a result, the state of b_2 remains constant during this period. This phase ends when $b_1$and b_2 contains the same charge for the first time. The duration of this phase is

$T_1 = \dfrac{y_1(0) - y_2(0)}{\widetilde{r}}$.

At the end of this phase, we have

$y_1(T_1) = y_2(0)$, $y_2(T_1) = y_2(0)$, $y_3(T_1) = -2y_2(0)$.

Next, battery b_3 continues to be charged with rate $\widetilde{r}$, while b_1 and b_2 are both being discharged but with rate $\tilde{r}/2$ until $y_1(t) = y_2(t) = y_3(t) = 0$. The duration of this phase is

$T_2 = \dfrac{|y_3(T_1)|}{\widetilde{r}} = \dfrac{2y_2(0)}{\widetilde{r}}$.

Therefore, the equalization time T_e can be calculated as

$T_e^{series} = T_1 + T_2 = \dfrac{y_1(0) + y_2(0)}{\widetilde{r}} = \dfrac{|y_3(0)|}{\widetilde{r}}$.

Case 3: $y_2(0) > y_1(0) > 0 > y_3(0)$. Based on (2.15), we have $y_3(0) = -[y_1(0) + y_2(0)]$.

Using similar approach as above, we can divide the equalization process under this initial formation into two phases. During the first phase, both b_1 and b_2 are charged by b_2 with rate $\widetilde{r}$, while b_2 discharges with rate $\widetilde{r}$. This phase ends when b_1 and b_2 contains the same charge for the first time. The duration of this phase is

$T_1 = \dfrac{y_2(0) - y_1(0)}{3\widetilde{r}}$.

At the end of this phase, we have:

$y_1(T_1) = \dfrac{2y_1(0) + y_2(0)}{3}$, $y_2(T_1) = \dfrac{2y_1(0) + y_2(0)}{3}$, $y_3(T_1) = -\dfrac{4y_1(0) + 2y_2(0)}{3}$.

Next, battery b_3 continues to be charged with rate $\widetilde{r}$, while b_1 and b_2 are both being discharged but with rate $\tilde{r}/2$ until $y_1(t) = y_2(t) = y_3(t) = 0$. The duration few this phase is

$T_2 = \dfrac{|y_3(T_1)|}{\widetilde{r}} = \dfrac{4y_1(0) + 2y_2(0)}{3\widetilde{r}}$.

Therefore, the equalization time

$T_e^{series} = T_1 + T_2 = \dfrac{y_1(0) + y_2(0)}{\widetilde{r}} = \dfrac{|y_3(0)|}{\widetilde{r}}$.

Proof of Theorem 5 By contradiction, first assume that all BNBGs are internal BGs. Without loss of generality, let $BG(i_0, g_0)$ denote an internal battery group (with a total of i_0 batteries starting with battery b_{g_0}), which is a BNBG. Then along with this group, consider two boundary battery groups surrounding it: $BG(i_0 - 1, 1)$ and $BG(B - i_0 - g_0 + 1, i_0 + g_0)$. Clearly, these three battery groups form a partition into the entire system, and

$g_0 > i_0$, $g_0 > B - i_0 - g_0 + 1$.

The ideal equalization time of these BGs are given by:

$$t_{ideal}(i_0 - 1, 1) = \frac{\left|\sum_{j=1}^{i_0-1} y_j(0)\right|}{r},$$

$$t_{ideal}(g_0, i_0) = \frac{\left|\sum_{j=i_0}^{i_0+g_0-1} y_j(0)\right|}{2r},$$

$$t_{ideal}(B - i_0 - g_0 + 1, i_0 + g_0) = \frac{\left|\sum_{j=i_0+g_0}^{B} y_j(0)\right|}{r}.$$

Since

$$\left|\sum_{j=1}^{i_0-1} y_j(0) + \sum_{j=i_0+g_0}^{B} y_j(0)\right| = \left|\sum_{j=i_0}^{i_0+g_0-1} y_j(0)\right|,$$

we have

$$t_{ideal}(g_0, i_0) = \frac{\left|\sum_{j=i_0}^{i_0+g_0-1} y_j(0)\right|}{2r} = \frac{\left|\sum_{j=1}^{i_0-1} y_j(0) + \sum_{j=i_0+g_0}^{B} y_j(0)\right|}{2r}$$

$$\leq \frac{\left|\sum_{j=1}^{i_0-1} y_j(0)\right| + \left|\sum_{j=i_0+g_0}^{B} y_j(0)\right|}{2r} \leq max\ \{t_{ideal}(i_0 - 1, 1),\ t_{ideal}(B - i_0 - g_0 + 1, i_0 + g_0)\}$$

.

Therefore, either $BG(i_0 - 1, 1)$ or $BG(B - i_0 - g_0 + 1, i_0 + g_0)$ is the BNBG, which contradicts the assumption. In other words, there exist at least one boundary BG being BNBG.

Next, assume that the boundary BNBG with the smallest number of batteries is

$BG(g_1, i_1)$ with $g_1 > \lfloor B/2 \rfloor$. Without loss of generality, assume that $i_1 = 1$. Then, the ideal equalization time of the BG is given by:

$$t_{ideal}(g_1, i_1) = \frac{\left| \sum_{j=1}^{g_1} y_j(0) \right|}{r}.$$

In addition, the ideal equalization time of battery group $BG(B - g_1, g_1 + 1)$ is given by:

$$t_{ideal}(B - g_1, g_1 + 1) = \frac{\left| \sum_{j=g_1+1}^{B} y_j(0) \right|}{r} = \frac{\left| \sum_{j=1}^{g_1} y_j(0) \right|}{r} = t_{ideal}(g_1, i_1).$$

Therefore, $BG(B-g_1, g_1+1)$ is also a BNBG. Since $g_1 > \lfloor B/2 \rfloor$, we have $B-g_1 < \lfloor B/2 \rfloor$. In other words, a BNBG with fewer than $\lfloor B/2 \rfloor$ batteries exists, which is a contradiction.

Proof of Theorem 7 Follows immediately from the independent operation of equalizers in the layer-based structure.

Proof of Theorem 9 Let one battery string has two different permutations A and B. Without loss of generality, let permutation A be random ordered (strictly not monotonic) and B be ordered (either from smallest to largest or the opposite).

Series-based: Theorem 2 said that for a battery string defined by (I)-(V), among all BNBGs of the system, there exists at least one boundary BG with no more than $\lfloor B/2 \rfloor$ batteries. Let T^{series}_{random} denote the equalization time for battery string A and $T^{series}_{ordered}$ denote the time for B. Due to theorem 2, for the random order battery string, let $BG_{random}(g,\ j)$ be the BNBG:

$$T^{series}_{random} = max \quad t_{ideal}(g, i) = \frac{\left| \sum_{i=j}^{j+g-1} y_j(0) \right|}{r}, \quad g \in [1, ..., \llcorner B/2 \lrcorner], i \in [1, ...B - g + 1].$$

Let $BG_{ordered}(g,\ j)$ be the same battery group starts from jth position in the battery string B. Since $BG_{random}(g,\ j)$ and $BG_{ordered}(g,\ j)$ are boundary battery groups and $y_i(0) = x_i(0) - \bar{x}$,

$$\frac{\left|\sum_{i=j}^{j+g-1} y_j^{random}(0)\right|}{r} \leq \frac{\left|\sum_{i=j}^{j+g-1} y_j^{ordered}(0)\right|}{r}$$

Thus we have the following inequality:

$$T_{random}^{series} = \frac{\left|\sum_{i=j}^{j+g-1} y_j^{random}(0)\right|}{r} \leq \frac{\left|\sum_{i=j}^{j+g-1} y_j^{ordered}(0)\right|}{r} \leq T_{ordered}^{series}.$$

Layer-based: Due to theorem 3, the equalization time of a layer-based battery string equals to the equalization time of the BNEQ. According to equation (3.9), equalization time of any equalizer depends on the $\Delta(0)$,which is the difference of SOC between the two BGs balanced by this equalizer at the beginning of the process. Obviously, if the permutation is monotonic, the "top" equalizer would has the largest $\Delta(0)$. Thus the equalization time of such a battery string is the longest.

Module-based: Let $\widehat{T}_{random}^{intra}$ denote the intra-module equalization time of battery string A and $\widehat{T}_{ordered}^{intra}$ denote the one of battery string B. According the proof of series-based battery string above, for each module, $\widehat{T}_{random}^{intra} \leq \widehat{T}_{ordered}^{intra}$, thus

$$max\ \widehat{T}_{random}^{intra} \leq max\ \widehat{T}_{ordered}^{intra},$$

By applying the concept of aggregation in algorithm 4.1, the aggregated string B is still monotonic. Thus

$$\widehat{T}_{random}^{inter} \leq \widehat{T}_{ordered}^{inter},$$

Because of the inequalities above, we can conclude that

$T_{random}^{module} \leq T_{ordered}^{module}$.