LETTERS

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Growth of ideal magnetohydrodynamic modes driven slowly through their instability threshold: Application to disruption precursors

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The growth of an ideal magnetohydrodynamic (MHD) instability in a high-temperature plasma is calculated in the case where the plasma β is driven slowly through its instability threshold. The MHD perturbation grows faster than exponentially, approximately as $\exp[(t/\tau)^{3/2}]$. Its characteristic growth time $\tau \sim (\frac{3}{2})^{2/3} \hat{\gamma}_{\mathrm{MHD}}^{-2/3} \gamma_h^{-1/3}$ is a hybrid of the ideal MHD incremental growth rate $\hat{\gamma}_{\mathrm{MHD}}$ and the heating rate γ_h. This simple model agrees well with the observed growth of disruption precursors in high β DIII-D [J. L. Luxon and L. G. Davis, Fusion Technol. **8**, 441 (1985)] discharges having strongly peaked pressure profiles, where the observed growth times of $\geqslant 10^{-4}$ s are significantly slower than the typical ideal MHD time scale of $\leqslant 10^{-5}$ s. © *1999 American Institute of Physics.* [S1070-664X(99)01508-6]

Disruptions are of great concern[1] in tokamak plasmas and not well understood.[2] Here, by disruptions we mean rapid decreases in plasma confinement, plasma pressure collapses, etc. While the instability boundaries for global ideal magnetohydrodynamic (MHD) modes apparently demarcate[3] the limits of the achievable volume-average pressure $\langle P \rangle$ or $\beta \equiv \langle P \rangle / (B^2/2\mu_0)$ in tokamak plasmas, the origin of the precursors to β limiting disruptions is not clear, although models have been advanced for some precursors.[4–6]

The growing oscillations observed[6] to precede major disruptions in DIII-D[7] L(low)-mode negative central magnetic shear (NCS) plasmas occur in well diagnosed plasmas and are of particular interest. The growth times of these precursors ($\sim$100–500 μs) are much slower than the few μs growth times typical of ideal MHD instabilities. Thus, slower growing resistive modes[8] were proposed[6] as contributing to at least the early development of the disruption precursors. These modes are resistive double tearing modes coupled externally to the ideal external kink and modified by the presence of sheared plasma rotation and finite pressure effects, especially at the inner rational surface where the resistive instability criterion $(D_R > 0)$ was satisfied.

Although these resistive modes had many of the features expected for the observed precursors, questions remain concerning the growth rates of these modes; specifically, the typical growth rate is slow ($\gamma_R^{-1} > 1$ ms) and very sensitive to details of the rotation and pressure profiles. Also, diamagnetic flow frequency effects[9] ($\omega_* \sim 10^4 \gg \gamma_R$) should reduce the growth rate even more (to $\gamma \sim \gamma_R^3/\omega_*^2 \sim 10$ s^{-1}). In the instability code calculations[6] the growth rate appears to increase continually with increasing β from a slow resistive double tearing-like rate $\gamma \sim \eta^{1/3}$ to an ideal-like rate as β increases up to the ideal limit. In some discharges, the observed β approaches the calculated ideal β limit. We will assume the mode is essentially an ideal-like mode near the marginal point and driven by the finite pressure gradient.

Similarly, the precursors to minor disruptions (sawtooth crashes) in present high-temperature tokamak plasmas seem slow[10] compared to typical ideal MHD instability growth times, although perhaps ideal MHD in character,[11] but are much faster than resistive MHD models would predict.[10,12] Thus, precursors to both major and minor disruptions in tokamak plasmas remain an important unresolved conundrum.

Here, we develop a simple theoretical model for the temporal evolution of a pressure-gradient-driven, interchange-like, ideal MHD mode in a high temperature plasma. By interchange-like we mean that the mode is global and not dominated by a large gradient near a rational surface that would be subject to magnetic reconnection during its early evolution. The basic hypothesis of our model is that the

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plasma pressure or β is increased slowly (via, for example, energetic neutral beam heating and fueling, on the transport time scale) through a global ideal MHD instability threshold, and into instability. The key assumptions of the model are that: (1) The plasma is near the marginal stability state for a global ideal MHD instability; (2) the plasma β is increasing approximately linearly with time according to

$$\beta=\beta_c(1+\gamma_h t), \tag{1}$$

such that the plasma is stable ($\beta<\beta_c$) for $t<0$, marginal at $t=0$, and unstable for $t>0$; and (3) the disruption precursors are linear phenomena, up to the instant of the disruption when a large-scale redistribution of thermal energy occurs.

The growth rate of a linear ideal MHD instability is obtained from the energy principle[13] global mode dispersion relation $\omega^2=\delta W/\delta K$ in which δW is the change in potential energy of the plasma produced by a fluid perturbation $\tilde{\boldsymbol{\xi}}(\mathbf{x},t)$, and δK is the concomitant change in plasma kinetic energy, both integrated over the entire volume of the plasma. The potential energy change δW is composed of a destabilizing (negative) term that is linearly proportional to the plasma pressure or β, plus various stabilizing terms that represent the excitation of Alfvén and sound waves in the plasma. (For this simple model, the magnetic field is assumed to evolve slowly compared to β and hence to be fixed.) Thus, we propose that near the ideal MHD instability threshold the mode dispersion relation can be written as

$$\omega^2=-\hat{\gamma}^2_{\mathrm{MHD}}(\beta/\beta_c-1), \tag{2}$$

in which $\hat{\gamma}^2_{\mathrm{MHD}}$ is assumed to be nearly constant and represents the incremental change (hence the hat over γ_{MHD}) in the square of the ideal MHD instability growth rate as β is increased above the marginal stability value β_c. Calculations that support this model will be shown below. Assuming the linear increase of β with time indicated in Eq. (1), the temporal evolution of the growth rate is given by

$$\gamma(t)=\hat{\gamma}_{\mathrm{MHD}}\sqrt{\gamma_h t}, \quad t\geqslant 0. \tag{3}$$

The growth of a linear fluid perturbation $\tilde{\xi}$ is governed by the equation

$$d\tilde{\xi}/dt=\gamma(t)\tilde{\xi}\ . \tag{4}$$

Assuming an initial perturbation ξ_0 at $t=0$, the solution of this equation for the $\gamma(t)$ given in Eq. (3) is

$$\tilde{\xi}=\xi_0\exp\left[\int_0^t dt\ \gamma(t)\right]=\xi_0\exp[(t/\tau)^{3/2}], \tag{5}$$

in which

$$\tau\equiv(3/2)^{2/3}\hat{\gamma}_{\mathrm{MHD}}^{-2/3}\gamma_h^{-1/3}\ . \tag{6}$$

This model is a simplified version of that in Ref. 4 in which the ideal MHD instability eigenmode equation $\ddot{\tilde{\xi}}-(\hat{\gamma}^2_{\mathrm{MHD}}\gamma_h)t\tilde{\xi}=0$ was solved in terms of Airy functions[4] which asymptotically (for $t\geqslant\tau$) scale as $\exp[(t/\tau)^{3/2}]/(t/\tau)^{1/4}$. The extra $(t/\tau)^{1/4}$ factor is negligible in the fitting to experimental data, and the simplified model used here is more readily adaptable for exploring possible resistivity and nonlinear evolution effects.

Our model for disruption precursors differs from the usual presumptions for exponential ideal MHD instability growth of the form $\exp[\gamma t]$ in two important respects: (1) The temporal evolution is not a simple exponential in time, but rather of the form $\exp[(t/\tau)^{3/2}]$; and (2) the characteristic time τ for exponentiation is a geometric mean[4] of the ideal MHD instability growth time ($\hat{\gamma}^{-1}_{\mathrm{MHD}}\sim$ few μs) and the drive or heating time ($\gamma_h^{-1}=100$ s of ms), and hence typically of order a few hundred μs—much longer than $\hat{\gamma}^{-1}_{\mathrm{MHD}}$. The characteristic growth time τ is much longer than the incremental ideal MHD growth time $\hat{\gamma}^{-1}_{\mathrm{MHD}}$ because in the time τ the plasma only slightly exceeds the threshold condition and the mode growth rate is only a small fraction of $\hat{\gamma}_{\mathrm{MHD}}$. Taking, for example, $\hat{\gamma}_{\mathrm{MHD}}/\gamma_h\sim 10^5$, which is typical (see below) of DIII-D NCS plasmas,[6] within the time τ the plasma exceeds the threshold by only $\Delta\beta/\beta_c=\gamma_h\tau\sim(\gamma_h/\hat{\gamma}_{\mathrm{MHD}})^{2/3}\sim 4\times 10^{-4}$ and $\gamma(\tau)\sim\hat{\gamma}_{\mathrm{MHD}}(\gamma_h/\hat{\gamma}_{\mathrm{MHD}})^{1/3}\sim 2\times 10^{-2}\hat{\gamma}_{\mathrm{MHD}}\ll\hat{\gamma}_{\mathrm{MHD}}$.

Before comparing this driven ideal MHD global instability model for disruption precursors with experimental data, we note that this model can be generalized by considering the types of precursor growth that would be obtained from other instabilities driven slowly (at rate γ_h) through their threshold conditions. The growth rates of resistive-interchange modes[8] and drift-wave instabilities scale as $(d\beta/dr)^{2/3}$ and $d\beta/dr$, respectively. Thus, we propose a generic model for Eq. (3) of the form $\gamma(t)=\gamma_0(\gamma_h t)^\alpha$ with $\alpha=\frac{1}{2}$ (ideal MHD), $\frac{2}{3}$ (resistive-interchange), or 1 (drift-wave instabilities). Then, we obtain

$$\tilde{\xi}=\xi_0\exp[(t/\tau)^{1+\alpha}]\ \text{ with }\ \tau\sim(\gamma_0\gamma_h^\alpha)^{1/(1+\alpha)}, \tag{7}$$

which indicates faster than exponential growth for all types of instabilities driven through their instability thresholds. Note that while the power of t in the exponential depends on the model chosen ($1+\alpha=\frac{3}{2}$, $\frac{5}{3}$, or 2), the characteristic time τ is always a geometric mean of the fast instability growth and slow drive times, and is always significantly longer than γ_0^{-1}. Also, the type of instability whose threshold is being exceeded could in principle be determined from the observed temporal growth of the perturbation $\tilde{\xi}$ in the plasma.

A minor complication is introduced in the model given by Eqs. (1)–(6) if we consider the effects of plasma resistivity and resistive modes[8] in the vicinity of the instability threshold conditions for an ideal MHD global mode. Two modes are of concern here, the double tearing and resistive interchange modes. As we noted earlier, for β below the ideal MHD instability threshold ($\beta<\beta_c$, $t<0$), the double tearing mode has essentially a resistive double tearing character near the resistive mode threshold, but becomes ideal-like as β is increased up to the ideal limit. The resistive-interchange instabilities, on the other hand, are not global modes. Rather, they are resistive layer[6,8] modes concentrated near a low order rational surface, and, as indicated above, should have their growth rate diminished by diamagnetic flow frequency effects[9] for high temperature plasmas such as the DIII-D NCS plasmas.[6] At precisely the ideal MHD threshold condition ($\beta=\beta_c$, $t=0$), Coppi[14] has shown that the growth rate of a resistive mode using the ideal MHD global marginal stability eigenmode is a factor of $S^{1/3}$ smaller than $\hat{\gamma}_{\rm MHD}$; hence, the resistive growth rate at marginal ideal MHD instability is smaller than $\gamma(\tau)$ as long as $S \gg \hat{\gamma}_{\rm MHD}/\gamma_h$. Here, $S \equiv \tau_R/\tau_A \equiv [a^2/(\eta/\mu_0)]/[R_0/c_A]$ is the Lundquist or magnetic Reynolds number—the ratio of the resistive diffusion time (τ_R) to the Alfvén time (τ_A). Thus, while the ''initial condition'' perturbation ξ_0 might be growing on a slower resistive mode time scale, $\tilde{\xi}$ should grow temporally as indicated in Eq. (5) for all times $t \gtrsim \tau$ as long as $S \gg \hat{\gamma}_{\rm MHD}/\gamma_h$, which is well satisfied for strongly beam heated DIII-D plasmas[6] where $\hat{\gamma}_{\rm MHD}/\gamma_h \sim 10^5$ and $S \sim 3 \times 10^7$. In view of these points, in fitting experimental data from disruption precursors to Eq. (5) we should neglect the early growth ($t \lesssim \tau$) where a transition from resistive to ideal MHD instability occurs and Eq. (2) might not be a good model [in Eq. (7) α effectively varies with β there], and instead concentrate on the temporal evolution within the last few characteristic times before the disruption where robust, global ideal MHD mode growth occurs and, hence, our model becomes most applicable.

The possible nonlinear evolution of driven ideal MHD instabilities is also of interest. Internal, kink-like ideal MHD modes induced by current-gradient free energy tend to be highly localized near low order rational surfaces, particularly near marginal stability. As they grow, they quickly form current sheets and nonlinearly saturate at small amplitudes;[15] then, they are converted into tearing modes that grow slowly in time ($\sim \gamma_R$). However, pressure-gradient-driven, interchange-like ideal MHD modes (particularly those beyond marginal stability conditions) can grow linearly to much larger amplitudes (until the flux surface distortions they induce would break the nested flux surface topology: $d\tilde{\xi}_r/dr = -1$[16]) before inducing current sheets that nonlinearly modify their growth. Thus, the driven interchange-like ideal MHD instabilities of interest here can be expected to grow linearly according to Eq. (5) from an initial perturbation level (perhaps $\sim \rho_i \sim$ a few millimeters for applicability of MHD) to a macroscopic scale size (perhaps $\sim$10% of the plasma radius). Thereafter, current sheets and topology effects would develop and lead to dramatic macroscopic plasma redistributions that we associate with the observed ''fast crashes'' of the plasma pressure—i.e., plasma disruptions.

We now compare the model developed in Eqs. (1)–(6) with experimental data from disruption precursors in DIII-D NCS plasmas.[6] In these tokamak plasmas a combination of time-dependent programming of the poloidal magnetic field system and energetic neutral beam heating induces an internal transport barrier to develop.[17] The barrier provides very good (ion neoclassical level) central plasma confinement.[18] Continued neutral beam heating induces a central peaking of the plasma pressure profile and causes the plasma β to increase approximately linearly in time. This is shown in Fig. 1 for discharge 87009. For this case, $\gamma_h^{-1} \sim 650$ ms at $t = 1.6$ s.

The increase in plasma pressure and β is terminated by a major disruption at $t \simeq 1.683$ s which is preceded by the growth of an $n=1$ precursor oscillation in the plasma.[6] The temporal development of the disruption precursor for discharge 87009, as measured by an external Mirnov loop and a central ($\rho \sim 0.3$) soft x-ray (SXR) chord, is shown in Fig. 2(a). A semilog plot of the amplitudes of these two signals is shown in Fig. 2(b), along with fitted curves based on simple exponential growth, and the faster than exponential form given by Eq. (5). The specific (nonlinear) fit function used for the envelope of the disruption precursor, $\tilde{A} = A_0 \exp\{[(t - t_0)/\tau]^{1+\alpha}\}$, is based on Eq. (5) and has four parameters: Initial amplitude A_0 and time t_0, temporal growth power α, and characteristic time τ. The fitting procedure used in Fig. 2 fixes $1+\alpha$ at 3/2 and determines A_0, t_0, and τ for the best fit to the data. More general fits varying all four parameters give similar chi-squared values, and hence are no more definitive in determining τ and α. The solid curve shown in Fig. 2(b) is obtained from a fit to data from an array of six Mirnov loops around the midplane of the torus, but also provides a good fit to the individual Mirnov loop and SXR measurements shown in Fig. 2. The data are fitted over the

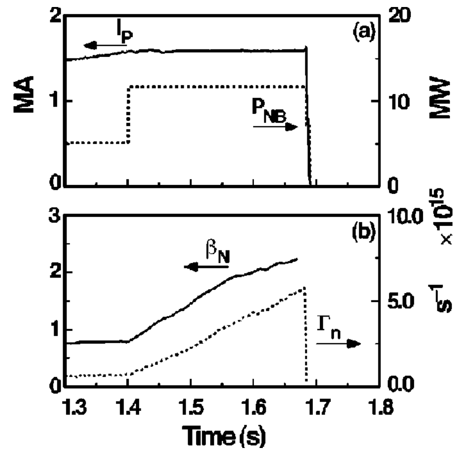

FIG. 1. Time evolution of DIII-D discharge 87009 (Ref. 6) showing (a) plasma current I_p and neutral beam power P_{NB}, and (b) normalized beta $\beta_N = \beta(aB/I)$ and D-D fusion neutron rate Γ_n.

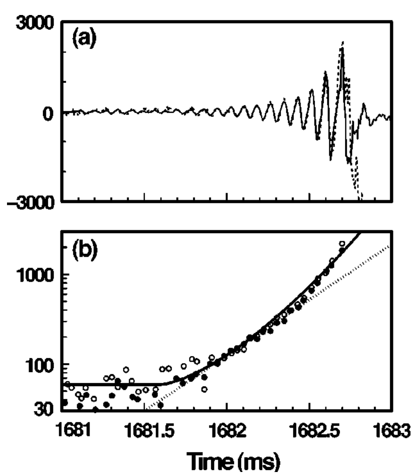

FIG. 2. Time development and growth of $n=1$ disruption precursor in DIII-D discharge 87009 observed via external (Mirnov coil) magnetic field perturbation $d\tilde{B}_\theta/dt$ and core ($\rho \sim 0.3$) SXR chord. The oscillating component of the SXR signal is scaled to match the Mirnov coil amplitude. (a) Raw data from Mirnov coil (solid curve) and SXR signals (broken curve). (b) Instantaneous amplitude of Mirnov coil (solid circles) and SXR (open circles) signals, with a curve fitted to the model of Eq. (5) (solid curve) and a representative exponential growth curve (broken curve).

time interval 1681.69–1682.69 ms; the initial time $t_0 = 1681.59$ obtained from the fit is consistent with this choice of time interval. The fit ignores the early phase, before clear growth occurs, where the oscillations are either due to noise or to an early resistive transition phase not represented by the model, as discussed above. Also, the model is inherently linear and cannot describe the final nonlinear phase of the disruption. In the intermediate range where the model should apply the fit is extremely good. In particular, the driven ideal MHD instability model developed above fits the data significantly better than the simple exponential also shown in Fig. 2(b). For this case, we infer a characteristic time $\tau = 490\ \mu$s.

Note the strong similarity in the phases and temporal development of the external (Mirnov loop) and internal (central SXR chord) measurements of the disruption precursors (Fig. 2). These central and edge measurements maintain the same ratio as their amplitudes increase by more than an order of magnitude, indicating that the mode structure remains constant throughout the growth. This observation coupled with the fact that the signals from all the other SXR chords through the plasma exhibit similar in-phase signals shows that these disruption precursors are robust global modes. In particular, they are not localized to small regions of the plasma as resistive-layer modes would be.

Having obtained experimental values for γ_h ($\sim 1.5\ \mathrm{s}^{-1}$) and τ ($\sim 490\ \mu$s), we now use Eq. (6) to determine an experimental value for the incremental ideal MHD instability growth rate: $\hat{\gamma}_{\mathrm{MHD}} \sim 1.1\times 10^5\ \mathrm{s}^{-1}$. Previously published[6] GATO code[19] calculations for this discharge indicate that the plasma should be stable to global ideal MHD modes, with $\beta^{\exp} \sim 0.8\,\beta_c$ at the disruption. It was conjectured[6] that for $0.8\beta_c < \beta < \beta_c$, the resistive double tearing mode noted earlier very quickly takes on essentially ideal character with $1+\alpha$ in Eq. (7) $\sim \frac{3}{2}$ as a result of the finite rotation shear, relatively high β, and vacuum boundary conditions, after passing $\beta = 0.8\,\beta_c$. More recent calculations have shown, however, that a stronger local pressure gradient in the low shear region can reduce β_c to near the experimental value and still yield a reasonable fit to the discharge equilibrium data. Because of uncertainties in the measured central density and in the calculated central fast ion pressure, such a steepening of the central pressure profile—approximately a 15% increase at $\rho = 0.3$—cannot be ruled out. In Fig. 3 we plot the GATO results for γ^2 versus $\beta_N / \beta_N^{\exp}$ for both the original equilibrium reconstruction and the steepened pressure profile. From the slope of the ideal MHD stability boundary for $\beta/\beta_c \gtrsim 1$ we infer $\hat{\gamma}_{\mathrm{MHD}} \sim 2-3\times 10^5\ \mathrm{s}^{-1}$, which agrees reasonably well with the value determined from Eq. (6) using experimentally determined parameters.

The combination of the good fit (Fig. 2) of the temporal evolution of the disruption precursor by Eq. (5), the distributed, global nature of the precursor, and the closeness of the experimental and theoretical values for $\hat{\gamma}_{\mathrm{MHD}}$ show that our driven ideal MHD global instability model provides a good description of the $n=1$ precursors to major disruptions in DIII-D L-mode NCS plasmas.[6] While current-driven external kink[4] or locally initiated pressure-gradient-driven[5] ideal MHD modes have been observed before, we have identified for the first time a plasma pressure-driven global ideal MHD instability and described its early (essentially linear) growth by the model embodied in Eqs. (1)–(6).

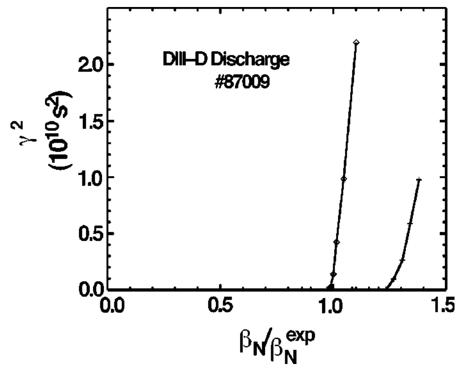

FIG. 3. Growth rate of a global ideal MHD instability in discharge 87 009 as a function of the normalized β calculated using the GATO code (Ref. 18) for the standard reconstruction (pluses) and the reconstruction with a steepened pressure profile (diamonds). The $\beta_N^{\exp}$ is 2.03 and 2.18 in these cases, respectively. The slope of the line near $\gamma^2 = 0$ determines the MHD stability growth parameter. This gives $\hat{\gamma}_{\mathrm{MHD}} \sim 2\times 10^5\ \mathrm{s}^{-1}$ for the standard reconstruction and $\hat{\gamma}_{\mathrm{MHD}} \sim 3\times 10^5\ \mathrm{s}^{-1}$ for the equilibrium with a steepened pressure profile.

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