Pricing a Student-Funded Incentive to Encourage Four-Year Graduation

AN ACTUARIAL ANALYSIS OF CONTINGENT PAYMENTS

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INTRODUCTION

The purpose of this project is to price a theoretical financial tuition incentive in order to increase the four-year graduation rate at the University of Wisconsin-Eau Claire. This project explores how such an incentive could be sustained through differential tuition alone. A financial tuition incentive like the one presented here is one way to facilitate the university’s goal of increased retention and graduation rates going forward.

ACTUARIAL CONNECTION

We priced this incentive as one would price a pure endowment, a life insurance product with a similar structure to our theoretical incentive. A pure endowment provides a payment to the insured at the conclusion of an n-year coverage period, provided that the insured survives to that time. Letting \( \nu \) represent the annualized effective discount factor and defining \( \mu_x, n \) to be the probability that a life aged \( x \) survives to age \( x+n \), the equation for the actuarial present value of a pure endowment is:

\[
\nu^n \cdot \sum_{x=0}^{n} \mu_x \cdot (1 + \nu)^{-x/2} \cdot \nu^{x/2} \mu_x
\]

This actuarial present value is calculated using both interest theory and principles of probability.

FINANCIAL MATHEMATICS

To determine the appropriate effective interest rate for pricing, we examined current and historic CD rates for varying term lengths. We used the current rates as a base for our models and the historical rates to allow for variability.

PROBABILITY

In our case, the incentive pays out if the student graduates within four years. The probability component of each contribution to the incentive comes from the initial student numbers and predicted retention rates. The payouts use the predicted four-year graduation rate instead. For an individual student, the actuarial present value of their contributions is:

\[
\sum_{i=1}^{n} (C \cdot (1 + i_{x,i})^{-i/2} \cdot \nu^{i/2} \mu_x)
\]

In this formula \( C \) is the contribution to the incentive and \( i_{x,i} \) is the appropriate annual effective interest rate for the \( i \)-th semester of a student starting in time period \( x \). The actuarial present value of the payouts is calculated in a similar manner, modified so that the payments are contingent upon graduation instead of retention.

THE CUBIC SPLINE

We chose to use the cubic spline for our simulations since it matched current growth trends the best of the three while still providing a reasonable pattern of long-term growth. Defining \( C_n(x, t) \) to be the predicted four-year graduation rate in year \( t \), we used the following equations to construct the cubic spline:

\[
\varphi(x,t) = \frac{N_n(x, t)}{N_n(x, t - 1)}
\]

\[
C_n(x,0) = \varphi(x,2013) = d
\]

\[
C_n(x,0) = \varphi(x,2013) - \varphi(x,2012) = c
\]

\[
C_n(x,27) = \varphi(x,2040) = 19683a + 729b + 27c + d
\]

\[
C_n(x,27) = 2187a + 54b + c = 0
\]

SIMULATION RESULTS

For our simulation we used a payout value of $1000 and used the cubic spline model to estimate both graduation and retention rates through the 2040 cohort. The results of our simulation are shown here. Prices are per semester.

<table>
<thead>
<tr>
<th>Price Data</th>
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<tbody>
<tr>
<td>Mean</td>
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<td>Standard Deviation</td>
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<td>Minimum</td>
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<td>95% Confidence</td>
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<td>99% Confidence</td>
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We determined that a reasonable estimate of the four-year graduation rate for the 2040 cohort would be 60%.

After determining this target rate, we had to decide on a model to give estimates of the four-year graduation rates between now and then. We looked at using three different models: a simple linear model, a quadratic model, and a cubic model. The graph of the models’ results is shown below.

REFERENCES AND ACKNOWLEDGEMENTS


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Tables made using Microsoft Excel. Graph made using the ggplot2 package in R.

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