NON-DESTRUCTIVE TESTING TECHNIQUES OF HIGHWAY GUARDRAIL POSTS:

STRESS-WAVE PROPAGATION AND MAGNETIC CHARACTERIZATION

by

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Executive Summary

The number of highway guardrail systems with posts of unvalidated depth is unknown to the Wisconsin Department of Transportation. These guardrail posts provide the support for barrier systems designed to protect motorists and infrastructure in the event of a crash. Non-Destructive Testing in the form of stress-wave propagation and magnetic characterization is explored as a method of length analysis for guardrail posts.

Longitudinal waves introduced to the post through an impulse were evaluated for natural frequency and wave velocity to estimate the post length. Field testing revealed length prediction error of 0.8 m for wood (length range: 1.2 m to 2.1 m) and 0.3 m for steel (length range: 1.8 m) when the posts are in-situ. Lab testing was conducted on a variety of wood posts under conditions of free, rubber-fixed, and fixed boundaries. Predicted length error varied from 0.2 m to 0.4 m (length range: 1.2 m to 3.1 m). To increase understanding of waveform characteristics through the wood medium, the attenuation, frequency dependent attenuation, and phase were analyzed. In the field, wood and steel posts were analyzed when attached and unattached to the guardrail. The information from this analysis further enhanced the understanding of energy loss, reflected waves, and the wave velocity, and the effect of frequency on the waveform with respect to the limitations imbedded in the stress-wave propagation technique for length prediction.

Characterization of the magnetic anomalies produced by the interaction between the magnetic field and ferrous materials was also evaluated as a methodology to determine the length of guardrail posts. The geometry, material characteristics, and distance of the material from the instrument affected the recorded magnetic anomaly. Magnetic models of expected anomalies compared to measured results provide a method to evaluate the expected length to the real length of the post.
Improving the methodical collection of data may help reduce the intrinsic error associated with magnetic surveying and further increase the relative impact of the base of the post. A methodology was devised to analyze wood posts with steel base plates to demonstrate the potential implementation.

Under the current state of the devised techniques, neither stress-wave propagation of posts nor the magnetic characterization of magnetic material can accurately predict length within the specified error tolerance of 5 to 10 cm.
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1. Introduction

The use of non-destructive testing (NDT) to evaluate the integrity and property of materials has been a technique widely researched and used in practice. If applied appropriately, the implementation of NDT methodology allows for the evaluation of various kinds of systems without damaging or compromising serviceability (Dwivedi et al., 2018). Furthermore, NDT techniques provide cost effective solutions for quality control (Dwivedi et al., 2018). Choosing a specific NDT technique to effectively evaluate the system’s properties or integrity is dependent on the target of interest.

Geotechnical engineering has used NDT integrity and dimension analysis of in-situ foundations with different levels of success. The need to assess the integrity of structural elements has led to the development of techniques utilizing physical properties such as stress-wave propagation, electromagnetic radiation, induced magnetics, thermal properties and more (Olson, 2003; Piscsalko et al., 2017). Depending on the target of interest, techniques can be used individually or in combination to evaluate parameters such as foundation length, depth to impedance change, distribution of concrete hydration, and overall structural integrity (Paikowsky & Chersauskas, 2003; Piscsalko et al., 2017; Rausche et al., 1994, 2002). A summary of several prominent NDT techniques related to foundation analysis can be found in Table 1.1.
Table 1.1. Summary of common foundation Non-Destructive Testing methods.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Property</th>
<th>Principle</th>
<th>Capabilities</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse Echo (PE)</td>
<td>Stress-wave propagation</td>
<td>Longitudinal wave in time domain to determine impedance depth from reflections to top receiver</td>
<td>Impedance change depths</td>
<td>Wave velocity of material must be known, force is not considered</td>
</tr>
<tr>
<td>Ultraseismic (US)</td>
<td>Stress-wave propagation</td>
<td>Longitudinal/flexural wave in time domain to determine impedance depth with multiple receivers</td>
<td>Impedance change depths</td>
<td>Wave velocity of material must be known, force is not considered</td>
</tr>
<tr>
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<td>Stress-wave propagation</td>
<td>Longitudinal wave and force records in frequency domain give mobility spectrum to identify dominant frequencies</td>
<td>Impedance change depths</td>
<td>Wave velocity of material must be known</td>
</tr>
<tr>
<td>Bending Waves (BW)</td>
<td>Stress-wave propagation</td>
<td>Horizontal impact on pile to identify bending wave energy from pile tip</td>
<td>Impedance change depths</td>
<td>Confined to short piles with soft soil, force is not considered</td>
</tr>
<tr>
<td>Parallel Seismic (PS)</td>
<td>Stress-wave propagation</td>
<td>Longitudinal wave in time domain recorded in adjacent borehole at various depths</td>
<td>Impedance change depths &amp; wave velocity determination</td>
<td>Requires borehole drilling &amp; Snell’s law correction, force is not considered</td>
</tr>
<tr>
<td>Ground Penetrating Radar (GPR)</td>
<td>Electromagnetic radiation</td>
<td>Reflection of electromagnetic waves from differences in dielectric permittivity</td>
<td>Wave reflection change depths, detailed images within materials</td>
<td>Requires borehole drilling for deep foundations, low conductivity soils</td>
</tr>
<tr>
<td>Induction Field (IF)</td>
<td>Induced magnetics</td>
<td>Magnetic field induction in adjacent borehole records change in intensity in the time domain for ferrous materials</td>
<td>Magnetic material depth change</td>
<td>Requires borehole drilling, results complicated by presence of other ferrous materials</td>
</tr>
<tr>
<td>Thermal Integrity Profiling (TIP)</td>
<td>Thermal gradient</td>
<td>Temperature measurements through the entire length of drilled</td>
<td>Integrity testing of drilled shafts</td>
<td>Requires prior construction planning</td>
</tr>
</tbody>
</table>

Sources: Olson (2003); Paikowsky & Chernauskas (2003); Robinson & Webster (2008); Huang & Ni (2012); Piscalko et al., (2017).

Foundations support the civil infrastructure which society depends on every day. These can be as large as the deep cast-in-place concrete piles required for high-rise buildings, or as small as the posts required to support highway guardrails. In recent years, acknowledgements have been made for the gaps in foundation knowledge. In the United States, many older bridges have no original contract documents. In 2000, the National Bridge Inventory estimated that at least 91,094 bridges over water have unknown foundations (Olson, 2003). Approximately 26,000 bridges are evaluated as scour critical (removal of sediment around bridge abutments is a concern) and have foundations
with unknown condition or geometry (Hossain et al., 2013). Understanding the scale of the issue further emphasizes the need to improve on inspection techniques. NDT offers a low-cost option to evaluate various types and aspects of in-situ foundations and is used in quality control and assurance today. The NDT techniques used in industry and being researched for geotechnical engineering range in physical principles but maintain the common goal of evaluating foundation safety.

1.1. Motivation of the Research

The question of interest for this research is to predict the depth of highway guardrail posts in the field. Highway guardrails are extensively used in the United States’ transportation systems. These safety structures improve motorist safety in the event of a crash by redirecting vehicles back into the roadway and reducing the likelihood of a second crash. In addition to human safety, the barriers help protect transportation structures, such as bridges, from damage (Yin et al., 2017).

In the state of Wisconsin, there are an unknown number of highway guardrail posts with unverified embedment depth. These posts are an integral part of the barrier system, transferring horizontal load from an impact, to the foundational soil. NDT techniques offer a potential method to rapidly explore post length determination for transportation agencies. Conventional methods of unknown length determination require the post to be unscrewed from the guardrail, pulled from the ground, and directly measured. This (destructive) method can be time-intensive, costly and may compromise the performance of the structures. If found plausible, NDT techniques should allow for analysis of guardrail posts under less time and with less resources than the conventional direct measurements.
1.2. Highway Guardrail Design and Post Embedment Depth

A number of different types of road barrier systems are used in the United States, which are practically considered based on the design speed of the road and the hazard offset from the road and the edge of the traveled way (Stephens, Jr., 2005). The most common guardrail design in the United States employs a strong beam connected to posts, creating a semi-rigid system (Michie et al., 1971). The typical mount of the guardrail beam is between 61 cm and 69 cm (24 in and 27 in) and the typical embedment depths of posts is between 91 cm and 122 cm (36 in and 48 in) (Michie et al., 1971). The embedment depth of the posts strongly affects the structural integrity of the guardrail. A schematic figure of a guardrail design is shown in Figure 1.1.

![Figure 1.1. Schematic diagram of a highway guardrail post. The deeper the post is embedded, the larger soil resistance available in the event of an impact or crash.](image)

Minimum distances to the edge of a slope hinge are specified to ensure sufficient soil resistance and to resist deflection (Stephens, Jr., 2005). Furthermore, the slope gradient at the guardrail installation site factors into the total length of pile required for design (WSDOT, 2019). This is illustrated in the variation of post length with respect to slope seen in Figure 1.2.
The deeper the post is embedded into the ground, the greater the available resistance force present in the soil. The available resistance force by a soil deposit at an arbitrary depth can be determined using the effective stress at a point and the horizontal stress condition. Using the effective stress term, the effective horizontal pressure ($\sigma'_{h1}$) can be calculated:

$$\sigma'_{h1} = k \sigma'_{v1} \tag{1.1}$$

where the term $k$ is the lateral earth pressure coefficient and it is dependent on the soil property (i.e., friction angle and stress history) and the horizontal strain, and is the $\sigma'_{v1}$ vertical effective stress. At rest ($k_o$), passive ($k_p$), and active ($k_a$) condition coefficients can be calculated if the friction angle ($\phi$) and over consolidation ratio ($OCR$) are known:
\[ k_o = (1 - \sin \phi)\sqrt{OCR} \]  

\[ k_p = \frac{1 + \sin \phi}{1 - \sin \phi} \quad \text{(soil at large positive strain)} \]  

\[ k_a = \frac{1 - \sin \phi}{1 + \sin \phi} \quad \text{(soil at large negative strain)} \]

Upon impact of the guardrail, the point of rotation of the post governs passive and active horizontal loading. Despite the condition at hand, the underlying principle governing the resistant force available by the post is dependent on the ability of the material to resist breaking, and the length of post embedment (Bohnhoff, 2015).

1.3. Testing Materials

The two dominant materials to be evaluated for length under NDT are steel wide flange beams and wooden prismatic posts. The specifications of a typical Wisconsin Midwest Guardrail System call for wood posts with 15 cm by 20 cm (6 in by 8 in) nominal cross-section or wide flange steel beams with 15 cm depth and 10 cm width (i.e., W 6 X 9 shape) (WisDOT, 2018). Both materials have different intrinsic properties reflecting the individual challenges of testing the differing systems.

**Wood.** Wood is a multicomponent, hygroscopic, anisotropic, heterogeneous, discontinuous, inelastic, fibrous, porous, and biodegradable material (Martin & Berger, 2001). These characteristics influence results, revealing individual traits of the sample itself, and its environment. Typical models of wood behavior under elastic theory assumes an orthotropic approach, resulting in the characterization using time-varying, heterogeneous, multiple moduli of elasticity and Poisson’s ratios (Martin & Berger, 2001; Tallavo et al., 2012). In addition, wood properties are affected by the bound and free water content of the wood and the temperature,
resulting in a moisture-temperature factor for wave velocity (Tallavo et al., 2012). The water content and the temperature inevitably effect the degree of dynamic (elastic wave) anisotropy as well. Wood guardrail posts are continuously exposed to environmental conditions (i.e., moisture content and temperature fluctuation) which promotes material deterioration.

**Steel.** Structural steel is an isotropic, homogenous, and elastic material. The mechanical properties of steel are less susceptible to environmental changes when compared to wood. Structural steels are assumed to be isotopically elastic, represented by only a single Poisson’s ratio and Young’s Modulus (Luecke et al., 2005; Steels, General Properties, 2020). The beams used for guardrail construction are of known weight (1.24 kg/m), cross-sectional dimensions (depth: 15 cm, width: 10 cm), and area (17.4 cm\(^2\)) (American Wide Flange Beams - W Beam, 2008).

The conducted research works within the constraints of wood and steel materials to more accurately predict lengths of posts in the field and in the laboratory. Due to more consistent mechanical and material properties of steel as compared to wood, it is anticipated that steel post length prediction error is less than that of wood.

**1.4. Organization of the Thesis**

Chapter 1 presents an introduction to Non-Destructive Testing and the research problem at hand. Chapter 2 discusses the principles of signal processing using time and frequency domain analysis to improve the understanding of the waveform. Chapter 3 analyzes stress propagation through a post and the accompanying natural frequencies and wave velocities. Chapter 4 analyzes the stress-wave through the post medium to assess the attenuation, coherence, and phase velocity. Chapter 5 introduces the implementation of magnetic anomaly characterization of posts with magnetic susceptibility. Chapter 6 provides the main takeaways of the thesis as a conclusion. Appendix A
includes the developed MATLAB scripts used to analyze data and build models. Subsequent appendices present the results from supplemental data analysis.
2. Discrete Signal Processing and Analysis of Waves

2.1. Discrete Signals in Geoengineering

The implementation of signal processing has led to developed techniques in the realm of *in-situ* soil and foundation analysis. Dynamic soil parameters may be characterized using stress-waves, signal receivers, and a data collection system to predict ground movement during seismic events (Campanella & Stewart, 1991). Imaging of the subsurface may be carried out with Ground Penetrating Radar (GPR) and the reflected waves are collected at a sampling rate dependent on the depth of survey desired, and processed into an image (Yelf, 2007; Robinson et al., 2013). Additional forms of subsurface imaging may be completed with seismic reflection and refraction (Telford et al., 1998), spectral analysis of surface waves (SASW - Olson, 2003), and/or a magnetometer (Reynolds, 2011). These techniques all rely on discrete sampling to collect data.

In this research, the response of stress propagation through a medium is a signal with wave characteristics. The received signal is a function of several variables such as the input signal, the mechanical material properties, the transfer function, and boundary conditions. The recorded signal can then be processed to capture important characteristics, such as travel time, natural frequency, phase, coherence, attenuation, etc.

2.2. Time and Frequency Domain Analyses

The analysis of the waveforms takes place in both the time and frequency domains. The signal is collected digitally in the time domain with discrete sampling interval $\Delta t$ over a period $n$ leading to a total waveform of length $n\Delta t$. The waveform may be converted into the frequency domain by
using the Fourier Transform, which represents the signal as a summation of sine and cosine functions with increasing frequency.

### 2.2.1. Stacking

Stacking is a technique used to improve the Signal-to-Noise Ratio (SNR) of the collected signals. In this technique, the data from several impulses of the same system are averaged to reduce the incoherent noise. The assumption is that noise is not coherent with respect to the input signal, therefore averaging multiple signals reduces the noise level in the records. From Gaussian statistics principles, the SNR of the signal increases with $\sqrt{M}$ where $M$ is the number of stacked signals (Santamarina & Fratta, 2005). Depending on the quality of the input signal, the signals generated are not identical, and this reduces the effectiveness of stacking as a noise reduction tool. Stacking emphasizes the dominant pulses while removing variations caused by inevitably differing impulses.

### 2.2.2. Truncation and Windowing

Through truncation and windowing, a signal may be broken up into discretized cyclical responses (i.e. when a pulse is seen returning to the top of post under stress-wave propagation analysis). Upon proper implementation, analysis can be completed regarding individual windows power spectra density, attenuation of the reflections, phase velocity, and reflection coherence. This type of windowing is typically completed in the time domain. Various methods exist to create a window for the signal (such as the Hanning and Hamming windows) (Santamarina & Fratta, 2005). Implementation of signal windowing in this research utilized a sinusoidal curve from “valley to peak” to adjust the window edges to converge to zero. With individual windowed reflections
generated, additional signal processing techniques such as coherence and phase analysis may be carried out.

2.2.3. Discrete Fourier Transform

Data collected by the system is recorded digitally in time. As previously stated, this signal may be transformed into the frequency domain by expressing the waveform as a summation of sine and cosine waves. The Fourier transform of a discrete signal can be defined as:

\[ X(\omega) = \sum_{n=0}^{N-1} x(n\Delta t) e^{-j(\omega n\Delta t)} \]  

(2.1)

where \( N \) is the number of samples in the recorded signal, \( x(n\Delta t) \) is the function of the signal, \( j \) is the imaginary number \( \pm \sqrt{-1} \), and \( \omega \) is the angular frequency. Upon transforming the signal, sine and cosine components for respective frequencies are determined. Utilizing the complex number notation:

\[ X(\omega) = \text{Re}(X(\omega)) + j \text{Im}(X(\omega)) \]  

(2.2)

where \( \text{Re}(X(\omega)) \) represents the real (cosine) component and \( j \text{Im}(X(\omega)) \) represents the imaginary (sine) component of the signal (Santamarina & Fratta, 2005). The complex conjugate can be determined as:

\[ X(\omega)^* = \text{Re}(X(\omega)) - j \text{Im}(X(\omega)) \]  

(2.3)

The amplitude of the system reflects the spectrum of waves contained in the signal with respect to frequency:

\[ amp = |X(\omega)| = \sqrt{[\text{Re}(X(\omega))]^2 + [\text{Im}(X(\omega))]^2} \]  

(2.4)
The phase of the system reflects the phase relative to the start of the time domain signal with respect to frequency. For this reason, a sine wave shows a phase of -90° at the sine wave frequency and a cosine wave shows a phase of 0° at the cosine wave frequency (Cerna & Harvey, 2000). The phase is calculated in radians as:

$$\varphi(\omega) = \tan^{-1}\left(\frac{\text{Im}(X(\omega))}{\text{Re}(X(\omega))}\right)$$

To synthesize the frequency signal, the inverse Fourier Transform is computed to transform the signal from the frequency domain back to the time domain:

$$x(n\Delta t) = \frac{1}{N} \sum_{\omega=0}^{N-1} X(\omega) e^{j(\omega n\Delta t)}$$

2.2.4. Power Spectral Density

The power spectral density (also known as autospectral density) represents the energy density of the signal as a function of frequency combining both the real and imaginary components. The formulation is similar to amplitude, with minor variation:

$$|X(\omega)|^2 = \frac{\text{abs}(X(\omega))^2}{N}$$

where $N$ is the number of points in the discrete signal.

2.2.5. Impulse Response and Frequency Response

Assuming a linear time-invariant (LTI) system, the impulse response $h(t)$ is the signal generated by the impulse with input signal $x(t)$ in the time domain. The output signal $y(t)$ is the convolution of the impulse response and the input signal:

$$y(n\Delta t) = \sum_{k=0}^{N-1} x(k\Delta t) h(n\Delta t - k\Delta t)$$
For the same LTI system, the frequency response $H(\omega)$ relates the input $X(\omega)$ and the output $Y(\omega)$ signals in the frequency domain. The output signal $Y(\omega)$ is the multiplication of the frequency response and the input:

$$Y(\omega) = H(\omega) \cdot X(\omega) \quad (2.9)$$

The impulse response and the frequency response effectively accomplish the same transformation in differing domains, demonstrating that the frequency response is the discrete Fourier Transform of the impulse response (Santamarina & Fratta, 2005).

### 2.2.6. Cross Correlation and Autocorrelation Functions

Analysis of signal similarity can be completed in the time domain or the frequency domain with correlation. Comparable signals (or transfer signals) are analyzed against one another through the cross-correlation function, and the same signal can be analyzed against itself through the autocorrelation function. This data processing technique allows for pattern recognition and provides a valuable tool for reflection identification. The correlation function $C_{xy}$ of two signals:

$$C_{xy}(k\Delta t) = \sum_{n=0}^{N-1} x(n\Delta t)y(n\Delta t + k\Delta t) \quad (2.10)$$

The discrete time cross correlation can be applied to two differing signals, or the same signal (which is the autocorrelation function). This process allows for interpretation of dominant repetition in signals patterns (Fratta, 1995).

In the frequency domain, the cross-power spectrum $G_{XY}(\omega)$ and the auto-power spectrum $G_{XX}(\omega)$ are determined as:

$$G_{XY}(\omega) = X(\omega)Y(\omega)^* \quad (2.11)$$

$$G_{XX}(\omega) = X(\omega)X(\omega)^* = |X(\omega)|^2 \quad (2.12)$$
The multiplication of frequency domain signals corresponds to the convolution of time domain signals. Therefore, cross correlation or autocorrelation may be performed in the time domain, however frequency domain analysis proves simple after determination of the spectrums. The cross correlation $C_{xy}(\tau)$ and autocorrelation $C_{xx}(\tau)$ functions are:

$$C_{xy}(\tau) = IFFT\left(G_{XY}(\omega)\right)$$ (2.13)

$$C_{xx}(\tau) = IFFT\left(G_{XX}(\omega)\right)$$ (2.14)

2.2.7. Padding

Increasing the duration of a signal improves the frequency resolution during spectral analysis due to $\Delta f = \frac{1}{N \Delta t}$. Therefore, simple padding measures of increasing the number of zeros at the end of the signal (zero padding), increases the frequency resolution in the frequency domain (Santamarina & Fratta, 2005). This technique is applied in testing low frequency impulses with the goal of reducing the difference between cross-correlation determined frequency and Fourier Transform frequency.

2.2.8. Phase Velocity

The phase velocity is determined by assessing the speed of varying frequency waves through the medium. The signal is windowed, converted into the frequency domain, and the frequency response $H$ is taken:

$$H(\omega) = \frac{R_{Y+1}(\omega)}{R_X(\omega)}$$ (2.15)

where $R(\omega)$ is the Fourier transform of the reflected signal with subscripts referring to different reflected signals. The phase $\varphi(\omega)$ is then calculated as described by Equation 2.5 and is
unwrapped to remove the bounds of $-\pi$ to $\pi$. The phase $P$ of the natural frequency pulse is then determined by:

$$P(\omega) = 2\pi \omega t_o$$

(2.16)

where $t_o$ is the time required for the longitudinal wave to travel to and from the reflection boundary. The value difference at the central (strongest) frequency between the calculated phase and the phase of the pulse is recorded as $dif$. The phase difference $\Delta \varphi$ and the phase velocity $V_{ph}$ are:

$$\Delta \varphi(\omega) = \varphi(\omega) - P(\omega) - dif$$

(2.17)

$$V_{ph}(\omega) = \frac{2L}{t_o + [\Delta \varphi(\omega)/2\pi \omega]}$$

(2.18)

2.2.9. Attenuation

As stress-waves propagate through a medium, the amplitude of the wave energy decays over time and with distance. The propagation of the elastic wave material converts part of the energy in the signal to heat. Part of the amplitude may also be lost at the material boundaries where energy is leaked from the system. The dissipation of energy from the wave is reflected in the rate at which the amplitude decreases. For spherical spreading (Green’s Theorem):

$$\frac{A(r_2)}{A(r_1)} = (r_1/r_2)^2$$

(2.19)

where $A(r)$ is the amplitude at distance $r$ and subscripts refer to the location. The amplitude and distance must be measured at two separate locations. For cylindrical spreading (Green’s Theorem):

$$\frac{A(r_2)}{A(r_1)} = \frac{r_1}{r_2}$$

(2.20)
2.2.10. Coherence

To assess the noise associated with individual reflections, the coherence of the signal is computed to determine the workable range of usable frequencies during signal analysis when compared across two windowed reflections for the number of signals tested:

\[
\gamma_{XY}^2(\omega) = \frac{|G_{XY}(\omega)|^2}{G_{XX}(\omega)G_{YY}(\omega)} = \frac{G_{YY}(\omega)G_{YY}(\omega)^*}{G_{XX}(\omega)G_{YY}(\omega)} \tag{2.21}
\]

in which the bar over the function indicates an average regarding the number of signals tested, and the subscripts \(X\) and \(Y\) refer to the windowed reflection signals. The coherence function quantifies the energy present in the signal output that was caused by the signal input (Santamarina & Fratta, 2005). When the coherence is equal to a value of one, all energy caused by the output is a direct result of the input. Coherence values less than one indicate potential output noise, system inputs which are unaccounted for, nonlinear behavior of the system, and/or lack of frequency resolution and leakage (Santamarina & Fratta, 2005). The SNR may be calculated using the coherence to assess the strength of the signal at specific frequencies:

\[
SNR(\omega) = \frac{\gamma_{XY}^2(\omega)}{1 - \gamma_{XY}^2(\omega)} \tag{2.22}
\]

where SNR may range from zero to infinity, and the larger the value, the stronger the signal (Fratta, 1995). The stronger the signal-to-noise ratio, the better the system coherence across inputs and outputs, resulting in a simpler waveform for analysis, emphasizing the importance of reproducible signals.
2.2.11. The Hilbert Transform, Analytical Signal, and Instantaneous Parameters

Assessment of instantaneous amplitude, phase, and frequency at location \( n \) of the signal may be determined through the Hilbert transform and the analytical signal. The Hilbert transform \( x^{(ht)} \) is orthogonal (perpendicular) to an original signal \( x \) such that:

\[
\sum_{n=1}^{N} x^{(ht)}(n)x(n) = 0
\]  
(2.23)

The Hilbert transform \( x^{(ht)} \) of a frequency domain signal \( X_u \) is determined as:

\[
X_u^{(ht)} = \begin{cases} 
-j \cdot X_u & 0 \leq u \leq N/2 \\
-j \cdot X_u & N/2 \leq u \leq N - 1 
\end{cases}
\]

\[
x^{(ht)} = \text{IFFT}(X^{(ht)})
\]  
(2.24)

where array \( X^{(ht)} \) is the Hilbert transform in the frequency domain (Santamarina & Fratta, 2005). The analytical signal can be determined by combining the signal as the real component and the Hilbert transform as the imaginary component.

\[
x^{(A)} = x + jx^{(ht)}
\]  
(2.25)

A simpler method to determine the analytical signal is transformation of the processed frequency domain to the time domain through the Inverse Discrete Fourier Transform (IDFT):

\[
x^{(A)} = \text{IFFT}(X)
\]  
(2.26)

where \( X \) is the Discrete Fourier Transform (DFT) of the original signal with a single-side array. Determination of the analytical signal allows for evaluation of instantaneous parameters within the signal. These parameters reveal dominating characteristics of amplitude and phase with respect to time, not frequency, which differentiate the instantaneous parameters from the non-
instantaneous parameters. Correct processing of the signal to the analytical signal must first be performed.

The instantaneous amplitude $amp_i$, the instantaneous phase $\varphi_i$, and the instantaneous frequency $\omega_i$ are:

\[
amp_i = \sqrt{\text{Re}(x_i^{(A)})^2 + \text{Im}(x_i^{(A)})^2} \quad (2.27)
\]

\[
\varphi_i = \tan^{-1}\left[\frac{\text{Im}(x_i^{(A)})}{\text{Re}(x_i^{(A)})}\right] \quad (2.28)
\]

\[
\omega_i = \frac{\varphi_i - \varphi_{i+1}}{\Delta t} \quad (2.29)
\]
3. Pulse Echo and Wave Velocity

A portion of the research to determine unknown foundation depth is focused on the Pulse Echo (PE) method and estimating the velocity of the introduced wave. This approach estimates the time taken for a stress-strain wave to travel through the length of the post. The wave is reflected at the post base and returned to the top of the post. Depending on the energy introduced into the system, the number of detected reflections varies, however the time between consecutive reflections informs the natural frequency of the system (Davis & Dunn, 1974; Finno et al., 1997). The wave velocity is assessed by estimating arrival times of waves as they travel between two specific points along the system. This method is explored as a practical and cost-effective way to predict length for both wooden and steel posts in the field.

3.1. Stress-Strain Waves

An impulse creates a stress-wave with multiple propagation modes. The propagation is dependent on the method of force introduction, boundary conditions, and material properties. In an infinite media, the only waves possible are P- and S-waves. Particle motion along the P-wave is in the propagation direction, whereas in the S-wave it is perpendicular to the propagation direction (Kolsky, 1963). A bounded, elastic media (such as a thin rod or bar) experiences longitudinal, torsional and lateral vibrations (Kolsky, 1963). The difference between P-waves and longitudinal waves is the medium of propagation (P-wave in infinite media and longitudinal in bounded media). These bounded waveforms inform the results obtained from data collection.

The velocities for P-wave (infinite space), longitudinal wave (bounded medium – post), S-wave, and Rayleigh wave in isotropic materials are \((V_P, V_L, V_S, V_R)\):
\[ V_p = \sqrt{\frac{M}{\rho}} = V_L \sqrt{\frac{1-\nu}{(1+\nu)(1-2\nu)}} \]  
\[ V_L = \sqrt{\frac{E}{\rho}} \quad \text{(for infinitely large wavelength/radius ratio)} \]  
\[ V_S = \sqrt{\frac{G}{\rho}} = V_L \sqrt{\frac{1}{2(1+\nu)}} \]  
\[ V_R = \frac{\kappa_S}{\kappa_R} V_S = \frac{\kappa_S}{\kappa_R} V_L \sqrt{\frac{1}{2(1+\nu)}} \]

where \( M \) is the constrain modulus, \( \rho \) is the material density, \( \nu \) is the material Poisson’s ratio, \( E \) is the Young’s modulus, \( G \) is the shear modulus, \( \kappa_S = \frac{\omega}{V_S} \), \( \kappa_R = \frac{\omega}{V_R} \), and \( \omega \) is the angular frequency (rad/s) of the wave. Maintaining a constant testing direction, measured values of \( V_S \) and \( V_L \) in a rod can be used to find Poisson’s ratio:

\[ \nu = \frac{1}{2} \left[ \frac{V_L}{V_S} \right]^2 - 1 \]

which then allow for computation of the P-wave. It is possible to measure various combinations of wave velocities to extract materials properties (i.e. determining the P and longitudinal wave velocities allow for computation of Poisson’s Ratio and the S-wave velocity).

Dispersive materials exhibit both phase and group velocities that are dependent on frequency, whereas non-dispersive materials display constant velocity with respect to frequency (Telford et al., 1998). An example of phase and group behavior is seen in Figure 3.1.
Mathematically, phase velocity $V_{ph}$ and group velocity $V_g$ are represented as follows:

$$V_{ph} = \frac{\omega}{\kappa} = 2\pi \frac{f}{\kappa} \quad (3.6)$$

$$V_g = \frac{d\omega}{d\kappa} = 2\pi \frac{df}{d\kappa} \quad (3.7)$$

where $\kappa$ is the wave number (1/m). Analysis of the phase and group velocity provide information regarding material parameters, particularly dispersity. The approximation expression for the phase velocity $V_{ph}$ is dependent on the shape of the bounded media. For a cylindrical rod, the Rayleigh derived equation is as follows:

$$V_{ph} = V_{L} \left[ 1 - v^2 \pi^2 \left( \frac{r}{\lambda} \right)^2 \right] \quad (3.8)$$

where $r$ is the radius of the cylinder, and $\lambda = \frac{V_{L}}{f}$ is the wavelength (Kolsky, 1963).
3.2. **Pulse-Echo (PE) Testing**

The Pulse-Echo (PE) method is a surface reflection NDT technique reliant on stress-wave propagation through a medium and reflected waves from interfaces with impedance mismatch. The PE method assumes that the material is elastic and non-dispersive (i.e., speed of wave propagation is independent of frequency), requiring the wavelength of the pulse to be small with respect to the diameter of the foundation (Tu et al., 1955; Finno et al., 1997). The PE method in piles and posts assumes longitudinal wave velocity through a non-infinite media with minimal reliance on torsional or flexural waves. This is predominantly attributed to longitudinal waves being the simplest to analyze. Flexural waves are known to be unfavorable for analysis due to their highly dispersive nature (Yu, 2019). Torsional waves are not dispersive but, correctly determining the speed of torsional waves is difficult as longitudinal waves arrive first (Timoshenko et al., 1974). Therefore, if longitudinal analysis is possible (i.e. the top of the foundation is exposed), this method is preferred to torsional or flexural wave analysis.

However, it is also important to consider that by choosing the method of longitudinal analysis, this does not remove the effects of torsional and bending excitation on the system. The inevitable presence of the multiple modes of vibration distort the longitudinal wave and limit the ability to accurately assess the natural frequency of the system. Additionally, material imperfections (such as damage, warping, and cracking which are present in wood) resulting in heterogeneity and anisotropic behavior limit the application of the longitudinal wave.

Energy leakage of the signal occurs during propagation. As the wave travels through the medium it is reflected and transmitted at boundaries and dissipates through the surrounding soil. This results in dispersion and distortion of the signal. The presence of heterogeneities in a material (such as
damage, warping, or cracking in wood) also impacts the dispersion and distortion. This signal disruption reduces the ability to analyze the wave propagation correctly.

The crossover effects of torsion and bending excitation, signal dispersion and distortion, and heterogeneities collectively affect the propagation of the longitudinal wave. Furthermore, the presence of these multiple disruptions together makes it difficult to separate how each individually impacts the signal.

To test deep foundation systems, a high frequency signal receiver (accelerometer, geophone, etc.) is secured to the top of a post and an impulse is applied to the top of the post (often with a plastic hammer to minimize high frequencies generated by steel) to generate a stress-strain wave through the pile (Rausche et al., 1994; Paikowsky & Chernauskas, 2003). The longitudinal wave travels through the medium and is reflected at impedance changes, such as the boundary between the pile and the soil. The signal receiver at the top of the pile records the initial impact and additional wave reflections.

The PE method is commonly used to evaluate the structural integrity of drilled shafts, cast-in-place, and driven concrete and wooden piles (Rausche et al., 2002) and is particularly useful when the wave velocity of the material and the pile length are known. Under these conditions, NDT techniques allow for analysis of pile defects by locating impedance changes other than the boundary between pile tip and soil/rock. Waves are reflected if there are defects in the post medium, such as air pockets in cast-in-place concrete foundations. The signal receiver at the top of the pile records the initial impact and reflections at the defect and pile-soil boundary. Figure 3.2 shows the idealized effect of the PE method when a defect is present. Traditionally, the PE
The governing principle related to wave reflection is acoustic impedance. Impedance $Z$ as it relates to a pile or drilled shaft can be defined as:

$$Z = \frac{EA}{V} = A\sqrt{E\rho} = AV\rho$$  \hspace{1cm} (3.9)

where $E$ is Young’s modulus, $A$ is the cross-sectional area of the foundation, $V$ is the longitudinal wave speed, and $\rho$ is the material density (Rausche et al., 1994). This equation highlights the importance of material properties when conducting stress-wave analysis. If there is no density, longitudinal wave speed, or area difference between materials, there is no wave reflection. The
reflection coefficient \( R \) determines the amount of energy/amplitude in the reflected signal. For media with a normal incident wave, this term is defined as:

\[
R = \frac{A_2 \rho_2 V_2 - A_1 \rho_1 V_1}{A_2 \rho_2 V_2 + A_1 \rho_1 V_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1}
\]  

(3.10)

where the subscript numbering refers to the boundary layers, with the wave traveling through layer 1 and being reflected at the interface between layer 1 and layer 2.

3.3. Description of Testing Techniques

In pursuing this research both laboratory and field testing was carried out. Laboratory and field testing relied on the use of wooden and steel posts.

Testing Setup and Electronics. To gather information, four accelerometers were attached to the post to collect waveform data of the impulse in combination with a power supply and signal gain, oscilloscope, and personal computer. A diagram of the lab testing set up can be seen in Figure 3.3, with an example of the set up shown in Figure 3.4.

![Figure 3.3. Schematic diagram of the data collection system. Accelerometers are connected to the amplifier with electrical wiring (blue), and the amplifier is connected to the oscilloscope with male to male assembly cables (black).](image-url)
Figure 3.4. Example set up of post testing in laboratory setting. Low frequency pulses created by dropping a steel ball from a known height attached to a string creating a pendulum. Accelerometers convert the mechanical waveform to an electrical signal with the help of the amplifier. The oscilloscope and the computer are used in tandem to visualize the collected data and adjust the systems data collection.

When testing wood, steel nails must be inserted into the post so that the accelerometers may be attached. ChrisNik Stainless Steel MAG Nail (3/4 in. length by 1/8 in. diameter) was selected for the accelerometer attachment in the wood posts. PCB/ICP Model 353B16, high frequency (1 to 10000 Hz) quartz shear accelerometer was selected to sense the wave propagation. Accelerometers are attached to magnets which are then connected to the magnetic nails or directly to steel posts. PCB/ICP Model 482A2 eight-channel charger amplifier was used to power and amplify the signal from the accelerometers. Rigol DS1054Z, a multifunctional and high-performance oscilloscope was selected to read and save the accelerometer signals. Hereafter, the configuration of accelerometers, amplifier, oscilloscope, computer, and cables shall be referred to as the data collection system.

Data processing is required to interpret and analyze the waveforms collected by the instruments. Computer scripts were written to estimate the wave velocity of the impulse, and the time (or
frequency) of the pulse as it travels through the medium, is reflected at the base, and returned to the top of the post. Data collected in the lab and in the field was conducted in a similar manner; however, it is important to differentiate between the collection methods and tested materials.

### 3.3.1. Field Testing

Testing posts in the field was completed in a similar manner to that in the laboratory. Several methods of pulse introduction were tested including low frequency hammer impulses, low frequency steel ball drop impulses, and high frequency pencil lead flexure and break. A 12V battery supplied power to the electronic data collection system. Field testing was completed on wooden and steel posts throughout the Wisconsin region. Four field tests were completed in 2019, with testing summarized in Table 3.1 and supplementary data presented in Appendix B.

<table>
<thead>
<tr>
<th>City</th>
<th>Location</th>
<th>Date</th>
<th>Wood Posts</th>
<th>Steel Posts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pewaukee</td>
<td>County Highway F</td>
<td>06/06/2019</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Madison</td>
<td>Forest Products Lab</td>
<td>07/07/2019</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>County Highway F</td>
<td>07/12/2019</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>County Highway F</td>
<td>10/04/2019</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: Additional tests were completed prior to the author’s involvement in the research.

When testing the posts in field conditions, the surrounding soil impacts the results. Any impulse leads to substantial energy damping due to the soil, when compared to testing posts with an air boundary (Davis & Dunn, 1974). Additionally, the denser the soil and/or the longer the pile, the larger the expected attenuation (Davis & Dunn, 1974).

Additional field tests were completed prior to the author’s involvement in the research. These tests were completed utilizing low frequency hammer strikes to determine natural frequency and high frequency pencil lead flexure breaks to determine wave velocity. Data was analyzed under a
different processing technique for natural frequency, only considering the power spectra density. The results of the additional tests are presented in Appendix C.

3.3.2. Laboratory Testing

Laboratory testing was performed on several posts of varying length, cross sectional area, and defects. It is assumed that all posts are the same type of wood. A summary of the properties of the several wood posts tested in the lab are shown in Table 3.2.

Table 3.2. Summary of wooden posts tested in laboratory experiments

<table>
<thead>
<tr>
<th>Post Length</th>
<th>Cross-Sectional Dimensions</th>
<th>Cracking</th>
<th>Warping</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.216 m</td>
<td>8.9 cm by 14.0 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.221 m</td>
<td>8.9 cm by 13.8 cm</td>
<td>Minor</td>
<td>-</td>
</tr>
<tr>
<td>1.236 m</td>
<td>13.7 cm by 13.8 cm</td>
<td>Major</td>
<td>-</td>
</tr>
<tr>
<td>1.832 m</td>
<td>8.6 cm by 14.0 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.834 m</td>
<td>8.6 cm by 13.4 cm</td>
<td>Minor</td>
<td>-</td>
</tr>
<tr>
<td>2.423 m</td>
<td>8.9 cm by 13.8 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.435 m</td>
<td>8.3 cm by 8.6 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.442 m</td>
<td>13.7 cm by 14.6 cm</td>
<td>Major</td>
<td>-</td>
</tr>
<tr>
<td>2.444 m</td>
<td>8.7 cm by 13.7 cm</td>
<td>Some</td>
<td>Torsional</td>
</tr>
<tr>
<td>3.040 m</td>
<td>8.7 cm by 13.4 cm</td>
<td>-</td>
<td>Bending</td>
</tr>
<tr>
<td>3.065 m</td>
<td>8.7 cm by 8.9 cm</td>
<td>-</td>
<td>Torsional</td>
</tr>
</tbody>
</table>

PE testing was performed using a multitude of sources including low frequency hammer blows, low frequency pendulum impulses, and high frequency pencil lead flexure and break. Preliminary testing programs exposed wood posts to dry, wet, and freezing conditions. The results of this analysis are presented in Appendix D. This research was performed to demonstrate the impact moisture content has on wave velocity (Wacker, 2010; Tallavo et al., 2012). Laboratory testing presented exposed posts to free (no clamp), rubber fixed (rubber clamp), and fixed (clamp) conditions. These measures were taken to simulate the coupling of posts with the guardrail and soil in the field. Posts in the lab were tested using a reproducible steel ball pendulum impulse.
**No clamp.** A wood post was placed on a lab bench with free top (where the impulse is applied) and bottom (where the wave is reflected) boundaries in a loose state. The post is not fixed to the table. If the post length $L_{post}$ exceeds 1.5 m, the post is placed so the distance from the last sensor to the edge of the lab bench ($\Delta L$) is 0.1 m, as shown in Figure 3.5. This system matches the expected condition of wooden posts in the field in which the height of the post above ground is approximately 70 cm.

If the post length $L_{post}$ is less than 1.5 m, the post is placed so that the middle sensor is aligned with the edge of the lab bench. The distance from the last sensor to the edge of the table ($\Delta L'$) is 0.25 m, as shown in Figure 3.6.
Under the no clamp condition, results are dependent on the impulse introduction method and the individual characteristics of the post. There will be minor coupling of the post with the lab bench. This testing setup does not match field conditions closely, in which posts are coupled with the guardrail and the soil.

**Rubber clamp.** Two 3 mm thick rubber sheets were placed on the lab bench, creating a 6 mm layer. A wood post was placed on the rubber with free top and bottom boundaries. The post is placed so the distance from the last sensor to the edge of the lab bench ($\Delta L$) is 0.1 m, as shown in Figure 3.7.

![Figure 3.7. Schematic of rubber clamp testing setup ($\Delta L = 0.10$ m). Bottom rubber boundary separates post from table. Top rubber boundary separates clamp from post. Three clamps are placed equidistant along the length of post section resting on the rubber/table. The first clamp is placed at the start of the lab bench, and the last clamp is placed at the end of the wood post. The middle clamp is placed between the first and last clamp. Reproducible steel ball pendulum used to provide impulse. Based on this configuration, $L_{\text{cantilever}} = 70$ cm.](image)

Three 6 mm rubber boundaries are placed equidistant from one another along the top of the post. These boundaries provide the separation from the post and the clamps. Three clamps are used to secure the post at the rubber boundaries creating a rubber fixed condition. Under the rubber clamp condition, results are dependent on the impulse method, the individual characteristics of the post, and the coupling of the post with the rubber boundary. The rubber allows the wood to vibrate semi-freely, isolated from the lab bench, and more damping occurs compared to the no clamp condition. This testing setup more closely matches field conditions, as the post is in a semi-fixed state.
**Clamp.** A wood post was placed on the lab bench with free top and bottom boundaries. The post is placed so the distance from the last sensor to the edge of the table ($\Delta L$) is 0.1 m, as shown in **Figure 3.8.** Three clamps are located equidistant from one another and are used to secure the post creating a fixed condition.

![Figure 3.8](image)

**Figure 3.8.** Schematic of clamp testing setup ($\Delta L = 0.10$ m). Three clamps are placed equidistant along the length of post section resting on the table. The first clamp is placed at the start of the lab bench, and the last clamp is placed at the end of the wood post. The middle clamp is placed between the first and last clamp. Reproducible steel ball pendulum used to provide impulse. Based on this configuration, $L_{\text{cantilever}} = 70$ cm.

Under the clamp condition, results are dependent on the impulse method, the individual characteristics of the post, and the coupling of the post with the clamp and lab bench boundary. The wood is not allowed to vibrate freely, and more damping occurs compared to the no clamp condition or rubber clamp condition as energy leaves through the boundaries. This testing setup more closely matches field conditions, as the post is in a fixed state.

### 3.4. Methodology

To estimate post length with the PE and wave velocity technique the components determined are the velocity $V$ of the wave propagating through the medium, and the frequency $f$ of the stress-wave reflections as the signal travels from post top, to post bottom, and back to post top. If both the wave velocity and the frequency (or time $t$) can be accurately determined, the length of the post $L$ can be solved by:
\[ 2L = V \cdot t = \frac{V}{f} \]

\[ L = \frac{V}{2f} \]  

(3.11)  

(3.12)

The test must use instruments able to detect small-scale elastic vibration levels, such as accelerometers. For the purpose of testing, the stress-wave propagation is a dynamic force causing small movements and vibrations through the post. During testing, four accelerometers are secured to the post as shown in Figure 3.9.

![Figure 3.9](image)

**Figure 3.9.** Schematic diagram representing the four sensors (S1, S2, S3 and S4) positioning during testing. Typical post testing completed with \( d_1 = 10 \) cm and \( d_2 = 25 \) cm.

Sensor 1 is placed on the top of the post to determine the frequency of wave reflections. Sensors 2 – 4 are placed on the side of the post, equidistant from one another. Typical testing of wood and steel posts was completed with Sensor 2 placed 10 cm from Sensor 1. Sensors 2 – 4 were equidistant with 25 cm spacing.
3.4.1. Impulse

The first consideration of the system is the energy source introduced into the system. A provided impulse creates the stress-strain wave:

\[
\text{Impulse} = F \Delta t = mV
\]  

(3.13)

where \( F \) is equal to force, \( \Delta t \) is equal to the time in which the force acts on the object, \( m \) is the mass of the object, and \( V \) is the velocity of the object (Giancoli, 1995). This equation can be represented graphically as seen in Figure 3.10.

![Figure 3.10](image-source)

**Figure 3.10.** Force as function of time during a typical collision, representing impulse. Image sourced after Giancoli, 1995.

The waveforms developed through the sample are dependent on the impulse generated. The more force delivered over a greater time period introduces a larger impulse. A larger impulse is generated with a hammer or pendulum strike. The less force delivered over a lesser time period results in a smaller impulse. Smaller impulse is tested with pencil lead breakage.

3.4.2. Pulse Natural Frequency

The natural frequency of the pulse through a post is determined by creating an impulse on the top of the post. A single accelerometer sensor located at the top of the post records the impact and the
accompanying stress-waves of the post after impact for a specified timeframe. The initial stress-wave impulse travels through the length of the post and is reflected at the boundary between the post tip and the surrounding medium (air in the lab, soil in the field). The reflection travels back to the top of the post and is registered by the accelerometer with greater energy, when compared to prior vibrations. The introduced impulse must be large enough to overcome the natural attenuation of the signal so that reflections can be registered by the accelerometer. If there is limited attenuation of the signal energy, the signal continues to reflect from the post top-to-tip-to-top and so on. The time required for the signal to travel from impulse location to post tip and back is deemed the two-way travel time, \( \Delta t \), of the stress-wave. Utilizing this travel time, the natural frequency of the system’s stress-wave can be found by:

\[
f = \frac{1}{\Delta t}
\]  

(3.14)

The natural pulse frequency is assessed in data processing through autocorrelation and power spectra density means. Natural frequency assessment through autocorrelation is denoted as AUTO. Natural frequency assessment through power spectra density is denoted as PSD.

**3.4.3. Wave Velocity**

The speed \( V \) at which a stress-wave travels through the medium must be measured (or assumed) to determine post length. Assuming the system propagates the stress-wave linearly through the medium, with the system acting as a longitudinal waveguide, the velocity may be calculated by measuring the time an impulse takes to propagate from the source location, to a receiver some specified distance away.

**Wave Arrival.** Assessment of wave velocity arrival time is done through application of the Akaike Information Criterion (AIC), which is used to determine the order of an autoregressive model.
(Akaike, 1974). The autoregressive model is a representation of a specific type of random processes in which future data is predicted or compared to past data. The basis of this equation is as follows:

$$AIC = -2 \log(\text{maximized likelihood function}) + 2P$$  \hspace{1cm} (3.15)

where $P$ is the number of independently estimated parameters and equals autoregressive order $M$ in the model (Akaike, 1974; Sleeman & van Eck, 1999). The development of the AIC picker has led to various methods for evaluation of wave arrival times, predominantly applied to seismograms. One approach applies the AIC to the waveform signal $x(k)$ of length $N$ to assess the merging point $k$ of a two-part model (Sleeman & van Eck, 1999; Zhang et al., 2003):

$$AIC(k) = (k - M) \log(\sigma_{1,max}^2) + (N - M - k) \log(\sigma_{2,max}^2) + C_2$$  \hspace{1cm} (3.16)

where $M$ is the order of the AR process to fit the data, $C_2$ is a constant, and $\sigma_{1,max}^2$ and $\sigma_{2,max}^2$ represent the two interval variance. This method delivers a robust performance, however determination of the autoregressive process must be completed through trial and error and with coefficients delivered by the Yule-Walker equations (Haykin, 1996; Zhang et al., 2003).

For a waveform $x(k)$, the AIC picker method used to assess the onset of the wave velocity in this research does not fit the recorded data to an autoregressive process (Maeda, 1985; Zhang et al., 2003). This algorithm is robust in that it provides immediate analysis of the data itself.

$$AIC(k) = k \cdot \log\{\text{var}(x[1,k])\} + (N - k - 1) \cdot \log\{\text{var}(x[k + 1, N])\}$$  \hspace{1cm} (3.17)

where $k$ is the range of the waveform samples and $N$ is the number of samples in the window. This methodology is limited to high signal-to-noise ratio (SNR), when the arrival is evident, and a limited sample window is specified (Zhang et al., 2003).
High Frequency (HF). To test the wave velocity of a high frequency source, a small impulse is generated by breaking the tip of pencil lead on the top of the post. The pencil lead is extended from a mechanical pencil and the tip of the lead is placed near the edge of the post, closest to the accelerometers along the side. The pencil is flexed and snapped, generating a sharp impulse that travel through the post (Figure 3.11).

Figure 3.11. High frequency source testing methodology. (a) Pencil lead extended from mechanical pencil and flexed near top sensor. (b) Extreme lead flexure causes break and imparts energy into the system with small impulse through a stress-wave.

Provided that the energy generated by breaking the pencil lead is large enough to be sensed by the top Sensor 1, all sensors collect data from the signal. After time $t_1$, the stress-wave reaches Sensor 2 (located 10 cm from the post top), showing an increase in signal amplitude. After time $t_2$, the stress-wave reaches Sensor 3 (located 35 cm from the post top). And after time $t_3$, the stress-wave reaches Sensor 4 (located 60 cm from the post top). Using the travel time to each sensor, the high frequency wave velocity ($V_{HF}$) can be calculated:

$$V_{HF} \approx \frac{0.1 \text{m}}{t_1} \approx \frac{0.35 \text{m}}{t_2} \approx \frac{0.60 \text{m}}{t_3}$$

(3.18)
For individual high frequency tests, the wave velocity is determined by assuming a constant wave velocity through the system and finding the slope of the “best fit” line with the time and the distance to each sensor.

**Low Frequency (LF).** To test the wave velocity of a low frequency source, a large impulse is generating by striking the top of the post with a hammer or steel ball. A hammer is raised and should strike the post in an area with consistent structural integrity. Or a steel ball should strike the post in a similar manner with a pendulum or known drop distance with guiding mechanism such as a tuber. If the material is wood, the strike location should ideally be an area with limited rotting or degradation. The strike generates a stress-wave impulse through the post (Figure 3.12). The impact of the hammer has high contact time with the post and a large force imparted, generating a larger impulse, when compared with the pencil lead breakage.

![Figure 3.12](image)

**Figure 3.12.** Low frequency source testing methodology. (a) A strike location is chosen for the hammer in an area with limited material degradation. (b) Hammer impact imparts energy with large impulse into the system through a stress-wave.

Sensors 2 through 4 capture travel time data in the same manner as the high frequency method. For individual low frequency tests, the wave velocity is determined by assuming a continuous and
constant wave velocity through the system. The velocity is estimated through linear regression of a constant speed with respect to arrival time and distance.

3.5. **Length Prediction Results**

The results revealed valuable information regarding Pulse Echo (PE) and wave velocity determination as a field-ready technique to evaluate the length of posts in the field. Field testing results are provided to discuss the immediate applicability of the technique. Lab testing results are provided to demonstrate length estimations and limitations under controlled conditions.

3.5.1. **Field Testing**

**Typical Results.** Graphical outputs from testing a 1.83-m long wide flange steel post are shown in Figures 3.13 to 3.17. The post is tested with low frequency hammer blow impulses while attached to guardrail to estimate natural frequency and the wave velocity. Individual waveform traces are presented in Figure 3.13 to demonstrate the effect of the hammer blow strikes and lack of self-similarity depending on the impulse applied. By not introducing reproducible signals in the field, this negatively effects the prediction of the natural frequency. The application of stacking the signal is diminished and much more variability is introduced into the analysis. Figure 3.14 shows the stacked results of the 10 normalized signals collected from the top receiver. Figure 3.15 shows the autocorrelation (AUTO) assessment of the natural frequency in the time domain. Figure 3.16 shows the power spectra density (PSD) assessment of the natural frequency in the frequency domain. The waveforms collected from Sensors 1 to 4 and the AIC picker results to determine the arrival of the waves at Sensors 2 to 4 are shown in Figure 3.17. Travel time points are used to calculate the wave velocity for the post length determination.
Figure 3.13. Top receiver signals from 10 impulses. Impulses generated from low frequency reproducible pendulum steel ball strikes.

Figure 3.14. Stacked top receiver signals from 10 impulses. Impulses generated from low frequency reproducible pendulum steel ball strikes.
Figure 3.15. Autocorrelation of stacked top receiver signal. Results from maximum amplitudes denote signal repetition. Average distance between peaks denoted by blue triangles and black circles determines travel time, with the inverse of travel time being the natural frequency as shown: 1399 Hz. Intermediate peaks are disregarded as they are reflective of higher energy modes of vibration.

Figure 3.16. Power spectra density (PSD) of stacked top receiver signal. Maximum values (peaks) denote dominant frequencies. Peak with frequency closest to that determined through autocorrelation selected as natural frequency, as shown: 1417 Hz. The max energy peak is not selected as it is the second mode of vibration, and not reflective of the time it takes the impulse to travel from post top to tip and back to top.
Figure 3.17. Results for wave velocity picker from one impulse using Akaike Information Criterion (AIC) picker method. Picker is used to find the wave velocity of ten impulses, with the median wave velocity used to determine the length. **TOP:** Waveforms from Sensor 1 (black), Sensor 2 (blue), Sensor 3 (red), and Sensor 4 (green). Wave arrival denoted by black circle which is determined AIC results. Wave velocity is equal to the slope of the wave arrivals: 5258 m/s. **BOTTOM:** Results from AIC picker of Sensor 2 (blue), Sensor 3 (red), and Sensor 4 (green). Wave arrival times are shown by minimum values.

Data processing is used to compute the natural frequency through autocorrelation or power spectra density, and the median wave velocity is taken from ten impulses in which wave arrivals are found using the Akaike Information Criterion (AIC) picker method. The length of the post may then be solved using **Equation 3.12.** Results from this test showed an autocorrelation natural frequency of 1399 Hz, a power spectra density natural frequency of 1417 Hz, and a median wave velocity of 5258 m/s. The autocorrelation method predicted a length of 1.88 m (2.6% error) and the power spectra density method predicted a length of 1.86 m (1.3% error).
Real and Predicted Length. The results of post length determination in the field are summarized in Figures 3.18 to 3.21. Tested posts were of known geometry, with values of length provided by WisDOT. All wood posts tested in the field showed nominal cross-sectional dimensions of 15 cm by 20 cm (6 in by 8 in). The approximate Root Mean Square Error (RMSE) in length prediction of wood posts is 0.8 m as shown by Figures 3.18 to 3.19.

![Figure 3.18](image)

Figure 3.18. Real post length compared to predicted post length for wood posts under high frequency (HF) pencil flexure waves. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.80 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.84 m.
Figure 3.19. Real post length compared to predicted post length for wood posts under low frequency (LF) hammer blow waves. *LEFT:* Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.77 m. *RIGHT:* Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.83 m.

Low frequency wave velocity determination improves the post length determination as indicated by the RMSE calculations. The natural frequency calculated with autocorrelation function improved the results over the power spectra density natural frequency determination. Still, all methods tested tend to overpredict the length of wood posts.

All steel posts tested in the field were standard wide flange steel beams, 15 cm depth and 10 cm width (i.e. W 6 X 9). The approximate RMSE in length prediction of steel posts is 0.3 m as shown by Figures 3.20 to 3.21.
Figure 3.20. Real post length compared to predicted post length for steel posts under high frequency (HF) pencil flexure waves. **LEFT**: Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.29 m. **RIGHT**: Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.31 m.

When compared to results for wood posts, it is shown that steel posts have a smaller RMSE (~0.5 m lower) indicating a higher degree of post length determination. However, all tested steel posts
have only one length, biasing the results. Low frequency wave velocity determination lowered the RMSE compared to the high frequency wave velocity determination. Natural frequencies calculated with the autocorrelation function yield lower RMSE when compared to the power spectra density results. All methods tested tend to underpredict the length of steel posts. Both wood and steel posts showed the lowest RMSE when using low frequency hammer blow waves and autocorrelation natural frequency determination.

3.5.2. Laboratory Testing

Laboratory testing was completed on several wooden posts to assess the PE/wave velocity determination methodology and attempt to model the effect of coupling conditions on the length prediction results. Laboratory analysis may be used to explain the limitations of the data interpretation from field results. Within the lab, post conditions can be manipulated and controlled to simulate the field conditions.

**Typical Results.** Graphical outputs from testing a 2.435-m long wood post (8.3 cm by 8.6 cm) are shown in Figures 3.22 to 3.26. The post is tested with low frequency reproducible steel ball impulses under rubber clamp conditions to determine natural frequency, wave velocity and thus, the length. Individual waveform traces are presented in Figure 3.22 to demonstrate the effect of the reproducible steel ball pendulum impulses. By introducing reproducible signals in the lab, the ability to assess the natural frequency is improved. The application of stacking the signal is amplified and the signal-to-noise ratio improves. Figure 3.23 shows the stacked results of the 10 signals collected from the top receiver. The autocorrelation (AUTO) assessment of the signal recorded at the top sensor to determine natural frequency is shown in Figure 3.24. The power spectra density (PSD) assessment of the signal recorded at the top sensor to determine natural
frequency is shown in Figure 3.25. The waveform traces from Sensors 1 to 4 and the AIC picker results to determine the arrival of the waves at Sensors 2 to 4 are shown in Figure 3.26.

![Figure 3.22](image1.png)

**Figure 3.22.** Top receiver signals from 10 impulses. Impulses generated by low frequency reproducible pendulum steel ball strikes.

![Figure 3.23](image2.png)

**Figure 3.23.** Stacked top receiver signals from 10 impulses. Impulses generated from low frequency reproducible pendulum steel ball strikes.
Figure 3.24. Autocorrelation of stacked top receiver signal. Results from maximum amplitudes denote signal repetition. Average distance between peaks denoted by blue triangles and black circles determines travel time, with the inverse of travel time being the natural frequency as shown: 993 Hz.

Figure 3.25. Power spectra density (PSD) of stacked top receiver signal. Maximum values (peaks) denote dominant frequencies. Peak with frequency closest to that determined through autocorrelation selected as natural frequency, as shown: 1083 Hz.
Results for wave velocity picker from one impulse using Akaike Information Criterion (AIC) picker method. Picker is used to find the wave velocity of ten impulses, with the median wave velocity used to determine the length. **TOP:** Waveforms from Sensor 1 (black), Sensor 2 (blue), Sensor 3 (red), and Sensor 4 (green). Wave arrival denoted by black circle which is determined AIC results. Wave velocity is equal to the slope of the wave arrivals: 4553 m/s. **BOTTOM:** Results from AIC picker of Sensor 2 (blue), Sensor 3 (red), and Sensor 4 (green). Wave arrival times are shown by minimum values.

Data processing was used to determine the natural frequency through autocorrelation or power spectra density, and the median wave velocity is taken from 10 impulses in which wave arrivals are found using the Akaike Information Criterion (AIC) picker method. The length of the post is calculated using **Equation 3.12.** Results from this test showed an autocorrelation natural frequency of 933 Hz, a power spectra density natural frequency of 1083 Hz, and a median wave velocity of 4992 m/s. The autocorrelation method predicted a length of 2.513 m (3.2% error) and the power spectra density method predicted a length of 2.304 m (5.4% error).
**Real and Predicted Lengths.** 222 reproducible steel ball low frequency (LF) impulse tests were completed for various post cross-sections, lengths, and boundary conditions and are presented in the following results. The reproducible steel ball pendulum is used in the lab to increase the similarity of signals compared to low frequency hammer blows. The three boundary conditions employed included placing the post directly on the lab bench with no applied clamp (No Clamp – NC), placing the post on the lab bench and clamping with rubber pad boundaries (Rubber Clamp – RC), and using direct clamps on the post (Clamp – CL). The conditions under no clamp, rubber clamp and clamp are presented in Figure 3.27, Figure 3.28, and Figure 3.29, respectively.

![Graph](image_url)

**Figure 3.27.** Results from testing all laboratory wood posts under low frequency steel ball pendulum impulse with no clamp (NC) conditions. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.38 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.40 m.
Figure 3.28. Results from testing all laboratory wood posts under low frequency steel ball pendulum impulse with rubber clamp (RC) conditions. **LEFT**: Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.35 m. **RIGHT**: Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.22 m.

Figure 3.29. Results from testing all laboratory wood posts under low frequency steel ball pendulum impulse with clamp (CL) conditions. **LEFT**: Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.36 m. **RIGHT**: Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.29 m.

The results show the posts attached to the lab bench with clamp and rubber pads having the lowest error (either for autocorrelation and power spectra density natural frequency calculation methods)
and the no clamp condition having the highest error (either for autocorrelation and power spectra density natural frequency calculation methods). The power spectra density natural frequency calculation method reduces the error in of the rubber clamp and clamp conditions and increases the error of the no clamp condition.

When examining the results from testing the post under no clamp conditions, strong data outliers are observed when post length is greater than 2 m. As post length increases, prismatic post warping through benching and torsion are observed to a higher degree. Additionally, cracking in posts at post lengths less than 2 m result in outlier length predictions as well. These material imperfections result in signal distortion due heterogeneity of the propagation medium. When no lab bench clamping occurs, there is less alignment of the post compared to clamped posts. Additionally, the negative effects of cracking are strongest under the no clamp condition. The lack of continuous medium across the length of the post may distort the propagation of the longitudinal wave. The lack of post alignment and cracking imperfections may be a cause of the increasing length prediction error.

The average percent error and the coefficient of variation with respect to individual posts and testing condition is summarized in the Table 3.3. The average percent error demonstrates prediction accuracy. The coefficient of variation demonstrates prediction precision. The coefficient of variation (COV) is:

\[ COV = \frac{\sigma}{\mu} \]  

(3.19)

where \( \sigma \) is the standard deviation of the predicted length and \( \mu \) is the average of the predicted length for an individual post and condition.
Table 3.3. Average percent error (% Error) and percent coefficient of variance (% COV) of length prediction for laboratory tested posts using low frequency reproducible steel ball impulses and either autocorrelation (AUTO) or power spectra density (PSD) for natural frequency determination.

<table>
<thead>
<tr>
<th>Post</th>
<th>Area</th>
<th>Condition</th>
<th>AUTO % Error</th>
<th>AUTO % COV</th>
<th>PSD % Error</th>
<th>PSD % COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.216 m</td>
<td>8.9 cm by 14.0 cm</td>
<td>No Clamp</td>
<td>10.0</td>
<td>1.8</td>
<td>13.0</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rubber Clamp</td>
<td>4.8</td>
<td>0.6</td>
<td>8.4</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
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<td>1.0</td>
<td>8.4</td>
<td>1.0</td>
</tr>
<tr>
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<td>8.9 cm by 13.8 cm</td>
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<td>8.2</td>
<td>6.1</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>0.8</td>
<td>14.6</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>6.3</td>
<td>3.4</td>
<td>8.9</td>
<td>4.2</td>
</tr>
<tr>
<td>1.236 m</td>
<td>13.7 cm by 13.8 cm</td>
<td>No Clamp</td>
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<td>14.5</td>
<td>11.5</td>
<td>14.6</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>4.5</td>
<td>10.9</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
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<td>26.0</td>
<td>25.9</td>
<td>26.7</td>
</tr>
<tr>
<td>1.832 m</td>
<td>8.6 cm by 14.0 cm</td>
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<td>1.5</td>
<td>3.8</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rubber Clamp</td>
<td>9.5</td>
<td>0.0</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>8.2</td>
<td>1.2</td>
<td>4.7</td>
<td>1.4</td>
</tr>
<tr>
<td>1.834 m</td>
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<td>4.6</td>
<td>4.2</td>
<td>5.3</td>
</tr>
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<td></td>
<td></td>
<td>Rubber Clamp</td>
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<td>1.5</td>
<td>4.6</td>
<td>1.5</td>
</tr>
<tr>
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<td>5.6</td>
<td>6.3</td>
<td>3.5</td>
</tr>
<tr>
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<td>8.9 cm by 13.8 cm</td>
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<td>11.4</td>
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<td>11.5</td>
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</tr>
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<td>1.9</td>
</tr>
<tr>
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<td>2.2</td>
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<tr>
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<td></td>
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<td>2.2</td>
<td>7.2</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>2.5</td>
<td>1.8</td>
<td>10.6</td>
<td>2.7</td>
</tr>
<tr>
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<td>13.7 cm by 14.6 cm</td>
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<td>13.5</td>
<td>11.1</td>
<td>13.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rubber Clamp</td>
<td>5.5</td>
<td>4.1</td>
<td>6.1</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>5.7</td>
<td>6.4</td>
<td>11.8</td>
<td>6.8</td>
</tr>
<tr>
<td>2.444 m</td>
<td>8.7 cm by 13.7 cm</td>
<td>No Clamp</td>
<td>7.4</td>
<td>7.4</td>
<td>7.2</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rubber Clamp</td>
<td>10.3</td>
<td>2.6</td>
<td>10.1</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>1.6</td>
<td>0.9</td>
<td>6.9</td>
<td>0.9</td>
</tr>
<tr>
<td>3.040 m</td>
<td>8.7 cm by 13.4 cm</td>
<td>No Clamp</td>
<td>17.5</td>
<td>2.9</td>
<td>14.2</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rubber Clamp</td>
<td>20.9</td>
<td>2.4</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>13.9</td>
<td>4.6</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>3.065 m</td>
<td>8.7 cm by 8.9 cm</td>
<td>No Clamp</td>
<td>22.2</td>
<td>15.4</td>
<td>20.5</td>
<td>23.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rubber Clamp</td>
<td>23.3</td>
<td>7.3</td>
<td>12.0</td>
<td>11.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Clamp</td>
<td>31.1</td>
<td>6.6</td>
<td>22.7</td>
<td>6.3</td>
</tr>
</tbody>
</table>
**Data Adjustment.** Upon completion of data collection for post testing in the laboratory for several posts under clamped, no clamp, and rubber clamped conditions, median frequency values were taken for the autocorrelation and the power spectral density ($f_{AUTO}$ and $f_{PSD}$) for all posts under each clamped condition. The median value was taken from a minimum of 5 tests per post per test condition. The average wave velocity ($V_M$) for each post and condition was taken from a minimum of 50 velocity measurements (i.e., population sample greater than 30) (Corder & Foreman, 2009). These values created a bulk error from all tests to assess the quality of the results obtained and the use of statistical methods to determine frequency and velocity. A summary of the results can be found in Table 3.4 and Table 3.5 for the autocorrelation method and power spectra density method for natural frequency determination, respectively.

**Table 3.4.** Bulk error associated with test results for individual environments, post lengths, and autocorrelation (AUTO) method. Median frequency ($f_{AUTO}$) taken from a minimum of 5 measurements and average velocity ($V_M$) taken from a minimum of 50 measurements across all results. Predicted length ($L_p$) and associated error denoted.

<table>
<thead>
<tr>
<th>L (m)</th>
<th>Clamp</th>
<th>No Clamp</th>
<th>Rubber Clamp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{AUTO}$ (Hz)</td>
<td>$V_M$ (m/s)</td>
<td>$L_p$ (m)</td>
</tr>
<tr>
<td>1.216</td>
<td>1805</td>
<td>4165</td>
<td>1.154</td>
</tr>
<tr>
<td>1.221</td>
<td>1841</td>
<td>4200</td>
<td>1.141</td>
</tr>
<tr>
<td>1.236</td>
<td>1815</td>
<td>4992</td>
<td>1.375</td>
</tr>
<tr>
<td>1.328</td>
<td>1128</td>
<td>4392</td>
<td>1.946</td>
</tr>
<tr>
<td>1.344</td>
<td>1042</td>
<td>3918</td>
<td>1.881</td>
</tr>
<tr>
<td>2.423</td>
<td>836</td>
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</tr>
<tr>
<td>3.040</td>
<td>578</td>
<td>3911</td>
<td>3.385</td>
</tr>
<tr>
<td>3.065</td>
<td>779</td>
<td>5877</td>
<td>3.771</td>
</tr>
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</table>
Table 3.5. Bulk error associated with test results for individual environments, post lengths, and power spectra density (PSD) method. Median frequency (f_{PSD}) taken from a minimum of 5 measurements and average velocity (V) taken from a minimum of 50 measurements across all results. Predicted length (L_p) and associated error denoted.

<table>
<thead>
<tr>
<th>L (m)</th>
<th>f_{PSD} (Hz)</th>
<th>V_m (m/s)</th>
<th>L_p (m)</th>
<th>Error (%)</th>
<th>f_{PSD} (Hz)</th>
<th>V_m (m/s)</th>
<th>L_p (m)</th>
<th>Error (%)</th>
<th>f_{PSD} (Hz)</th>
<th>V_m (m/s)</th>
<th>L_p (m)</th>
<th>Error (%)</th>
</tr>
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<tr>
<td>1.216</td>
<td>1875</td>
<td>4165</td>
<td>1.111</td>
<td>9%</td>
<td>1875</td>
<td>4002</td>
<td>1.067</td>
<td>12%</td>
<td>1875</td>
<td>4177</td>
<td>1.114</td>
<td>8%</td>
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<tr>
<td>1.221</td>
<td>1875</td>
<td>4200</td>
<td>1.120</td>
<td>8%</td>
<td>1875</td>
<td>4253</td>
<td>1.134</td>
<td>7%</td>
<td>1875</td>
<td>3916</td>
<td>1.044</td>
<td>14%</td>
</tr>
<tr>
<td>1.236</td>
<td>1875</td>
<td>4992</td>
<td>1.331</td>
<td>8%</td>
<td>1875</td>
<td>4748</td>
<td>1.266</td>
<td>2%</td>
<td>1875</td>
<td>5037</td>
<td>1.343</td>
<td>9%</td>
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<td>1.832</td>
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<td>4392</td>
<td>1.882</td>
<td>3%</td>
<td>1167</td>
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<td>1167</td>
<td>4477</td>
<td>1.919</td>
<td>5%</td>
</tr>
<tr>
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<td>3918</td>
<td>1.808</td>
<td>1%</td>
<td>1167</td>
<td>4101</td>
<td>1.758</td>
<td>4%</td>
<td>1167</td>
<td>4449</td>
<td>1.907</td>
<td>4%</td>
</tr>
</tbody>
</table>

Results using autocorrelation frequency determination lead to higher error in 14 out of 33 post lengths and conditions compared to the power spectra density frequency determination. Another characteristic to note is the success of the 1.834-m long post under all conditions and analysis methods. Furthermore, prediction of the 1.832-m long post shows consistent success in length prediction, however not as successful as prediction of the 1.834-m long.

Additional work was completed to characterize the results for individual test environments, post lengths, and analysis method by assessing the over or under prediction of wave velocity with respect to the expected value. This characterization was achieved by calculating the wave velocity adjustment (correction) to obtain a precise length prediction, when assuming the frequency is accurately determined. Table 3.6 and Table 3.7 show the measured value of velocity used in length prediction (V_m) and the expected velocity regarding post length and measured frequency (V_r) for autocorrelation and power spectra density methods, respectively. The error represents the value the measured velocity would need to be multiplied by to achieve the correct length prediction.
There is an adjustment to length correction for individual posts which may be independent of the clamp environment.

**Table 3.6.** Bulk error associated with velocity measurement \((V_M)\) compared to expected velocity \((V_R)\) for autocorrelation (AUTO) method. Adjustment factor \((\chi)\) is average error for individual posts under all clamp conditions (i.e. adjustment factor for 1.216-m long post is the average of the error from the clamp, no clamp, and rubber clamp conditions).

<table>
<thead>
<tr>
<th>Clamp</th>
<th>L [m]</th>
<th>(f_{AUTO}) [Hz]</th>
<th>(V_M) [m/s]</th>
<th>(V_R) [m/s]</th>
<th>Error [ ]</th>
<th>(f_{AUTO}) [Hz]</th>
<th>(V_M) [m/s]</th>
<th>(V_R) [m/s]</th>
<th>Error [ ]</th>
<th>(f_{AUTO}) [Hz]</th>
<th>(V_M) [m/s]</th>
<th>(V_R) [m/s]</th>
<th>Error [ ]</th>
<th>(\chi) [ ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUTO</td>
<td>1.216</td>
<td>1805</td>
<td>4165</td>
<td>4390</td>
<td>1.05</td>
<td>1814</td>
<td>4002</td>
<td>4411</td>
<td>1.10</td>
<td>1803</td>
<td>4177</td>
<td>4385</td>
<td>1.05</td>
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<td>1.221</td>
<td>1841</td>
<td>4200</td>
<td>4495</td>
<td>1.07</td>
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<td>4420</td>
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<td>1825</td>
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<td>4503</td>
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**Table 3.7.** Bulk error associated with velocity measurement \((V_M)\) compared to expected velocity \((V_R)\) for power spectra density (PSD) method. Adjustment factor \((\chi)\) is average error for individual posts under all clamp conditions (i.e. adjustment factor for 1.216-m long post is the average of the error from the clamp, no clamp, and rubber clamp conditions).

<table>
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<tr>
<th>Clamp</th>
<th>L [m]</th>
<th>(f_{PSD}) [Hz]</th>
<th>(V_M) [m/s]</th>
<th>(V_R) [m/s]</th>
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<th>(f_{PSD}) [Hz]</th>
<th>(V_M) [m/s]</th>
<th>(V_R) [m/s]</th>
<th>Error [ ]</th>
<th>(f_{PSD}) [Hz]</th>
<th>(V_M) [m/s]</th>
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<td>1.08</td>
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<tr>
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<td>5731</td>
<td>5364</td>
<td>0.94</td>
<td>0.90</td>
</tr>
</tbody>
</table>

The errors of each post under all conditions are averaged to determine an adjustment factor \(\chi\) as:
\[
\chi = AVERAGE(\text{Error}_{\text{Clamp}}, \text{Error}_{\text{No Clamp}}, \text{Error}_{\text{Rubber Clamp}})
\]

(3.20)

The adjusted velocity \((V_{ADJ})\) is calculated by multiplying the measured velocity \((V_M)\) by the adjustment factor \((\chi)\) for that specific post:

\[
V_{ADJ} = V_M \times \chi
\]

(3.21)

This led to the results seen in Table 3.8 and Table 3.9. With comparison to Table 3.4 and Table 3.5, the error is reduced, limiting the number of predictions with over 10% error to zero.

**Table 3.8.** Bulk error associated with test results for individual environments, post lengths, using autocorrelation (AUTO) method after adjustment factor applied to velocity measurement.

<table>
<thead>
<tr>
<th>Clamp</th>
<th>No Clamp</th>
<th>Rubber Clamp</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L [m]</strong></td>
<td><strong>f_{AUTO} [Hz]</strong></td>
<td><strong>V_{ADJ} [m/s]</strong></td>
</tr>
<tr>
<td>1.216</td>
<td>1805</td>
<td>4451</td>
</tr>
<tr>
<td>1.221</td>
<td>1841</td>
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</tr>
<tr>
<td>3.065</td>
<td>779</td>
<td>4798</td>
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</tbody>
</table>
Table 3.9. Bulk error associated with test results for individual environments, post lengths, using power spectra density (PSD) method after adjustment factor applied to velocity measurement.

<table>
<thead>
<tr>
<th>Clamp</th>
<th>No Clamp</th>
<th>Rubber Clamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>L [m]</td>
<td>f_{PSD} [Hz]</td>
<td>V_{ADJ} [m/s]</td>
</tr>
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</tr>
<tr>
<td>3.065</td>
<td>833</td>
<td>5276</td>
</tr>
</tbody>
</table>

Error increases in 10 of 66 conditions, and error increases by more than 1.5% in only four cases.

When using the autocorrelation function to calculate the natural frequency:

- the 2.444-m long post experiences an increase in error of 5.4% when clamped directly to the lab bench under the new adjustment.

When using the power spectra density (PSD) to calculate the natural frequency:

- the 2.423-m long post experiences an increase in error of 5.4% when clamped directly to the lab bench under the new adjustment,
- the 3.040-m long post experiences an increase in error of 3.3% when clamped directly to the lab bench under the new adjustment, and
- the 2.435-m long post experiences an increase in error of 5.9% when no clamps are applied under the new adjustment.

Data adjustment demonstrates that testing wood posts reflects individual heterogeneities and discontinuities of the material which may or may not be present in the next post tested. It reveals
that there may be more satisfactory length estimations when considering and factoring in the inherent inconsistencies of the material rather than attempting to deliver perfect results from the measurements alone.

3.6. Lessons Learned and Engineering Recommendations

Lessons learned:

- In the field, predicting steel post length led to less error than predicting wood post length. Steel is more homogenous than wood and presents less challenges when attempting to measure a constant natural frequency and wave velocity through the entire medium. The heterogeneity of wood increases the error associated with the technique. Furthermore, the wood post length is consistently overpredicted whereas steel post length is consistently under predicted.

- The low frequency hammer blow utilized in the field resulted in a non-reproducible signal. Analysis of waveforms collected at the top sensor indicated a limited degree of similarity on an impulse by impulse basis. The lack of a reproducible signal limits the ability of applied stacking and the accurate assessment of the natural frequency, thus increasing the error in the field.

- When predicting the length in the field, it is possible that either the natural frequency determination or the wave velocity determination negatively affects the result. By improving the reproducible signal in the field, the error assessed with the wave velocity may be more accurately constrained.

- Testing under controlled conditions lowered Root Mean Square Error for wood posts by more than 0.4 m from field conditions. It is expected that a major component of the
reduction in error is associated with the reproducible steel ball pendulum utilized in the lab to introduce the impulse. By increasing the self-similarity of the signal on an impulse by impulse basis, the ability to accurately assess the natural frequency of the wave increase, and the length prediction technique is improved. Additionally, signal energy leakage is reduced when the post is not coupled to the surrounding soil. This energy leakage reduces the ability to analyze the natural frequency as dispersion and distortion occur.

- Coupling the post with rubber pad boundaries or directly to the lab bench as an approximation for field post embedment and guardrail fixture does not limit the predictability of length. Tests indicated an increased length prediction accuracy under the rubber clamp or direct clamp compared to the no clamp condition. This may suggest a reduction in the effect of material imperfection when clamped. Imperfections due to warping of the post or cracking negatively affect the ability to estimate the natural frequency of the wave in the system. By applying a clamp at the boundaries, cracks may be closed, and warping may be straightened. These applications likely increase the ability of the wave to propagate through the system with minimal energy dispersion or distortion and increase the accuracy of the length prediction tool.

- The intrinsic heterogeneity and orthotropic nature of wood (independent of coupling condition) is demonstrated by the data adjustment of wood. Boundary conditions affected length prediction; however, properties of the wood appear to affect the results for length determination the strongest.

- Achieving a Root Mean Square Error of less than or equal to 10 cm was not possible in the field for wood or steel posts, and achieving an average Root Mean Square Error below 10 cm or an average percent error below 10% for wood posts in controlled conditions was not
possible. This suggests limited applicability of the technique for length assessment of guardrail posts in the field as currently devised.

Engineering Recommendations:

- As the length estimation technique is presented it is not currently recommended to enact a testing method to deploy natural frequency and wave velocity determination to predict highway guardrail post length.
- Increasing the understanding of stress-wave behavior through posts may inform the reasoning of why the technique does not achieve a high degree of accuracy. Methods should be explored to analyze the longitudinal wave through the system and assess the attenuation, signal coherence, and phase velocity.
- Future lab testing should be carried out to assess error of steel posts in controlled conditions. Results from field testing suggest that the error encountered may be reduced to a reasonable value if more research is applied to understanding the waveform and the impulse applied.
- Natural frequency measurements may be considered to detect the change across post lengths without measuring the absolute length. This may be more successful for steel posts as wave velocity is more consistent as a homogenous material.
- The results of testing and the large amount of data collected suggests that this may be an opportunity for machine learning to analyze frequency of posts (and potential wave velocity) across a large data pool.
- Future research of the technique should utilize a reproducible signal in the field. The lack of self-similarity in individual waveform traces recorded from the top sensor in the field limits the analysis of the natural frequency. The reproducible signal devised must also
provide enough impulse to allow for multiple reflections before dissipation of the wave energy. A proposed reproducible signal introduction methodology is presented in Figure 3.30. Under this testing procedure a steel ball is dropped from a known height and guided by plastic tubing to impact a specified location. An attached string to the steel ball would allow the user to “grab” the steel ball before it double hits the post. This is a very rudimentary idea of how to introduce a reproducible signal on an impulse by impulse basis.

**Figure 3.30.** Mechanism 1 for proposed reproducible impulse in the field with a steel ball drop and guiding tubing.

- An additionally proposed reproducible signal may involve the use of a torsional spring attached to a rod with a steel ball mounted at the end. The contraption may be fixed to the block separating the guardrail from the post. The torsional spring may be loaded by pulling the rod with ball back, and then releasing. The applied load and the strike location would be consistent, and the torsional spring would limit double hits to the post, as resistance to
rotation occurs after the ball passes the rest location. The schematic figure is presented in

**Figure 3.31.**

*Figure 3.31. Mechanism 2 for proposed reproducible impulse in the field with a torsional spring striking contraption.*
4. Analysis of Stress Waveforms

4.1. Testing Techniques

Testing techniques were implemented to collect data beyond that obtained from length determination methods. These techniques received signal impulses from one of two ways: hammer blow or a reproducible steel ball strike. The method of analysis was in one of two ways: longitudinal or torsional and bending.

4.1.1. Hammer Blow

The hammer blow is utilized to provide impulse in the field due to the large energy needed due to overcome the attenuation of the posts. These posts are coupled to the guardrail and the ground, resulting in large dissipation of energy through the surrounding medium. The hammer blow limits repeatability, that is, the signals generated vary depending on the magnitude of the strike, the location of the strike, and the orientation of the hammer as it contacts the posts.

4.1.2. Reproducible Steel Ball Impulse

The use of a steel ball in the laboratory allowed for simple signal repetition by maintaining use of consistent pendulum mechanics as seen in Figure 4.1. The ball was continually released from the same location allowing for the impulse to occur in the same place on the post and with the same force. This reproducible steel ball impulse was also brought to the field with limited results. The lack of energy imparted by the steel ball to the post reduced analysis of the reflection pattern and the natural frequency for posts.
4.1.3. **Longitudinal Excitation**

To ascertain information regarding the stress-strain wave patterns from the post, additional testing was completed to characterize the longitudinal waves through the system. By adjusting the length of the acquisition period, data can be collected over a timescale in which multiple reflection signals were acquired, and the phase velocity and attenuation of the waves were estimated. The post is impacted in a similar manner as a Hammer Blow or Reproducible Impulse – Steel Ball.

The longitudinal testing technique allows for the evaluation of waves and signal parameters, including attenuation, phase velocity, coherence. Entire signal attenuation may be evaluated with limited processing, provided the time scale is larger enough, however windowing allows for assessment of attenuation, phase velocity and coherence between individual reflections/windows.

4.1.4. **Torsional and Bending Excitation**

In addition to longitudinal testing, preliminary testing was completed to analyze torsional and bending waves to a minor extent. Accelerometer location and the post striking differs from the longitudinal method; however, the timescale remains the same. The torsional testing method
requires the addition of a bar connected to the top of the post to introduce torsional deformation. An impulse is introduced to the bar and the sensors monitor acceleration about the center of rotation (Figure 4.2). The resulting impulse of the strike is oriented to introduce torsional waves; however, the excitation symmetry inevitable create flexural waves. The use of two opposite located sensors allows the separation of the torsional waves from the flexural bending waves. If the sensors are equidistant from the center of torsional rotation and the attached rod is perfectly straight, adding the signals obtained from the two sensors removes the acceleration due to bending, and subtracting the signals removes the acceleration due to torsion.

![Figure 4.2. Top view of post with torsional testing set-up (length of post extends into page). The torsional bar is secured to the post with nails, and two sensors are attached to the ends of the bar. A torsional impulse is introduced, and the sensors monitor acceleration.](image)

Due to the orientation of the sensors, the accelerations being measured by signals are dependent on the orientation. If the post vibrates (bends) laterally along the z-axis, the resulting amplitudes from Sensor 1 are opposite that of Sensor 2. Therefore, adding the sensors together removes the effect of bending. The same principles are applied to remove the effect of torsion through subtraction of sensor amplitudes.

### 4.2. Methodology

**Longitudinal Excitation Testing.** Measures were explored to understand the response of the waveguide under dynamic longitudinal. Attenuation, natural frequency, coherence, and phase
velocity were assessed for longitudinal waves. Several posts were used to assess the impact of cross-sectional area of wood and the length of wood posts. Limited tests were also performed on steel posts in the field. Field testing was completed with low frequency hammer blows and low frequency steel ball impulses. Two wood posts of the same geometry and two steel posts of the same geometry were used as field test specimens. Additionally, posts were tested as attached and unattached to the highway guardrail to reduce interference of different elements in the system.

**Torsional and Bending Excitation Testing.** Measurements were collected to understand the response of the waveguide material under torsional and bending wave excitations. Attenuation, natural frequency, coherence, phase velocity, and analytical signal were assessed for torsional waves. Lab testing was completed with controlled pendulum impulses. One 1.834-m long wood post was tested. Field testing was completed with low frequency hammer blows. Two wood posts of the same geometry were used as field test specimens. The posts were tested unattached to the highway guardrail to reduce interference of different elements in the system and allow for rotation of the post. Information gained from torsional and bending testing was limited. Example results from the field are documented in Appendix E.

### 4.3. Spectral Analysis Results

#### 4.3.1. Laboratory Testing

Laboratory testing was completed with three boundary conditions under longitudinal, reproducible steel ball impulses. The boundary conditions are as follows: post placed on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), or post clamped directly to the lab bench (Clamp). This testing procedure led to an increased understanding of the intrinsic properties of the wood with respect to coupling. Tests were also
completed to show the impact of cross-sectional area when length is constant (2.44 m ± 0.005 m), and the impact of length when cross-sectional area is constant (120 cm² ± 5 cm²).

**Attenuation.** A decay function is fit to the peak values obtained to estimate the energy attenuation of the introduced signal as seen in **Figures 4.3 to 4.8**. As seen in all the attenuation plots, the attenuation is greatest with the direct clamp and least with no clamp, demonstrating how limited mobility and the fixed boundary lead to signal dissipation. This dissipation of energy shown in the clamp condition is expected to dominate field testing results, in which the post is embedded into the ground, and strong coupling with the surrounding soil increases energy loss. **Figures 4.3 to 4.5** shows effect of increasing cross-sectional area on attenuation when length is kept constant.

![Attenuation Graph](image-url)

**Figure 4.3.** Attenuation results from testing 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Attenuation is highest under clamp conditions and lowest under no clamp conditions denoted by exponential fits (shown as black lines).
Figure 4.4. Attenuation results from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Attenuation is highest under clamp conditions and lowest under no clamp conditions denoted by the exponential fits (shown as black lines).

Figure 4.5. Attenuation results from testing 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm$^2$, density: 520 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Attenuation highest under clamp conditions and lowest under no clamp conditions denoted by the exponential fits (shown as black lines).
As cross-sectional area increases, exponential decay lessens under individual boundary testing conditions (i.e., post on lab bench with no clamps, post clamped to the bench with rubber pad boundaries, or post clamped directly to the bench). All tests show the unclamped post with the least amount of decay and the post clamped to the bench with the highest decay when testing individual posts. The results for attenuation analysis in the time domain imply decreasing attenuation as cross-sectional area increases. As the system medium increases, less energy is dissipated at the boundaries, allowing for a longer retention of signal energy.

Figures 4.6. to 4.8 shows the effect of increasing post length on attenuation when cross-sectional area is kept constant.

Figure 4.6. Attenuation results from testing 1.216 m long wooden post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Attenuation highest under clamp conditions and lowest under no clamp conditions denoted by the exponential fits (shown as black lines).
Figure 4.7. Attenuation results from testing 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm², density: 600 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Attenuation highest under clamp conditions and lowest under no clamp conditions denoted by the exponential fits (shown as black lines).

Figure 4.8. Attenuation results from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm², density: 660 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Attenuation highest under clamp conditions and lowest under no clamp conditions denoted by the exponential fits.
As the length increases, there is not a noticeable trend with respect to decreasing or increasing attenuation under individual boundary conditions (i.e., post on lab bench with no clamps, post clamped to the lab bench with rubber pad boundaries, or post clamped directly to the lab bench). When testing individual posts, the post with no clamp exhibits the least amount of amplitude decay and the post clamped to the lab bench exhibits the highest amount of decay.

Immediate observable values of attenuation with respect to length do not reveal noticeable trends. However, normalizing attenuation with respect to length (i.e., decay coefficient divided by the length), results in the largest attenuation values observed in the 1.216-m long post (no clamp: 337 m\(^{-1}\), rubber clamp: 378 m\(^{-1}\), direct clamp: 518 m\(^{-1}\)) and the lowest attenuation values observed in the 2.423-m long post (no clamp: 132 m\(^{-1}\), rubber clamp: 252 m\(^{-1}\), direct clamp: 260 m\(^{-1}\)). The normalization emphasizes the effect of energy dissipation at the post boundaries. Across the same period, the shorter post experiences more wave reflections at the top and bottom of the post, which increases attenuation.

**Natural Frequency.** To assess the natural frequency of the posts excited with an impulse, the autocorrelation of the signal is taken and the average distance between peak maximums is the travel time as seen in Figures 4.9 to 4.14. The natural frequency of the first mode of vibration is the inverse of the travel time. Figures 4.9 to 4.11 shows the effect of increasing cross-sectional area on the autocorrelation when length is kept constant. The difference in natural frequency is assumed to be due to changing longitudinal velocity (a function of density and Young’s modulus in pure longitudinal mode - Equation 3.2). The 2.423-m long post (cross-section: 8.9 cm by 13.8 cm) has a density of 660 kg/m\(^3\) and the lowest frequency (i.e., lowest wave velocity). The 2.423-m long post is 110 kg/m\(^3\) denser than the 2.435-m long post (cross-section: 8.3 cm by 8.6 cm) and 140 kg/m\(^3\) denser than the 2.442-m long post (cross-section: 13.7 cm by 14.6 cm).
Figure 4.9. Autocorrelation results from testing 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Natural frequency is the highest when post is clamped to lab bench with rubber pads, or when directly clamped to lab bench.

Figure 4.10. Autocorrelation results from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm², density: 660 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Natural frequency determined as distance from zero to first peak (shown between $10^{-3}$ seconds and 1.5·$10^{-3}$ seconds) with other peaks ignored. Natural frequency is highest when the wood is clamped to lab bench.
Figure 4.11. Autocorrelation results from testing 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm², density: 520 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Natural frequency is the highest when the wood is clamped to lab bench.

The lab determined longitudinal-radial Young’s modulus for the 2.435-m long post is 13000 MPa, the 2.423-m long post is 10000 MPa, and the 2.442-m long post is 10000 MPa. These results conform with literature reported values (Green et al., 1999). As the Young’s modulus is approximately equal for each post, this demonstrates the relationship between density and longitudinal wave velocity.

Figures 4.12 to 4.14 shows the effect of increasing length on natural frequency when cross-sectional area is kept constant. The difference in natural frequency is due to changes in length and wave velocity, with length change being the largest factor for varying frequency. The results for natural frequency analysis through autocorrelation show increasing lengths decrease the measured natural frequency as expected for constant wave velocity medium.
Autocorrelation results from testing 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Natural frequency is constant under all boundary conditions.

Natural frequency determined as average distance from zero to first peak and from first to second peak with third peaks ignored (shown after $2.5 \times 10^{-3}$ seconds). Natural frequency is the highest when the wood is clamped to lab bench.

Figure 4.12. Autocorrelation results from testing 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm², density: 600 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Natural frequency determined as average distance from zero to first peak and from first to second peak with third peaks ignored (shown after $2.5 \times 10^{-3}$ seconds). Natural frequency is the highest when the wood is clamped to lab bench.

Figure 4.13.
Figure 4.14. Autocorrelation results from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm², density: 660 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Natural frequency determined as distance from zero to first peak (shown between 1E-3 seconds and 1.5E-3 seconds) with other peaks ignored. Natural frequency is the highest under clamp condition.

When the post is direct clamped to the lab bench, the natural frequency is greater than or equal to the natural frequency for the post not clamped to the lab bench, or when clamped with rubber pad boundaries. When the post is clamped to the lab bench with rubber boundaries, the natural frequency is always equal to the natural frequency of the post when directly clamped and/or the natural frequency of the post when not clamped to the lab bench. When the post is not clamped to the lab bench, the natural frequency is always less than or equal to the natural frequency for the post when directly clamped or clamped with rubber pads.

**Power Spectra Density (PSD).** Upon signal transformation from the time domain to the frequency domain, the natural frequency of wave reflections is shown as the lowest frequency peak in Figures 4.15 to 4.20. The distribution of energy with respect to frequency is represented in a power spectra density plot. Figures 4.15 to 4.17 show the effect of cross-sectional area increase on power
spectra density when length remains constant. As the cross-sectional area increases, the dominant low frequencies (as shown by peaks) sharpen and the energy density at higher frequencies is reduced. The power spectra density is also affected if the post is laid on the lab bench in a free state, clamped with rubber pads, or directly clamped. When the post is placed on the bench with no clamps or is clamped with rubber pads, higher amplitude peaks at low frequencies are observed when compared to the post being directly clamped to lab bench. This demonstrates a noticeable relationship in which larger values of energy density are observed at higher frequencies if the post is directly clamped to the lab bench. Under this condition, the post has fixed boundaries and is coupled to both the clamps and the bench with an inability to vibrate freely. This boundary restriction and coupling likely leads to increased energy at high frequencies while reducing the amplitude to low frequency peaks.

Figure 4.15. Power spectra density results from testing 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).
Figure 4.16. Power spectra density results from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).

Figure 4.17. Power spectra density results from testing 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm$^2$, density: 520 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).
Figures 4.18 to 4.20 show the effect of increasing length on power spectra densities when cross-sectional area remains constant. As lengths increase, dominant frequency peaks occur at lower frequencies (i.e., the range of dominant frequencies for the 1.216-m long post is approximately from 1800 to 7200 Hz, whereas the range of dominant frequencies for the 2.423-m long post is approximately from 800 to 3200 Hz). This reduction in frequency range with increasing length is expected as natural frequency decreases. The first mode of longitudinal vibration (i.e., natural frequency or the fundamental mode) occurs at the peak with the lowest frequency, and increasing modes of longitudinal vibrations occur at subsequent peaks, which is a multipliable integer of the first mode \( f_n = n f_1 \) (Timoshenko et al., 1974).

![Power spectra density results from testing 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).](image-url)

**Figure 4.18.** Power spectra density results from testing 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).
Figure 4.19. Power spectra density results from testing 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm², density: 600 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).

Figure 4.20. Power spectra density results from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm², density: 660 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp).
The results for power spectra density analysis show increasing cross-sectional areas limits the amount of energy at high frequencies. Length increase leads to larger energy density at higher frequencies. When the post is clamped directly to the lab bench, larger energy is present at higher frequencies when compared to no clamp being applied to the post or when the post is clamped with rubber pad boundaries.

**Windowing.** Signals can be windowed into individual reflections to assess attenuation and phase velocity. An example of a windowed signal can be seen in **Figure 4.21** with remaining results presented in Appendix F. Four windows are shown and denoted by alternating solid and dashed lines.

![Figure 4.21](image_url)

**Figure 4.21.** Windowed reflections from testing 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm$^2$, density: 600 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Alternating solid and dashed lines denote reflection windows.

Window time span is directly the inverse of the natural frequency (i.e., the travel time of the wave from post top to bottom boundary and back to top).
Window Power Spectra Density. The power spectra density (PSD) of individually windowed waveforms can be compared against one another. As seen in Figures 4.22 to 4.27, the power spectra density of the second and third windowed signals are taken. Figures 4.22 to 4.24 show the effect of increasing cross-sectional areas on the power spectra densities of the windowed signals. Windowed power spectra densities show the strongest similarity when testing individual posts, regardless of boundary condition. There is little similarity in window power spectra density of posts with the same boundary condition but different cross-sectional area.

![Power spectra densities of windows two (PSD2) and windows three (PSD3) from 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).](image)

**Figure 4.22.** Power spectra densities of windows two (PSD2) and windows three (PSD3) from 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
Figure 4.23. Power spectra densities of windows two (PSD2) and windows three (PSD3) from 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).

Figure 4.24. Power spectra densities of windows two (PSD2) and windows three (PSD3) from 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm$^2$, density: 520 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
Figures 4.25 to 4.27 show the effect of length increase on windowed signals power spectra densities when cross-sectional area remains constant. Comparable to the results found when cross-sectional area varied, windowed power spectra densities show the strongest similarity when testing individual posts, regardless of boundary condition.

Figure 4.25. Power spectra densities of windows two (PSD2) and windows three (PSD3) from 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
Figure 4.26. Power spectra densities of windows two (PSD2) and windows three (PSD3) from 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm$^2$, density: 600 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).

Figure 4.27. Power spectra densities of windows two (PSD2) and windows three (PSD3) from 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
The power spectra density is minimally affected by boundary conditions and is largely dependent on properties of the post. This demonstrates the importance of individual post geometry, heterogeneity, anisotropy, etc.

**Frequency Dependent Attenuation and Phase Velocity.** The signal attenuation as function of frequency may be calculated by comparing the reduction in amplitude with respect to frequency across windowed PSDs. This is represented as PSDX divided by PSD(X+1), where X is the window of concern. Linear best fit lines are used to estimate the approximate attenuation from 0 Hz to approximately 4500 Hz. The best fit line is not extended past 4000 Hz due to the decrease in coherence after this frequency.

The phase and the phase velocity are processed in the frequency domain. The phase is calculated for the frequency response of two reflections. The phase is then unwrapped to remove the bounds of $\pm \frac{\pi}{2}$. The phase may then be transformed into the phase velocity. Wrapped and unwrapped signal phases are presented in Appendix F. The phase velocity is determined to assess the speed of waves at varying frequencies (or wavelengths). If the velocity of the wave changes with respect to frequency (i.e., the medium is dispersive), the signal may be highly distorted, and the ability to correctly measure the natural frequency and/or the longitudinal wave velocity of the system becomes limited. The phase velocity is compared to the velocity of the longitudinal wave (determined from the natural frequency and the inputted known length of the post).

**Figures 4.28 to 4.30** show the effect of the cross-sectional area increase on the frequency dependent attenuation, phase velocity, and coherence when the post length is kept constant.
Figure 4.28. Attenuation with respect to frequency, phase velocity, and coherence results from 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm$^2$, density: 550 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3). Bold lines show the attenuation linear fit.
Figure 4.29. Attenuation with respect to frequency, coherence, and phase velocity results from 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).
Figure 4.30. Attenuation with respect to frequency, coherence, and phase velocity results from 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm², density: 520 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).
As cross-sectional area increases, frequency dependent attenuation decreases. The post with the largest cross-sectional area also demonstrates similar values of attenuation irrespective of boundary condition. This may be directly linked to the reduction in energy dissipation. It is also possible that the increase in cross-sectional area, has resulted in the wavelength becoming sufficiently small with respect to the effective radius of the post, and thus an improved signal (Finno et al., 1997). When the post is not clamped to the lab bench, the lowest attenuation with respect to frequency occurs.

The post with the largest cross-sectional area shows the most constant phase velocity in all boundary conditions compared to posts with medium and small cross-sectional area. This may be similarly linked to the results shown for attenuation with respect to frequency. The largest cross-sectional area may exhibit a large enough effective radius compared to the wavelength to reduce signal distortion, resulting in phase velocities consistent with the group velocity (i.e., natural frequency velocity). Furthermore, the effect of the boundary condition is lessened as the cross-sectional area increases, which may reduce the dispersity of the medium.

The equivalent radius \( r_{\text{equivalent}} \) of the post cross-section can be compared to the wavelength of the stress-wave to distinguish the boundary between longitudinal wave and P-wave frequencies:

\[
r_{\text{equivalent}} = \sqrt{A/\pi}
\]

(4.1)

where \( A \) is the cross-sectional area of the post. Frequency values smaller than the boundary conform to longitudinal wave and the wavelength is not small with respect to the equivalent radius of the post; therefore, the post is treated as a bounded, finite media. Frequency values larger than the boundary conform to the P-wave and the wavelength is small with respect to the equivalent
radius of the post; therefore, and the post is treated as an infinite media. The frequency boundary \( (f_b) \) is calculated as:

\[
f_b = \frac{V}{r_{\text{equivalent}}}
\]

where \( V \) is the natural frequency wave velocity. This equation finds the value at which the wavelength and equivalent radius are equal. A summary of the frequency boundaries of posts with varying cross-sectional area and constant length is in Table 4.1.

Table 4.1. The equivalent radius and the velocity of the wave through the medium are used to assess the frequency boundary between longitudinal waves (frequencies less than boundary) and P-waves (frequencies greater than boundary) for posts which range in cross-section and constant length.

<table>
<thead>
<tr>
<th>Length</th>
<th>Cross-Section</th>
<th>Equivalent Radius</th>
<th>Velocity</th>
<th>Frequency Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.435 m</td>
<td>8.3 cm by 8.6 cm</td>
<td>4.8 cm</td>
<td>4854 m/s</td>
<td>102000 Hz</td>
</tr>
<tr>
<td>2.423 m</td>
<td>8.9 cm by 13.8 cm</td>
<td>6.3 cm</td>
<td>3929 m/s</td>
<td>63000 Hz</td>
</tr>
<tr>
<td>2.442 m</td>
<td>13.7 cm by 14.6 cm</td>
<td>8.0 cm</td>
<td>4447 m/s</td>
<td>54000 Hz</td>
</tr>
</tbody>
</table>

The frequency boundary is very large with respect to the recorded frequency range (0 Hz to 10000 Hz); therefore, the wave regime always exists in the longitudinal domain.

The divide between high and low coherence becomes more distinct as cross-sectional area increases. This is particularly evident in Figure 4.30c, with the sudden drop off from a coherence of one at approximately 5000 Hz. The drastic change from a coherence other than one is best recognized in posts with larger cross-sectional area.

Figures 4.31 to 4.33 show the effect of frequency dependent attenuation, phase velocity, and coherence as post length increase and cross-sectional area remains constant.
Figure 4.31. Attenuation with respect to frequency, coherence, and phase velocity results from 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).
Figure 4.32. Attenuation with respect to frequency, coherence, and phase velocity results from 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm$^2$, density: 600 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).
Figure 4.33. Attenuation with respect to frequency, coherence, and phase velocity results from 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).
As length increases, the frequency dependent attenuation increases in posts which are not clamped to the lab bench and those that are directly clamped to the lab bench. When the post is directly clamped to the bench with rubber pad boundaries, the 1.216-m long post has the smallest attenuation, followed by the 2.423-m long post, and lastly the 1.832-m long post. Further testing of this relationship would likely reveal error in these results. It is expected that the attenuation with respect to frequency for the post clamped with rubber pad boundaries would see an increase in attenuation as post length increases, similar to the pattern demonstrated with the post not clamped to the lab bench, and the post which is directly clamped to the lab bench. Signal damping increases as the post length increases (Davis & Dunn, 1974) indicating that increased attenuation with respect to frequency should be present in posts with longer length, irrespective of boundary condition.

The phase velocity of posts clamped directly to the lab bench or with rubber pad boundaries sees increased deviation from the natural frequency velocity as length increases. The 1.216-m long post shows the least changes in phase velocity with respect to frequency under all boundary conditions. This may be due to the lack of signal distortion across the time span. Reflections two and reflections three for the 1.216-m long post occur earlier in time after the initial impact than reflections two and three for the 1.832-m long post, respectively (or the 2.423-m long post). When the post is not directly clamped to the lab bench, the phase velocity remains relatively constant from 0 to 10000 Hz for all lengths. This indicates that when the post is clamped directly to the lab bench or with rubber pad boundaries, signal distortion is more prominent. Signal energy is dissipated through the boundaries and the phase velocity is increasingly altered. Nonconstant phase velocity negatively affects values determined for natural frequency or for wave velocity in length determination.
A summary of the frequency boundaries of posts with varying length and constant cross-section is found in Table 4.2.

Table 4.2. The equivalent radius and the velocity of the wave through the medium are used to assess the frequency boundary between longitudinal waves (frequencies less than boundary) and P-waves (frequencies greater than boundary) for posts which range in cross-section and constant length.

<table>
<thead>
<tr>
<th>Length</th>
<th>Cross-Section</th>
<th>Equivalent Radius</th>
<th>Velocity</th>
<th>Frequency Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.216 m</td>
<td>8.9 cm by 14.0 cm</td>
<td>6.3 cm</td>
<td>4406 m/s</td>
<td>70000 Hz</td>
</tr>
<tr>
<td>1.832 m</td>
<td>8.6 cm by 14.0 cm</td>
<td>6.2 cm</td>
<td>4148 m/s</td>
<td>67000 Hz</td>
</tr>
<tr>
<td>2.423 m</td>
<td>8.9 cm by 13.8 cm</td>
<td>6.3 cm</td>
<td>3929 m/s</td>
<td>63000 Hz</td>
</tr>
</tbody>
</table>

The frequency boundary is very large with respect to the recorded frequency range; therefore, the wave regime always exists in the longitudinal domain.

The coherence of the signal at high frequencies becomes more variable as length increases. The increase in length of post likely increases signal distortion, introduces more noise, which are dominantly present in high frequencies. The coherence from the 1.216-m long post signals lacks defined boundaries of high coherence and low coherence. As length increases, the boundaries between low and high begin to sharpen.

From the laboratory analysis of frequency dependent attenuation and phase velocity:

- Frequency dependent attenuation is largest when the post is directly clamped to the lab bench, followed by the post being clamped with rubber pad boundaries, and is the smallest when the post is not clamped to the lab bench, expect for the 2.442-m long wood post (largest cross-sectional area). This exception is likely due to the minimal attenuation of the 2.442-m long post in all conditions. The introduction of the clamp boundary allows for large signal energy dissipation through the boundaries. Increasing the cross-sectional area
reduced the attenuation with respect to frequency. Increasing the length of post increased the attenuation with respect to frequency.

- The phase velocity analysis demonstrated the effect of boundary conditions, cross-sectional area, and length. Posts not directly clamped to the lab bench consistently demonstrated the most constant phase velocity for all tested posts, suggesting a phase alteration due to the introduction of coupling the wood post system with rubber boundaries or with clamps and the table. The post with the largest cross-section displays the most constant phase velocity in all boundary conditions. As the cross-sectional area increases, the effect of the boundary condition is less impactful on signal distortion or phase velocity alteration. Increasing post length results in less constant phase velocity when the post is clamped to the lab bench with rubber pads or directly clamped to the lab bench. Post length increase does not largely affect the phase velocity deviation when the post is not clamped to the lab bench. Introduction of system coupling distorts signals and leads to phase velocity changes with respect to frequency, if the cross-sectional area is not large with respect to the wavelength.

- Signal coherence shows a strong relationship with cross-sectional area. Frequencies with low values of coherence are more easily recognizable in posts with large cross-sectional area. The relationship between coherence and number of signals is demonstrated, with results indicating nonlinearity or heterogeneity of the system. The effect of the boundary condition on coherence is not clear; however, normalized values of coherence suggest values closest to one when the post is clamped to the lab bench with rubber pad boundaries. The coherence results are also used to justify the attenuation best fit line. The highest values of coherence from every tested post under any boundary condition occurs between the
range of 0 to 4500 Hz indicating a verifiable link between the attenuation with respect to frequency estimates.

The principles of signal number increase and the effect on coherence are shown in Appendix G.

4.3.2. Field Testing

Longitudinal pulse field testing was completed on two wood posts (of equivalent geometry) and two steel posts (of equivalent geometry). The testing was completed on posts attached and detached from the highway guardrail. Typical results from wood post 11-38 and steel post 11-28 of County Highway F in Pewaukee, Wisconsin are presented.

**Attenuation.** The attenuation of steel and wood posts in the detached and attached state can be seen in Figures 4.34 to 4.35. Attached posts have larger values of decay compared to detached, and steel posts experience faster decay rates compared to the wood posts.

*Figure 4.34.* Attenuation results from testing 1.83-m long wide flange steel post. Testing completed under detached and attached conditions. Attenuation highest under attached conditions.
Figure 4.35. Attenuation results from testing 1.83-m long wood post, nominal cross-sectional dimensions: 15 cm by 20 cm. Testing completed under detached and attached conditions. Attenuation highest under attached conditions.

Coupling the post with the guardrail increases signal damping, and thus the attenuation. The longitudinal wave velocity of steel is greater than that of wood. The steel wave travels a greater distance over a shorter time, which may explain the larger decay rate when compared to wood posts in the field.

**Autocorrelation.** The applied autocorrelation function of the stacked signals of steel and wood posts in the detached and attached state can be seen in Figures 4.36 to 4.37. Steel posts show higher natural frequency values due to an increase in wave velocity. Steel is denser than wood, however steel has a much greater Young’s modulus than wood, resulting in a faster wave speed (Green et al., 1999; Luecke et al., 2005). In addition to peaks associated with the natural frequency of steel, peaks of lower amplitude occur (such as the peaks at approximately 0.3\cdot 10^{-3} s in Figure 4.36) which likely represent the second mode of vibration). These lower amplitude peaks do not occur in the wood post.
Figure 4.36. Autocorrelation results from testing 1.83-m long wide flange steel post. Testing completed under detached and attached conditions. Natural frequency determined as average distance between peaks from time zero with first peaks ignored. Intermediate peaks are likely reflective of higher modes of vibration present in the system.

Figure 4.37. Autocorrelation results from testing 1.83-m long wood post, nominal cross-sectional dimensions: 15 cm by 20 cm. Testing completed under detached and attached conditions. Natural frequency determined as average distance between peaks from time zero.
**Power Spectra Density.** The power spectra density of steel and wood posts in the detached and attached state can be seen in Figures 4.38 to 4.39.

![Power Spectra Density](image)

**Figure 4.38.** Power spectra density results from testing 1.83-m long wide flange steel post. Testing completed under detached and attached conditions.

![Power Spectra Density](image)

**Figure 4.39.** Power spectra density results from testing 1.83-m long wood post, nominal cross-sectional dimensions: 15 cm by 20 cm. Testing completed under detached and attached conditions.

Attached posts show lower amplitude peaks compared to detached posts. Steel posts show dominant higher frequency peaks compared to dominant lower frequency peaks for wood. This is
expected due to the natural frequency difference between the two posts. Maximum peak amplitude for wood posts is approximately four times greater than for steel posts. Steel posts experience larger energy densities at higher frequencies (not associated with dominant peaks) compared to the wood posts.

**Windowing.** The windowed results of steel and wood posts in the detached and attached state can be seen in Figures 4.40 to 4.41. Steel posts have smaller windows compared to wood posts due to the differences in natural frequency/wave velocity. Irrespective of detach or attach conditions, the window sizes remain relatively constant for each post.

![Figure 4.40](image_url)

**Figure 4.40.** Windowed reflections from testing 1.83 m wide flange steel post. Testing completed under detached and attached conditions. Alternating solid and dashed lines denote reflection windows.
Figure 4.41. Windowed reflections from testing 1.83-m long wood post, nominal cross-sectional dimensions: 15 cm by 20 cm. Alternating solid and dashed lines denote reflection windows.

**Window Power Spectra Density.** The windowed power spectra density results of steel and wood posts in the detached and attached state can be seen in Figures 4.42 to 4.43. The results for windowed power spectra density are heavily dependent on the material tested, demonstrating strong frequency domain signal similarly for steel posts or for wood posts.
Figure 4.42. Power spectra densities of windows two (PSD2) and windows three (PSD3) from testing 1.83-m long wide flange steel post. Testing completed under detached and attached conditions.

Figure 4.43. Power spectra densities of windows two (PSD2) and windows three (PSD3) from testing 1.83-m long wood post, nominal cross-sectional dimensions: 15 cm by 20 cm. Testing completed under detached and attached conditions.
Frequency Dependent Attenuation and Phase Velocity. The frequency dependent attenuation, phase velocity, and coherence of steel and wood posts in the detached and attached state can be seen in Figures 4.44 to 4.45.

(a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit.

(b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency.

(c) Coherence of reflections two (R2) with respect to reflections three (R3).

Figure 4.44. Attenuation with respect to frequency, coherence, and phase velocity results from testing 1.83-m long wide flange steel post. Testing completed under detached and attached conditions. (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).
Figure 4.45. Attenuation with respect to frequency, coherence, and phase velocity results from testing 1.83-m long wood post, nominal cross-sectional dimensions: 15 cm by 20 cm. Testing completed under detached and attached conditions. (a) Frequency attenuation of reflections two power spectra density (PSD2) divided by reflections three power spectra density (PSD3). Bold lines show the attenuation linear fit. (b) Phase velocity of reflections two (R2) with respect to reflections three (R3). Bold lines show the wave velocity of the natural frequency. (c) Coherence of reflections two (R2) with respect to reflections three (R3).

For steel posts, the detached condition results in a linear best fit with a negative attenuation coefficient. This is not theoretically possible indicating an error in the analysis or the data.
collection. For both posts, the attenuation of the detached condition post is less than the attached post. Unlike the attenuation with respect to time, the attenuation with respect to frequency is larger for the wood posts compared to the steel posts. This further emphasizes the concept of attenuation being strongly dependent on the distance traveled. Wood material characteristics may also lead to stronger energy dissipation, such as biodegradability.

The phase velocity assesses the variability of the wave speed with respect to frequency, and potential signal distortion. The detached steel post shows relatively constant phase velocity with respect to the natural frequency velocity from 3500 to 10000 Hz. The attached steel post shows phase velocity relatively constant from 3500 to 10000 Hz, however phase velocity is less than natural frequency velocity by approximately 1500 m/s. The variation of phase velocity from 0 to 3500 Hz of the steel posts in either the detached or attached states indicates potential signal distortion, which may limit the ability to accurately determine the natural frequency or the wave velocity. The wood post shows phase velocity centered about the natural frequency velocity in the detached and attached state; however, there is deviation from the natural frequency throughout the entire frequency spectrum. This also indicates potential signal distortion.

A summary of the frequency boundaries of steel and wood posts for longitudinal waves and P-waves is presented in Table 4.3.

Table 4.3. The equivalent radius and the velocity of the wave through the medium are used to assess the frequency boundary between longitudinal waves (frequencies less than boundary) and P-waves (frequencies greater than boundary) for posts which range in cross-section and constant length.

<table>
<thead>
<tr>
<th>Material</th>
<th>Length</th>
<th>Cross-Section Area</th>
<th>Equivalent Radius</th>
<th>Velocity</th>
<th>Frequency Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>1.83 m</td>
<td>0.0017 m²</td>
<td>2.4 cm</td>
<td>5272 m/s</td>
<td>224000 Hz</td>
</tr>
<tr>
<td>Wood</td>
<td>1.83 m</td>
<td>0.0300 m²</td>
<td>9.8 cm</td>
<td>3128 m/s</td>
<td>32000 Hz</td>
</tr>
</tbody>
</table>
The frequency boundary is large with respect to the recorded frequency range. The frequency boundary is larger for the steel post compared to the wood post due to the small cross-section area and the larger wave velocity. The wave regime always exists in the longitudinal domain.

Both steel and wood show poor results of coherence (does not equal one) across the entire frequency range. The is to be expected considering the testing method employed (hammer blows) resulting in limited signal similarly across individual signals collected. While it is expected that coherence decrease in the field, the results are worse when compared to the values obtained in laboratory testing. Strong coupling of the system to the soil with or without attachment to the guardrail may increase unwanted noise or introduce unwanted inputs into the system. Wood coherence results are observably better than that of steel which is surprising considering the discussed heterogeneity of wood materials, whereas steel exhibits homogeneity. This may indicate a low signal-to-noise ratio of steel which reduces coherence.

4.4. Lessons Learned and Engineering Recommendations

Lab and Field Testing:

- **Attenuation.** Attenuation decreases with increasing cross-sectional area. It may or may not increase with length increase. It is largest under when the post is directly clamped to the lab bench, followed by clamping with rubber boundaries and then no clamp directly applied to the post. Attenuation increases for steel compared to wood. Attenuation also increases when attached to guardrails.

- **Natural frequency.** Natural frequency increases as length increases. It is dependent on wave velocity, which is dependent on density and Young’s modulus. The natural frequency increases for steel posts (as wave velocity increases) compared to wood posts.
• **Power spectra density.** Larger amplitudes of high frequency energy are exhibited for increasing cross-sectional area or decreasing length. When the post is directly clamped to the lab bench, higher energies at higher frequencies are also experienced. Attached posts and steel posts exhibit more energy at higher frequencies compared to detached posts and wood posts, respectively. Peak amplitudes are larger for wood compared to steel.

• **Windowed power spectra density.** The windowed power spectra density is most dependent on the post tested, with similar results irrespective of boundary condition or attached/detached condition.

• **Frequency Dependent Attenuation.** The frequency dependent attenuation decreases with increasing cross-sectional area or decreasing length. It is largest when the post is directly clamped to the lab bench, followed by the post being clamped with rubber boundaries and then when no clamp is applied. The frequency dependent attenuation is greater when a post is attached to guardrail, as opposed to detached, and it is greater for steel compared to wood.

• **Phase velocity.** Phase velocity is less dependent on the effects of boundary condition (i.e., post place on lab bench, post secured to lab bench with rubber boundaries, or post directly clamped to lab bench) as the cross-sectional area increases. The phase and phase velocity are less constant as the length is increased, particularly when posts are clamped directly to the lab bench or clamped with rubber boundaries. Length increase has a minimal effect on phase when the post is not clamped to the lab bench.

• **Coherence.** Coherence improvement is most dependent on increasing cross-sectional area. There is some evidence to suggest increased coherence when the post is clamped with
rubber boundaries resulted in a semi-fixed state. Field testing indicates severe limitations in retrieving output signals from inputs under low frequency hammer blows.

Lessons Learned:

- Reproducible signals have a strong impact on results. Stacking of the signal removes unnecessary noise; however, when the recorded waveform has little similarity, the application of stacking has minimal effect. Low frequency hammer blows in the field limit signal reproducibility, indicating a need to improve the source in future field testing.

- Increasing cross-sectional area likely increases the coherence of the waveform by reducing the effects of dispersion and unwanted high frequency noise. As the cross-sectional area increase, the medium becomes closer to an “infinite” material. While increasing the area does not realistically create an infinite material, it does reduce the effect of signal distortion at the surrounding boundaries and the effective radius is increased with respect to the wavelength.

- Clamping posts directly to the bench or with rubber boundaries increases the signal attenuation and frequency attenuation compared to posts not clamped to the bench as energy is dissipated through the surrounding boundaries to a greater degree. Reduction in signal energy limits the analysis of the true natural frequency in the field.

Engineering Recommendations:

- Methods to increase signal reproducibility in the field should be explored. This is recommended in the previous chapter as well. The current method of hammer blows limits signal self-similarity and coherence compared to reproducible steel ball pendulum drops in
the laboratory. The lack of reproducibility negatively affects the prediction of length in the field.

- If wood posts of a variable cross-sectional area exist in the field, these should be tested to evaluate the results determined in the laboratory. It would be ideal to determine how the signal travels in the field under posts with large and small effective radius when coupled to the surrounding soil.

- Longitudinal propagation testing should be completed on steel posts in the lab to improve the understanding of the waveform during field testing with respect to the attenuation and the phase velocity.

- It is recommended to continue longitudinal propagation testing for field posts. Understanding and correcting field length prediction may improve if additional longitudinal information is incorporated into the analysis.
5. Magnetic Determination of Post Lengths

5.1. The Magnetic Phenomenon

The Earth’s magnetic field resembles that of a magnetic dipole with an axis slightly misaligned with respect to the direction to the axis of the Earth’s rotation. The directions of the magnetic poles are opposite that of geographic coordinates, in which geographic north is the south magnetic pole and vice versa. The origin of the Earth’s magnetism is due to the convection current of material in its liquid outer core. This convection current leads to the induction of a magnetic field that is commonly described as a permanent magnet (Reynolds, 2011). The first recorded scientific investigation into the Earth’s magnet was carried out by Sir William Gilbert (1540-1603) following the introduction of the lodestone to Europe from China by Marco Polo (Telford et al., 1998). The lodestone, and then the compass, allowed for improved navigation techniques. The development of magnetic tools was mostly limited to the magnetic balance until the start of the 20th century. The introduction of the magnetometer during WWII led to improved understanding of measuring magnetic techniques including in the detection of submarines (Muffly, 1946).

The Earth’s magnetic field magnitude ranges from 25,000 to 70,000 nT on the surface and changes with latitude and longitude. The magnetic field is a vector and is described with several components. In a compass, a magnetic needle orients itself in alignment with the direction of the geomagnetic field if no other magnetic field is present. The magnitude \( F_e \), the needle’s inclination angle \( I \) from the horizontal, and the declination angle \( D \) made from north allow for the complete definition of the geomagnetic vector (Telford et al., 1998). The horizontal \( H_e \) and the vertical \( Z_e \) components of the Earth’s field are the simplest way to deconstruct the Earth’s magnetic field. The
horizontal component may also be classified as positive north $X_e$ and positive east $Y_e$ components.

The geomagnetic components are summarized as:

\[
F_e^2 = H_e^2 + Z_e^2 = X_e^2 + Y_e^2 + Z_e^2
\]  
(5.1)

\[
H_e = F_e \cos I
\]  
(5.2)

\[
Z_e = F_e \sin I
\]  
(5.3)

\[
X_e = H_e \cos D
\]  
(5.4)

\[
Y_e = H_e \sin D
\]  
(5.5)

The values for the magnetic field $Z_e$, $X_e$, and $Y_e$ are scalars. The field may be converted into a vector through the following expression:

\[
\mathbf{F}_e = F_e (\cos D \cos I \mathbf{i} + \sin I \mathbf{j} + \sin D \cos I \mathbf{k})
\]  
(5.6)

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the N-S, E-W, and vertical directions.

The presence of the geomagnetic field has an impact on materials with magnetic properties through the induction of a local magnetic field (Hu et al., 2012). This results in a change in the local Earth’s magnetic field close to the anomaly (magnetic material). These changes in the local Earth’s magnetic field can be observed through a magnetometer.

When considering magnetic anomalies, the major components governing the response of the local magnetic field are dependent on the magnetic susceptibility $\kappa$ and the geometry of the anomaly. The magnetic susceptibility $\kappa$ is an intrinsic property of the material which can be characterized by the ability of the object to induce magnetization provided an external magnetic field. If no
remnant magnetization exits in the anomaly, the resulting induced magnetization $J$ and the geomagnetic field $F_e$ are linearly related as:

$$J = \kappa \left( \frac{F_e}{\mu_0} \right) = \kappa H$$

(5.7)

where $\mu_0$ is the magnetic permeability of free space $4\pi \times 10^{-7} \frac{H}{m}$ [or $\frac{Wb}{A\cdot m}$] and $H$ is the magnetizing force. Induced magnetization units are A-m$^2$/m$^3$ (= A/m). If no remnant magnetization is present, resultant magnetization is equal to the induced magnetization (Reynolds, 2011).

The magnetic susceptibility is related to the relative permeability $\mu_r$ through the expression:

$$\kappa = \mu_r - 1$$

(5.8)

In common geophysical analysis, $\mu_r$ is greater than one, however materials with weak susceptibility are analyzed in fields such as healthcare with magnetic resonance imaging (MRI) (Schenck, 1996). A list of common magnetic materials with their respective magnetic susceptibility and relative permeability is in Table 5.1.

**Table 5.1.** Magnetic susceptibility of common materials. Magnetic susceptibility of materials much greater than one strongly impact the surrounding magnetic field which may be easily recorded with magnetometers.

<table>
<thead>
<tr>
<th>Material</th>
<th>Magnetic Susceptibility, $\kappa$</th>
<th>Relative Permeability, $\mu_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>$-2 \times 10^{-7}$ to $-4 \times 10^{-7}$</td>
<td>0.99999998 to 0.9999996</td>
</tr>
<tr>
<td>Glass</td>
<td>$-1 \times 10^{-6}$</td>
<td>0.999999</td>
</tr>
<tr>
<td>Water (37°C)</td>
<td>$-9 \times 10^{-6}$</td>
<td>0.999991</td>
</tr>
<tr>
<td>Stainless Steel (nonmagnetic, austenitic)</td>
<td>$3.5 \times 10^{-3}$ to $6.7 \times 10^{-3}$</td>
<td>1.0035 to 1.0067</td>
</tr>
<tr>
<td>Magnetite (Fe$_3$O$_4$)</td>
<td>70</td>
<td>71</td>
</tr>
<tr>
<td>Stainless Steel (magnetic, martensitic)</td>
<td>400 to 1,100</td>
<td>401 to 1,101</td>
</tr>
<tr>
<td>Iron</td>
<td>200,000</td>
<td>200,001</td>
</tr>
</tbody>
</table>

Sources: (Ilic, 2001; Schenck, 1996)

Upon determination of the induced magnetization $J$, the dipole moment or magnetic moment $M$ is calculated as:
\[ M = J V = pl \]  

(5.9)

where \( V \) is the volume of the magnetic dipole, \( p \) is the magnetic pole strength, and \( l \) is the length separating the two poles (Telford et al., 1998; Reynolds, 2011). The units of dipole moment are A-m\(^2\). Classification of the dipole moment is simpler to assess in magnetic surveys when accounting for the material volume and the effect of magnetic susceptibility, as these values can be readily determined. Upon determination of the dipole moment \( M \), the field anomaly of a traverse along the declination line can be expressed as:

\[
F = \left( \frac{cM}{r^3} \right) \left[ (3\cos^2 I - 1)x^2 - 6xz_m \sin I \cos I + (3\sin^2 I - 1)z_m^2 \right]
\]  

(5.10)

\[
F = \left( \frac{cM}{r^3} \right) (3\cos^2 \theta - 1)
\]  

(5.11)

where \( c = \frac{\mu_0}{4\pi} \), \( r \) is the distance from the magnetized body, \( I \) is the inclination angle, \( x \) is the horizontal distance from magnetized body, \( z_m \) is the vertical distance from the magnetized body, and \( \theta \) is the angle from inclination to the direction vector \( r \). The vertical and horizontal components of the field may also be determined as:

\[
Z = \left( \frac{cM}{r^3} \right) \left[ (2z_m^2 - x^2) \sin I - 3xz_m \cos I \right]
\]  

(5.12)

\[
H = \left( \frac{cM}{r^3} \right) \left[ (2x^2 - z_m^2) \cos I - 3xz_m \sin I \right]
\]  

(5.13)

Utilizing Equation 5.11, the magnetic field anomaly may be determined if the traverse is completed along the declination line, and no remnant magnetism remains in the body of question. The declination line is achieved by orienting the survey in the direction of the declination angle. A schematic diagram of the traverse across a magnetic dipole moment is shown in Figure 5.1.
Under many magnetic surveys, the size and magnetic susceptibility of the anomaly are not definitively known, but must they be determined through data observation, characterizing an inverse problem. The understanding of body geometry, magnetic susceptibility, and the generation of magnetic anomalies informs the direction of this research. Data are collected for a known system, the expected results for the system are modelled using Equations 5.7 to 5.11, and comparison between the real and expected is carried out.

### 5.2. Magnetic Measurements

Magnetic techniques are predominantly utilized for surveying localized magnetic anomalies due to the presence of diamagnetic (slight negative susceptibility), paramagnetic (small susceptibility), ferromagnetic (high susceptibility), and/or ferrimagnetic (weak susceptibility) materials. Diamagnetic materials (no susceptibility) are not recognizable. The expansion of magnetic surveying occurred during WWII as means to detect submarines from aircrafts (Muffly, 1946).
This methodology eventually transitioning into an exploratory geophysical technique for geological surveying (Telford et al., 1998).

One of the first usages of magnetism for foundation evaluation was done utilizing a direct probe magnetometer with jetting to determine lengths of steel piles (Forrest, 1977). This technique relied on the principles of induced magnetization as previously discussed, measuring the change in magnetic field between sensors. Enacted for use in deep foundations, the inability to further the probe in the ground limited the results during testing, but the “probe appeared to perform adequately, and it did not have its performance impaired” suggesting a robust method for ferrous material detection within the earth (Forrest, 1977).

Nowadays, Induction Field (IF) testing is the predominant NDT magnetic method to determine depths of deep steel pile or steel-reinforced concrete foundations. This process requires borehole drilling in which an induction sensor is lowered near the foundation. The induction sensor creates a magnetic field which is disrupted by the presence of metal objects (Robinson & Webster, 2008). The sensor records the response of the magnetic field as it travels the length of the foundation, resulting in a strong signal change where a ferrous/non-ferrous material boundary exists.

Magnetic techniques are often dependent on the heterogenous distribution of magnetic properties. The consistent (with carefully controlled geometry) surveys of highway guardrail posts with induced magnetization provide the basis of the nonhomogeneous system of question, with changes in the magnetic field recorded by a high precession sensor. The concept of arrival and passing of relative permeability inhomogeneity has been applied to integrity testing of magnetic materials, such as thin sheet aluminum alloys, with success (Hu et al., 2012).
The focus of this research study is to use magnetic survey testing to characterize the length of posts with strong magnetic susceptibility markers.

5.3. Description of the Magnetic Testing Technique

Testing was completed using a Geometric G-858 cesium vapor magnetometer. A single sensor, mounted at 45°, was used to collect magnetic data next to guardrail posts. Depending on the survey, the magnetic sensor location with respect to the survey line varied both vertically and horizontally. The sensor is mounted on a long horizontal rod to maintain a horizontal level positioning, or the sensor can be mounted on an accessory backpack.

When surveying with a G-858, magnetic data are recorded at a rate as high as 10 samples per second. This requires the user to use some form of positioning procedure or instrument to properly align the collected data with geometric location. A magnetometer can be paired with a Global Position System (GPS) receiver to combine magnetic results with positioning. While the GPS system can be a powerful tool for magnetic surveys, GPS sensitivity limitations can negatively impact survey results. Within this research, the GPS demonstrated low coordinate resolution with respect to the magnetic material of interest: the highway guardrail posts. Therefore, a valuable mode of location positioning can be completed by manually marking positions throughout the survey and ending lines. This type of magnetic data collection may be called a simple survey.

A simple survey employs a pre-mapped grid, which the magnetometer user uses to correctly mark locations and end line. A limitation of this method is the inability to effectively change mark spacing or line spacing when processing the data with MagMap (the software used to read and analyze the data). The software pulls the results from the data acquisition system and requires immediate processing of the survey path by specifying a grid layout. The grid layout is created by
selecting a walking pattern, the position of the starting location, the mark spacing, and the line spacing. Thus, keeping constant spacings through the survey allows for a simpler analysis of the data. The use of linear interpolation in the software processing urges a constant walking speed during data collection to remove positioning error. Therefore, the post-processing of the magnetic data must be carefully considered before conducting the survey.

When testing highway guardrail posts, the simplest method of constraining the results to position is obtained by placing marks as the sensor encountered individual guardrail posts. This approach improves location resolution.

5.4. Methodology

5.4.1. Data Collection

To begin a guardrail magnetic survey, the distance from the start of the survey to the individual posts is measured and recorded. A user is then outfitted with the magnetometer sensor, data acquisition system, and battery pack. Sensor positioning and GPS coupling are dependent on the survey and the anticipated results. A simple survey should be specified on the system for data collection. The user then slowly walks at a constant speed (< 1 m/s) alongside the length of the guardrail survey in one direction. Upon finishing the line, the guardrail line is re-surveyed several times. It is likely that magnetic noise is inevitable due to the proximity to roadways and passing by vehicles (with ferromagnetic metals). Therefore, completing several magnetic surveys of the same guardrail under similar sensor testing setup allows for data stacking and noise reduction.

Properly collected data, with minimal error and consistent results, was processed through a combination of the MAGMAP software and MATLAB data manipulation. The data were then
compared to constructed models (which are reliant on magnetic theory). The models attempt to fully represent the influence of magnetic anomalies on the tested site.

5.4.2. Data Processing

During data collection, a mark was placed whenever the sensor passed next to a guardrail post. By measuring the distance from the start of the magnetic survey line to the individual guardrail posts, the marks inputted during data collection are then linked to the correct positioning. After completing data processing of individual survey lines, correctly surveyed lines (such as those without magnetic gradient failures) are stacked to reduce noise. To isolate the effect of the magnetic anomalies due to the steel posts, the magnetic regional gradient is removed from the total field. The regional gradient is the variation in the magnetic field with respect to position (Hornak, 1996). The regional gradient is largely due to the natural magnetic susceptibility of the subsurface. In the case of the surveys from this research, the regional gradient captured is also due to the guardrail, electrical wiring, magnetic material from nearby bridges, and so on. To remove the regional gradient, a moving average is subtracted from the stacked data.

5.4.3. Magnetic Modeling

To assess the data collected in the field, magnetic models are built to compare expected and real anomalies. If the model matches results obtained in the field, in which the specifications of the highway guardrail are known, the method may be successful in predicting variations in steel post length for guardrails with unconfirmed specifications.

In building the model, proper characterization of the magnetic inclination and declination with respect to the survey line of the guardrail is the first step required. If the degree of difference between the angle of declination and the survey line is small (less than 5°), Equation 5.11 may be
used with limited error. If the degree of difference between the angle of declination and the survey line is greater than 5°, the magnetic field and inclination must be matched to successfully use the modelling equations. Irrespective of survey orientation, the effect of the vertical magnetic field remains constant, however the horizontal magnetic field changes. If the survey is completed ±90° from the declination angle, the horizontal magnetic field has no effect on the induced magnetic field as it is perpendicular. The effective inclination $I'$ and the effective field strength $F'$ allows for the correct interpretation of the observed magnetic anomaly. When considering a survey direction $\alpha$ with respect to declination $D$:

$$I' = I + \left| \frac{\alpha - D}{90^\circ} \right| (90^\circ - I)$$  \hspace{1cm} (5.14)

$$H' = F \cos I'$$  \hspace{1cm} (5.15)

$$F' = \sqrt{H'^2 + Z^2}$$  \hspace{1cm} (5.16)

where north is 0° or 360°, $0^\circ \leq \alpha \leq 360^\circ$ and $0^\circ \leq D \leq 360^\circ$ to limit angle calculation errors. Typically, declination to the west is reported as a negative angle which must be converted to the positive angle. As the effective inclination angle approaches 90°, the effect of the horizontal magnetic field is minimized. As the angle between the survey direction and the declination exceeds 90°, the contribution of the horizontal magnetic field reverses.

**Steel Post Anomaly.** To model the steel post anomaly, the geometry and positioning of the guardrail with respect to the magnetic sensor must be known as seen in **Figure 5.2.** The location of sensor monitoring height $H_t$, and known (or assumed) height segments of the post $H_a$ and $H_b$ are used to assess the distance from the sensor to the base of the post $z$. 

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The steel beam is divided into 50 equilateral segments along the length of the post. The purpose of this division is to assess the distribution of dipoles and the effect on the magnetic field. Segments located closer to the sensor have a stronger effect on the anomaly, whereas segments located further have a lesser effect, as evidenced by the distance $r$ in Equation 5.11. The volume of the individual segments and the position with respect to the sensor determines the anomaly of single segment magnetic dipole. The effect of all segments is summed to determine the total anomaly due to the post. The standard W 6 X 9 wide flange beam cross-sectional area (17.4 cm$^2$) used in the field is employed to calculate segment volume. Known geometric properties of this post are denoted in Figure 5.3.
The model is compared to field results. Therefore, the total field strength, the inclination, and the declination must be determined at the magnetic survey location used for comparison. The survey angle direction from declination is also recorded and used in the model. A summary of parameters used in the magnetic model of the site is listed in Table 5.2.

Table 5.2. Parameters used in magnetic anomaly modeling of steel posts.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total field strength</td>
<td>$F$</td>
<td>47,255.0</td>
<td>nT</td>
</tr>
<tr>
<td>Inclination (+ D</td>
<td>– U )</td>
<td>$I$</td>
<td>60.8403</td>
</tr>
<tr>
<td>Declination (+ E</td>
<td>– W )</td>
<td>$D$</td>
<td>−14.5774</td>
</tr>
<tr>
<td>Survey direction from declination</td>
<td>$\alpha$</td>
<td>14</td>
<td>deg</td>
</tr>
<tr>
<td>Sensor height</td>
<td>$H_i$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Post height above ground</td>
<td>$H_a$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Post height below ground</td>
<td>$H_b$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Distance from instrument to post base</td>
<td>$z$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Steel magnetic susceptibility</td>
<td>$\kappa$</td>
<td>15,000</td>
<td>unitless</td>
</tr>
<tr>
<td>Number of segments</td>
<td>$n$</td>
<td>50</td>
<td>unitless</td>
</tr>
<tr>
<td>Segment length</td>
<td>$l$</td>
<td>$(H_a + H_b)/n$</td>
<td>m</td>
</tr>
<tr>
<td>Steel post area</td>
<td>$A$</td>
<td>17.4</td>
<td>cm²</td>
</tr>
</tbody>
</table>

**Wooden Post with Steel Base Plate.** To model the anomaly of a wooden post with steel base plates, the same procedure for steel posts is employed, with a schematic shown in Figure 5.4.
However, as the steel base plate is a small, prismatic volume compared to the steel post, the magnetic dipole of concern is not split into segments. The magnetic susceptibility of the steel base plate is assumed to be equivalent to that of steel posts. The area of the steel plate is equal to the cross-sectional area of the timber post. Nominal 15 cm by 20 cm posts typically used for guardrails have cross-sectional area of 300 cm$^2$. The thickness of the base plates is assumed. In addition to steel base plate considerations, the model must also incorporate the assumed steel screws used to secure the wooden post to the guardrail. The steel screw has an impact on the magnetic anomaly and must be separately accounted for in the model. The susceptibility of the screw is assumed to be equal to that of other steel materials. The total anomaly is equal to the anomaly due to base plate and the anomaly due to the screws. The effect of the screw is substantial when the steel plate increases in distance from the sensor. Depending on the length of the wood post, prediction of low embedment depth is simpler than large embedment depth. The closer the steel plate to the sensor, the larger the effect of the anomaly due to the base plate. A summary of parameters from the steel base plate magnetic model is provided in Table 5.3.
Table 5.3. Parameters used in magnetic anomaly modeling of wooden posts with steel base plates.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total field strength</td>
<td>$F$</td>
<td>47,255.0</td>
<td>nT</td>
</tr>
<tr>
<td>Inclination (+ D</td>
<td>– U )</td>
<td>$I$</td>
<td>60.8403</td>
</tr>
<tr>
<td>Declination ( + E</td>
<td>– W )</td>
<td>$D$</td>
<td>−14.5774</td>
</tr>
<tr>
<td>Survey direction (CW from N)</td>
<td>$\alpha$</td>
<td>14</td>
<td>deg</td>
</tr>
<tr>
<td>Sensor height</td>
<td>$H_i$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Post height above ground</td>
<td>$H_a$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Post height below ground</td>
<td>$H_b$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Distance from instrument to post base</td>
<td>$z$</td>
<td>varies</td>
<td>m</td>
</tr>
<tr>
<td>Steel magnetic susceptibility</td>
<td>$\kappa$</td>
<td>1,000</td>
<td>unitless</td>
</tr>
<tr>
<td>Number of segments</td>
<td>$n$</td>
<td>1</td>
<td>unitless</td>
</tr>
<tr>
<td>Base plate area</td>
<td>$A_{plate}$</td>
<td>300</td>
<td>cm²</td>
</tr>
<tr>
<td>Base plate thickness</td>
<td>$t$</td>
<td>2.5</td>
<td>cm</td>
</tr>
<tr>
<td>Steel screw radius</td>
<td>$r_{screw}$</td>
<td>0.5</td>
<td>cm</td>
</tr>
<tr>
<td>Steel screw length</td>
<td>$L_{screw}$</td>
<td>23</td>
<td>cm</td>
</tr>
</tbody>
</table>

5.4.4. Field Data and Model

The goal of collecting field data and building a magnetic model to encompass the characteristics of the field is for comparative measures. Assuming the geometry and spatial distribution of posts is known, and the magnetic model is accurate, the model should match the field results to a high degree of similarity. If the model matches the magnetic survey, it may then be used to predict when post length deviates from specifications.

Magnetic cleanliness was practiced, the survey was carried out systematically, and the generated models have utilized all known information of the guardrail system to build a robust prediction tool. Despite the care taken to ensure the best collection of data and modelling process, there were errors in the magnetic surveys and the inputs of the model may not have fully represented all magnetic activity of the area. The magnetometer height from the base of the post is assumed constant in the model, yet the walking process and sloping terrain ensure that the sensor height is not perfectly constant throughout the survey. Regional filtering of the data is completed to allow
for anomaly determination; but information might be lost in the process if over-filtering occurs. The flux of magnetic cars passing also imparted unwanted results. The model requires an input for magnetic susceptibility; however, susceptibility is only determined through the application of the inverse problem and is not explicitly known. Ultimately, there are many factors which influence the results presented in the following section.

5.5. Results

Steel posts data collection and the resulting anomalies are compared to a model built to encompass the spatial geometry of the guardrail system. Wood posts with steel base plates and steel screws are modelled to show the impact of changing distance from magnetometer to baseplate on depth determination.

5.5.1. Vertical Steel Posts Survey

The guardrail magnetic survey presented in this section was conducted on County Highway F in Pewaukee, WI. Location information regarding the location of the guardrail surveyed is shown in Figure 5.5. Magnetic surveying was completed with the magnetometer sensor mounted on a Geometric backpack (the height of the instrument is 2.0 m). During testing, care was taken to allow the sensor to pass as closely as possible over the individual metal posts, with as little horizontal distance change from the post to the sensor. Four lines, denoted by different colors, were travelled from south to north. The survey began at 0 m and encountered the metallic guardrail at 10 m distance. Metal posts were not encountered until after 25 m. The end of the survey line is located at 75 m.
Figure 5.5. Location information for magnetic guardrail surveys. Latitude, longitude: 43.076474 N, 88.205978 W. (a) Red marker shows location of guardrail on state-scale. (b) Red marker shows location of guardrail on city-scale. (c) Red line shows the guardrail surveyed, and the direction of the survey, yellow box denotes the guardrail.

The end of the survey line is also marked by the onset of a concrete bridge with a steel embedded (i.e., rebar, steel beams, etc.). This is reflected in the results as the magnetic field begins to increase substantially, which also reduces the effect of the individual steel posts. Processed field results from County Highway F guardrail in Pewaukee, WI are shown in Figure 5.6.
The data profiles are stacked to improve the signal-to-noise ratio. Then, a 200-point moving average across the data set is applied to characterize the regional magnetic gradient across the guardrail. Assessment of the regional gradient is required to remove the effect of undesirable magnetic bodies (such as the magnetic guardrail, steel from the bridge, subsurface material, etc.). Data stacking and the moving mean are shown in Figure 5.7. The location of the steel posts along the length of the survey are denoted as blue dashed vertical lines to demonstrate the signal peaks as magnetic anomaly is encountered. Removing the regional gradient from the stacked data isolates steel posts anomaly peaks, as shown in Figure 5.8. The magnetic field anomaly results are used for comparison to the model results. Results for peak amplitude ranges from approximately 500 to 1000 nT. This variation in peaks may be due to changing post geometry, the effects of filtering out the regional gradient, or intrinsic error during the surveying process.
Figure 5.7. Magnetic field results from Pewaukee, WI, County Highway F stacked vertical survey lines. Moving mean characterizes the magnetic regional gradient. Steel post locations denoted.

Figure 5.8. Magnetic anomaly results from Pewaukee, WI, County Highway F vertical survey steel posts after removing 200-pt moving mean from total magnetic anomaly. Steel post locations denoted.

With data provided by the Wisconsin Department of Transportation (WisDOT), the spatial distribution of steel posts, the length of steel posts, and the embedment depths of posts are used to model the expected individual steel post anomalies as seen in Figure 5.9.
Along the guardrail, the embedment depth and length of nine out of the 33 steel posts is known. Interpolation of embedment depth and length between known posts is required to estimate the spatial geometry of the remaining 24 unknown posts. Individual post anomalies are denoted by different colors in Figure 5.9. Due to the small distance of the steel posts, overlap between the anomalies is to be expected. When anomaly overlap is too great, such as the posts grouped around the 70 m mark (±5 m), individual anomaly isolation is not possible with the final model. The individual anomalies are summed to generate a total field anomaly, the results of which are shown in Figures 5.10. The total field anomaly is a representation of the magnetic effect the steel post arrangement would have on a magnetometer in a perfectly constant magnetic field. That is, the only effect on the magnetic field is due to the steel posts, and the regional gradient is only due to the constant magnetic field. The magnetic field is not constant at the location of interest due to the guardrail itself, potential nearby or near-surface magnetic objects, and the continuous presence of cars. The results obtained from the magnetic survey are affected by all changes in the magnetic field (not only changes due to the steel posts).
Figure 5.10. Modeled total magnetic anomaly for Pewaukee, WI, County Highway F vertical survey steel posts. 500-pt moving mean shown to demonstrate the isolation of peaks and valleys within the model. Steel post locations denoted.

Application of a moving mean to isolate the peaks and valleys from the model is required as the large scale (±2.5 m) magnetic field change is filtered out from the magnetic survey data. Removal of the moving average profile isolates the steel post magnetic anomaly peaks and valleys, resulting in Model 1 (Figure 5.11).

Figure 5.11. Model 1 of magnetic anomaly for Pewaukee, WI, County Highway F vertical survey steel posts after removing 500-pt moving mean from total magnetic anomaly. Results to be used in comparison with magnetic survey magnetic anomaly. Steel post locations denoted.
In locations with small steel post spacing, anomaly effects overlap (i.e., the results from distances 65 m to 75 m). As previously stated, decreasing steel post spacing limits detection of individual anomalies. This is evidenced by the lack of peaks at steel post locations near the 70 m mark. As posts distance decreases, the anomaly overlap is too large, and individual post anomalies are unrecognizable.

The final model is compared to the processed field data (Figure 5.12). From the comparison, Model 1 captures the location of the steel posts well when the distance between posts is large enough to prevent anomaly overlap (as shown from 20 to 60 m).

![Figure 5.12](image)

**Figure 5.12.** Comparison of Model 1 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 1 is filtered with 500-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 15000$. Field data is filtered with 200-pt regional gradient removal.

The value of magnetic susceptibility found to most accurately fit the field results is approximately 15,000, which is considerably larger than values found in the literature for stainless steel (Schenck, 1996). In Figure 5.13, the length of the magnetic survey shown is from 20 to 60 m, where the distance between posts was large enough to show individual peak anomalies. To capture the error of the model and data comparison, the Root Mean Square Error (RMSE) is calculated:
\[ RMSE = \sqrt{\sum_{i=1}^{N} (y_{pi} - y_{oi})^2 / N} = \sqrt{\sum_{i=1}^{N} (error_i)^2 / N} \]  

(4.17)

where \( i \) is the sample, \( y_{pi} \) is the model predicted value, \( y_{oi} \) is the observed value, \( N \) is number of samples, and \( error_i \) is the difference from the model and field \((y_{pi} - y_{oi})\).

**Figure 5.13.** Comparison of Model 1 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 1 is filtered with 500-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as \( \kappa = 15000 \). Field data is filtered with 200-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 1 compared to the field data, from post 1 (27 m) to post 17 (59 m) is 224 nT.

As **Figure 5.13** shows, the locations of steel posts 1 through 17 are captured by the model and the data to a high degree. The general geometry and spatial orientation of the posts can also be inferred. Posts 1, 4, 6, 8, 9, 10, 11, and 14 show similar peaks in the model and the field results. Posts with varying model and field results peak amplitudes indicate a divergence. Assuming the testing protocol was followed correctly with a constant height of instrument above ground, the field results were processed correctly, and the geometry of the system has been correctly accounted for, posts 2, 3, 5, 7, 12, 13, 15, and 17 would indicate a potential field post geometry discrepancy.
An attempt to model the discrepancies by altering the distance from the instrument to the top of the posts is shown in Figure 5.14 as Model 2.

![Figure 5.14](image)

**Figure 5.14.** Comparison of Model 2 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 2 adjusts distance from instrument to post and is filtered with 500-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 15000$. Field data is filtered with 200-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 2 compared to the field data, from post 1 (27 m) to post 17 (59 m) is 191 nT (an error reduction of 33 nT from Model 1). The resulting Model 2 data magnetic anomaly peaks more closely align with the field data peaks when compared to Model 1. Model 2 demonstrates the importance of constant distance between the sensor and the ground and taking accurate measurements of the distance from the ground to the top of steel posts. The changes made to the height of the steel posts are summarized in Table 5.4. The height change is used to alter the distance from the sensor to the top of the steel posts while maintaining the length of the steel posts. The Root Mean Square (RMS) misfit may be determined for the height change. The RMS misfit of the 17 posts height changes is 5.6 cm.
Table 5.4. Height change of individual steel posts to create Model 2. Negative height change: increase in distance from the sensor to the top of the post. Positive height change: decrease in distance from sensor to top of post.

<table>
<thead>
<tr>
<th>Steel Post</th>
<th>Height Change</th>
<th>Steel Post</th>
<th>Height Change</th>
<th>Steel Post</th>
<th>Height Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5 cm</td>
<td>7</td>
<td>-8 cm</td>
<td>13</td>
<td>-4 cm</td>
</tr>
<tr>
<td>2</td>
<td>-15 cm</td>
<td>8</td>
<td>-3 cm</td>
<td>14</td>
<td>-4 cm</td>
</tr>
<tr>
<td>3</td>
<td>+3 cm</td>
<td>9</td>
<td>+3 cm</td>
<td>15</td>
<td>-5 cm</td>
</tr>
<tr>
<td>4</td>
<td>+1 cm</td>
<td>10</td>
<td>+3 cm</td>
<td>16</td>
<td>+0 cm</td>
</tr>
<tr>
<td>5</td>
<td>+5 cm</td>
<td>11</td>
<td>+3 cm</td>
<td>17</td>
<td>-6 cm</td>
</tr>
<tr>
<td>6</td>
<td>-1 cm</td>
<td>12</td>
<td>+8 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In addition to altering the distance between the instrument and the post, the length of the steel posts may also be changed to indicate potential anomalies.

Model 3, shown in Figure 5.15, captures the deviations in length from the model and the field results, while maintaining the expected distance from the instrument to the ground.

Figure 5.15. Comparison of Model 3 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 3 adjusts post lengths and is filtered with 500-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 15000$. Field data is filtered with 200-pt regional gradient removal. Locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 3 compared to field data, from post 1 (27 m) to post 17 (59 m) is 199 nT (an error reduction of 25 nT from Model 1). The resulting Model 3 data magnetic anomaly peaks more closely align with the field data peaks when compared to Model 1. Matching post length to large
peak differences where field data is greater than the model has limitations (such as in post 12) due to the relationship between distance and magnetic anomaly. The changes made to steel post length are summarized in Table 5.5. Length change is used to increase or decrease the volume of steel at the base of the post by adjusting the total length. The height of the instrument is maintained and the distance from the sensor to the post top is maintained. The RMS misfit of the 17 posts length changes is 1.25 m.

**Table 5.5.** Length change of individual steel posts to create Model 3. Negative length change: decrease in post length. Positive length change: increase in post length.

<table>
<thead>
<tr>
<th>Steel Post</th>
<th>Length Change</th>
<th>Steel Post</th>
<th>Length Change</th>
<th>Steel Post</th>
<th>Length Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.76 m</td>
<td>7</td>
<td>-1.07 m</td>
<td>13</td>
<td>-1.07 m</td>
</tr>
<tr>
<td>2</td>
<td>-1.37 m</td>
<td>8</td>
<td>-0.91 m</td>
<td>14</td>
<td>-1.07 m</td>
</tr>
<tr>
<td>3</td>
<td>+0.61 m</td>
<td>9</td>
<td>+0.00 m</td>
<td>15</td>
<td>-1.22 m</td>
</tr>
<tr>
<td>4</td>
<td>-0.61 m</td>
<td>10</td>
<td>+0.15 m</td>
<td>16</td>
<td>-0.91 m</td>
</tr>
<tr>
<td>5</td>
<td>+2.74 m</td>
<td>11</td>
<td>-0.30 m</td>
<td>17</td>
<td>-1.07 m</td>
</tr>
<tr>
<td>6</td>
<td>-0.61 m</td>
<td>12</td>
<td>+2.74 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Changing the distance from the instrument to the post to the sensor results in a lower degree of error in the model (191 nT) compared to changing the lengths of the steel posts (199 nT), however the difference in error between the two model alterations is negligible (<10 nT difference). Both model alterations improve the accuracy of the prediction tool, indicating intrinsic error in data collection or modelling.

**Regional Gradient Change.** Alteration of the regional gradient produces different comparison results. In the previous data and models, the field data is filtered with a 200-pt moving mean and the modelled data is filtered with a 500-pt moving mean. Changing regional gradients to a smaller moving mean emphasizes the small-scale variation of magnetic anomalies. The resulting anomaly comparison under new 100-pt moving mean regional gradients is shown in Figure 5.16 from distance 20 m to 60 m, as Model 4.
Figure 5.16. Comparison of Model 4 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 4 is filtered with 100-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 15000$. Field data is filtered with 100-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 4 compared to the field data, from post 1 (27 m) to post 17 (59 m) is 77 nT.

Reducing the number of points in the moving average decreases the amplitude of the captured anomaly, resulting in a lower RMSE from that determined in Model 1. Discrepancies comparing Model 4 to the field data are like that of Model 1.

Similar to Model 2, the distance from the instrument to the top of the posts is altered to reduce error. The result is shown in Figure 5.17 as Model 5.
Figure 5.17. Comparison of Model 5 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 5 adjusts distance from instrument to post is filtered with 100-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 15000$. Field data is filtered with 100-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 5 compared to the field data, from post 1 (27 m) to post 17 (59 m) is 67 nT (an error reduction of 10 nT from Model 4). The changes made to the height of the steel posts are summarized in Table 5.6. The RMS misfit of the 17 posts height changes is 7.2 cm.

Table 5.6. Height change of individual steel posts to create Model 5. Negative height change: increase in distance from the sensor to the top of the post. Positive height change: decrease in distance from sensor to top of post.

<table>
<thead>
<tr>
<th>Steel Post</th>
<th>Height Change</th>
<th>Steel Post</th>
<th>Height Change</th>
<th>Steel Post</th>
<th>Height Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+10 cm</td>
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<td>-8 cm</td>
<td>13</td>
<td>-6 cm</td>
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<td>-18 cm</td>
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<tr>
<td>3</td>
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<td>16</td>
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<td>-1 cm</td>
<td>12</td>
<td>+10 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Similar to Model 3, the length of steel posts is altered to reduce error. The result is shown in Figure 5.18 as Model 6.
Figure 5.18. Comparison of Model 6 to field collected vertical survey data. Instrument height 2.0 m above ground with no horizontal offset from guardrail posts. Model 6 adjusts post lengths with 100-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 15000$. Field data is filtered with 100-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 6 compared to field data, from post 1 (27 m) to post 17 (59 m) is 68 nT (an error reduction of 9 nT from Model 4). The changes made to steel post length are summarized in Table 5.7. The RMS misfit of the 17 posts length changes is 1.27 m.

Table 5.7. Length change of individual steel posts to create Model 6. Negative length change: decrease in post length. Positive length change: increase in post length.

<table>
<thead>
<tr>
<th>Steel Post</th>
<th>Length Change</th>
<th>Steel Post</th>
<th>Length Change</th>
<th>Steel Post</th>
<th>Length Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.76 m</td>
<td>7</td>
<td>-1.07 m</td>
<td>13</td>
<td>-1.07 m</td>
</tr>
<tr>
<td>2</td>
<td>-1.37 m</td>
<td>8</td>
<td>-0.91 m</td>
<td>14</td>
<td>-1.07 m</td>
</tr>
<tr>
<td>3</td>
<td>+0.61 m</td>
<td>9</td>
<td>+0.00 m</td>
<td>15</td>
<td>-1.22 m</td>
</tr>
<tr>
<td>4</td>
<td>-0.61 m</td>
<td>10</td>
<td>+0.15 m</td>
<td>16</td>
<td>-0.91 m</td>
</tr>
<tr>
<td>5</td>
<td>+2.74 m</td>
<td>11</td>
<td>-0.30 m</td>
<td>17</td>
<td>-1.07 m</td>
</tr>
<tr>
<td>6</td>
<td>-0.61 m</td>
<td>12</td>
<td>+2.74 m</td>
<td>\</td>
<td></td>
</tr>
</tbody>
</table>

Changing the lengths of the steel posts results in a lower degree of error in the model (65 nT) compared to changing the distance from the instrument to the post to the sensor (67 nT), however the difference in error between the two model alterations is negligible (< 3 nT difference).
model alterations (due to changing the distance from the instrument to the post or changing the length of the posts) improve the accuracy of the prediction tool, indicating intrinsic error in data collection or modelling.

A summary of the six models presented, the applied regional gradient moving point average removals, the error in distance from instrument to post, the error in post length, and the total RMSE can be found in Table 5.8.

Table 5.8. Results of models of vertical magnetic survey data, with height of instrument 2.0 m above ground and no horizontal offset.

<table>
<thead>
<tr>
<th>Model</th>
<th>Model Moving Point Average</th>
<th>Data Moving Point Average</th>
<th>Geometry Adjustment</th>
<th>Misfit</th>
<th>Model RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>224 nT</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>200</td>
<td>Instrument to Post Distance</td>
<td>5.6 cm</td>
<td>191 nT</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>200</td>
<td>Post Length</td>
<td>125 cm</td>
<td>199 nT</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>77 nT</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100</td>
<td>Instrument to Post Distance</td>
<td>7.2 cm</td>
<td>67 nT</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>100</td>
<td>Post Length</td>
<td>127 cm</td>
<td>68 nT</td>
</tr>
</tbody>
</table>

The assessed error in vertical survey steel post models suggests limited workability of magnetic anomaly prediction tool for field length predictions as is currently devised. Small-scale variation in the length at the base of the steel post is likely too difficult to assess in the model provided for vertical surveys. Due to the exponential relationship of distance on the effect of magnetic material of the magnetic field, adjusting the survey to have horizontal offset from the guardrail, while lowering the height of the instrument may provide a more ideal approach to predicting post length.

5.5.2. **Horizontal Steel Posts Survey**

In addition to the vertical survey completed at County Highway F, in Pewaukee, WI, a horizontal offset survey of the same location and type was conducted. The instrument was placed closer to the ground without exceeding the tolerable magnetic gradient. Similar methodology was
completed to process and compare field data and modelled data as discussed in the previous section. Magnetic surveying was completed with the magnetometer sensor mounted on a horizontal rod 1.1 m above the ground and located approximately 0.35 m to 0.40 m west of the guardrail posts. Three survey lines are stacked to improve the signal-to-noise ratio. Then, a 300-point moving average across the data set is applied to characterize the regional magnetic gradient across the guardrail. Data stacking and the moving mean are shown in Figure 5.19.

![Magnetic field results from Pewaukee, WI, County Highway F stacked for horizontal offset survey lines. Moving mean characterizes the magnetic regional gradient. Steel post locations denoted.](image)

**Figure 5.19.** Magnetic field results from Pewaukee, WI, County Highway F stacked for horizontal offset survey lines. Moving mean characterizes the magnetic regional gradient. Steel post locations denoted.

Removing the regional gradient from the stacked data isolates steel posts anomaly peaks which are compared to model results. The spatial distribution of the system can be modelled with individual steel post anomalies summed to generate a total field anomaly, the results of which are shown in **Figures 5.20.**
Figure 5.20. Modeled total magnetic anomaly for Pewaukee, WI, County Highway F steel posts for horizontal offset survey. 550-pt moving mean shown to demonstrate the isolation of peaks and valleys within the model. Steel post locations denoted.

Removal of the moving mean isolates the steel post magnetic anomalies for comparison, generating Model 7. The value of magnetic susceptibility found to most accurately fit the field results is approximately 700, which is within the range of values found in the literature for stainless steel (Schenck, 1996). In Figure 5.21, the length of the magnetic survey shown is from 20 to 60 m, where the distance between posts was large enough to show individual peak anomalies.
Figure 5.21. Comparison of Model 7 to field collected horizontal offset survey data. Instrument height 1.1 m above ground with 0.35 m horizontal offset from guardrail posts. Model 7 is filtered with 550-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 700$. Field data is filtered with 300-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 7 compared to the field data, from post 1 (27 m) to post 17 (59 m) is 406 nT.

As Figure 5.21 shows, the locations of steel posts 1 through 17 are captured by the model and the data to a high degree. The general geometry and spatial orientation of the posts can also be inferred. Posts 1, 5, and 9 show similar peaks in the model and the field results. The remaining posts with varying model and field results for peak amplitudes indicate a divergence.

The horizontal offset of the instrument with respect to the post may be adjusted to assess the potential variation in horizontal distance from the sensor to the post. The result is shown in Figure 5.22 as Model 8.
Figure 5.22. Comparison of Model 8 to field collected horizontal offset survey data. Instrument height 1.1 m above ground with 0.35 m horizontal offset from guardrail posts. Model 8 adjusts horizontal offset from post with 550-pt regional gradient removal. The magnetic susceptibility of steel posts is modelled as $\kappa = 700$. Field data is filtered with 300-pt regional gradient removal. The locations of the first 17 steel posts along the length of the survey are shown.

The RMSE of Model 8 compared to the field data, from post 1 (27 m) to post 17 (59 m) is 364 nT (an error reduction of 42 nT from Model 7). The changes made to the horizontal offset are summarized in Table 5.9. The RMS misfit of the 17 posts height changes is 11 cm.

<table>
<thead>
<tr>
<th>Steel Post</th>
<th>Offset Change</th>
<th>Steel Post</th>
<th>Offset Change</th>
<th>Steel Post</th>
<th>Offset Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+0 cm</td>
<td>7</td>
<td>+15 cm</td>
<td>13</td>
<td>+15 cm</td>
</tr>
<tr>
<td>2</td>
<td>+20 cm</td>
<td>8</td>
<td>+4 cm</td>
<td>14</td>
<td>+13 cm</td>
</tr>
<tr>
<td>3</td>
<td>+10 cm</td>
<td>9</td>
<td>+0 cm</td>
<td>15</td>
<td>+15 cm</td>
</tr>
<tr>
<td>4</td>
<td>+15 cm</td>
<td>10</td>
<td>+4 cm</td>
<td>16</td>
<td>+3 cm</td>
</tr>
<tr>
<td>5</td>
<td>-3 cm</td>
<td>11</td>
<td>+11 cm</td>
<td>17</td>
<td>+6 cm</td>
</tr>
<tr>
<td>6</td>
<td>+15 cm</td>
<td>12</td>
<td>-5 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The assessed horizontal offset error is entirely possible due to the existing slope of the survey line and inevitable deviation from the line. Relatively small adjustments to the horizontal offset results in large changes to the model data, demonstrating the strong ability to model peak behavior along
the horizontal offset survey line. The valleys of the field data are not modelled well due to the lack of valleys in the total field anomaly model.

The assessed error in the horizontal offset steel post models suggests limited workability of magnetic anomaly prediction tool for field length predictions. Peak behavior is easily captured by deviation of the horizontal distance offset; however, valley behavior alteration is limited due to the expected strong peaks and low valleys of modelled posts. Small-scale variation in the length at the base of the steel post may be measurable for horizontal offset surveys, however the current results are inconclusive.

5.5.3. **Horizontal Offset Numerical Analysis**

To assess the relative effect of increasing the horizontal survey offset from the survey line, a numerical analysis was performed. **Figure 5.23** depicts the model of the first 17 guardrail steel posts (from the guardrail previously presented), with constant height of instrument (1.1 m) and increasing horizontal offset from the guardrail posts (the location range of the survey spans from 20 m to 60 m). As horizontal offset increases, the amplitude of the modelled anomalies is visibly reduced. At a horizontal offset of 2.0 m, peaks are no longer visible in the graph, however the expected amplitude (25 nT or greater), would still be detectable by the G-858 magnetometer. In addition to reduction of peaks, valleys are also reduced as the horizontal offset increases.
Figure 5.23. Modelled magnetic anomalies of steel posts as horizontal offset distance increases. The magnetic susceptibility of steel posts is modelled as $\kappa = 1000$. The locations of the first 17 steel posts along the length of the survey are shown.

To compare the expected horizontal offset surveys against each other, the model is normalized to the maximum anomaly recorded across the length range (from 20 to 60 m – Figure 5.24). This normalization more clearly shows the relative attributes of the horizontal offset. Differences in the width of anomaly peaks and the amplitude of anomaly valleys are amplified with the normalization. When the horizontal offset is 0.1 m, the lowest valleys are presented. When the horizontal offset is 1.0 m, the largest valleys are consistently generated.
Figure 5.24. Modelled normalized magnetic anomalies of steel posts as horizontal offset distance increases. The magnetic susceptibility of steel posts is modelled as $\kappa = 1000$. The locations of the first 17 steel posts along the length of the survey are shown.

Different peaks may be normalized against one another to compare the width of the anomaly based on horizontal offset. Figure 5.24 shows the relative effect of increasing the horizontal offset distance on the magnetic anomaly created by Post 9. As the offset increases, the effect of the post on the normalized anomaly increases the width of the peak. As the offset increases, the relative impact of the top of the post on the overall magnetic anomaly is reduced and the relative impact of the base of the post is increase. This indicates that the post length change may be more easily determinable if the horizontal offset distance is large (greater than 0.4 m) and if the magnetic susceptibility is also acceptably large.
Figure 5.25. Modelled normalized magnetic anomaly of Post 9 (length 1.83 m) as horizontal offset from guardrail posts increases.

The 1.0 m horizontal offset curve crosses the $y = 0$ axis at approximately 42.87 m and 43.66 m (a width of 0.79 m). The 2.0 m horizontal offset curve crosses the $y = 0$ axis at approximately 42.84 m and 43.66 m (a width of 0.82 m).

The impact of adding and removing length to post 9 is demonstrated in Figure 5.25 and Figure 5.26, respectively. Increasing the post length from 1.83 m to 2.13 m or decreasing post length from 1.83 m to 1.53 m has minimal visible effect when the horizontal offset is 0.1 to 0.4 m; however, when the horizontal offset is 1.0 m or 2.0 m, there is a noticeable change in the width of the peak.

When the post length increases by 30 cm, the 1.0 m horizontal offset curve crosses the $y = 0$ axis at approximately 42.85 m and 43.67 m (a width of 0.82 m). The 2.0 m horizontal offset curve crosses the $y = 0$ axis at approximately 42.80 m and 43.69 m (a width of 0.89 m).
Figure 5.26. Modelled normalized magnetic anomaly of Post 9 with 30 cm added to post base (length of post now 2.13 m) as horizontal offset from guardrail posts increases.

Figure 5.27. Modelled normalized magnetic anomaly of Post 9 with 30 cm removed from post base (length of post now 1.53 m) as horizontal offset from guardrail posts increases.
When the post length decreases by 30 cm, the 1.0 m horizontal offset curve crosses the $y = 0$ axis at approximately 42.89 m and 43.65 m (a width of 0.76 m). The 2.0 m horizontal offset curve crosses the $y = 0$ axis at approximately 42.91 m and 43.63 m (a width of 0.72 m).

Decreasing or increasing the length of the 1.83-m wide flange steel post by 0.30 cm changes the width of the intersection of the 1.0 m horizontal offset curve and the $y = 0$ axis by ± 0.03 m. Decreasing or increasing the length of the 1.83-m wide flange steel post by 0.30 cm changes the width of the intersection of the 2.0 m horizontal offset curve and the $y = 0$ axis by -0.10 m and +0.07 m, respectively.

It is unlikely that the testing method would be able to accurately detect variations in curve width due to the current need to walk and manually mark locations; however, the results do indicate the potential advantages of increasing the horizontal offset. Provided that data collection is more methodological, this may lead to improved length estimation of steel posts.

5.5.4. **Wood Posts with Steel Base Plates**

**Model.** The following model demonstrates the applicability of determining length of wood posts with ferromagnetic steel base plates and the known presence of one steel screw attaching the post to the guardrail. The expected magnetic field anomaly of the steel base plate and the screw are determined independently and summed as a total dipole. The magnetic susceptibility $\kappa$ of the steel is modelled as 1000 (Schenck, 1996). The volume of the steel base plate is 750 cm$^3$. The volume of the single modelled screw is 18 cm$^3$. The height of the instrument in the modelled survey is 1.1 m and the height of the screw is 0.6 m. As the depth increases, the peak due to the steel base plate decreases, however the results from the steel base plate contribute substantially to the width of the anomaly. Should the parameters of the model remain constant in a field test (i.e., magnetic
susceptibility, general geometry, location, etc.), the information obtained would allow for length prediction by estimating the magnitude of the anomaly and determining the horizontal distance between the curves at some percentage of the peak maximum.

When the distance from the instrument to the baseplate is 1.5 m, the effect of the baseplate amplitude is larger than the effect of the screw amplitude (Figure 5.28) The baseplate also has a dominant effect on controlling the width of the total dipole anomaly. The peak of the modelled magnetic anomaly is 2139 nT. To use the model for length estimation a method of horizontal line fitting may be applied. Under the prescribed conditions, a 1.5 m horizontal line (representing the distance from the baseplate to the sensor) may be fit under the curve at approximately 670 nT (31.3% of the anomaly maximum). Should field data be collected for a system matching that shown in the model, the magnetic anomaly should reach peak amplitude at approximately 2139 nT. At approximately 31.3% of this maximum value, a 1.5 m line should be able to be fit under the line to represent the distance from the instrument to sensor.

![Magnetic anomaly graph](image)

**Figure 5.28.** Modeled magnetic anomaly due to steel base plate 1.5 m below sensor and (1) steel screw 0.5 m below sensor. Plate to instrument line shown at magnetic field anomaly of 670 nT (31.3% of anomaly maximum)
Increasing the distance of the plate to 2.0 m results in a reduction in the magnetic anomaly associated with the baseplate (Figure 5.29). The effect of the screw remains constant, resulting in a larger effect on the peak than the baseplate. The peak of the magnetic anomaly is 1376 nT and the 2.0 m distance from the baseplate to the sensor may be fit under the curve at approximately 270 nT (19.6% of the anomaly maximum).

Figure 5.29. Modelled magnetic anomaly due to steel base plate 2.0 m below sensor and (1) steel screw 0.5 m below sensor. Plate to instrument line shown at magnetic field anomaly of 270 nT (19.6% of anomaly maximum).

Increasing the distance of the plate to 2.5 m continues to highlight the decreasing baseplate effect (Figure 5.30). The peak of the magnetic anomaly is 1142 nT and the 2.5 m distance from the baseplate to the sensor may be fit under the curve at approximately 135 nT (11.8% of the anomaly maximum).
This analysis may be applied to increasing baseplate distances to understand the relationship between anomaly maximum and the plate to instrument horizontal line fitting. A summary of distance, maximum anomalies, fit values, and percentages of maximum are shown in Table 5.10.

Table 5.10. Summary of wood post with baseplate and screw modelling. Distance from instrument to plate impacts the magnetic anomaly maximum ($F_{\text{max}}$) and the magnetic anomaly value used for horizontal fitting ($F_{\text{fit}}$).

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>$F_{\text{max}}$ (nT)</th>
<th>$F_{\text{fit}}$ (nT)</th>
<th>$F_{\text{fit}}/F_{\text{max}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>2139</td>
<td>670</td>
<td>31.3%</td>
</tr>
<tr>
<td>2.0</td>
<td>1376</td>
<td>270</td>
<td>19.6%</td>
</tr>
<tr>
<td>2.5</td>
<td>1142</td>
<td>135</td>
<td>11.8%</td>
</tr>
<tr>
<td>3.0</td>
<td>1033</td>
<td>75</td>
<td>7.3%</td>
</tr>
<tr>
<td>3.5</td>
<td>965</td>
<td>45</td>
<td>4.7%</td>
</tr>
<tr>
<td>4.0</td>
<td>909</td>
<td>31</td>
<td>3.4%</td>
</tr>
<tr>
<td>4.5</td>
<td>858</td>
<td>21</td>
<td>2.4%</td>
</tr>
</tbody>
</table>

The values presented for maximum magnetic anomaly and the values for magnetic anomaly fit value are well above the minimum sensitivity of the G-858 magnetometer (0.1 nT) which indicates a strong ability to detect these values in the field. Figure 5.31 shows the effect of base plate to sensor distance increase on the maximum anomaly value and the percentage of the maximum anomaly to fit a horizontal line under and within the bounds of the anomaly.
**Figure 5.31.** Effect of base plate distance on anomaly maximum and length determination from magnetic field results. Power trendlines fit to data shown as dotted lines. Geometry and material properties of model: one steel baseplate (750 cm³), one steel screw (18 cm³), steel magnetic susceptibility of 1000. **LEFT:** Anomaly maximum as a function of plate distance. **RIGHT:** Percentage of anomaly maximum value to fit horizontal line equal to plate distance under anomaly curve (see red line in Figures 4.19 to 4.24).

The graphs shown in Figure 5.31 may be used in tandem to estimate baseplate distance from field data. However, to utilize the generated model, field data is required for verification. No field sites were tested matching the conditions described in the model; however, data was collected for a base plate survey.

**Field Data.** Magnetic surveys were completed in Camp Randall Memorial Park, Madison, WI to improve understanding of baseplate magnetic anomalies (without screws present). A metal baseplate with volume of 1,370 cm³ was surveyed from the South to North direction with various levels of horizontal offset (0 cm, 25 cm, 50 cm, and 75 cm) from the magnetic sensor line. A schematic diagram of the setup is shown in Figure 5.32. The results of the survey are shown in Figure 5.33 as bold lines, with a 300-pt moving mean applied to the data set to determine the regional magnetic gradient (shown as thin lines).
Figure 5.32. Schematic diagram of baseplate surveying. Surveying direction denoted by black arrow from South to North. Surveys were completed with increasing horizontal offset of the baseplate from the survey line by 0.25 m increments.

Figure 5.33. Results from magnetic field testing of metal baseplate in Camp Randall Memorial Park, WI shown in bold with two survey paths (red and blue) per offset condition. Thin lines show the magnetic regional gradient. Dashed line shows the location of the baseplate. a) Offset from sensor path: 0 cm. b) Offset from sensor path: 25 cm. c) Offset from sensor path: 50 cm. d) Offset from sensor path: 75 cm.
Removing the regional gradient from the collected data allows for analysis of the magnetic anomaly, the results of which are shown in Figure 5.34. Lines from two surveys per horizontal offset are presented.

The maximum anomaly value of the baseplate with no offset is 53 nT. The maximum anomaly value with 25 cm of offset is 34 nT. The maximum anomaly value with 50 cm of offset is 19 nT. The maximum anomaly value with 75 cm of offset is 10 nT. The effect of distance on the maximum anomaly value is further illustrated by Figure 5.35.
Figure 5.35. Results for maximum anomaly value from baseplate testing. Trendlines fit to data to estimate the empirical relationship between distance or offset on the magnetic anomaly. **LEFT**: Total distance from instrument to plate (instrument height: 1.1 m, offset varies). **RIGHT**: Horizontal distance varied from 0 to 0.75 m by 0.25 m.

To estimate the magnetic susceptibility of the material, the results from the field testing are used to model a magnetic anomaly (**Figure 5.36**) approximating the peaking behavior of the baseplate when there is no horizontal offset (i.e., offset equals 0 cm).

Figure 5.36. Modelled magnetic anomaly due to 1,370 cm$^2$ base plate 1.1 m below sensor with magnetic susceptibility of 8. Model is compared to two runs conducted of the base plate with no horizontal offset.
The generated model assumes a volume of 1,370 cm³, located 1.1 m from the magnetic sensor, with a peak magnetic anomaly of approximately 50 nT. This results in a magnetic susceptibility of 8. This value is less than the value used to estimate the magnetic susceptibility of steel post materials in the field (15,000 and 700). This value is also less than literature values for stainless steel suggesting a baseplate of different material.

Should a program be enacted to assess the length of wood posts with steel baseplates, the magnetic susceptibility of the plate should be very large to increase the effect on the magnetic field. The noticeable effect of the tested base plate material on the magnetic field diminishes after a horizontal offset of 75 cm (total distance 1.33 m) indicating the difficulty in potential length detection. Furthermore, reducing the magnetic susceptibility of other steel material (such as the attaching screw), would increase the ability to detect the individual anomaly of the base plate.

One difficult aspect of comparing baseplate models to field data is the distance between posts. The usefulness of the model relies on an ability to detect anomalies from independent posts. Overlapping anomalies reduce the effectiveness of the model, particularly for large embedment depths when baseplate anomalies are wider with small amplitude.

5.6. Lessons Learned and Engineering Recommendations

Lessons Learned:

- Magnetic gradient tolerance for magnetometers plays a role during data collection. If the magnetometer is located too close to a highly magnetic material, the gradient of measured total field may exceed the gradient allowable by the sensor. For this reason, the surveyor must take care to locate the magnetometer far enough from the material to remove the likelihood of failure.
• Geometric position of guardrail (i.e., the distance from sensor to post in the vertical direction) has a stronger impact on the results of the survey. This geometric position has a larger effect than the length of metal posts, indicating the required need for constant and known distance from sensor to top of guardrail post during the entire duration of the survey.

• The main information desired by testing and modelling is the distance from the ground to the base of the post. The closer the sensor is to the ground, the larger the magnetic anomaly of the post base, increasing the viability of length determination via magnetic means.

• The emphasize and strength of the magnetic characterization technique is the ability to detect changes when post length is decreased, not increased. The purpose of the research is to predict posts with under-designed post length, not over-designed. The effect of post length on the recorded magnetic anomaly has a stronger effect on varying the recorded anomaly when the post is under-designed due to the inverse relationship of the cubed distance’s effect on the total anomaly.

• Increasing the horizontal offset from the steel post to the sensor will reduce the relative effect of the top of the steel post and increase the relative effect of the bottom of the steel post. The numerical analysis for horizontal offset indicates that determining the length of the post will be most easily detectable if the offset is large and the magnetic susceptibility of the post material is large.

• Determining the distance to a steel base plate attached to the base of a wood post may be feasible if the magnetic susceptibility of the material is large.

Engineering Recommendations:

• While it is not currently recommended to enact a magnetic surveying method to determine the length of highway guardrail steel posts, the technique has great potential.
• Future research should allow magnetometer testing at consistent height between sensor and top of steel post. This could be implemented with a guardrail attachment using a rolling mechanism to reduce inconsistent instrument position with respect to guard rail and post. Error in the height and the horizontal distance of the instrument due to the gait of the operator is inevitable, especially on sites with inconsistent slopes along the length of the guardrail. Implementation of a testing methodology in which the height of the instrument is explicitly known would increase the accuracy of the field collected data and reduce the error of model comparison. A potential mechanism to improve the consistent measurement height is shown in Figure 5.37.

![Figure 5.37](image)

**Figure 5.37.** Potential frame devised for magnetic sensors which may be used to maintain a constant height of instrument across the entire surveyed guardrail.

The position of the magnetometer sensor would be maintained using a frame which would roll along the length of the guardrail W beam. The rolling portion of the frame would be connected to a horizontal beam/rod which would then connect to the “sensor rod” and the
“balance rod.” The “sensor rod” would be used to attach the magnetometer sensor to the frame. The balance rod would be used to improve the balancing of the entire frame and allow for easier “pushing” of the frame across the entire guardrail.

- Future research should explore the possibility of placing the sensor as close as possible to the furthest distance of interest without exceeding the magnetic gradient tolerance of the sensor (i.e., the magnetometer instrument height above the ground should be as little as possible to detect small scale variations at the base of the post).

- Future research should also explore the effect of increasing horizontal offset on the field data results. It is expected that assessing the length of the post will be simplest when the horizontal offset is sufficiently large, increasing the relative effect of the base of steel posts on the anomaly.

- Testing wood posts with steel baseplates (or a similar geometric system), would allow for verification or dismissal of generated model. Anomalies must be detectable at depths greater than 2 m. Any potential methods should use baseplates with large magnetic susceptibility ($\kappa \gg 500$)
6. Conclusions

The goal of this research was to determine a method to accurately determine the length of highway guardrails through non-destructive testing. This was not achieved. Determining the length of posts within an acceptable error tolerance of 5 to 10 cm specified by the Wisconsin Department of Transportation was not possible through stress-wave propagation or magnetic characterization methods. However, the information presented demonstrates that increased research and testing on the subject matter may further decrease the error in the prediction.

Literature was reviewed for non-destructive testing of in-situ foundations which led into a detailed assessment of the Pulse Echo (PE) method. The use of discrete signal processing in soil engineering was studied as a necessary tool to evaluate stress-waves generated from the Pulse Echo test. The use of magnetics to perform quality control and assessment was explored to provide background for magnetic anomaly field data collection and modelling.

Principles of discrete signal processing were reviewed to provide a framework for waveform analysis. The processing techniques discussed predominately applied to analysis of stress-wave propagation NDT. Some of the principles, such as stacking, were applied to the magnetic characterization technique.

With respect to stress-wave propagation, wood and steel posts of known length were tested in the field to assess the error of the Pulse Echo method with wave velocity determination, resulting in a Root Mean Square Error (RMSE) of 0.8 m for wood posts and 0.3 m for steel posts. The length range of wood posts in the field was 1.2 m to 2.1 m. Only 1.83-m long steel posts were tested in the field. Testing in the laboratory under free (no clamp), rubber fixed (rubber clamp), and fixed (clamped) conditions was completed for wood posts of several lengths and cross-sectional areas.
under low frequency reproducible steel ball pendulum drops. No clamp conditions resulted in the highest RMSE (0.38 – 0.40 m), clamp conditions resulted in medium RMSE (0.29 – 0.36 m), and rubber clamp conditions resulted in the smallest RMSE (0.22 – 0.35 m). Lab tested wood posts ranged in length from 1.2 m to 3.1 m.

Methods to improve understanding of wood posts were carried out through longitudinal wave analysis. Posts of varying cross-sectional area and constant length and posts of varying length and constant cross-sectional area were compared against one another. A testing criterion to assess the post posting with boundary conditions was also implemented. The signal attenuation, autocorrelation, power spectra density, windowed power spectra density, frequency attenuation, coherence, phase, and phase velocity were determined for the varying posts. Larger cross-sectional area reduces attenuation, attenuation with respect to frequency, increases coherence, results in more constant phase velocity, and reduces the overall effect of boundary conditions (i.e., coupling) on the post.

In magnetic anomaly characterization, methods were explored to predict the length to the base of steel posts or the length to a steel plate at the base of a wood post. Field results for a magnetic survey of a guardrail with steel posts were compared to modelled data. Two different survey methods (1 - instrument height of 2.0 m with no horizontal offset, and 2 - instrument height of 1.1 m with 0.35 m horizontal offset) were tested with systems built to most accurately model the collected, filtered data. A numerical analysis was also completed to analyze the relative effect of increasing the horizontal offset. The effect of increasing steel base plate distance for wood posts on magnetic anomaly results was assessed when a steel screws was also present to attach the post to the highway guardrail. Results for the anomaly of a baseplate were presented as the horizontal offset was increased, indicating a need for a baseplate with large magnetic susceptibility.
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https://doi.org/10.1016/j.ndteint.2019.102174

https://doi.org/10.1785/0120020241
Appendix A: MATLAB Scripts

A1. PE Method: Wave Velocity and Natural Frequency Determination

%% Pulse Echo and Wave Velocity: Length Determination
% March 31, 2020
% Luke Anderson
% Graduate Student
% Civil & Environmental Engineering
% University of Wisconsin-Madison
clc; clear; close all;
files = 10;
range = 50:280; %set the AIC picker range to properly encompass signal arrival
auto_val = 1:4;
dc = 0.0006;
zeroval = 1;
prompt = 'What is the true pile length (m)? '
tpl = input(prompt);

%% Data import
TL = csvread('LF_1.csv',1,6,[1,6,1,6]); % sampling time interval, seconds
TH = TL;
for i = 1:files
    LF_file = ['LF_' num2str(i) '.csv'];
    LF(:,:,i) = csvread(LF_file,2,0);
    HF(:,:,i) = csvread(LF_file,2,0);
end
LF = LF1(:,1:2,:);
clear LF_datafile HF_datafile

%% Signal Normalization
% Low Frequency
NL = size(LF); LFn = zeros(NL(1), NL(2)-1, NL(3));
for i1 = 1:NL(3)
    for i2 = 1:NL(1)
        LFn(i2,1,i1) = LF(i2,2,i1) / max(abs(LF(:,2,i1)));
    end
end
XLF=TL*LF(:,1,1); % time series of recorded data

% High Frequency
NH = size(HF); HFn = zeros(NH(1), NH(2)-1, NH(3));
for i1 = 1:NH(3)
    for i2 = 1:NH(2)-1
        for i3 = 1:NH(1)
            HFn(i3,i2+1,i1) = HF(i3,i2+1,i1) / max(abs(HF(:,i2+1,i1)));
        end
    end
end
XHF=TH*HF(:,1,1); % time series of recorded data
%% Signal Stacking to reduce noise and concentrate dominate pulse echo

% Low Frequency
LAvg = zeros(NL(1),NL(2)-1);
for i1 = 1:NL(2)-1
    for i2 = 1:NL(1)
        LAvg(i2,i1) = sum(LFn(i2,i1,:)) / NL(3);
    end
end

% High Frequency
HAvg = zeros(NH(1),NH(2)-1);
for i1 = 1:NH(2)-1
    for i2 = 1:NH(1)
        HAvg(i2,i1) = sum(HFn(i2,i1,:)) / NH(3);
    end
end

% zero padding
zeropad1 = zeros(zeroval*NL(1), 1);
LAvg = [LAvg;zeropad1];
X2 = transpose(TL*(NL(1)+1 : 1 : (zeroval+1)*(NL(1)) )); % time series of recorded data
XLF = [XLF;X2];

% FFT/Natural Frequency (Low Frequency)
Fs = 1/TL; % Sampling Frequency, hz
L = length(XLF);
f = Fs*(0:(L/2))/L;
Y = fft(LAvg);
Yc = conj(Y); % complex conjugate of fast fourier transform
G = Y.*Yc; % remove phase
C = ifft(G); % autocorrelation
P2 = abs(Y).^2/L;
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);

% Autocorrelation of top receiver
[Cy,Cx] = findpeaks(C,XLF(1:L/2+1),'MinPeakDistance',dc);
t_O = mean(diff([0;Cx(auto_val)])); % signal travel time
nf = 1/t_O;
[locind,a] = findpeaks(P1,f);

% Plotting
% Figure 1: NATURAL FREQUENCY
figure
subplot(2,2,1)
hold on
for i = 1:NL(3)
    plot(XLF(1:NL(1)),LFn(:,1,i))
end
title ('Top receiver signals')
xticklabel ('Time (s)')
yticklabel ('Amplitude ( )')

subplot(2,2,3)
plot(XLF(1:NL(1)),LAvg(1:NL(1)))
title ('Stacked top receiver signals')
xticklabel ('Time (s)')
yticklabel ('Amplitude ( )')

subplot(2,2,2)
findpeaks(C,XLF(1:L/2+1), 'MinPeakDistance', dc);
hold on
plot(Cx(auto_val), Cy(auto_val), 'blacko')
title ('AutoCorrelation')
xlabel ('Time (s)')
ylabel ('Amplitude ( )')
txt = ['$
\text{num2str(round(nf,0))}$ ' Hz'];
text(max(XLF)*2/6,max(C(:,1))*0.90,txt,'HorizontalAlignment','center')

subplot(2,2,4)
findpeaks(P1,f, 'MinPeakDistance',50)
title('FFT of signal with peaks')
xlabel('Frequency (Hz)')
ylabel('PSD')
txt  = ['
\leftarrow ' num2str(round(nf1,0)) ' Hz'];
idxP=find(f==nf1);
text(nf1,P1(idxP),txt)
xlim([0 5000])

%% AIC Picker Algorithm - AIC(k)=k*log(variance(x[1,k]))+(n-k-1)*log(variance(x[k+1,n]))
AIC = zeros(NH(1),NH(2)-1, NH(3)); AICavg = zeros(NH(1),NH(2)-1);
for i1 = 1:NH(3)
   for i2 = 1: (NH(2)-1)
      for i3 = 1:NH(1)
         var1(i1) = var(HF(1:i3,i2,i1)); var2(i1) = var(HF(i3+1:NH(1,1),i2,i1));
         var1_avg = var(HAvg(1:i3,i2)); var2_avg = var(HAvg(i3+1:NH(1,1),i2));
         AIC(i3,i2,i1) = i3*log(var1(i1))+(NH(1,1)-i3-1)*log(var2(i1));
         AICavg(i3,i2) = i3*log(var1_avg)+(NH(1,1)-i3-1)*log(var2_avg);
      end
   end
end

aicx = 1:length(XHF);

%% Wave velocity determination
idxHF = zeros(NH(2)-1,NH(3)); idxavg = zeros(NH(2)-1,1);
for i1 = 1:NH(3)
   for i2 = 1: (NH(2)-1)
      [~,idx] = min(AIC(1:i2,i1)); idxHF(i2,i1) = idx;
      [~,idx] = min(AICavg(i2,i1)); idxavg(i2,1) = idx;
   end
end

%% AIC Error Check --> Ensure correct picking of data
for i = 1:NH(3)
   if length(idxHF(2:4,i)) > length(unique(idxHF(2:4,i)))
      disp(['AIC' num2str(i) ' ERROR'])
   end
end

y1 = [-0.1,-0.35,-0.6]; % Sensor locations
x = zeros(5,4); P = zeros (5,2); vel = zeros(5,1);
for i = 1:NH(3)
   x(i,:) = idxHF(:,i)*TH;
end
for i = 1:NH(3)
   P(i,:) = polyfit(x(i,2:4),y1,1);
   vel(i) = -1*P(i,1);
end
vel1 = 1:length(vel);

for i = 1:length(vel)
    if isnan(vel(i))
        vel1(i) = vel(i);
    else
        vel1(i) = NaN;
    end
end

% velocity bounds for waves --> should not be used in data collection,
% should be used to correct field data
if vel(i) > 6000
    vel1(i) = NaN;
elseif vel(i) < 3000
    vel1(i) = NaN;
end

velmed(1:length(vel1)) = nanmedian(vel1);

pl = (velmed(1)/nf)/2;
pe = abs(tpl - pl)/tpl * 100;
pl2 = (velmed(1)/nf1)/2;
pe2 = abs(tpl - pl2)/tpl * 100;

disp('-----')
disp(['Wave Velocity = ', num2str(velmed(1)), ' m/s'])
disp('-----')
disp(['Natural Frequency (SIG) = ', num2str(nf), ' Hz'])
disp(['Pile Length = ', num2str(pl), ' m'])
disp(['Percent Error = ', num2str(pe), '%'])
disp('-----')
disp(['Natural Frequency (FFT/SIG) = ', num2str(nf1), ' Hz'])
disp(['Pile Length = ', num2str(pl2), ' m'])
disp(['Percent Error = ', num2str(pe2), '%'])

% Plotting
% Figure 2: INDIVIDUAL WAVE VELOCITIES
figure
for i1 = 1:5
    subplot(2,5,i1)
    hold on
    plot(XHF,HFn(:,1,i1),XHF,HFn(:,2,i1)-1,XHF,HFn(:,3,i1)-3.5,XHF,HFn(:,4,i1)-6)
    plot(idxHF(2,i1)+range(1)-1)*TH,-1,'blackO',
         (idxHF(3,i1)+range(1)-1)*TH,-3.5,'blackO',
         (idxHF(4,i1)+range(1)-1)*TH,-6,'blackO')
    hold off
    title(['Impulse ', num2str(i1), ' Waves'])
    xlabel('Time (s)')
    ylabel('Location (dm)')
    xlim([range(1)*TH range(end)*TH])
    ylim([-7 1])
    subplot(2,5,i1+5)
    hold on
    for i2 = 1:4
        plot(aicx,AIC(:,i2,i1))
    end
    hold off
    title(['Impulse ', num2str(i1), ' AIC'])
    xlim([range(1)*TH range(end)*TH])
end

% Figure 3: INDIVIDUAL WAVE VELOCITIES
if NH(3)>5
    figure
    for i1 = 6:10
        subplot(2,5,i1-5)
        hold on
        plot(XHF,HFn(:,1,i1),XHF,HFn(:,2,i1)-1,XHF,HFn(:,3,i1)-3.5,XHF,HFn(:,4,i1)-6)
end
plot((idxHF(2,i1)+range(1)-1)*TH,-1,'blackO',(idxHF(3,i1)+range(1)-1)*TH,-3.5,'blackO',(idxHF(4,i1)+range(1)-1)*TH,-6,'blackO')
hold off
title(['Impulse ' num2str(i1) ' Waves'])
xlabel('Time (s)')
ylabel('Location (dm)')
xlim([range(1)*TH range(end)*TH])
ylim([-7 1])

subplot(2,5,i1)
hold on
for i2 = 1:4
    plot(aicx,AIC(:,i2,i1))
end
hold off
title(['Impulse ' num2str(i1) ' AIC'])
xlim([range(1) range(end)])
end
A2. Longitudinal Wave Analysis

% Longitudinal PE/NF, Windowing, Attenuation, Phase Velocity, Coherence
% April 13th, 2020
% Luke Anderson
% Graduate Student
% Civil & Environmental Engineering
% University of Wisconsin-Madison

cclc; clear; close all;

%% variables
% figure1
dx = 0.0009; %set appx. range between autocorrelation peaks
auto_val = 1:2; %set autocorrelation peaks to determine t_O

% figure2
windows = 4; %plotted window reflections with PSD (max. 4)
alpha_atten = 2; %reflected signal for attenuation PSDX/PSDX+1 (max. 3)

% figure3
phase = alpha_atten; %windows - 1; %phase plots
phase_vel = ([3500 5500]);

% figure4
reflection = alpha_atten; reflection2 = reflection + 1; %coherence windows
freqmax = 10000; %use coherence to determine
alpha_freq = 55; %cutoff mark for linear attenuation

winbound(1,1,1) = 99; %picked window bounds for signal
winbound(1,1,2) = 99; %picked window bounds for signal
winbound(1,1,3) = 99; %picked window bounds for signal
peak_val = 2; %select starting peak for attenuation decay function as seen in figure 1

%side variables
%cutoff = 0.10; %remove all amplitudes less than the max amplitude*cutoff
signals = 10; %number of signals for LF/HF impulses
smoothnumber = 1; %value of moving average for smoothing top sensor data
cvalue = 8; %cvalue for sine window implementation

% Rayleigh's model
nu = 0.30; %poissons ratio?
d = 0.10; %m diameter of cylinder, how does it relate to rectangular bar?
prompt = 'What is the true pile length (m)? ';
tpl = input(prompt);

%% Data import
T = csvread('C_LF_1.csv',1,6,[1,6,1,6]); %sampling time interval, seconds
for i = 1:signals
    NC_LF_datafile = ['NC_LF_' num2str(i) '.csv'];
    MC_LF_datafile = ['MC_LF_' num2str(i) '.csv'];
    LNC_1(:,i,1) = csvread(NC_LF_datafile,2,0);
    LRC_1(:,i,1) = csvread(MC_LF_datafile,2,0);
    LNC_1(:,i,2) = csvread(NC_LF_datafile,2,0);
    LRC_1(:,i,2) = csvread(MC_LF_datafile,2,0);
    LNC_1(:,i,3) = csvread(NC_LF_datafile,2,0);
end

clear NC_LF_datafile MC_LF_datafile

%time series of recorded data and sensor "size"
X=T*LNC_1(:,1,1);
N = size(S_1);

% adjust signal to return to starting position "0"
med1 = zeros(1,N(2),N(3));
for i = 1:N(3)
    for m = 1:N(2)
        med1(1,m,i) = median(S_1(1:21,m,i));
        for m1 = 1:N(1)
            S_1(m1,m,i) = S_1(m1,m,i) - med1(1,m,i);
        end
    end
end

clear med1

% Stacking signal
S1n = zeros(N(1),1,N(3));
for i = 1:N(3)
    for m1 = 1:N(1)
        S1n(m1,1,i) = sum(S_1(m1,:,i))/N(2);
    end
end

% Normalize signal
for i = 1:N(3)
    S1ns(:,:,i) = S1n(:,:,i)./max(S1n(:,:,i));
end

%% Smoothing data series
for i = 1:N(3)
    S1ns(:,1,i) = movmean(S1ns(:,1,i),smoothnumber); % smooth data to reflect dominate frequency
    end
    for i = 1:N(3)
        [-,,locVal] = findpeaks(S1ns(:,1,i),X,'MinPeakDistance',dx);
        echo = mean(diff(locVal));
        nf(i) = 1/(echo);
    end
    clear echo

%% Longitudinal Pulse Testing. Phase Velocity
Fs = 1/T;
%fSampling frequency, hz
L = length(LF_C(:,1,1));
%nNumber of samples
T = (0:L-1)*T;
%time period
f = Fs*(0:(L/2))/L;
%frequency of PSD
for i = 1:N(3)
    Y(:,1,i) = fft(S1ns(:,1,i)); %fast fourier transform of signal
    Yc(:,1,i) = conj(Y(:,1,i)); %complex conugate of fast fourier transform
    G(:,1,i) =Y(:,1,i).*Yc(:,1,i); %remove phase
    Cc(:,1,i) = ifft(G(:,1,i)); %autocorrelation
    C(:,1,i) = Cc(1:L/2+1,1,i);
end

clear Cc

%% Power Spectra Density, normalized
PSD = zeros(N(1),1,N(3));
for i = 1:N(3)
    for m = 1:N(1)
        PSD(m,1,i) = sqrt(real(Y(m,1,i))^2 + imag(Y(m,1,i))^2);
    end
    PSD(:,1,i) = PSD(:,1,i)/L;
    PSD1(:,1,i) = PSD(1:L/2+1,1,i);
    PSD1(2:end-1,1,i) = 2*PSD1(2:end-1,1,i);
end
% Exponential decay function of peak values

for i = 1:N(3)
    [y,x] = findpeaks(S1ns(:,1,i),X,'MinPeakDistance',dx);
    expfit = fit(x(peak_val:end),y(peak_val:end),'exp1');
    decayval(1,:,:) = coeffvalues(expfit);
    decayfunc(:,1,i) = decayval(1,1,i).*exp(X.*decayval(1,2,i));
end

% Autocorrelation peaks and travel time velocity with total pile length

for i = 1:N(3)
    [Cy,Cx] = findpeaks(C(:,1,i),X(1:L/2+1),'MinPeakDistance',dx);
    t_O(1,1,i) = mean(diff([0;Cx(auto_val)]));  % signal travel time
    V_O(1,1,i) = 2*tpi/t_O(1,1,i);  % velocity of the main frequency
    Q(1,1,i) = round(t_O(1,1,i)/T);
end

% Windowing

for i = 1:N(3)
    winboundl(:,1,i) = [winbound(1,1,i) winbound(1,1,i)+Q(1,1,i) winbound(1,1,i)+2*Q(1,1,i) winbound(1,1,i)+3*Q(1,1,i) winbound(1,1,i)+4*Q(1,1,i)];
    Wnum(1,1,1) = length(winboundl(:,1,i)) - 1;
end

window = zeros(N(1),Wnum(1), N(3));

for i = 1:N(3)
    for m = 1:1:
        for ml = 1:(winboundl(:,m+1,i)-winboundl(:,m,i))
            if ml < cvalue
                curve(ml,m,i) = (sin((ml/cvalue)*pi()) - pi()/2) + 1/2;
            elseif ml <= (winboundl(:,m+1,i)-winboundl(:,m,i) - cvalue)
                curve(ml,m,i) = 1;
            else
                curve(ml,m,i) = (sin(((ml - (winboundl(:,m+1,i)-winboundl(:,m,i) - cvalue))/cvalue)*pi() + pi()/2) + 1)/2;
            end
        end
    end
    for m = 1:1:(Wnum)
        for ml = winboundl(:,m,i):( winboundl(:,m+1,i)-1)
            window(ml,m,i) = curve((ml-winboundl(:,m,i)+1),1,i);
        end
    end
end

% reflection windowing

r = zeros(N(1),Wnum,N(3)); rc = r; R = r;

for i = 1:N(3)
    for m=1:Wnum
        for ml = 1:N(1)
            r(ml,m,i) = window(ml,m,i)*S1ns(ml,1,i);
        end
        for ml = 1:N(1)
            rc(ml,m,i) = r(ml,m,i) - mean(r(:,m,i));
        end
    end
end
\[
\text{R}(r,m,i) = \text{fft}(\text{rc}(r,m,i));
\]
end

% Phase velocity

% Frequency Response
H = zeros(length(f),Wnum-1,N(3)); phi = H;
for i = 1:N(3)
    for m=1:Wnum-1
        for m1 = 1:length(f)
            H(m1,m,i) = R(m1,m+1,i)/R(m1,m,i);
            phi(m1,m,i) = atan(imag(H(m1,m,i))./real(H(m1,m,i)));
        end
    end
end

% Cumulative phase change
k = zeros(length(f),Wnum-1, N(3)); Pha = k;
for i = 1:N(3)
    for m1 = 1:length(f)
        if abs(phi(m1,m,i) - phi(m1,m+1,i)) > pi()/2
            k(m1,m,i) = k(m1,m+1,i)+1;
        else
            k(m1,m,i) = k(m1,m+1,i);
        end
        Pha(m1,m,i) = abs(phi(m1,m,i) - pi*k(m1,m,i));
    end
end
CP(:,:,i) = 2*pi()*f.*t_O(:,:,i);
P(:,:,i) = CP(:,:,i)';
end

% Central frequency

% [maxy, maxind] = max(PSD1);
for i = 1:N(3)
    [~, mi] = max(PSD1(:,:,i));
    maxind(:,:,i) = mi;
    cen_freq(:,:,i) = f(maxind(:,:,i));
end

% Calculation of phase velocity
dif = zeros(1,1,Wnum-1,N(3)); deltaphi = zeros(length(f),Wnum-1,N(3)); V_ph = zeros(length(f)-1,Wnum-1,N(3));
for i = 1:N(3)
    for m = 1:length(f)
        dif(:,m,i) = Pha(maxind(:,:,i),m,i) - P(maxind(:,:,i),t,O(:,:,i));
        deltaphi(:,m,i) = Pha(:,m,i) - P(:,t,O(:,:,i));
        if m = 2:length(f)
            V_ph(m1-1,m,i) = (2*tpl)/(t_O(:,:,i) + (deltaphi(m1,m,i)/(f(m1)*2*pi())));
        end
    end
end

V_phR=zeros(length(f)-1,1,N(3));
for i = 1:N(3)
    for m = 2:length(f)
        V_phR(m1-1,m1,i) = (1 - ((nu*pi()*(d*f(m1))/2*(V_O(:,:,i))))^2)*V_O(:,:,i);
    end
end

% Longitudinal Pulse Testing. Attenuation

% Power spectra density

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RPSD = zeros(length(f),Wnum,N(3));
for i = 1:N(3)
    for m=1:Wnum
        RPSD(m1,m,i) = sqrt(real(R(m1,m,i))^2 + imag(R(m1,m,i))^2);
    end
    RPSD(:,m,i) = RPSD(:,m,i)/L;
end

% Attenuation coefficient
alpha_r = zeros(length(f),Wnum-1,N(3)); alpha_avg = zeros(length(f),1,N(3));
for i = 1:N(3)
    for m1 = 1:length(f)
        alpha_r(m1,m,i) = 1/(2*tpl) * log(RPSD(m1,m,i)/RPSD(m1,m+1,i));
    end
    alpha_avg(m1,1,i) = mean(alpha_r(m1,:,i));
end

% Attenuation parameters
% Boundary effects
for i = 1:N(3)
    rho_1 = 700; % medium 1 density, kg/m^3
    V_1 = V_O; % medium 1 velocity, m/s
    I1(:,i) = rho_1*V_1(:,i); % medium 1 mechanical impedance
    rho_air = 1.2; % medium 2 density, kg/m^3
    V_air = 343; % speed of sound in air
    I2 = rho_air*V_air; % medium 2 mechanical impedance
    RC(:,i) = (I2-I1(:,i))/(I2 + I1(:,i)); % reflection coefficient
end

% Linear regression of attenuation
freq = f';
alpha_fit(:,i) = freq(2:alpha_freq)*alpha_r(2:alpha_freq,alpha_atten,i);
D(:,i) = alpha_fit(:,i)*V_O(:,i)/(2*pi());
end

% Longitudinal Pulse Testing. Coherence
Ip1 = zeros(N(1),N(2),N(3)); Ip2 = Ip1; LP1 = Ip1; LP2 = Ip1;
for i = 1:N(3)
    for m = 1:N(2)
        Ip1(:,m,i) = window(:,reflection,i).*S_1(:,m,i);    LP1(:,m,i) = fft(Ip1(:,m,i));
        Ip2(:,m,i) = window(:,reflection2,i).*S_1(:,m,i);   LP2(:,m,i) = fft(Ip2(:,m,i));
    end
end

% calculation of gamma parameters
gamma1 = zeros(N(1),N(2),N(3)); gamma2 = gamma1; gamma3 = gamma1; gamma4 = gamma1;
for i = 1:N(3)
    for m = 1:N(2)
        gamma1(m1,m,i) = LP2(m1,m,i)*conj(LP1(m1,m,i));
        gamma2(m1,m,i) = conj(gamma1(m1,m,i));
        gamma3(m1,m,i) = LP1(m1,m,i)*conj(LP1(m1,m,i));
        gamma4(m1,m,i) = LP2(m1,m,i)*conj(LP2(m1,m,i));
    end
end

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gamma3(m1,1,i) = sum(gammai3(m1,:,i));
gamma4(m1,1,i) = sum(gammai4(m1,:,i));
gamma(m1,1,i) = (gamma1(m1,1,i)*gamma2(m1,1,i))/(gamma3(m1,1,i)*gamma4(m1,1,i));
end
end

%% Torsional Testing: Analytical Signal
% compute the DFT of the signal:
for i = 1:N(3)
    AS1(:,1,i) = S1ns(:,1,i);
    AS(:,1,i) = fft(AS1(:,1,i));
end

% set all values above the nyquist frequency to zero:
for i = 1:N(3)
    for m = 1
        for m1 = (N(1)/2) - 1 : 1 : N(1)
            AS(m1,1,i) = 0;
        end
        for m2 = 1:((N(1)/2))
            AS(m2,1,i) = 2*AS(m2,1,i);
        end
        AnSi(:,1,i) = ifft(AS(:,1,i));
    end
end

for i = 1:N(3)
    for m = 1
        for m1 = 1:N(1)
            AnSi_amp(m1,1,i) = sqrt((real(AnSi(m1,1,i))^2) + imag(AnSi(m1,1,i))^2);
            AnSi_phi(m1,1,i) = atan( imag(AnSi(m1,1,i)) / real(AnSi(m1,1,i)));
        end
    end
    k = zeros(size(AnSi_phi)); AnSi_pha = k;
    for i = 1:N(3)
        for m = 1:length(AnSi_phi)
            if abs(AnSi_phi(m-1,1,i) - AnSi_phi(m,1,i)) > pi/2
                k(m,1,i) = k(m-1,1,i)+1;
            else
                k(m,1,i) = k(m-1,1,i);
            end
            AnSi_pha(m,1,i) = abs(AnSi_phi(m,1,i) - pi*k(m,1,i));
        end
    end
    for i = 1:N(3)
        for m = 1:length(AnSi_omega)
            AnSi_omega(m,1,i) = abs((AnSi_pha(m,1,i) - AnSi_pha(m+1,1,i)))/(2*pi*T);
        end
    end

% despiking
for i = 1:N(3)
    for m = 1:length(AnSi_omega)-1
        if AnSi_omega(m,1,i) > 50000
            AnSi_omega(m,1,i) = (AnSi_omega(m-1,1,i) + AnSi_omega(m+1,1,i))/2;
        end
    end
end

hit = 1;
%% Attenuation, frequency, etc. plots

colors = [0,0,1
1,0,0
0, 0.5, 0];

% figure 1
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    plot(X,Sins(:,1,i)+2*i,'Color',colors(i,:));
end

% Figure 2
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    C_max(i) = max(C(:,:,i));

    findpeaks((C(:,:,i)/C_max(i))+3*(i-1),X(1:L/2+1),'
    MinPeakDistance',dx);
    plot(X(1:L/2+1),(C(:,:,i)/C_max(i))+3*(i-1),'Color', colors(i,:))
end

% Figure 3
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    plot(f,PSD1(:,:,i),'Color',colors(i,:))
end
linS = {..., '--', 'linestyle', linS{m}};
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    for m = 1:windows
        plot(t, r(:,m,i)+2*i, 'Color', colors(i,:), 'linestyle', linS{m})
    end
    text(t(end/2)*0.75, 0.5+2*i, txt1(i), 'fontname', 'times', 'fontsize', 12)
end
for i = 1:N(3)
    plot(t, zeros(length(t))+2*i, 'k-')
end
xlabel('Time (s)')
ylabel('Amplitude ( )')
xlim([0 t(end/2)])
ylim([0 8])
box on
set(gca, 'fontname', 'times') % Set it to times

%% FIGURE 5
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    for m = alpha_atten
        plot(f, RPSD(:,m,i) + 0.01*(i-1), 'Color', colors(i,:), 'LineWidth', 1.5)
    end
end
for i = 1:N(3)
    for m = alpha_atten
        plot(f, RPSD(:,m+1,i) + 0.01*(i-1), 'Color', colors(i:))
    end
end
xlim([0 freqmax])
xlabel('Frequency (Hz)')
ylabel('Amplitude ( )')
box on
legend('
NC PSD' num2str(alpha_atten)),
'RC PSD' num2str(alpha_atten+1)),
'CPL PSD' num2str(alpha_atten+1)),
'NumColumns',2,...
'fontsize',12)
legend box off
set(legend, 'Box', 'on', 'EdgeColor', get(legend, 'Color'));
set(gca, 'fontname', 'times') % Set it to times
ylim([0 0.035])

%% FIGURE 6
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    plot(f, smoothdata( alpha_r(:,alpha_atten,i) ), 'Color', colors(i:))
end
for i = 1:N(3)
    plot(freq(1:alpha_freq), alpha_fit(:,:,i)*freq(1:alpha_freq), 'Color',
        colors(i,:), 'linewidth',1.5)
end
legend('
NC PSD' num2str(alpha_atten) '/PSD' num2str(alpha_atten+1)),
'RC PSD' num2str(alpha_atten) '/PSD' num2str(alpha_atten+1)),
'CPL PSD' num2str(alpha_atten) '/PSD' num2str(alpha_atten+1)),
'NC \alpha = \ num2str(round(alpha_fit(:,:,1),2,'significant'))
'RC \alpha = \ num2str(round(alpha_fit(:,:,2),2,'significant'))
'CPL \alpha = \ num2str(round(alpha_fit(:,:,3),2,'significant'))
'NumColumns',2,'location', 'NW', 'fontsize',12)
legend box off
set(legend, 'Box', 'on', 'EdgeColor', get(legend, 'Color'));
xlim([0 10000])
xlabel('Frequency (Hz)')
ylim([-0.1 0.7])
box on
set(gca,'fontname','times') % Set it to times

%%% phase plots

figure(7)
figure('DefaultAxesFontSize',14)
subplot(2,1,1)
hold on
for i = 1:N(3)
    for m = phase
        plot(f, phi(:,m,i) - 5 + 5*i, 'Color', colors(i,:)) %+ 5*m
    end
end
xlabel('Frequency (Hz)', 'FontSize',14)
ylabel('Phase (rads)', 'FontSize',14)
xlim([0 freqmax])
ylim([0 30])
box on
set(gca,'fontname','times') % Set it to times

subplot(2,1,2)
hold on
for i = 1:N(3)
    for m = phase
        plot(f, Pha(:,m,i), 'Color', colors(i,:))
    end
end
legend(['NC R' num2str(phase) '/R' num2str(phase+1)],...
       ['RC R' num2str(phase) '/R' num2str(phase+1)],...
       ['CL R' num2str(phase) '/R' num2str(phase+1)],...
       'location','SE','fontsize',12)
legend box off
xlabel('Frequency (Hz)', 'FontSize',14)
ylabel('Phase (rads)', 'FontSize',14)
xlim([0 freqmax])
box on
set(gca,'fontname','times') % Set it to times
clear
txt

%%% FIGURE 8
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    for m = phase
        plot(f(2:end),movmean(V_ph(:,m,i),1), 'Color', colors(i,:))
    end
end
for i = 1:N(3)
    plot([0 f(end)], [V_O(:,:,i) V_O(:,:,i)], 'Color', colors(i,:), 'linewidth',1.5)
end
for i = 1:N(3)
    txt(i,:) = [num2str(round(V_O(:,:,i))) ' m/s '];
end
xlabel('Frequency (Hz)', 'FontSize',14)
ylabel('Phase Velocity (m/s)', 'FontSize',14)
legend(['NC R' num2str(phase) '/R' num2str(phase+1)],...
       ['RC R' num2str(phase) '/R' num2str(phase+1)],...
       ['CL R' num2str(phase) '/R' num2str(phase+1)],...
       txt(1,:), txt(2,:), txt(3,:),...
       'location','NE',...
       'numcolumns',2, 'fontsize',12)
set( legend, 'Box', 'on', 'EdgeColor', get( legend, 'Color' ));
ylim(phase_vel)
xlim([0 freqmax])
box on
```matlab
set(gca,'fontname','times') % Set it to times

%% coherence plots

% figure 9
figure('DefaultAxesFontSize',14)
hold on
for i = 1:N(3)
    plot(f,gamma(1:length(f),:,i), 'Color', colors(i,:))
end
hold off

ylabel('Coherence ( )
xlabel('Frequency (Hz)')
legend(['NC R' num2str(reflection) '/R' num2str(reflection2)],'RC R' num2str(reflection) '/R' num2str(reflection2)','location','NE','fontsize',12);
set( legend,'Box','on','EdgeColor', get( legend,'Color')) ;
xlim([0 10000])
ylim([0 1.2])
box on

%% attenuation, coherence, phase velocity

% figure 10
figure('DefaultAxesFontSize',14)
subplot(3,1,1)
hold on
for i = 1:N(3)
    plot(f, smoothdata( alpha_r(:,alpha_atten,i) ), 'Color', colors(i,:))
end
for i = 1:N(3)
    plot(freq(1:alpha_freq), alpha_fit(:,:,i)*freq(1:alpha_freq),'Color',colors(i,:),'linewidth',1.5)
end
legend(['NC PSD' num2str(alpha_atten) '/PSD' num2str(alpha_atten+1)],'RC PSD' num2str(alpha_atten) '/PSD' num2str(alpha_atten+1)],'location','NE','fontsize',10)
legend box off
xlim([0 10000])
ylabel('Attenuation ( )
xlabel('Frequency (Hz)')
ylim([-0.1 0.7])
box on

set(gca,'fontname','times') % Set it to times

```

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subplot(3,1,2)
hold on
for i = 1:N(3)
    for m = phase
        plot(f(2:end),movmean(V_ph(:,m,i),15), 'Color', colors(i,:))
    end
end
for i = 1:N(3)
    plot([0 f(end)],[V_O(:,:,i) V_O(:,:,i)], 'Color', colors(i,:), 'linewidth',1.5)  
%     plot(f(2:601),V_phR(:,:,i), 'black-')
    txt(i,:) = [num2str(round(V_O(:,:,i))) ' m/s'];
end
xlabel('Frequency (Hz)', 'FontSize',14)
ylabel('Phase Velocity (m/s)', 'FontSize',14)
legend(['NC R' num2str(phase) '/R' num2str(phase+1)],...
      ['RC R' num2str(phase) '/R' num2str(phase+1)],...
      ['CL R' num2str(phase) '/R' num2str(phase+1)],...
      txt(1,:), txt(2,:), txt(3,:),...
      'Rayleigh',...
      'location', 'NE',...
      'numcolumns',2, 'fontsize',10)
set(legend, 'Box', 'on', 'EdgeColor', get(legend, 'Color')) ;
ylim(phase_vel)
xlim([0 freqmax])
box on
set(gca, 'fontname', 'times')  
% Set it to times
set(gcf, 'Position', [100, 50, 625, 750])

% Window picking
% winbound picks
figure
box on
hold on
for i = 1:N(3)
    findpeaks(S1ns(:,1,i)+2*i,1:length(X),'MinPeakDistance',(dx/T));
    plot(1:length(X),S1ns(:,1,i)+2*i,'black')
    plot(1:length(X),movmean(S1ns(:,1,i),35)+2*i)
    plot(1:length(X),zeros(length(X))+2*i,'black')
    for m = 1:Wnum+1
        plot([winbound1(1,m,i) winbound1(1,m,i)],[0.8+2*i 0.8+2*i],'red')
    end
    text(length(X)*(4/6), 0.8+2*i, txt1(i))
end
grid off
xlabel('Discrete Unit')
ylabel('Amplitude')
title('Window Bounds Picking')
A3. Torsional and Bending Waves Analysis

% Torsional Attenuation and Phase Velocity Modeling
% April 15, 2020

% Luke Anderson
% Graduate Student
% Civil & Environmental Engineering
% University of Wisconsin-Madison

clc; clear; close all;

%% true variables
dx = 0.0040;    %set appx. range between peak signals
winbound = 98;  %picked window bounds for signal
alpha_freq = 100;  %cutoff mark for linear attenuation
auto_val = 2:4;  %auto VAL
peak_val = 5;    %peak VAL

%% side variables
freqmax = 2500;  %max frequency
phase_vel = [400 1200];  %phase velocity bounds
windows = 4;    %plotted window reflections with PSD
alpha_atten = 2;  %reflected signal for attenuation PSDX/PSDX+1
cvalue = 8;  %cva for sin window implementation
phase = windows - 1;  %phase plots
reflection = alpha_atten;  %reflection
reflection2 = reflection + 1;  %coherence windows

prompt = 'What is the true pile length (m) ?

.tpl = input(prompt);

%% Data import
T = csvread('LF_1.csv',1,4,[1,4,1,4]);  %sampling time interval, seconds
L1 = csvread('LF_1.csv',2,0); sensor1(:,1) = L1(:,2); sensor2(:,1) = L1(:,3);  %signal 1, accel. 1-2 output, volts
L2 = csvread('LF_2.csv',2,0); sensor1(:,2) = L2(:,2); sensor2(:,2) = L2(:,3);  %signal 2, accel. 1-2 output, volts
L3 = csvread('LF_3.csv',2,0); sensor1(:,3) = L3(:,2); sensor2(:,3) = L3(:,3);  %signal 3, accel. 1-2 output, volts
L4 = csvread('LF_4.csv',2,0); sensor1(:,4) = L4(:,2); sensor2(:,4) = L4(:,3);  %signal 4, accel. 1-2 output, volts
L5 = csvread('LF_5.csv',2,0); sensor1(:,5) = L5(:,2); sensor2(:,5) = L5(:,3);  %signal 5, accel. 1-2 output, volts
L6 = csvread('LF_6.csv',2,0); sensor1(:,6) = L6(:,2); sensor2(:,6) = L6(:,3);  %signal 6, accel. 1-2 output, volts
L7 = csvread('LF_7.csv',2,0); sensor1(:,7) = L7(:,2); sensor2(:,7) = L7(:,3);  %signal 7, accel. 1-2 output, volts
L8 = csvread('LF_8.csv',2,0); sensor1(:,8) = L8(:,2); sensor2(:,8) = L8(:,3);  %signal 8, accel. 1-2 output, volts
L9 = csvread('LF_9 (REVERSE HIT).csv',2,0); sensor1(:,9) = L9(:,2); sensor2(:,9) = L9(:,3);  %signal 9, accel. 1-2 output, volts
L10 = csvread('LF_10 (REVERSE HIT).csv',2,0); sensor1(:,10) = L10(:,2); sensor2(:,10) = L10(:,3);  %signal 10, accel. 1-2 output, volts

X = T.*L1(:,1);  %time series of recorded data
N = size(sensor1);

for m=1:N(2)
  med1(m) = median(sensor1(1:21,m));
end

for m = 1:N(2)
  med2(m) = median(sensor1(1:21,m));
end

for m = 1:N(2)
  % return signal starting position to "0"
sensor1(m1,m) = sensor1(m1,m) - med1(1,m);
sensor2(m1,m) = sensor2(m1,m) - med2(1,m);
end
end

add = zeros(N); sub = zeros(N);
for m = 1:N(2)
    for m1 = 1:N(1)
        add(m1,m) = sensor1(m1,m)/max(sensor1(:,m)) + sensor2(m1,m)/max(sensor2(:,m));
        sub(m1,m) = sensor1(m1,m)/max(sensor1(:,m)) - sensor2(m1,m)/max(sensor2(:,m));
    end
end

% Normalize signal
S1n = zeros(N(1),1);
for m = 1:N(2)
    for m1 = 1:N(1)
        S1n(m1,m) = add(m1,m)/max(add(:,m));
    end
end

addst = zeros(N(1),1); subst = zeros(N(1),1);
for m1 = 1:N(1)
    addst(m1,1) = sum(add(m1,:))/N(2);
    subst(m1,1) = sum(sub(m1,:))/N(2);
end

% % zero padding
zeroval = 1;
zeropad1 = zeros(zeroval*length(S1n), length(S1n(:,1)));
zeropad2 = zeros(zeroval*length(addst), 1);
add = [add;zeropad1];
sub = [sub;zeropad1];
S1n = [S1n;zeropad1];
addst = [addst;zeropad2];
subst = [subst;zeropad2];
X2= transpose(T* (L1(end,1)+1 : 1 : (zeroval+1)*(1+L1(end,1)) - 1) ); % time series of recorded data
X = [X;X2];
N = size(S1n);

% % moving average of signal
addstack = movmean(addst,1); % torsional
addstackmean = movmean(addst,10);
addstackhf = addstack - addstackmean;
addstacklf = addstack - addstackhf;

substack = movmean(subst,1); % bending

% Frequency Analysis
Fs = 1/T; % Sampling frequency, Hz
L = length(addst(:,1)); % Number of samples
t = (0:L-1)*T; % time period
f = Fs*(0:(L/2))/L; % Frequency of PSDgf
Yadd = zeros(N); Ysub = zeros(N);
for m = 1:N(2)
    Yadd(:,m) = fft(add(:,m));
    Ysub(:,m) = fft(sub(:,m));
end
Padd = zeros(N); Psub = zeros(N); PSDadd = (zeros(length(f),N(2))); PSDsub = (zeros(length(f),N(2)));
for m = 1:N(2)
    for m1 = 1:N(1)
        Padd(m1,m) = sqrt(real(Yadd(m1,m))^2 + imag(Yadd(m1,m))^2);    Padd(m1,m) = Padd(m1,m)/L;
        Psub(m1,m) = sqrt(real(Ysub(m1,m))^2 + imag(Ysub(m1,m))^2);    Psub(m1,m) = Psub(m1,m)/L;
    end
    PSDadd(:,m) = Padd(1:L/2+1,m);    PSDadd(2:end-1,m) = 2*PSDadd(2:end-1,m);
    PSDsub(:,m) = Psub(1:L/2+1,m);    PSDsub(2:end-1,m) = 2*PSDsub(2:end-1,m);
end

PSDaddstack = zeros(length(f),1); PSDsubstack = zeros(length(f),1);
for m1 = 1:length(f)
    PSDaddstack(m1,1) = sum(PSDadd(m1,:)/N(2));
    PSDsubstack(m1,1) = sum(PSDsub(m1,:)/N(2));
end

% removal of high frequency component of torsional waves
Yaddhf = fft(addstackhf);
Yaddlf = fft(addstacklf);
for m1 = 1:N(1)
    Paddhf(m1,1) = sqrt(real(Yaddhf(m1,1))^2 + imag(Yaddhf(m1,1))^2);    Paddhf(m1,1) = Paddhf(m1,1)/L;
    Paddlf(m1,1) = sqrt(real(Yaddlf(m1,1))^2 + imag(Yaddlf(m1,1))^2);    Paddlf(m1,1) = Paddlf(m1,1)/L;
end
PSDaddhf(:,1) = Paddhf(1:L/2+1,1);    PSDaddhf(2:end-1,1) = 2*PSDaddhf(2:end-1,1);
PSDaddlf(:,1) = Paddlf(1:L/2+1,1);    PSDaddlf(2:end-1,1) = 2*PSDaddlf(2:end-1,1);
for m=1:length(f)-1
    PSDasa(m) = (PSDaddstack(m)+PSDaddstack(m+1))/2;
    PSDaahf(m) = (PSDaddhf(m)+PSDaddhf(m+1))/2;
    PSDaalf(m) = (PSDaddlf(m)+PSDaddlf(m+1))/2;
end
PSDaddstackarea = sum(PSDasa);
PSDaddareahf = sum(PSDahf);
PSDaddarealf = sum(PSDalf);
for m = 1:length(f)
    PSDaddstackn(m) = PSDaddstack(m)/PSDaddstackarea;
    PSDaddhfn(m) = PSDaddhf(m)/PSDaddareahf;
    PSDaddlfn(m) = PSDaddlf(m)/PSDaddarealf;
end

% Exponential decay function of peak values for SUBTRACTION
% figure
% findpeaks(substack(:,1),X,'MinPeakDistance',dx);
% [y,x] = findpeaks(substack(:,1),X,'MinPeakDistance',dx);
expf1 = fit(x(peak_val:end),y(peak_val:end),'exp1');
deayval1=coeffvalues(expf1);
deayfunc1 = decayval1(1).*exp(X.*decayval1(2));
[locind,a] = findpeaks(PSDaddstack(:,1),f); %Determine peak energy densities of PSD
[-a1] = max(locind);
f1 = a(a1);
% Damping
D = (decayval1(2)-1)/(2*pi()*f1);
% Velocity
V = 4*f1*tpl;

% Exponential decay function of peak values for SUBTRACTION
% figure
% findpeaks(substack(:,1),X,'MinPeakDistance',dx);
% [y,x] = findpeaks(substack(:,1),X,'MinPeakDistance',dx*5);
expfit = fit(x(2:end),y(2:end),'exp1');

decayval2=coeffvalues(expfit);
decayfunc2 = decayval2(1).*exp(X.*decayval2(2));

% Cross Correlation
% torsion
TP = fft(addstack);
TPconj = conj(TP); %complex conjugate of fast fourier transform
G = TP.*TPconj; %remove phase
C = ifft(G); %autocorrelation
C = C(1:L/2+1);

% Autocorrelation peaks
[Cy,Cx] = findpeaks(C,X(1:L/2+1),'MinPeakDistance',dx);

% Travel time, velocity

% Windowing
winbound = [winbound(1) winbound(1)+Q winbound(1)+2*Q winbound(1)+3*Q winbound(1)+4*Q];
P = winbound(1);
P_t = t(P);
Wnum = length(winbound)-1;

for m = 1:Wnum
    for m1 = 1:(winbound(m+1)-winbound(m))
        if m1 < cvalue
            curve(m1,m) = (sin((m1/cvalue)*pi() - pi()/2) + 1)/2;
        elseif m1 <= (winbound(m+1)-winbound(m) - cvalue)
            curve(m1,m) = 1;
        else
            curve(m1,m) = (sin(((m1 - (winbound(m+1)-winbound(m) - cvalue))/cvalue)*pi() + pi()/2) + 1)/2;
        end
    end
end

window = zeros(N(1),Wnum);
for m = 1:(Wnum)
    for m1 = winbound(m): (winbound(m+1)-1)
        window(m1,m) = curve((m1-winbound(m)+1),m);
    end
end

% reflection windowing
% window = zeros(N(1),Wnum);
r = zeros(N(1),Wnum); rc = r; R = r;
for m=1:Wnum
    for m1 = 1:N(1)
        r(m1,m) = window(m1,m)*addstack(m1,1);
    end
    for m1 = 1:N(1)
        rc(m1,m) = r(m1,m) = mean(r(:,m));
    end
    R(:,m) = fft(rc(:,m));
end

% Phase velocity

H = zeros(length(f),Wnum-1); phi = H;
for m=1:Wnum-1
    for m1 = 1:length(f)
        H(m1,m) = R(m1,m+1)/R(m1,m);
        phi(m1,m) = atan(imag(H(m1,m))./real(H(m1,m)));
    end
end

% Cumulative phase change
k = zeros(length(f),Wnum-1); Pha = k;
for m1 = 1:Wnum-1
    for m = 2:length(phi(:,1))
        if abs(phi(m-1,m1) - phi(m,m1)) > pi()/2
            k(m,m1) = k(m-1,m1)+1;
        else
            k(m,m1) = k(m-1,m1);
        end
        Pha(m,m1) = abs(phi(m,m1) - pi*k(m,m1));
    end
end

P = 2*pi()*f.*t_O;
P = P';

% Central frequency
[maxy, maxind] = max(PSDaddstack);
cen_freq = f(maxind);

% Calculation of phase velocity
dif = zeros(1,Wnum); deltaphi = zeros(length(f),Wnum-1); V_phi = zeros(length(f)-1,Wnum-1);
for m = 1:Wnum-1
    dif(m) = Pha(maxind,m) - P(maxind);
    deltaphi(:,m) = Pha(:,m) - P(:) - dif(m);
    for m1 = 2:length(f)
        V_phi(m1-1,m) = (2*tpl)/(t_O + (deltaphi(m1,m)/(f(m1)*2*pi())));
    end
end

% Torsional Pulse testing. Attenuation

% Power spectra density
RPSD = zeros(length(f),Wnum);
for m=1:Wnum
    for m1 = 1:length(f)
        RPSD(m1,m) = sqrt(real(R(m1,m))^2 + imag(R(m1,m))^2);
    end
    RPSD(:,m) = RPSD(:,m)/L;
end

% Attenuation coefficient
alpha_r = zeros(length(f),Wnum-1); alpha_avg = zeros(length(f),1);
for m1 = 1:length(f)
    for m=1:Wnum-1
        alpha_r(m1,m) = 1/(2*tpl) * log(RPSD(m1,m)/RPSD(m1,m+1));
    end
    alpha_avg(m1,1) = mean(alpha_r(m1,:));
end

% Attenuation parameters
% Boundary effects

rho_1 = 700; % medium 1 density, kg/m^3
V_1 = V_O; % medium 1 velocity, m/s
I1 = rho_1*V_1; % medium 1 mechanical impedance
rho_air = 1.2; % medium 2 density, kg/m^3
V_air = 343; % medium 2 velocity, m (speed of sound in air)
I2 = rho_air*V_air;           %medium 2 mechanical impedance

RC = (I2-I1)/(I2 + I1);       %reflection coefficient

% Linear regression of attenuation
freq = f';
alpha_fit = freq(1:alpha_freq)
alpha_r(1:alpha_freq,alpha_atten);
D = alpha_fit*V_O/(2*pi());

% Torsional Testing. Coherence
Ip1 = zeros(N(1),N(2));  Ip2 = Ip1;  LP1 = Ip1;  LP2 = Ip1;
for m = 1:N(2)
    Ip1(:,m) = window(:,reflection).*S1n(:,m);  LP1(:,m) = fft(Ip1(:,m));
    Ip2(:,m) = window(:,reflection2).*S1n(:,m);  LP2(:,m) = fft(Ip2(:,m));
end

% calculation of gamma parameters
gamma1 = zeros(N(1),N(2));  gamma2 = gamma1;  gamma3 = gamma1;  gamma4 = gamma1;
gamma1 = zeros(N(1), 1);  gamma2 = gamma1;  gamma3 = gamma1;  gamma4 = gamma1;  gamma = gamma1;
for m1=1:N(1)
    for m = 1:N(2)
        gamma1(m1,m) = LP2(m1,m)*conj(LP1(m1,m));
        gamma2(m1,m) = conj(LP2(m1,m)*conj(LP1(m1,m)));
        gamma3(m1,m) = LP1(m1,m)*conj(LP1(m1,m));
        gamma4(m1,m) = LP2(m1,m)*conj(LP2(m1,m));
    end
    gamma1(m1,1) = sum(gamma1(m1,:));
    gamma2(m1,1) = sum(gamma2(m1,:));
    gamma3(m1,1) = sum(gamma3(m1,:));
    gamma4(m1,1) = sum(gamma4(m1,:));
    gamma(m1,1) = (gamma1(m1,1)*gamma2(m1,1))/(gamma3(m1,1)*gamma4(m1,1));
end

% Torsional Testing: Analytical Signal
% compute the DFT of the signal:
AS1 = addstacklf;
AS = fft(AS1);
% set all values above the nyquist frequency to zero:
for m = 1:N(2)
    for m1 = (N(1)/2)-1 : 1 : N(1)
        AS(m1,m) = 0;
    end
    for m2 = 1:((N(1)/2))
        AS(m2,m) = 2*AS(m2,m);
    end
    AnSi(:,m) = ifft(AS(:,m));
end

for m = 1
    for m1 = 1:N(1)
        AnSi_amp(m1,m) = sqrt((real(AnSi(m1,m))^2) + imag(AnSi(m1,m))^2 );
        AnSi_phi(m1,m) = atan( imag(AnSi(m1,m)) / real(AnSi(m1,m)) ) ;
    end
end

k = zeros(size(AnSi_phi));  AnSi_pha = k;
for m = 2:length(AnSi_phi)
    if abs(AnSi_phi(m-1) - AnSi_phi(m)) > pi/2
        k(m) = k(m-1)+1;
    else
        k(m) = k(m-1);
    end
    AnSi_pha(m) = abs(AnSi_phi(m) - pi*k(m));
end
for i=1:(N(1)-1)
AnSi_omega(i,1) = abs((AnSi_pha(i) - AnSi_pha(i+1)))/(2*pi*T);

end

for i = 2:length(AnSi_omega)-1
    if AnSi_omega(i,1) > 1000
        AnSi_omega(i,1) = (AnSi_omega(i-1) + AnSi_omega(i+1))/2;
    end
end

hit = 1;

%% Plotting

% Figure 1: Analytical Signal
figure('DefaultAxesFontSize',14)
% plot(X, sensor1(:,hit))
plot(X, AS1, 'blue')
xlim([0 X(end)*(2/5)])
ylabel('Amplitude')
xlabel('Time (s)')
set(gca, 'fontname', 'times') % Set it to times

%% Figure 2
figure('DefaultAxesFontSize',14)
subplot(2,1,1)
plot(X, AnSi_amp(:,hit), 'blue')
xlim([0 X(end)*(2/5)])
ylabel('Instant. Amp. ( )')
xlabel('Time (s)')
set(gca, 'fontname', 'times') % Set it to times

subplot(2,1,2)
plot(X(1:end-1), AnSi_omega(:,hit), 'blue')
xlim([0 X(end)*(2/5)])
% ylim([0 1000])
ylabel('Instant. Freq. (Hz)')
xlabel('Time (s)')
ylim([0 1000])
set(gca, 'fontname', 'times') % Set it to times

% Figure 3: Torsional vs. Bending of Beam
figure('DefaultAxesFontSize',14)
% sensor addition --> Torsion (stacked)
subplot(2,1,1)
plot(X, addstack, 'blue')
hold on
plot(X, decayfunc1, 'black')
txt = ['y = ' num2str(round(decayval1(1,1),2, 'significant')) '*exp(' num2str(round(decayval1(1,2),2, 'significant'))) ')']
	ext{(0.15,0.7,txt, 'fontname', 'times', 'fontsize',12)
xlim([0 T*L1(end,1)])
ylim([-2 2])
% title('Torsional time series')
xlabel('Time (s)')
ylabel('Amplitude ( )')
set(gca, 'fontname', 'times') % Set it to times

% sensor subtraction --> bending (stacked)
subplot(2,1,2)
plot(X, substack, 'red')
hold on
plot(X, decayfunc2, 'black')
txt = ['y = ' num2str(round(decayval2(1,1),2, 'significant')) '*exp(' num2str(round(decayval2(1,2),2, 'significant'))) ')']
	ext{(0.15,0.7,txt, 'fontname', 'times', 'fontsize',12)
xlim([0 T*L1(end,1)])
ylim([-2 2])
% title('Bending time series')
xlabel('Time (s)')
ylabel('Amplitude ( )')
set(gca,'fontname','times') % Set it to times

%% Figure 4
figure('DefaultAxesFontSize',14)
subplot(2,1,1)
plot(f,PSDaddstack,'blue')
% title('Torsional power spectra density')
xlabel('Frequency (Hz)')
ylabel('Amplitude ( )')
xlim([0 freqmax/2])
set(gca,'fontname','times') % Set it to times

subplot(2,1,2)
plot(f,PSDsubstack,'red')
% title('Bending power spectra density')
xlabel('Frequency (Hz)')
ylabel('Amplitude ( )')
xlim([0 freqmax/2])
set(gca,'fontname','times') % Set it to times

% set(findall(gcf,'-property','FontName'),'FontName','Times New Roman')

% Figure 5: Attenuation, Windowing, and PSD
figure('DefaultAxesFontSize',14)
plot(X,addstack,'black')
hold on
plot(X,decayfunc1,'red')
plot(X,zeros(N(1,1)),'
Color','Black')
xlim([0 T*L1(end,1)*(2/5)]
ylim([-2 2])
title('Stacked time series torsion (sensor addition)')
xlabel('Time (s)')
ylabel('Amplitude ( )')
set(gca,'fontname','times') % Set it to times

figure('DefaultAxesFontSize',14)
findpeaks(C,X(1:L/2+1),'
MinPeakDistance',dx);
hold on
plot(X(1:L/2+1),C,'blue')
xlim([0 T*round(L1(end,1)/2*(2/5)]
ylim([-10 10])
title('Autocorrelation')
xlabel('Time (s)')
ylabel('Amplitude ( )')
grid off
set(gca,'fontname','times') % Set it to times

figure('DefaultAxesFontSize',14)
plot(f(1:end/1),PSDaddstack(1:end/1),'black')
title('Power Spectrum Density')
xlabel('Frequency (Hz)')
ylabel('Amplitude ( )')
xlim([0 freqmax])
set(gca,'fontname','times') % Set it to times

% set(findall(gcf,'-property','FontName'),'FontName','Times New Roman')

%% figure 4: Windowing imaging
figure('DefaultAxesFontSize',14)
hold on
for m = 1:windows
plot(t, rc(:,1), 'blue')
plot(t, rc(:,2), 'blue--')
plot(t, rc(:,3), 'blue--')
end
plot(t, rc(:,4), 'blue--')
end
plot(t,zeros(length(t)),'black')
ylim([-1.2, 1.2])
xlim([0 0.06])
box on
xlabel('Time (s)')
ylabel('Amplitude ( )')
% title('Stacked Reflections (Windowed)')
xlim([0 T*L1(end,1)*(2/5)])
xlim([0 0.06])
set(gca,'fontname','times') % Set it to times

figure('DefaultAxesFontSize',14)
hold on
plot(f, RPSD(:,2), 'blue', 'linewidth',1.5)
plot(f, RPSD(:,3), 'blue')
box on
legend('PSD2','PSD3','location','NE','fontname','times')
legend box off
xlabel('Frequency (Hz)')
ylabel('Amplitude ( )')
% title('Power Spectra Density')
xlim([0 freqmax])
set(gca,'fontname','times') % Set it to times

figure('DefaultAxesFontSize',14)
hold on
plot(f, alpha_r(:,alpha_atten), 'blue')
plot(freq(1:alpha_freq), alpha_fit*freq(1:alpha_freq), 'blue--', 'Linewidth',1.5)
xlim([0 freqmax])
ylim([-0.1 inf])
xlabel('Frequency (Hz)')
% title('Attenuation Fitting')
legend('PSD' num2str(alpha_atten) '/PSD' num2str(alpha_atten+1) ', '','
   ['\alpha - num2str(round(alpha_fit,2,'significant'))'],'
   'location','SE','fontname','times')
legend box off
box on
set(gca,'fontname','times') % Set it to times

set(findall(gcf, '-property','FontName'),'FontName','Times New Roman')

% Figure 5
figure('DefaultAxesFontSize',14)
subplot(2,1,1)
hold on
for m = 2
   plot(f, phi(:,m)+5*m - 5, 'blue')
end
box on
% title('Phase')
xlabel('Frequency (Hz)')
ylabel('Phase (rads)')
xlim([0 freqmax])
ylim([0 10])
set(gca,'fontname','times') % Set it to times

subplot(2,1,2)
hold on
for m = 2
   plot(f, Pha(:,m), 'blue')
end
box on
% title('Cumulative phase')
xlabel('Frequency (Hz)')
ylabel('Phase (rads)')
xlim([0 freqmax])
% box on
set(gca,'fontname','times') % Set it to times

%% figure('DefaultAxesFontAngle','slanted','FontName','times')
hold on
for m = 2
plot(f(2:end),V_ph(:,m), 'blue')
end
plot([0 f(end)],[V_O V_O],'blue','linewidth',1.5)
% title('Phase velocity')
xlabel('Frequency (Hz)')
ylabel('Phase Velocity (m/s)')
txt = num2str(round(V_O));
legend('R2/R3',[txt ' m/s'])
legend box off
xlim([0 freqmax])
box on
set(gca,'fontname','times') % Set it to times
% set(findall(gcf,'-property','FontName'),'FontName','Times New Roman')

% ------ COHERENCE PLOTS
% figure 6
% plot(f,gamma(1:length(f)),'blue')
xlabel('Frequency (Hz)')
ylabel('Amplitude')
% title([Reflection ' num2str(reflection) ' - Reflection ' num2str(reflection2) ' Coherence'])
set(gca,'fontname','times') % Set it to times

% Window picking
% winbound picks
findpeaks(addstack(:,1),1:length(X),'MinPeakDistance',(dx/T));
hold on
plot(1:length(X),addstack,'black')
plot(1:length(X),addstackmean,'red')
plot(1:length(X),zeros(length(X)),'black')
for m = 1:length(winbound)
plot([winbound(m) winbound(m)],[-2 2],'red')
end
xlabel('Discrete Unit')
ylabel('Amplitude')
title('Window Bounds Picking')
xlim([0 L1(end,1)*(2/5)])
ylim([-2 2])
grid off
box on
A4. Magnetic Field Data and Model Generation

%% Magnetic Data Model
% April 15, 2020
% Luke Anderson
% Graduate Student
% Civil & Environmental Engineering
% University of Wisconsin-Madison
clc; clear; close all;
%% location of guardrail marks with respect to beginning of survey
% first steel post at index = 12
surveyposts = [0 10 10.56 12.45 14.31 16.24 18.13 20.01 21.96 23.84 25.75 27.73 29.63 31.54 33.45 35.37 37.25 39.18 41.08 42.94 44.90 46.78 48.68 50.58 52.45 54.42 56.33 58.20 60.90 62.81 63.89 65.76 66.72 67.72 68.64 69.56 70.52 71.47 71.98 72.42 72.92 73.42 74.80 75.27];

% steel posts
postnumber = [32 28 24 20 16 12 8 4 2];
postembed  = [36 34 34.5 34 33 37 38 39.25 53.5]; % inches
postlength = (72 72 72 72 72 72 72 72 88); % inches
postheight = postlength - postembed; % inches
% steel post number
poststeel = 33:1:1;
% steel post height
poststeel(2,:) = interp1(postnumber,postheight,poststeel);
poststeel(2,1) = poststeel(2,2);
poststeel(2,end) = poststeel(2,end-1);
% steel post length
poststeel(3,:) = interp1(postnumber,postlength,poststeel(1,:));
poststeel(3,1) = poststeel(3,2);
poststeel(3,end) = poststeel(3,end-1);
% steel post position along guardrail
poststeel(4,:) = surveyposts(12:end);

%% Data Import
run1 = readtable('Run2.dat');
R1 = table2array(run1(:,[1,2,3,6,7]));
N1 = size(R1);

%% Data manipulation --> Split data into individual line files
R1L1=R1(R1(:,4)==0,:); R1L1=flip(R1L1);
R1L2=R1(R1(:,4)==1,:); R1L2=flip(R1L2);
R1L3=R1(R1(:,4)==2,:); R1L3=flip(R1L3);
R1L4=R1(R1(:,4)==3,:); R1L4=flip(R1L4);

% File 1, first line mark manipulation
for m = 1:length(R1L1(:,1))
    if R1L1(m,5)>0
        R1L1(m,5) = R1L1(m,5)+1;
    end
end
for m = 2:length(R1L1(:,1))
    if R1L1(m,5)>0 && R1L1(m-1,5) == 0
        R1L1(m,5) = R1L1(m-1,5)+1;
    end
end
R1L2=R1(R1(:,4)==1,:); R1L2=flip(R1L2);
R1L3=R1(R1(:,4)==2,:); R1L3=flip(R1L3);
R1L4=R1(R1(:,4)==3,:); R1L4=flip(R1L4);

%% Manipulate locations to match exact post location
for m=1:(length(surveyposts)-1)
    for m1 = 1:length(R1L1(:,1))
        if R1L1(m1,5) == m
            R1L1(m1,2) = surveyposts(1,m+1);
        end
        clear m1;
    end
    for m1 = 1:length(R1L2(:,1))
        if R1L2(m1,5) == m
            R1L2(m1,2) = surveyposts(1,m+1);
        end
        clear m1;
    end
    for m1 = 1:length(R1L3(:,1))
        if R1L3(m1,5) == m
            R1L3(m1,2) = surveyposts(1,m+1);
        end
        clear m1;
    end
    for m1 = 1:length(R1L4(:,1))
        if R1L4(m1,5) == m
            R1L4(m1,2) = surveyposts(1,m+1);
        end
        clear m1;
    end
end

%% linear interpolation to post locations

% File 1
line = 0;
[a,-] = hist(R1L1(:,5),unique(R1L1(:,5)));  
for m=1:(length(surveyposts)-1)  
    x = [surveyposts(m+1) surveyposts(m)];  
    v = x*0;  
    delta = (surveyposts(m)-surveyposts(m+1))/a(2,m);  
    xq = surveyposts(m+1):delta:surveyposts(m);  
    xq(end) = [];  
    line = [line flip(xq)];  
    clear delta, clear xq;
end
line = line';
for m1 = 1:length(line)  
    R1L1(m1,2) = line(m1);
end
R1L1=R1L1(1:length(line),:);

% File 2
line = 0;  
[a,-] = hist(R1L2(:,5),unique(R1L2(:,5)));  
for m=1:(length(surveyposts)-1)  
    x = [surveyposts(m+1) surveyposts(m)];  
    v = x*0;  
    delta = (surveyposts(m)-surveyposts(m+1))/a(2,m);  
    xq = surveyposts(m+1):delta:surveyposts(m);  
    xq(end) = [];  
    line = [line flip(xq)];  
    clear delta, clear xq;
end
line = line';
for m1 = 1:length(line)  
    R1L2(m1,2) = line(m1);
end
R1L2=R1L2(1:length(line),:);

% File 3
line = 0;  
[a,-] = hist(R1L3(:,5),unique(R1L3(:,5)));  
for m=1:(length(surveyposts)-1)  
    x = [surveyposts(m+1) surveyposts(m)];  
    v = x*0;  
    delta = (surveyposts(m)-surveyposts(m+1))/a(2,m);  
    xq = surveyposts(m+1):delta:surveyposts(m);  
    xq(end) = [];  
    line = [line flip(xq)];  
    clear delta, clear xq;
end
line = line';
for m1 = 1:length(line)  
    R1L3(m1,2) = line(m1);
end
R1L3=R1L3(1:length(line),:);

% File 4
line = 0;  
[a,-] = hist(R1L4(:,5),unique(R1L4(:,5)));  
for m=1:(length(surveyposts)-1)  
    x = [surveyposts(m+1) surveyposts(m)];  
    v = x*0;  
    delta = (surveyposts(m)-surveyposts(m+1))/a(2,m);  
    xq = surveyposts(m+1):delta:surveyposts(m);  
    xq(end) = [];  
    line = [line flip(xq)];  
    clear delta, clear xq;
end
line = line';
for m1 = 1:length(line)  
    R1L4(m1,2) = line(m1);
end
R1L4=R1L4(1:length(line),:);
x = [surveyposts(m+1) surveyposts(m)];
v = x*0;
delta = (surveyposts(m)-surveyposts(m+1))/a(1,m);
xq = surveyposts(m+1):delta:surveyposts(m);
xq(end) = [];
line = [line flip(xq)];
clear delta, clear xq;
end

clear a; clear b;
line = line';
for m1 = 1:length(line)
    R1L3(m1,2) = line(m1);
end
R1L3=R1L3(1:length(line),:);
clear line;

[a,-]=hist(R1L4(:,5),unique(R1L4(:,5)));
for m=1:length(surveyposts)-1
    x = [surveyposts(m+1) surveyposts(m)];
v = x*0;
delta = (surveyposts(m)-surveyposts(m+1))/a(1,m);
xq = surveyposts(m+1):delta:surveyposts(m);
xq(end) = [];
line = [line flip(xq)];
clear delta, clear xq;
end

clear a; clear b;
line = line';
for m1 = 1:length(line)
    R1L4(m1,2) = line(m1);
end
R1L4=R1L4(1:length(line),:);
clear line;

%% FIGURE 1 graph data, File 1
figure('DefaultAxesFontSize',14)
plot(R1L1(:,2),R1L1(:,3))
hold on
plot(R1L2(:,2),R1L2(:,3))
plot(R1L3(:,2),R1L3(:,3))
plot(R1L4(:,2),R1L4(:,3))
ylabel('Magnetic Field (nT)')
xlabel('Distance (m)')
legend('Line 1','Line 2','Line 3','Line 4','Location','Northwest',... 'FontSize',12,'NumColumns',2)
legend box off
ylim([50000 65000])
box on
set(gca,'fontname','times')
set(gcf, 'Position', [500, 400, 625, 300])

%% Interpolate data to allow for stacking
xq = 0:0.01:76;

% File 1
x1L1 = R1L1(:,2); v1L1 = R1L1(:,3); vq1L1 = interp1(x1L1,v1L1,xq);
x1L2 = R1L2(:,2); v1L2 = R1L2(:,3); vq1L2 = interp1(x1L2,v1L2,xq);
x1L3 = R1L3(:,2); v1L3 = R1L3(:,3); vq1L3 = interp1(x1L3,v1L3,xq);
x1L4 = R1L4(:,2); v1L4 = R1L4(:,3); vq1L4 = interp1(x1L4,v1L4,xq);

% data stacking
vq1stack = zeros(length(xq),1); 
vq2stack = vq1stack; vq3stack = vq1stack;
for m = 1:length(xq)
    vq1stack(m,1) = (vq1L2(m)+vq1L3(m)+vq1L4(m))/3;
end

smooth_data = 300;
for m = 1:length(xq)
    vq1mean(:,1) = movmean(vq1stack, smooth_data);
end

%% Figure 2
figure('DefaultAxesFontSize',14);
plot(xq,vq1stack,'black','Linewidth',1.5);
hold on
plot(xq,vq1mean,'red');
for n = 1:(length(surveyposts)-11)
    plot([surveyposts(11+n),surveyposts(11+n)],[0,70000],'blue--')
end
ylabel('Magnetic Field (nT)')
xlabel('Distance (m)')
ylim([50000 65000])
legend('Stacked Survey', [num2str(smooth_data) '-pt Moving Mean'],... 
'Stack Steel Post Location','Location','NW','FontSize',12)
box on
set(gca,'fontname','times')
set(gcf,'Position', [500, 400, 625, 300])

%% FIGURE 3
figure('DefaultAxesFontSize',14)
% yyaxis left
plot(xq,(vq1stack-vq1mean), 'black')
hold on
for n = 1:(length(surveyposts)-11)
    plot([surveyposts(11+n),surveyposts(11+n)],[-10000,70000],'blue--')
end
ylim([-2000 2500])
ylabel('Magnetic Anomaly (nT)')
xlabel('Distance (m)')
set(gca,'fontname','times')
set(gcf,'Position', [500, 400, 625, 300])

%% Magnetic Field Parameters

% Magnetic Field (October 4th, 2019)
% ----------
Fe = 54050.9; %nT magnetic field strength (input)
I = 69.8667; %degrees inclination (input)
Ze = Fe*sind(I); %nT vertical magnetic field
alpha = 14; %degrees survey direction (input)
D = -3.6582; %degrees declination (input)
I = I + (alpha - D)/90 * (90-1);

% ----------
He = Fe*cosd(I); %nT horizontal magnetic field
Fe = sqrt(He^2+Fe^2); %nT magnetic field strength
Xe = He*cosd(D); %nT E-W magnetic field component
Ye = He*sind(D); %nT N-S magnetic field component
mu_0 = 4*pi*(10^-7); %H/m magnetic permeability of a vacuum
C = mu_0/(4*pi); %H/m constant

%% Steel post geometry manipulation

% pile heights (200 pt & 500 pt)
poststeel(2,1) = poststeel(2,1)-2;
poststeel(2,2) = poststeel(2,2)-6;
poststeel(2,3) = poststeel(2,3)+1;
poststeel(2,4) = poststeel(2,4)+0.5;
% poststeel(2,5) = poststeel(2,5)+2;
% poststeel(2,6) = poststeel(2,6)+0.5;
% poststeel(2,7) = poststeel(2,7)-3;
% poststeel(2,8) = poststeel(2,8)-1;
% poststeel(2,9) = poststeel(2,9)+1;
% poststeel(2,10) = poststeel(2,10)+1;
% poststeel(2,11) = poststeel(2,11)+1;
% poststeel(2,12) = poststeel(2,12)+3;
% poststeel(2,13) = poststeel(2,13)-1.5;
% poststeel(2,14) = poststeel(2,14)-1.5;
% poststeel(2,15) = poststeel(2,15)-2;
% poststeel(2,16) = poststeel(2,16)+0;
% poststeel(2,17) = poststeel(2,17)-2.5;

% pile lengths (200 pt & 500 pt)
% poststeel(3,1) = poststeel(3,1)-30;
% poststeel(3,2) = poststeel(3,2)-54;
% poststeel(3,3) = poststeel(3,3)+24;
% poststeel(3,4) = poststeel(3,4)-24;
% poststeel(3,5) = poststeel(3,5)+108;
% poststeel(3,6) = poststeel(3,6)-24;
% poststeel(3,7) = poststeel(3,7)+42;
% poststeel(3,8) = poststeel(3,8)-36;
% poststeel(3,9) = poststeel(3,9)+0;
% poststeel(3,10) = poststeel(3,10)+6;
% poststeel(3,11) = poststeel(3,11)-12;
% poststeel(3,12) = poststeel(3,12)+108;
% poststeel(3,13) = poststeel(3,13)-42;
% poststeel(3,14) = poststeel(3,14)-42;
% poststeel(3,15) = poststeel(3,15)-48;
% poststeel(3,16) = poststeel(3,16)-36;
% poststeel(3,17) = poststeel(3,17)-42;

% -------------------------------
% pile heights (100-pt moving average)
% poststeel(2,1) = poststeel(2,1)+4;
% poststeel(2,2) = poststeel(2,2)-7;
% poststeel(2,3) = poststeel(2,3)+1.5;
% poststeel(2,4) = poststeel(2,4)-1;
% poststeel(2,5) = poststeel(2,5)+2;
% poststeel(2,6) = poststeel(2,6)-0.5;
% poststeel(2,7) = poststeel(2,7)-3;
% poststeel(2,8) = poststeel(2,8)-1;
% poststeel(2,9) = poststeel(2,9)+1;
% poststeel(2,10) = poststeel(2,10)+0.5;
% poststeel(2,11) = poststeel(2,11)+1;
% poststeel(2,12) = poststeel(2,12)+4;
% poststeel(2,13) = poststeel(2,13)-2.5;
% poststeel(2,14) = poststeel(2,14)-2;
% poststeel(2,15) = poststeel(2,15)-3;
% poststeel(2,16) = poststeel(2,16)+0.5;
% poststeel(2,17) = poststeel(2,17)-4;

% pile lengths (100-pt moving average)
% poststeel(3,1) = poststeel(3,1)-0;
% poststeel(3,2) = poststeel(3,2)-60;
% poststeel(3,3) = poststeel(3,3)+24;
% poststeel(3,4) = poststeel(3,4)-24;
% poststeel(3,5) = poststeel(3,5)+108;
% poststeel(3,6) = poststeel(3,6)-24;
% poststeel(3,7) = poststeel(3,7)-42;
% poststeel(3,8) = poststeel(3,8)-36;
% poststeel(3,9) = poststeel(3,9)+0;
% poststeel(3,10) = poststeel(3,10)+6;

% poststeel(3,11) = poststeel(3,11)-12;
% poststeel(3,12) = poststeel(3,12)+108;
% poststeel(3,13) = poststeel(3,13)-42;
% poststeel(3,14) = poststeel(3,14)-42;
% poststeel(3,15) = poststeel(3,15)-60;
% poststeel(3,16) = poststeel(3,16)-36;
% poststeel(3,17) = poststeel(3,17)-48;
poststeel(3,9)  = poststeel(3,1)-12;

for m= 1:length(poststeel(1,:))
    hdo(m,1) = 0.10;  % horizontal distance offset from guardrail (input)
    hdo(m,2) = 0.20;
    hdo(m,3) = 0.40;
    hdo(m,4) = 1.00;
    hdo(m,5) = 2.00;
end

% hdo(1)  = hdo(1)+0;
% hdo(2)  = hdo(2)+0.20;
% hdo(3)  = hdo(3)+0.10;
% hdo(4)  = hdo(4)+0.15;
% hdo(5)  = hdo(5)-0.03;
% hdo(6)  = hdo(6)+0.15;
% hdo(7)  = hdo(7)+0.03;
% hdo(8)  = hdo(8)+0.04;
% hdo(9)  = hdo(9)+0;
% hdo(10) = hdo(10)+0.04;
% hdo(11) = hdo(11)+0.11;
% hdo(12) = hdo(12)-0.05;
% hdo(13) = hdo(13)+0.15;
% hdo(14) = hdo(14)+0.13;
% hdo(15) = hdo(15)+0.15;
% hdo(16) = hdo(16)+0.05;
% hdo(17) = hdo(17)+0.06;

%% Steel H-Pile Parameters (W 6 X 9)

% --------
hp = poststeel(2,:)*0.0254;  %m pile heights above ground (input)
hi = 1.1;  %m instrument height above ground (input)
l = poststeel(3,:)*0.0254;  %m pile lengths (input)
n = 50;  % number of "dipoles" (input)
x = -80:0.01:80;  %m distance from post (input)

% --------
d = 5.90;  %in depth (y) (input)
w = 3.94;  %in flange width (x) (input)
A = 2.68;  %in^2 cross-section area (input)
% kappa = 700;  % steel magnetic susceptibility (input)
kappa = 1000;  % steel magnetic susceptibility (input)

% --------
J = kappa*Fe/mu_0;  %A/m induced magnetization

% Total Magnetic Anomaly Analysis

% --------
hd = l-hp;  %m depth of piles below ground
Vol = A.*l;  %m^3 pile volumes
Vn = Vol/n;  %m^3 steel pole "dipole" volume
Mn = Vn*J;  %A m^2 magnetic dipole moment

% --------
zm = 1:zeros(n);
F_sum = zeros(length(l),length(x));
xzm = zeros(length(n),length(x)); r1 = xzm; r = xzm; a = xzm; theta = xzm; F = xzm;

for i = 1:5
    for m = 1:length(l)
        for m1 = 1:n
            zm(m1) = ((l(m)/n)*m1)-(l(m)/(2*n))+(hi-hp(m));  %depth to sections of pile
            offset(m1) = hdo(m,i);
        end
    end
end
for m2 = 1:length(x)
    xzm(m1,m2) = x(m2)/zm(m1);
    r1(m1,m2) = (x(m2)^2 + zm(m1)^2)^0.5;
    r(m1,m2) = (r1(m1,m2)^2 + offset(m1)^2)^0.5;
    a(m1,m2) = atand(1/xzm(m1,m2));
    theta(m1,m2) = a(m1,m2)+1;
end

F(m1,m2) = (c*Mn(m)/(r(m1,m2)^3))\(3\cos(\theta(m1,m2))^2 - 1\); % F for every radius and every pile section
F_sum(m,m2,i) = sum(F(:,m2)); % sum of the F values at x distances
end

%%

for m = 1:length(l)
    X(m,:) = x(1,:)+poststeel(4,m);
end

%% Matrix and data manipulation
for m = 1:length(X(:,1))
    for m1 = 1:length(X(1,:))
        A(m,m1) = X(m,m1) < 0;
        B(m,m1) = X(m,m1) > 80;
    end
    C = A+B;
end
for i = 1:5
    F_s(:,:,i) = F_sum(:,:,i) - (F_sum(:,:,i).*C);
end
X = X - (X.*C);
idx=C(1,:)==0;
out=sum(idx(:));
for i = 1:5
    for m1 = 1:length(F_s(:,i,i))
        a = 0;
        for m = 1:length(F_s(1,:,i))
            if abs(F_s(m1,m,i)) > 0
                a = a + 1;
                F_edit(m1,a,i) = F_s(m1,m,i);
                X_edit(m1,a) = X(m1,m);
            end
        end
    end
end
F_anomaly = sum(F_edit)/33;

smooth_model = 550;
F_movmean = movmean(F_anomaly,smooth_model);
X_survey = sum(X_edit)/33;

%% Figure 4
figure('DefaultAxesFontSize',14)
hold on
for i = 1
    for m = 1:length(l)
        plot(X(m,:),F_sum(m,:,i),'LineWidth',0.5)
text(1.5,40000, ['Magnetic Susceptibility, \kappa = ' num2str(kappa)],...
    'fontname','times','fontsize',12)
xlabel('Distance (m)')
ylabel('Magnetic Anomaly (nT)')
set(gca,'fontname','times')
set(gcf, 'Position', [500, 400, 625, 300])

%% Figure 5
figure('DefaultAxesFontSize',14)
hold on
for i = 1:
    plot(X_survey, F_anomaly(:,:,i), 'black', 'linewidth',1.5)
    plot(X_survey, F_movmean(:,:,i), 'red')
end
for n = 1:(length(surveyposts)-11)
    plot([surveyposts(11+n),surveyposts(11+n)],
        [-1000,3000], 'blue--')
end
ylabel('Magnetic Field (nT)')
xlabel('Distance (m)')
legend('Total Magnetic Anomaly','Steel Post Location', 'location','Northwest','FontSize',12)
xlim([0 80])
ylim([-1000 7000])
set(gca,'fontname','times')
set(gcf, 'Position', [500, 400, 625, 300])

%% Figure 6
figure('DefaultAxesFontSize',14)
hold on
for i = 1:
    plot(X_survey, F_anomaly(:,:,i)-F_movmean(:,:,i), 'black')
end
for n = 1:(length(surveyposts)-11)
    plot([surveyposts(11+n),surveyposts(11+n)],
        [-1000,2000], 'blue--')
end
ylabel('Magnetic Anomaly (nT)')
xlabel('Distance (m)')
legend('Magnetic Anomaly','Steel Post Location', 'location','SW',...
    'FontSize',12)
xlim([0 80])
set(gca,'fontname','times')
set(gcf, 'Position', [500, 400, 625, 300])

%% Figure 7
figure('DefaultAxesFontSize',14)
hold on
for i = 1:
    plot(X_survey + 0.5, F_anomaly(:,:,1)-F_movmean(:,:,1), 'black', 'linewidth',1.5)
end
plot(xq,(vq1stack-vq1mean), 'red')
% ylim([-500 1000])
xlabel('Distance (m)')
ylabel('Magnetic Anomaly (nT)')
xlim([0 80])
legend(['Model, \kappa = ' num2str(kappa)],'Field Data',...
    'location','NW','FontSize',12)
set(gca,'fontname','times')
set(gcf, 'Position', [300, 300, 625, 300])

%% Figure 8
figure
F_model = F_anomaly - F_movmean;
hold on
for i = 1
end

202
plot(X_survey + 0.5, F_model(:, :, 1), 'black', 'linewidth', 1.5)
end
F_data = smoothdata((vq1stack - vq1mean));
plot(xq, F_data, 'red')

ylim([-600 600])
xlabel('Distance (m)')
ylabel('Magnetic Anomaly (nT)')

legend({'Model, kappa = ' num2str(kappa), 'Field Data', 'location', 'SW'})
legend box off
set(gca, 'fontname', 'times')
set(gca, 'fontsize', 14)
set(gcf, 'Position', [300, 300, 625, 300])

%% Figure 9
figure
F_model = F_anomaly - F_movmean;
hold on
plot(X_survey + 0.5, F_model(:, :, 1), 'blue', 'LineWidth', 0.5)
plot(X_survey + 0.5, F_model(:, :, 2), 'red', 'LineWidth', 0.75)
plot(X_survey + 0.5, F_model(:, :, 3), 'Color', [0.9290, 0.6940, 0.1250], 'LineWidth', 1.25)
plot(X_survey + 0.5, F_model(:, :, 4), 'Color', [0.2280, 0.2280, 0.2280], 'LineWidth', 1.5)
plot(X_survey + 0.5, F_model(:, :, 5), 'cyan', 'LineWidth', 1.5)

ylim([-2000 12000])
xlabel('Distance (m)')
ylabel('Magnetic Anomaly (nT)')
xlim([20 60])
legend({'0.1 m offset', '0.2 m offset', '0.4 m offset', '1.0 m offset', '2.0 m offset', 'NumColumns', 2}, 'location', 'NW')
legend box off
box on
set(gca, 'fontname', 'times')
set(gca, 'fontsize', 14)
set(gcf, 'Position', [300, 300, 625, 400])

%% Figure 10
for i = 1:5
    F_model2(:, :, i) = F_model(:, :, i) ./ max(F_model(:, 2551:6051, i));
end
figure
hold on
plot(X_survey + 0.5, F_model2(:, :, 1), 'blue')
plot(X_survey + 0.5, F_model2(:, :, 2), 'red')
plot(X_survey + 0.5, F_model2(:, :, 3), 'Color', [0, 0.5, 0])
plot(X_survey + 0.5, F_model2(:, :, 4), 'Color', [0.9290, 0.6940, 0.1250])
plot(X_survey + 0.5, F_model2(:, :, 5), 'cyan')

ylim([-1 2])
xlabel('Distance (m)')
ylabel('Normalized Anomaly (nT/nT)')
xlim([20 60])
legend({'0.1 m offset', '0.2 m offset', '0.4 m offset', '1.0 m offset', '2.0 m offset', 'NumColumns', 2}, 'location', 'NW')
legend box off
box on
set(gca, 'fontname', 'times')
set(gca, 'fontsize', 14)
set(gcf, 'Position', [300, 300, 625, 400])

%% Figure 11
for i = 1:5
    F_model2(:, :, i) = F_model(:, :, i) ./ max(F_model(:, 4251:4451, i));
end
figure
hold on
plot(X_survey + 0.5, F_model2(:,:,1), 'blue')
plot(X_survey + 0.5, F_model2(:,:,2), 'red')
plot(X_survey + 0.5, F_model2(:,:,3), 'Color',[0, 0.5, 0])
plot(X_survey + 0.5, F_model2(:,:,4), 'Color',[0.9290, 0.6940, 0.1250])
plot([0 80],[0 0], 'black')
% grid on
ylim([-1 2])
xlabel('Distance (m)')
ylabel('Normalized Anomaly (nT/nT)')
nxlim([42.6 44])
legend([0.1 m offset'],...['0.2 m offset'],...['0.4 m offset'],...['1.0 m offset'],...['2.0 m offset'],... 'NumColumns', 2, 'location','NW')
legend box off
box on
set(gca, 'fontname', 'times')
set(gca, 'fontsize', 14)
set(gcf, 'Position', [300, 300, 625, 350])

%% data fitting error

data_y = F_model(2701:5901);
model_y = F_data(2751:5951);
for i = 1:length(data_y)
    RMSEi(i) = (data_y(i) - model_y(i))^2;
end
RMSE = sqrt(sum(RMSEi)/length(RMSEi))
Appendix B: Stress wave propagation field testing results by the author.

Table B.1. Results (1) from field testing highway guardrail in Pewaukee, Wisconsin.

<table>
<thead>
<tr>
<th>Run-Post</th>
<th>Wood (W) / Steel (S)</th>
<th>LF/HF Waves</th>
<th>Frequency Analysis</th>
<th>Measured Length (m)</th>
<th>Calculated Length (m)</th>
</tr>
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<tbody>
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Table B.2. Results from field testing highway guardrail at Forest Product Laboratory (FPL), Madison, WI.

<table>
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<tr>
<th>Post</th>
<th>Wood (W) / Steel (S)</th>
<th>LF/HF Waves</th>
<th>Frequency Analysis</th>
<th>Measured Length (m)</th>
<th>Calculated Length (m)</th>
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Table B.3. Results (2) from field testing highway guardrail in Pewaukee, Wisconsin.

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<th>Calculated Length (m)</th>
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Table B.4. Results (3) from field testing highway guardrail in Pewaukee, Wisconsin.

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<th>Run-Post</th>
<th>Wood (W) / Steel (S)</th>
<th>LF/HF Waves</th>
<th>Frequency Analysis</th>
<th>Measured Length (m)</th>
<th>Calculated Length (m)</th>
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</tr>
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<td>1.50</td>
</tr>
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<td>1.46</td>
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<td>1.64</td>
</tr>
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<td>1.62</td>
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<td>LF - Ball Detach</td>
<td>PSD</td>
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Appendix C: Results from field testing wood and steel posts prior to the involvement of the author.

Field testing was completed in several locations in the state of Wisconsin, including Tomah, Chippewa Falls, Oxford, Janesville, Pewaukee, and Westfield (Edgewood Ct). Presented plots are from posts of known length compared to predicted length.

Figure C.1. Locations of testing sites in the state of Wisconsin for stress-wave propagation technique analysis.
Figure C.2. Results from field testing posts before the involvement of the author in Tomah, Chippewa Falls, Pewaukee, and Westfield. Low frequency hammer strikes used to evaluate natural frequency under power spectra density analysis and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT:** Results when testing steel. Root Mean Square Error (RMSE) equals 0.41 m. **RIGHT:** Results when testing wood. Root Mean Square Error (RMSE) equals 0.83 m.
Table C.1. Results from field testing highway guardrail in Tomah, Wisconsin.

<table>
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<th>Material</th>
<th>Measured length (m)</th>
<th>Calculated length (m)</th>
<th>Observations</th>
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<td>3</td>
<td>Steel</td>
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<td>2.24</td>
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<td>6</td>
<td>Steel</td>
<td>1.84</td>
<td>1.83</td>
<td>Not a very strong peak</td>
</tr>
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<td>1.86</td>
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</tr>
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<td>10</td>
<td>Steel</td>
<td>1.86</td>
<td>1.27</td>
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<td>11</td>
<td>Steel</td>
<td>1.86</td>
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<td>1.85</td>
<td>1.75</td>
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<td>Wood</td>
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<td>2.24</td>
<td>Peak before 1000 Hz</td>
</tr>
<tr>
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<td>Wood</td>
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<td>2.97</td>
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</tr>
<tr>
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<td>1</td>
<td>Wood</td>
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<td>1.55</td>
<td>Poor velocity determination</td>
</tr>
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<td>Chippewa Falls</td>
<td>1</td>
<td>3</td>
<td>Wood</td>
<td>1.93</td>
<td>2.87</td>
<td>Poor first mode determination, peak before 1000 Hz</td>
</tr>
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<td></td>
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<td>Wood</td>
<td>1.85</td>
<td>3.91</td>
<td>Poor velocity and first mode</td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
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<td>Wood</td>
<td>2.41</td>
<td>This post was not pulled, poor power spectral density</td>
<td></td>
</tr>
<tr>
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<td>1</td>
<td>7</td>
<td>Wood</td>
<td>1.91</td>
<td>2.24</td>
<td>Peak before 1000 Hz</td>
</tr>
<tr>
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<td>9</td>
<td>Wood</td>
<td>1.85</td>
<td>2.97</td>
<td></td>
</tr>
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<td>13</td>
<td>Steel</td>
<td>1.88</td>
<td>1.73</td>
<td></td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>16</td>
<td>Steel</td>
<td>1.83</td>
<td>1.63</td>
<td>Peak at 1500 Hz, good velocity calculation</td>
</tr>
<tr>
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<td>1</td>
<td>19</td>
<td>Steel</td>
<td>1.83</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
<td>Chippewa Falls</td>
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<td>22</td>
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</table>

Table C.2. Results from field testing highway guardrail in Chippewa Falls, Wisconsin.

<table>
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<tr>
<th>Site</th>
<th>Run</th>
<th>Post</th>
<th>Material</th>
<th>Measured length (m)</th>
<th>Calculated length (m)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>1</td>
<td>Wood</td>
<td>1.91</td>
<td>1.55</td>
<td>Poor velocity determination</td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>3</td>
<td>Wood</td>
<td>1.93</td>
<td>2.87</td>
<td>Poor first mode determination, peak before 1000 Hz</td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>4</td>
<td>Wood</td>
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<td>2.87</td>
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</tr>
<tr>
<td>Chippewa Falls</td>
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<td>5</td>
<td>Wood</td>
<td>1.85</td>
<td>3.91</td>
<td>Poor velocity and first mode</td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>6</td>
<td>Wood</td>
<td>2.41</td>
<td>This post was not pulled, poor power spectral density</td>
<td></td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>7</td>
<td>Wood</td>
<td>1.91</td>
<td>2.24</td>
<td>Peak before 1000 Hz</td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>9</td>
<td>Wood</td>
<td>1.85</td>
<td>2.97</td>
<td></td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>10</td>
<td>Steel</td>
<td>0.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>13</td>
<td>Steel</td>
<td>1.88</td>
<td>1.73</td>
<td></td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>16</td>
<td>Steel</td>
<td>1.83</td>
<td>1.63</td>
<td>Peak at 1500 Hz, good velocity calculation</td>
</tr>
<tr>
<td>Chippewa Falls</td>
<td>1</td>
<td>19</td>
<td>Steel</td>
<td>1.83</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
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<td>Steel</td>
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</tbody>
</table>
Table C.3. Results from field testing highway guardrail in Oxford, Wisconsin.

<table>
<thead>
<tr>
<th>Site</th>
<th>Run</th>
<th>Post</th>
<th>Material</th>
<th>Measured length (m)</th>
<th>Calculated length (m)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxford</td>
<td>2</td>
<td>54</td>
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<td>3.05</td>
<td>3.05</td>
<td>Poor power spectra</td>
</tr>
<tr>
<td>Oxford</td>
<td>2</td>
<td>54</td>
<td>Wood</td>
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<td>3.05</td>
<td>No bolt, slight peak change</td>
</tr>
<tr>
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<td>2</td>
<td>55</td>
<td>Wood</td>
<td>2.64</td>
<td>2.64</td>
<td>Early peak</td>
</tr>
<tr>
<td>Oxford</td>
<td>2</td>
<td>55</td>
<td>Wood</td>
<td>2.64</td>
<td>2.64</td>
<td>No bolt</td>
</tr>
<tr>
<td>Oxford</td>
<td>2</td>
<td>56</td>
<td>Wood</td>
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<td>2.67</td>
<td></td>
</tr>
<tr>
<td>Oxford</td>
<td>2</td>
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<td>Wood</td>
<td>2.64</td>
<td>2.64</td>
<td>Poor power spectra, very linear post length</td>
</tr>
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<td>58</td>
<td>Wood</td>
<td>2.57</td>
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<td>Early peak</td>
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<td>Oxford</td>
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<td>1.63</td>
<td>Bimodal peaks</td>
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Table C.4. Results from field testing highway guardrail in Janesville, Wisconsin.

<table>
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<th>Site</th>
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<th>Post</th>
<th>Material</th>
<th>Measured length (m)</th>
<th>Calculated length (m)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janesville</td>
<td>1</td>
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<td>Wood</td>
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<td>1.80</td>
<td>Early Peak</td>
</tr>
<tr>
<td>Janesville</td>
<td>1</td>
<td>3</td>
<td>Wood</td>
<td>2.44</td>
<td>2.44</td>
<td>Early Peak</td>
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<td>1</td>
<td>5</td>
<td>Wood</td>
<td>1.80</td>
<td>1.80</td>
<td>Early Peak</td>
</tr>
<tr>
<td>Janesville</td>
<td>1</td>
<td>7</td>
<td>Wood</td>
<td>2.31</td>
<td>2.31</td>
<td>Poor power spectra</td>
</tr>
<tr>
<td>Janesville</td>
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<td>9R</td>
<td>Wood</td>
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<td>2.31</td>
<td>Only HF</td>
</tr>
<tr>
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<td>13R</td>
<td>Wood</td>
<td>2.90</td>
<td>2.90</td>
<td>No block, Early and sharp peak</td>
</tr>
<tr>
<td>Janesville</td>
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<td>15</td>
<td>Wood</td>
<td>2.39</td>
<td>2.39</td>
<td>Early peak</td>
</tr>
<tr>
<td>Janesville</td>
<td>2</td>
<td>16</td>
<td>Wood</td>
<td>2.54</td>
<td>2.54</td>
<td>No block, Early peak</td>
</tr>
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<td>Wood</td>
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<td>No block</td>
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</tr>
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<td>19</td>
<td>Wood</td>
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<td>2.08</td>
<td></td>
</tr>
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<td>Janesville</td>
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<td>Wood</td>
<td>3.15</td>
<td>3.15</td>
<td>No block, Early peak</td>
</tr>
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<td>22</td>
<td>Wood</td>
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<td>2.18</td>
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<tr>
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<td>23</td>
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<td>No block</td>
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</tr>
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<td>Early peak</td>
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### Table C.5. Results (4) from field testing highway guardrail in Pewaukee, Wisconsin.

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<th>Material</th>
<th>Measured length (m)</th>
<th>Calculated length (m)</th>
<th>Observations</th>
</tr>
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</tr>
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<td>3</td>
<td>Wood</td>
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<td>0.91</td>
<td>Poor peak determination</td>
</tr>
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<td>Wood</td>
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<td></td>
<td>Early peak</td>
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<td>5</td>
<td>Steel</td>
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<td>2.13</td>
<td></td>
</tr>
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<td>Steel</td>
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<td>1.93</td>
<td></td>
</tr>
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<td>11</td>
<td>Steel</td>
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<td>Steel</td>
<td>2.13</td>
<td>0.97</td>
<td>Poor velocity determination</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>9</td>
<td>17</td>
<td>Steel</td>
<td>2.13</td>
<td>1.83</td>
<td>Poor power spectra, one signal lower in amplitude</td>
</tr>
<tr>
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<td>2</td>
<td>Steel</td>
<td>2.24</td>
<td>1.73</td>
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</tr>
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<td>4</td>
<td>Steel</td>
<td>1.83</td>
<td>1.47</td>
<td>Poor power spectra</td>
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<td>Steel</td>
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</tr>
<tr>
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<td>11</td>
<td>16</td>
<td>Steel</td>
<td>1.83</td>
<td>1.83</td>
<td>Low peak, accurate post length</td>
</tr>
<tr>
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<td>11</td>
<td>20</td>
<td>Steel</td>
<td>1.83</td>
<td>1.83</td>
<td>Low peak, accurate post length</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>11</td>
<td>24</td>
<td>Steel</td>
<td>1.83</td>
<td>1.96</td>
<td>Low peak</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>11</td>
<td>28</td>
<td>Steel</td>
<td>1.83</td>
<td>Error</td>
<td>Error when code runs</td>
</tr>
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<td>Steel</td>
<td>1.83</td>
<td>2.44</td>
<td>Poor velocity determination</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>11</td>
<td>34</td>
<td>Wood</td>
<td>1.83</td>
<td>1.17</td>
<td>4 peaks</td>
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<td>35</td>
<td>Wood</td>
<td>1.83</td>
<td>2.46</td>
<td>Bimodal peaks</td>
</tr>
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<td>Pewaukee</td>
<td>11</td>
<td>36</td>
<td>Wood</td>
<td>1.83</td>
<td>2.44</td>
<td>Early peak</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>11</td>
<td>37</td>
<td>Wood</td>
<td>1.83</td>
<td>0.79</td>
<td>Bimodal peaks</td>
</tr>
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<td>38</td>
<td>Wood</td>
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<td>2.06</td>
<td>Not a sharp peak</td>
</tr>
<tr>
<td>Pewaukee</td>
<td>11</td>
<td>39</td>
<td>Wood</td>
<td>1.77</td>
<td>2.36</td>
<td>Not a sharp peak</td>
</tr>
<tr>
<td>Pewaukee</td>
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<td>40</td>
<td>Wood</td>
<td>1.83</td>
<td>2.36</td>
<td>Bimodal peaks</td>
</tr>
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<td>41</td>
<td>Wood</td>
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<td>1.85</td>
<td></td>
</tr>
<tr>
<td>Pewaukee</td>
<td>11</td>
<td>42</td>
<td>Wood</td>
<td>1.17</td>
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</tr>
</tbody>
</table>

### Table C.6. Results from field testing highway guardrail in Westfield (Edgewood Ct), Wisconsin.

<table>
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<tr>
<th>Site</th>
<th>Run</th>
<th>Post</th>
<th>Material</th>
<th>Measured length (m)</th>
<th>Calculated length (m)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edgewood Ct.</td>
<td>10</td>
<td>4</td>
<td>Wood</td>
<td>1.83</td>
<td>2.67</td>
<td>No block</td>
</tr>
<tr>
<td>Edgewood Ct.</td>
<td>10</td>
<td>4</td>
<td>Wood</td>
<td>1.83</td>
<td>2.77</td>
<td>No block</td>
</tr>
<tr>
<td>Edgewood Ct.</td>
<td>10</td>
<td>6</td>
<td>Wood</td>
<td>1.83</td>
<td>2.06</td>
<td>No block</td>
</tr>
<tr>
<td>Edgewood Ct.</td>
<td>10</td>
<td>6</td>
<td>Wood</td>
<td>1.83</td>
<td>2.26</td>
<td>No block</td>
</tr>
<tr>
<td>Edgewood Ct.</td>
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<td>8</td>
<td>Wood</td>
<td>1.84</td>
<td>2.84</td>
<td></td>
</tr>
<tr>
<td>Edgewood Ct.</td>
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<td>8</td>
<td>Wood</td>
<td>1.84</td>
<td>2.57</td>
<td>No block</td>
</tr>
<tr>
<td>Edgewood Ct.</td>
<td>10</td>
<td>9</td>
<td>Wood</td>
<td>1.83</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>Edgewood Ct.</td>
<td>10</td>
<td>9</td>
<td>Wood</td>
<td>1.83</td>
<td>2.67</td>
<td>No block</td>
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<td>Wood</td>
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<td>2.62</td>
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<tr>
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<td>14</td>
<td>Wood</td>
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<td>2.49</td>
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<tr>
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<td>Wood</td>
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<td>2.21</td>
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<tr>
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<td>20</td>
<td>Wood</td>
<td>1.93</td>
<td>1.80</td>
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</table>
Appendix D: Results from testing posts when dry, exposed to freezing conditions, and soaked conditions, when clamped and not clamped to lab bench.

The testing program performed exposed posts to low frequency hammer strikes to evaluate the natural frequency, and high frequency pencil lead flexure breaks to evaluate the wave velocity.

Table D.1. Summary of wooden posts tested in moisture and freezing experiments

<table>
<thead>
<tr>
<th>Post Length</th>
<th>Cross-Sectional Dimensions</th>
<th>Cracking</th>
<th>Warping</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.216 m</td>
<td>8.9 cm by 14.0 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.236 m</td>
<td>13.7 cm by 13.8 cm</td>
<td>Major</td>
<td>-</td>
</tr>
<tr>
<td>1.834 m</td>
<td>8.6 cm by 13.4 cm</td>
<td>Minor</td>
<td>-</td>
</tr>
<tr>
<td>2.442 m</td>
<td>13.7 cm by 14.6 cm</td>
<td>Major</td>
<td>-</td>
</tr>
<tr>
<td>2.444 m</td>
<td>8.7 cm by 13.7 cm</td>
<td>Some</td>
<td>Torsional</td>
</tr>
<tr>
<td>3.040 m</td>
<td>8.7 cm by 13.4 cm</td>
<td>-</td>
<td>Bending</td>
</tr>
<tr>
<td>3.065 m</td>
<td>8.7 cm by 8.9 cm</td>
<td>-</td>
<td>Torsional</td>
</tr>
</tbody>
</table>

Figure D.1. Results from testing wood posts when dry and clamped (CL) to the lab bench directly. Low frequency hammer strikes used to evaluate natural frequency and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.42 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.38 m.
Figure D.2. Results from testing wood posts after being soaked in water and clamped (CL) to the lab bench directly. Low frequency hammer strikes used to evaluate natural frequency and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.61 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.60 m.

Figure D.3. Results from testing wood posts when exposed to outdoor freezing conditions after being soaked in water and clamped (CL) to the lab bench directly. Low frequency hammer strikes used to evaluate natural frequency and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.52 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.59 m.
Figure D.4. Results from testing wood posts when dry and not clamped (NC) to the lab bench. Low frequency hammer strikes used to evaluate natural frequency and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.51 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.46 m.

Figure D.5. Results from testing wood posts after being soaked in water and not clamped (CL) to the lab bench. Low frequency hammer strikes used to evaluate natural frequency and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT:** Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.78 m. **RIGHT:** Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.82 m.
Figure D.6. Results from testing wood posts when exposed to outdoor freezing conditions after being soaked in water and not clamped (CL) to the lab bench. Low frequency hammer strikes used to evaluate natural frequency and high frequency lead flexure breakages used to evaluate the wave velocity. **LEFT**: Results when using autocorrelation (AUTO) to determine the natural frequency. Root Mean Square Error (RMSE) equals 0.70 m. **RIGHT**: Results when using power spectra density (PSD) to determine natural frequency. RMSE equals 0.73 m.
Appendix E: Typical torsional and bending field-testing results for detached wood post (1.83 m, nominal 10 cm by 15 cm)

Figure E.1. Results for signal amplitude of torsional and bending waves. **TOP:** Torsional wave from sensor addition. **BOTTOM:** Bending wave from sensor subtraction.

Figure E.2. Results for power spectra density (PSD) of torsional and bending waves. **TOP:** Torsional wave PSD from sensor addition. **BOTTOM:** Bending wave PSD from sensor subtraction.
Figure E.3. Results for autocorrelation of torsional wave with peaks shown.

Figure E.4. Results for windowing torsional wave signal. Alternating solid and dashed lines denote windows. Four windows shown.
Figure E.5. Results for torsional wave power spectra density (PSD) of window 2 and window 3.

Figure E.6. Results for torsional wave attenuation with respect to frequency across windowed power spectra density (PSD) of window 2 and window 3.
Figure E.7. Results for torsional wave signal coherence across window 2 and window 3.

Figure E.8. Results for torsional wave phase analysis. **TOP:** Wrapped phase for window 2 compared to window 3.  
**BOTTOM:** Unwrapped phase for window 2 compared to window 3.
Figure E.9. Results for phase velocity of torsional wave with respect to windows 2 and window 3 compared to the velocity of the natural torsional velocity (bold line).

Figure E.10. Results for analytical analysis of torsional wave. **TOP:** Instantaneous amplitude of the signal. **BOTTOM:** Instantaneous frequency of the signal.
Appendix F: Windowing and phase results for spectral analysis of longitudinal waves of wood posts in the laboratory

Windowing. Signals are windowed into individual reflections to assess attenuation and phase velocity. Four windows are created, denoted by alternating solid and dashed lines. Figures F.1 to F.3 show the effect of cross-sectional area increase on windowing when length remains constant.

![Windowed Reflections](image)

**Figure F.1.** Windowed reflections from 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Solid and dashed lines denote reflection windows.
Figure F.2. Windowed reflections from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm², density: 660 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Solid and dashed lines denote reflection windows.

Figure F.3. Windowed reflections from testing 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm², density: 520 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Solid and dashed lines denote reflection windows.
Figures F.4 to F.6 show the effect of length increase on windowing when cross-sectional area remains constant.

**Figure F.4.** Windowed reflections from testing 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Solid and dashed lines denote reflection windows.

**Figure F.5.** Windowed reflections from testing 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm², density: 600 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Solid and dashed lines denote reflection windows.
Figure F.6. Windowed reflections from testing 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (No Clamp), post clamped to the lab bench with rubber pad boundaries (Rubber Clamp), and post clamped directly to the lab bench (Clamp). Solid and dashed lines denote reflection windows.

**Phase.** The phase is processed in the frequency domain and is calculated for the frequency response of two reflections. The phase is unwrapped to remove the bounds of $\pm \frac{\pi}{2}$. The phase may then be transformed into the phase velocity. **Figures F.7 to F.9** show the effect of cross-sectional area increase on phase when length remains constant.
Figure F.7. Wrapped phase (TOP) and unwrapped phase (BOTTOM) of reflections two (R2) with respect to reflections three (R3) from 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm$^2$, density: 550 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).

Figure F.8. Wrapped phase (TOP) and unwrapped phase (BOTTOM) of reflections two (R2) with respect to reflections three (R3) from 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm$^2$, density: 660 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
Figure F.9. Wrapped phase (TOP) and unwrapped phase (BOTTOM) of reflections two (R2) with respect to reflections three (R3) from 2.442-m long wood post, with cross-sectional dimensions of 13.7 cm by 14.6 cm, cross-sectional area: 200 cm$^2$, density: 520 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).

Figures F.10 to F.12 show the effect of length increase on phase when cross-sectional area remains constant.

Figure F.10. Wrapped phase (TOP) and unwrapped phase (BOTTOM) of reflections two (R2) with respect to reflections three (R3) from 1.216-m long wood post, cross-sectional dimensions: 8.9 cm by 14.0 cm, cross-sectional area: 120 cm$^2$, density: 550 kg/m$^3$. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
Figure F.11  Wrapped phase (TOP) and unwrapped phase (BOTTOM) of reflections two (R2) with respect to reflections three (R3) from 1.832-m long wood post, cross-sectional dimensions: 8.6 cm by 14.0 cm, cross-sectional area: 120 cm², density: 600 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).

Figure F.12. Wrapped phase (TOP) and unwrapped phase (BOTTOM) of reflections two (R2) with respect to reflections three (R3) from 2.423-m long wood post, cross-sectional dimensions: 8.9 cm by 13.8 cm, cross-sectional area: 120 cm², density: 660 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL).
Appendix G: Coherence analysis of signal number

The $SNR$ increases as a function of $\sqrt{M}$, therefore as the number of the signals increase, coherence values are expected to go to one. The effect of number of signals ($M$) on coherence for the post with the smallest cross-sectional area (8.3 cm by 8.6 cm cross-section, 2.435-m long post) is shown in Figure G.1. As the number of signals increases, the normalized coherence does not consistently increase for any of the boundary conditions as presented in Table G.1.

The best example of increasing coherence is shown with the rubber pad boundaries. When the post is clamped with rubber pads, the coherence increases from 4 to 8 to 16 signals, however, the coherence of the 32 signals is less than the coherence of the 16 signals. By increasing the number of signals, there is not a strongly identifiable relationship with increasing coherence. This indicates a lack of consistent input and output signals and introduces questions of system uniformity (such as nonlinear behavior of the system). Wood is a strongly anisotropic material, and like other cellular solids, often experiences a nonlinear stress-strain behavior (Holmberg et al., 1999) which may be the reasoning for inconsistent coherence expectations.
Figure G.1. The effect of signal number on coherence of reflection two (R2) and reflection three (R3) window for 2.435-m long wood post, cross-sectional dimensions: 8.3 cm by 8.6 cm, cross-sectional area: 70 cm², density: 550 kg/m³. Laboratory testing was completed under three boundary conditions: post on lab bench without clamps (NC), post clamped to the lab bench with rubber pad boundaries (RC), and post clamped directly to the lab bench (CL). TOP LEFT: Number of signals: 4. TOP RIGHT: Number of signals: 8. BOTTOM LEFT: Number of signals: 16. BOTTOM RIGHT: Number of signals: 32.

Table G.1. Effect of signal number on normalized coherence for 2.435-m post from 0 to 10000 Hz.

<table>
<thead>
<tr>
<th>Signals</th>
<th>No Clamp Coherence</th>
<th>Rubber Clamp Coherence</th>
<th>Clamp Coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.6976</td>
<td>0.7543</td>
<td>0.7597</td>
</tr>
<tr>
<td>8</td>
<td>0.7110</td>
<td>0.8575</td>
<td>0.7687</td>
</tr>
<tr>
<td>16</td>
<td>0.7078</td>
<td>0.8787</td>
<td>0.7457</td>
</tr>
<tr>
<td>32</td>
<td>0.7036</td>
<td>0.8708</td>
<td>0.6876</td>
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