University of Wisconsin-Synchrotron Radiation Center **TECHNICAL NOTE**	**File No.** **SRC-219**	**Page** 1 of 13
Subject: **Off-energy injection of Aladdin**	**Author(s):** **R. A. Bosch**	
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Injection of the Aladdin 800-MeV electron storage ring is performed at a low ring energy of 108 MeV, where the stacking time period of 0.8 s is much smaller than the radiation damping time. Electrons are injected from inside the ring at a location where the horizontal dispersion is large, using an injected beam whose energy is 0.7 MeV lower than the equilibrium energy of the storage ring.

We consider the initial horizontal offset of each injected electron to be the sum of an energy ("synchrotron") oscillation and a betatron oscillation, and assume that the oscillations of different electrons decohere in the stacking time period. This model correctly predicts the successful stacking for off-energy injection at a location with large horizontal dispersion, and the poor stacking that occurs when injecting into a lattice whose horizontal dispersion is small at the injection location.

The model also predicts that the maximum stacked current may be increased by nearly a factor of three if a 200-MeV linac is used for injection.

1. Introduction

For 108-MeV injection of the Aladdin electron storage ring, the horizontal and longitudinal radiation damping times of 14.1 s and 5 s are much longer than the stacking time period of 0.8 s (where the stacking time period is the time between firing of kickers to form an injection bump). Stacking of 300 mA is routinely obtained with the Aladdin "base" lattice, whose horizontal dispersion is 540 mm at the location of injection. The injected beam is a 4-mA beam from a microtron located inside the storage ring, whose energy is 0.7 MeV lower than the storage ring energy.

With the Aladdin-II lattice [1], whose horizontal dispersion at the location of injection is only 17 mm, the maximum stacked current is only 90–145 mA.

In a previous study [2], the possibility of fast horizontal damping due to collective effects ("head-tail damping") was disproved. The horizontal-vertical emittance coupling at injection is only 8%, so that coupling of horizontal motion into vertical motion appears to be a secondary effect. The energy of the injected beam was measured to be 0.7 MeV lower than the storage-ring equilibrium energy. It was suggested that off-energy injection at a location with non-zero horizontal dispersion increases the survival of the injected beam.

We present a model of the injection process that accounts for the experimental observations. In this model, the initial horizontal offset of an injected electron is considered as a sum of an energy ("synchrotron") oscillation and a betatron oscillation. In the time interval between firing of the kickers, the oscillations of different electrons are assumed to decohere from the variation of their initial positions and velocities. We thereby calculate the survival probability of the injected electrons.

In the base lattice, injecting a beam whose energy is 0.8% lower than the storage-ring energy increases the computed survival probability by a factor of 30 in comparison with on-energy injection, consistent with observations. A maximum stacked current of 390 mA is estimated, in agreement with experiment.

In computations for the Aladdin-II lattice, off-energy injection provides no benefit. A maximum stacked current of 100 mA is estimated, consistent with the measured stacked currents of 90–145 mA for this lattice. The model confirms that a large horizontal dispersion at the injection location improves the low-energy injection.

For the possibility of injection into the base lattice by a 200-MeV linac with current of 4 mA, the faster radiation damping is estimated to allow a higher maximum stacked current of 1100 mA.

2. Notation

Consider injection over a single turn during which the orbit is kicked near to the septum. We consider a septum located inside the orbit of the storage ring. When the septum is outside the ring, the same analysis applies if the one replaces the horizontal dispersion with its opposite. Let $x_i > 0$ equal the (horizontal) distance between the kicked orbit and the center of the injection hole from which beam is emitted. Let $x_{\text{loss}} > 0$ equal the distance between the kicked orbit and the limiting aperture of the septum, so that $x_{\text{loss}} < x_i$. This geometry is shown in fig. 1.

Let τ_x and τ_L equal the horizontal and longitudinal radiation damping time periods, and t_s equal the stacking time period. Let D equal the opposite of the horizontal dispersion at the location of injection. (If electrons are injected from a point outside of the kicked orbit, let D equal the horizontal dispersion at the location of injection.) Let $e > 0$ equal the magnitude of the electron's charge, V_{rf} equal the peak radiofrequency (rf) voltage, E_0 equal the ring energy (at injection), and α equal the momentum compaction. Let k be the harmonic number, equal to the number of rf buckets.

We assume that the synchronous energy at injection is much smaller than V_{rf}, and that the injected electrons are launched in a direction that minimizes their betatron oscillations. Table 1 gives parameters for the existing Aladdin injection at 108 MeV into the "base" lattice, injection at 108 MeV into the Aladdin II lattice, and the possibility of base-lattice injection at 200 MeV.

3. Initial oscillation amplitudes

Consider the injection of an electron with phase $\phi = \omega_{rf} t$, where ω_{rf} is the angular rf frequency and t is the injection time w.r.t. the synchronous time. Let ε equal the relative injection energy error of the electron w.r.t. the ring energy. The equivalent RF-potential allows the description of the energy ("synchrotron") oscillation to be described in terms of potential and kinetic energy. When the synchronous voltage is much smaller than the rf voltage, the initial potential and kinetic energy are [3]

$$PE = \frac{\alpha e V_{rf}}{2\pi k E_0}(1-\cos\phi) \quad , \qquad KE = \frac{1}{2}\alpha^2\varepsilon^2 . \tag{1}$$

In units of relative energy deviation $\Delta E / E_0$, the rf bucket height E_{b} is

$$E_{\mathrm{b}} = \sqrt{\frac{\alpha e V_{rf}}{\pi k E_0}} \Bigg/ \sqrt{\frac{1}{2}\alpha^2} = \sqrt{\frac{2 e V_{rf}}{\pi \alpha\, k E_0}} . \tag{2}$$

In units of relative energy deviation $\Delta E / E_0$, the initial oscillation amplitude E is

$$E = \sqrt{PE + KE} \Bigg/ \sqrt{\frac{1}{2}\alpha^2} = \sqrt{\frac{E_{\mathrm{b}}^2}{2}(1-\cos\phi) + \varepsilon^2} . \tag{3}$$

The energy oscillation is stable when the amplitude is smaller than or equal to the bucket height, i.e., when $E \le E_{\mathrm{b}}$. The initial horizontal amplitude of the energy oscillation is

$$r_E = | DE | \tag{4}$$

In addition to the energy oscillation, a horizontal betatron oscillation is excited of amplitude

$$r_\beta = | x_i - D\varepsilon | \tag{5}$$

4. Losses at the septum

Let us calculate the probability that an injected electron, whose energy oscillation is stable, will be lost on the *n*th firing of the kicker magnets after its injection. On the *n*th firing, the horizontal amplitude of its energy oscillation is decreased by radiation damping to equal

$$r_{E(n)} = r_E \exp(-n t_s / \tau_{\mathrm{L}}) , \tag{6}$$

while the horizontal amplitude of its betatron oscillation is

$$r_{\beta(n)} = r_\beta \exp(-n t_s / \tau_{\mathrm{x}}) . \tag{7}$$

Assuming that the energy and betatron oscillations decohere, the probability distributions for energy oscillations and for betatron oscillations are circles in polar coordinates, given by

$$P(r,\theta) = (2\pi r_{(n)})^{-1} \delta(r - r_{(n)}) \tag{8}$$

Therefore, each probability distribution of horizontal position is

$$P(x) = H(r_{(n)}^2 - x^2) \Big/ \left(\pi\sqrt{| r_{(n)}^2 - x^2 |}\right) , \tag{9}$$

where H denotes the Heaviside function. The probability that the betatron oscillation horizontal coordinate equals x_β while the energy oscillation horizontal coordinate equals x_E is

$$P(x_\beta, x_E) = \frac{H(r_{\beta(n)}^2 - x_\beta^2) H(r_{E(n)}^2 - x_E^2)}{\pi^2 \sqrt{| r_{\beta(n)}^2 - x_\beta^2 |} \sqrt{| r_{E(n)}^2 - x_E^2 |}} \tag{10}$$

so that the probability that the electron's horizontal coordinate equals x is

$$P(x_\beta + x_E = x) = \int_{-r_{\beta(n)}}^{r_{\beta(n)}} dx_\beta P(x_\beta, x - x_\beta) \tag{11}$$

The electron is lost when $x > x_{\text{loss}}$, so that the probability of loss on the *n*th firing is

$$P_{\text{loss}(n)} = \int_{x_{\text{loss}}}^{\infty} dx \int_{-r_{\beta(n)}}^{r_{\beta(n)}} dx_\beta \frac{H(r_{\beta(n)}^2 - x_\beta^2) H[r_{E(n)}^2 - (x - x_\beta)^2]}{\pi^2 \sqrt{| r_{\beta(n)}^2 - x_\beta^2 |} \sqrt{| r_{E(n)}^2 - (x - x_\beta)^2 |}} \tag{12}$$

Reversing the order of integration and letting $w = x - x_\beta$ gives

$$\begin{aligned} P_{\text{loss}(n)} &= \frac{1}{\pi^2} \int_{-r_{\beta(n)}}^{r_{\beta(n)}} \frac{dx_\beta}{\sqrt{r_{\beta(n)}^2 - x_\beta^2}} \int_{x_{\text{loss}} - x_\beta}^{\infty} dw \frac{H(r_{E(n)}^2 - w^2)}{\sqrt{| r_{E(n)}^2 - w^2 |}} \\ &= \frac{1}{\pi^2} \int_{-r_{\beta(n)}}^{r_{\beta(n)}} \frac{dx_\beta}{\sqrt{r_{\beta(n)}^2 - x_\beta^2}} \int_{\max(\min(x_{\text{loss}} - x_\beta, r_{E(n)}), -r_{E(n)})}^{r_{E(n)}} dw \frac{1}{\sqrt{r_{E(n)}^2 - w^2}} \\ &= \frac{1}{\pi^2} \int_{-r_{\beta(n)}}^{r_{\beta(n)}} \frac{dx_\beta}{\sqrt{r_{\beta(n)}^2 - x_\beta^2}} \arccos\left\{ \max\left[\min\left(\frac{x_{\text{loss}} - x_\beta}{r_{E(n)}}, 1 \right), -1 \right] \right\} \end{aligned} \tag{13}$$

The probability of survival on the *n*th firing is $1 - P_{\text{loss}(n)}$, so that the probability of surviving all firings of the kickers is

$$P = \prod_{n=1}^{\infty} \left(1 - \frac{1}{\pi^2} \int_{-r_{\beta(n)}}^{r_{\beta(n)}} \frac{dx_\beta}{\sqrt{r_{\beta(n)}^2 - x_\beta^2}} \arccos\left\{ \max\left[\min\left(\frac{x_{\text{loss}} - x_\beta}{r_{E(n)}}, 1 \right), -1 \right] \right\} \right) \tag{14}$$

where $r_{\beta(n)}$ and $r_{E(n)}$ are given by eqs. (4)–(7). Evaluation of eq. (14) is simplified by noting that no particles are lost on the *n*th firing when $n > n_{\max}$ where

$$n_{\max} = \text{floor}\left[\frac{\max(\tau_x, \tau_{\text{L}})}{t_{\text{s}}} \ln\left(\frac{r_\beta + r_E}{x_{\text{loss}}} \right) \right], \tag{15}$$

so that eq. (14) becomes

$$P = \prod_{n=1}^{n_{\max}} \left(1 - \frac{1}{\pi^2} \int_{-r_{\beta(n)}}^{r_{\beta(n)}} \frac{dx_\beta}{\sqrt{r_{\beta(n)}^2 - x_\beta^2}} \arccos\left\{ \max\left[\min\left(\frac{x_{\text{loss}} - x_\beta}{r_{E(n)}}, 1 \right), -1 \right] \right\} \right) \tag{16}$$

To evaluate the survival probability, we use the computer program given in Appendix A. The survival probability is given by eq. (16) for an electron whose energy oscillation is stable (i.e., when $E \le E_{\text{b}}$), while the survival probability equals zero when $E > E_{\text{b}}$ Since the microtron delivers an equal current for all values of the injection phase ϕ, the survival probability is averaged over the injection phase.

Figure 2 shows the survival probability for the existing injection scheme with ring energy of 108 MeV and rf voltage of 8 kV, in which case the rf bucket height is 0.97%. The survival probability is enhanced by a factor of 30 by using an energy offset of -0.8%, despite the fact that many of the injected electrons are outside of the rf bucket. Figure 2 agrees with measurements indicating that the optimized microtron energy is 0.7 MeV lower than the ring energy [2], and that a further decrease in microtron energy causes a dramatic dropoff in the survival of injected electrons [4].

Off-energy injection is also computed to increase the survival probability in base lattice if we specify equal horizontal and longitudinal damping times ($\tau_x = \tau_L = 5$ or $14.1\,\mathrm{s}$).

Figure 3 shows the survival probability for 108-MeV injection into the Aladdin-II lattice, where the dispersion at the injection location is very small. Off-energy injection offers no advantage, and the survival probability is low for all injection energy offsets. This is consistent with the poor stacking and low stacked currents obtained with Aladdin II.

Figure 4 shows the survival probability for an injection scheme in which a linac injects into the base lattice with ring energy of 200 MeV and rf voltage of 14.8 kV, in which case the rf bucket height is again 0.97%. For this case, the survival probability is much higher and off-energy injection offers little or no advantage.

5. Maximum stacked current

The maximum stacked current I_{max} may be estimated if the lifetime τ of the stored beam at the injection energy is known. When the kickers are fired to allow injection over a single turn, with repetition period t_s, the stored current is increased at a rate of $PI_{\mu\mathrm{tron}}/t_s$, where $I_{\mu\mathrm{tron}}$ is the microtron output current and P is the survival probability. Stored current is lost at the rate I_{max}/τ. These rates are equal when the stored current is at its maximum value, so that

$$I_{\mathrm{max}}/\tau = PI_{\mu\mathrm{tron}}/t_s \tag{17}$$

When the stored-beam lifetime is determined by Touschek scattering, we have approximately

$$I_{\mathrm{max}}\tau = (I\tau) \tag{18}$$

where $(I\tau)$ is the Touschek current-lifetime product. Multiplying eqs. (17) and (18) and then taking the square root gives the maximum stacked current as

$$I_{\mathrm{max}} = \sqrt{PI_{\mu\mathrm{tron}}(I\tau)/t_s}\,. \tag{19}$$

For existing Aladdin injection at 108 MeV with rf voltage of 8 kV, the kickers are fired at their maximum rate with $t_s = 0.8\,\mathrm{s}$. The microtron current is ~ 4 mA. Under good vacuum conditions, $(I\tau) \approx 140\mathrm{mA\text{-}hr}$ at injection energy [5]. With off-energy injection, fig. 2 indicates that the maximum survival probability P is 6.0%. Equation (19) predicts a maximum stacked current of 390 mA, consistent with observations.

For injection into the Aladdin-II lattice at 108 MeV, fig. 3 indicates a maximum survival probability of 0.41% for on-energy injection. For an estimated Touschek current-lifetime product of 140 mA-hr, eq. (19) gives an estimated maximum stacked current of 100 mA, in agreement with the maximum stacked currents of 90–145 mA obtained for Aladdin-II.

For injection at 200 MeV with rf voltage of 14.8 kV, linac current of 4 mA, and stacking time period of 0.8 s, let us assume that the Touschek current-lifetime product is again ~140 mA-hr. The maximum value of P given by fig. 4 is 0.48, so that eq. (19) estimates a maximum stacked current of 1100 mA. This is nearly three times the maximum stacked current estimated for 108-MeV injection.

6. Conclusion

We have presented a model for the low-energy off-energy injection of Aladdin. We assume that the initial horizontal offset of the injected beam is the sum of an energy oscillation and a betatron oscillation, and that each oscillation decoheres before the kickers are fired again. This model correctly predicts that 108-MeV injection into base lattice is optimized when the injected beam energy is ~0.8% lower than the storage ring energy, in which case the amplitudes of the betatron oscillations and energy oscillations are comparable. Our model estimates a maximum stacked current of 390 mA, consistent with observation.

For 108-MeV injection into the Aladdin-II lattice, whose horizontal dispersion at the injection location is very small, on-energy injection is predicted to provide the maximum stacked current. A maximum stacked current of 100 mA is predicted, in agreement with the experimental value of 90–145 mA.

Our model correctly predicts that a sufficiently large horizontal dispersion at the location of injection improves the low-energy injection.

For the possibility of injection into the Aladdin base lattice at 200 MeV, our model indicates that the maximum stacked current will be 1100 mA for a linac current of 4 mA. In comparison with injection at 108 MeV, injection at 200 MeV is expected to increase the maximum stacked current by nearly a factor of three.

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[1] W. S. Trzeciak and D. C. Morin, "Aladdin II," in *Proceedings of the 1991 IEEE Particle Accelerator Conference, San Francisco* (IEEE, Piscataway, NJ, 1991), p. 2643.

[2] Chihying Hsu, "Anomalous repetition rate, low energy injection study in Aladdin," Synchrotron Radiation Center Technical Note No. SRC-94, 1989.

[3] M. Sands, "The physics of electron storage rings, an introduction," Stanford Linear Accelerator Report No. SLAC-121, 1970, p. 91.

[4] K. J. Kleman, private communication (2007).

[5] R. A. Legg, private communication (2007).

Symbol	description	Base lattice 108 MeV	Aladdin-II 108 MeV	Base lattice 200 MeV
x_i	distance from injection hole to kicked orbit	9 mm	9 mm	9 mm
x_{loss}	distance from limiting septum aperture to kicked orbit	3 mm	3 mm	3 mm
τ_x	horizontal radiation damping time	14.1 s	12.4 s	2.2 s
τ_{L}	longitudinal radiation damping time	5.0 s	5.3 s	0.79 s
t_s	stacking time period	0.8 s	0.8 s	0.8 s
D	negative of horizontal dispersion at the injection point	-540 mm	-17 mm	-540 mm
V_{rf}	rf voltage	8 kV	3.7 kV	14.8 kV
E_0	ring energy	108 MeV	108 MeV	200 MeV
α	momentum compaction	0.0335	0.0154	0.0335
k	harmonic number	15	15	15
$E_{\text{b}} = \sqrt{\dfrac{2eV_{rf}}{\pi\alpha k E_0}}$	relative rf bucket height	0.97%	0.97%	0.97%

Table 1. Parameters for 108-MeV injection into the Aladdin "base" lattice and the Aladdin-II lattice, and for the possibility of 200-MeV injection into the Aladdin base lattice. Electrons are injected from a septum/inflector assembly located inside the storage-ring orbit.

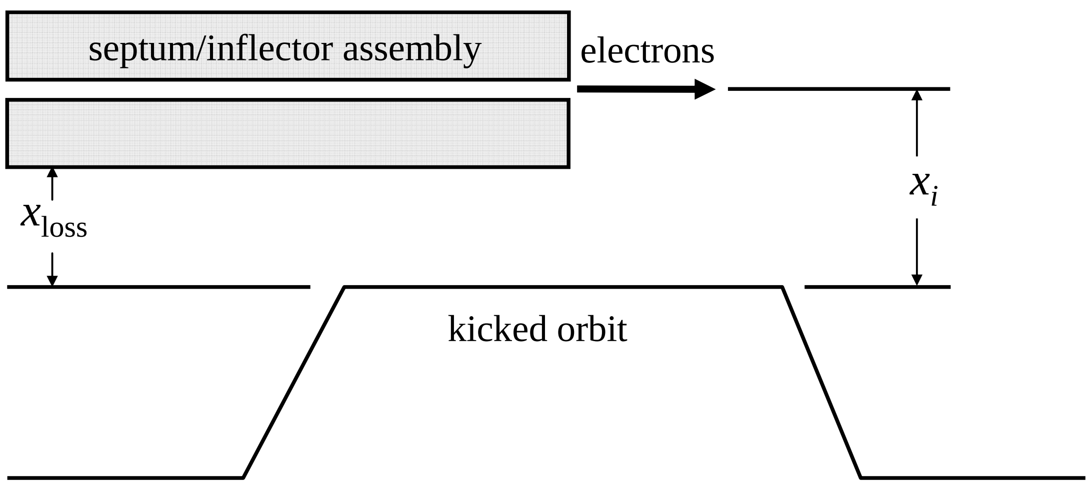

Figure 1. Electrons are emitted from the septum/inflector assembly at a distance of x_i from the kicked orbit. The distance from the septum/inflector assembly to the kicked orbit is x_{loss}.

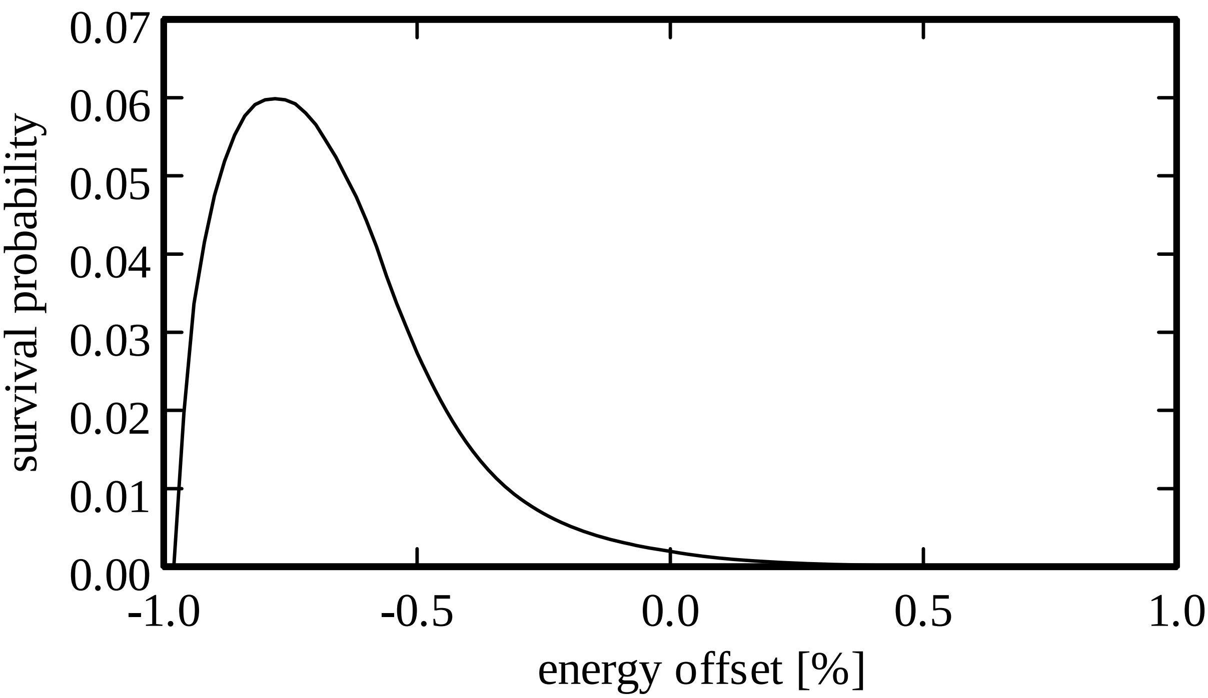

Figure 2. Probability that an injected electron survives the stacking process, for injection into the Aladdin base lattice at 108 MeV. The survival probability is averaged over the rf phases of the injected electrons.

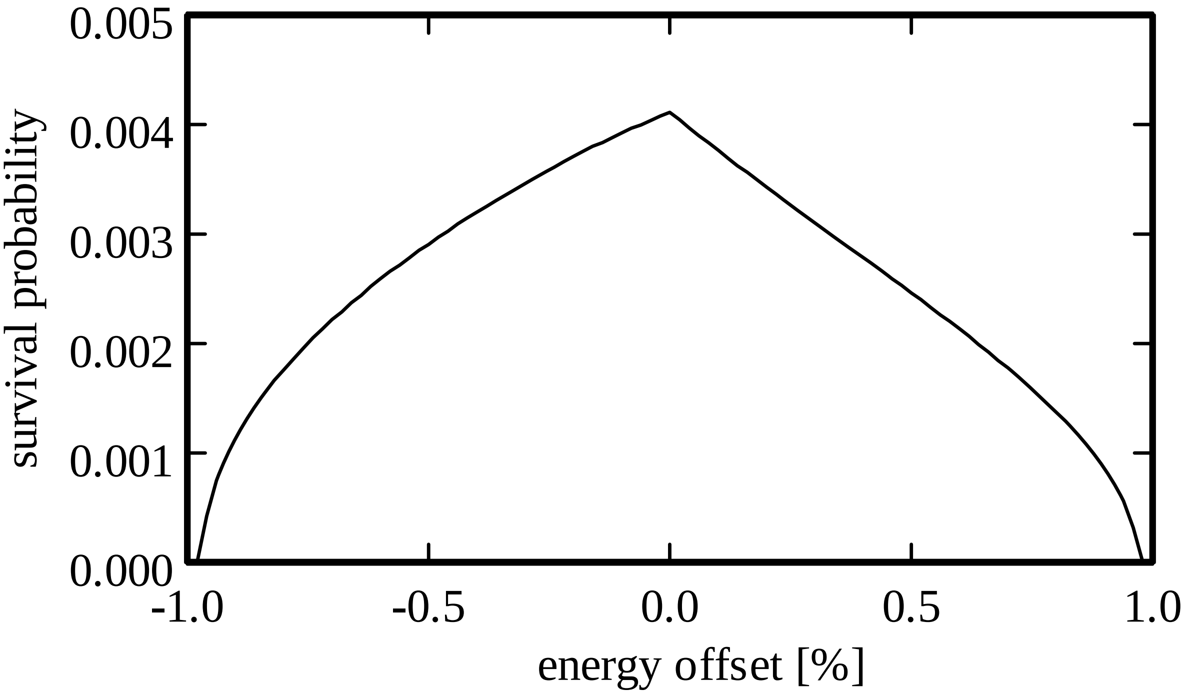

Figure 3. Probability that an injected electron survives the stacking process, for injection into the Aladdin-II lattice at 108 MeV. The survival probability is averaged over the rf phases of the injected electrons.

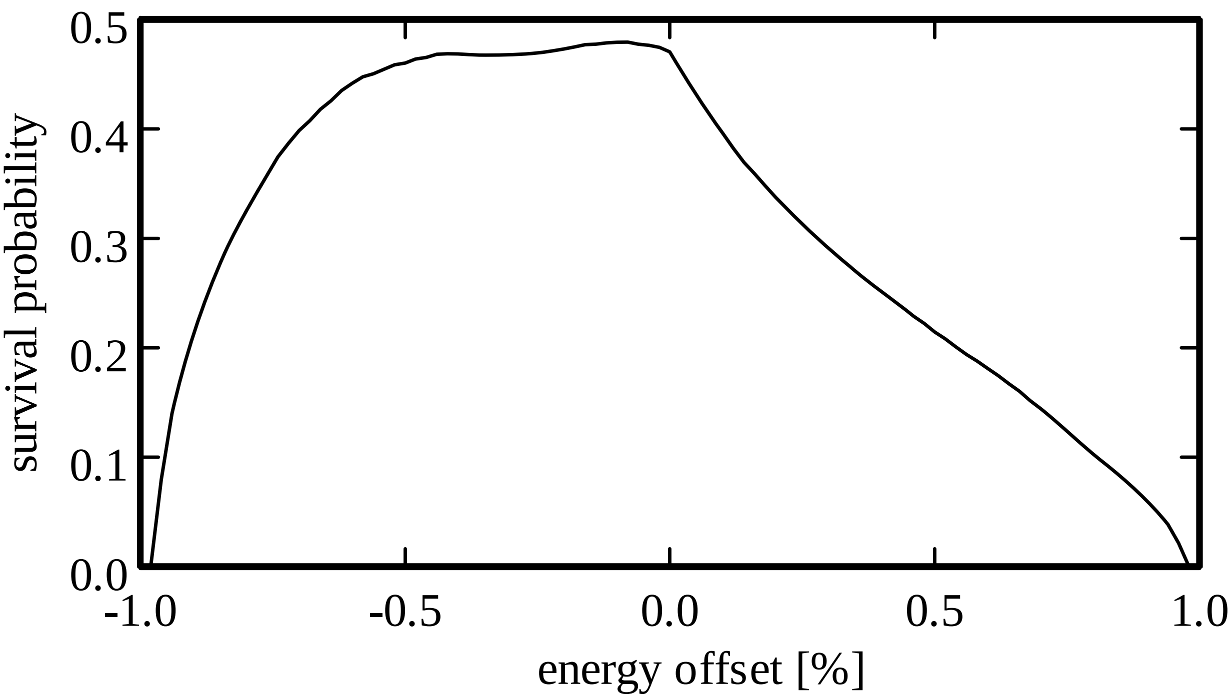

Figure 4. Probability that an injected electron survives the stacking process, for injection into the Aladdin base lattice at energy of 200 MeV. The survival probability is averaged over the rf phases of the injected electrons.

Appendix A. FORTRAN program for calculating the survival probability of an injected electron.

```
      program off_energy  ! calculates proportion of injected beam that contributes
                          ! to damped beam.  4/8/05 and 7/16/07

      integer imax
      parameter(imax=720)    ! number of RF phases sampled

      double precision phi(imax),xi,xloss,tx,tL,ts,Dx,e0,Eb,E,rb,re,dampx,
     * dampL,pi,Prob,xb,P(imax),deltaxb,sum,e0min,e0max,e0delta
      integer i,j,m,n,nmax,mmax,ii,iimax,jmax
      open (unit=8,status='unknown', file='survival.out',
     *  form='formatted')

      jmax = 400000 ! number of xb-positions sampled to calculate loss integral
                    ! Using jmax=200000 gives integration accuracy better than 1%.

      e0min = -.01 ! min relative energy offset
      e0max = .01  ! max relative energy offset
      e0delta = .0002 ! relative energy offset increment
      xi = 9.      ! injected beam offset from kicked orbit [mm], use positive value
      xloss = 3.   ! limiting distance from kicked orbit [mm], use positive value
      tx = 14.1    ! horizontal rad. damping time [s]
      tL = 5.0     ! longitudinal rad. damping time [s]
      ts = 0.8     ! stacking time (s) = time period between firing kickers
      D = -540.    ! Dx [mm] for injection from the outside, -Dx for inside injection
      Eb = 0.0097  ! relative RF bucket for Aladdin: 0.0097 for 8 kV RF at 108 MeV

      pi = 3.141592654

      write(8,*) " xi                  tx "
      write(8,*)   xi, tx
      write(8,*) " tL                     D                    Eb "
      write(8,*)  tL, D, Eb
      write(8,*) " xloss                 ts          "
      write(8,*)  xloss, ts
c     write(8,*) " phi(i)                 P(i)"
      write(8,*) " e0                  Prob "

      dampx = exp(-ts/tx)  ! horizontal damping factor per stacking time
      dampL = exp(-ts/tL)  ! longitudinal damping factor per stacking time

      iimax = floor((e0max - e0min)/e0delta) + 2. ! iterate over energy offsets
      do 4000, ii = 1, iimax
      e0 = e0min + float(ii-1)*e0delta
      Prob = 0.  ! initialize probability of survival for a given energy offset

      do 3000, i = 1, imax  ! i indexes the RF phase
      phi(i) = -pi + 2. *real(i)*pi/imax ! RF phase angle
      E = sqrt( (Eb**2/2.)*(1.-cos(phi(i)))+e0**2) ! energy oscillation amplitude
      rb = abs(xi - D*e0)+0.0000001  ! nonzero betatron oscillation amplitude (mm)
      re = abs(D*E)+0.0000001  ! nonzero energy oscillation hor. amplitude (mm)

      if (E.ge.Eb) then
          P(i) = 0.  ! no capture by the RF system
          nmax = 1.  ! no need to compute losses at each firing of the kickers
```

```
        else
           P(i) = 1.
           nmax = floor(max(tx,tL)/ts*log((rb+re)/xloss)) ! firings where loss can
                                                          ! occur
      endif
      do 2000, n = 1, nmax  ! n indexes the kick number
        sum = 0.
      do 1000, j = 1, jmax  ! j indexes rb for integration
        deltaxb = 2.*dampx**n*rb/real(jmax)
        xb = -dampx**n*rb+real(j-0.5)*deltaxb
        sum=sum+abs((rb**2*dampx**(2*n)-xb**2))**(-.5)
     *    * acos(max(min((xloss-xb)/(re*dampL**n),1.),-1.))
     *    * deltaxb/pi/pi
1000  continue
      P(i) = (1.-sum)*P(i)
2000  continue
c      write(8,*) phi(i), P(i)  ! probability of surviving for the ith RF phase
      Prob = Prob + P(i)/real(imax)  ! survival probability averaged over RF phase
3000  continue

      write(8,*) e0, Prob
4000  continue

      close(8)
      end
```