Chapter 3. LOAD ELECTRICAL CHARACTERISTICS

At a given irradiance and cell temperature, a PV system can produce power at voltages ranging between zero and the open circuit voltage, V OC" Between these limits, the output current, It is a function of voltage only. An I-V curve shows the possible points, or I-V pairs, at which the system may operate. Electrical loads also have a characteristic I-V curve. This chapter describes the I-V characteristics of three general types of electrical loads used in direct-coupled applications: fixed voltage, resistive, and inductive motor loads.

In a direct-coupled PV system, the load is connected so that the array and load voltage are the same. The intersection of the load I-V curve and the array I-V curve, if there is one, determines the operating voltage and current of the system. If the load and array I-V curves do not intersec4 there will be no power output from the array. To find the intersection of the load and array I-V curves, an expression for load voltage as an explicit function of load current ($V = f(I)$) is developed. This expression is substituted into the array I-V equation (Eqn. 2.67) for V. The operating current, I, can be calculated implicitly from the resulting equation. Appendix B details the operating point calculation procedure for each load type.

3.1 Fixed Voltage Loads

The I-V characteristic of a fixed voltage load is simple: For any current drawn by the load, the voltage is constant The I-V "curve" is a straight venica1line on a current-voltage coordinate scale. The vertical line extends from zero amps to some upper-rated current, usually limited by a fuse or other protective device. The magnitude of the load voltage depends on the specific application. Some fixed voltage applications include cathodic well protection, DC appliances such as television and radio, and idealized battery loads. The long-term performance model presented in Chapter 5 is, however, not applicable for systems with battery energy storage.

The operating current is found by substituting the load voltage into Eqn. 2.67 and then solving for I. To be used effectively, an array should be designed so that the number of modules in series produces a maximum power point voltage that, under typical summer operating conditions, is close to the load voltage. The load and maximum power point voltages should be designed to match at a high (summer) cell temperature. The reason is that the maximum power point voltage decreases by about O.4%/°C increase in cell temperature (at constant irradiance); if the load voltage is designed to match the maximum power point voltage at a low cell temperature, typical of winter operation, the load voltage may exceed the array open circuit voltage at high (summer) cell temperatures and the power output may drop to zero. Figure 31 shows how the power output and maximum power point voltage vary with cell temperature for a 30 W Solarex module. Both power vs. voltage curves are based on an irradiance of 1000 W/m^2, but one curve is representative of winter operation, at 10 °C, and the other represents summer operation at 67 °C.

The cell temperature does not vary as much within a day as it does from season to season. The daily variation in maximum power point voltage is less than the O.4%/°C cell temperature sensitivity would indicate. This is because the irradiance is not constant, and the sensitivity of maximum power point voltage to irradiance opposes (although not - as strongly) the temperature dependence. The net effect of varying irradiance and cell temperature is illustrated in Figure 32 in the following section.

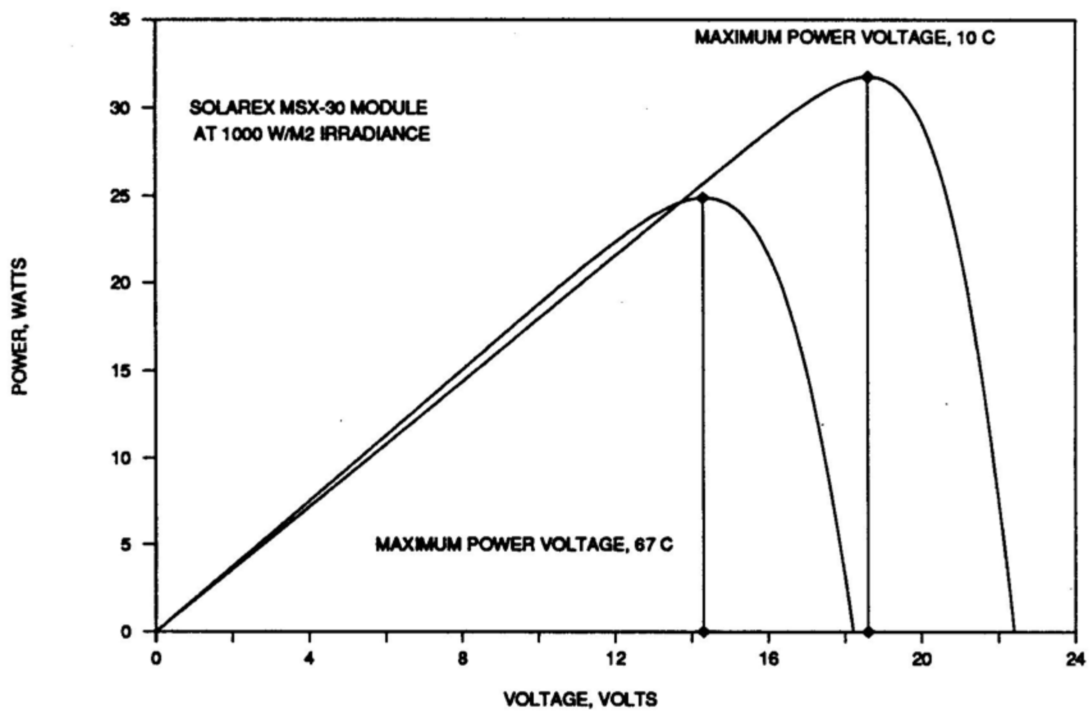


Figure 31. Variation of Optimal Voltage with Cell Temperature

3.2 Resistive Loads

Resistive loads are used for applications such as incandescent lighting, cooking, and heating. The I-V characteristic of a resistive load is governed by Ohms' law, V = I x R_L. where R_L is the load resistance. The I-V "curve" for a resistive load is a straight line beginning at the origin, with a slope of 1/ R_L. The load I-V line continues out to the maximum current and voltage of the device. The operating current is found by substituting the load voltage, $V = I \times R_L$, into Eqn. 2.67 and then solving for I.

A well matched direct-coupled resistive load and array will have I-V curves that intersect near the maximum power point of the array. Choosing an optimal fixed resistance load is more difficult than choosing an optimal fixed voltage load. The optimal resistive load is equal to the ratio V_{MP}/I_{MP}. While the maximum power point voltage is relatively constant over a typical day's operation, the maximum power point current is not. The resistive load which yields the highest long-term output lies closer to an optimal resistance at high irradiance, because the potential electric generation is greater than at low irradiance levels. Figure 32 illustrates the typical hourly variation of optimal resistive loads for a 75 W Applied Solar module. At each time shown, the optimal resistive load passes through the maximum power point.

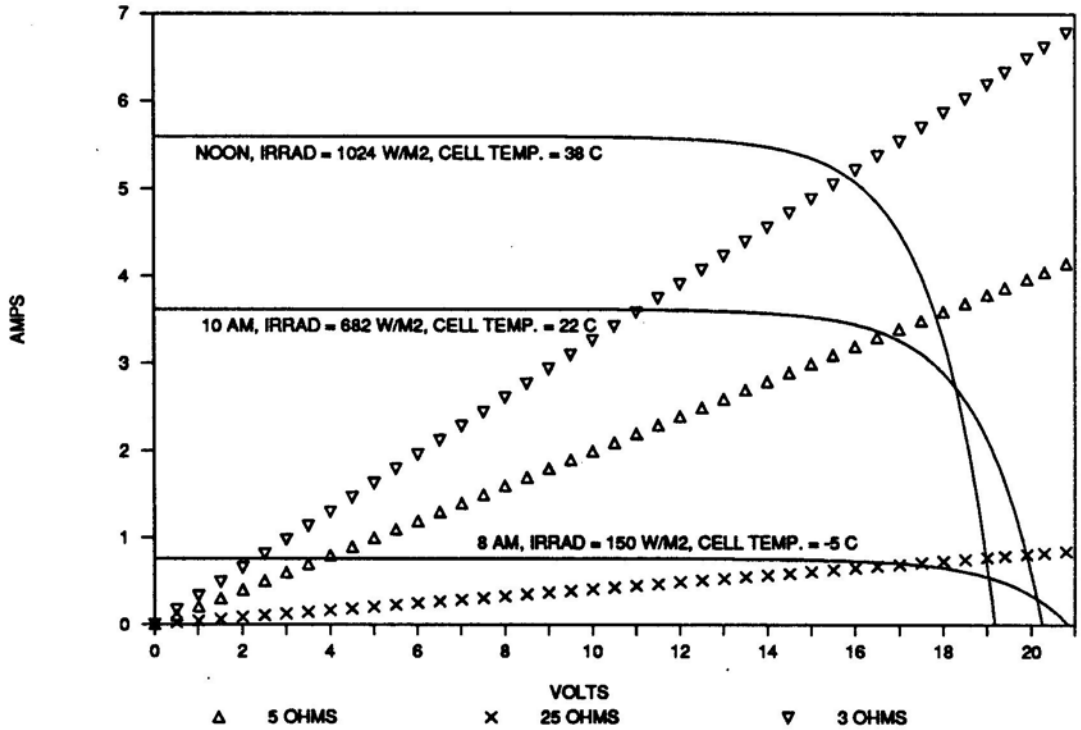


Figure 32. Hourly Variation of Optimal Resistive Load

The maximum power point voltage decreases from 17.3 V at 8 AM to 15.8 V at noon as the cell temperature rises from -5 °C to 38 °C. The variation of maximum power point voltage over this temperature range is less than O.4%/°C, but, as discussed in the previous section, this is because the maximum power voltage also increases slowly with increasing irradiance. In this example, the effect of irradiance on voltage is significant, because the irradiance at noon is about 7 times higher than the 8 AM irradiance. The net effect is that the locus of maximum power points over a typical day's operation occurs over a fairly narrow voltage range.

3.3 Inductive Motor Loads

Unlike fixed voltage and resistive loads, the I-V characteristics of motor loads are non-linear. Motor I-V curves differ considerably among motor types, and are also dependent on the torque and speed characteristics of the mechanical load being driven by the motor. In Sections 3.3.1 thru 3.3.3, I-V equations for three DC motor types are described: series, shunt, and separately excited (with permanent magnet). In Section 3.3.4, three types of mechanical loads commonly used in direct-coupled systems are described: a centrifugal water pump, centrifugal ventilator fan, and a positive displacement water pump.

Continuous functions describing pump and fan performance are not ordinarily provided by manufacturers. More often, data must be read from a single performance curve which relates pressure (head), flowrate, efficiency, and speed. Therefore, the non-linear shape of motor/pump I-V curves must be approximated by a number of separate straight line segments. The endpoints of each I-V line segment are derived from points on the pump/fan performance curve. The I-V curve may be divided into any number of segments of any length. To solve for the operating point a set of I-V pairs are required as inputs. Then, a linear interpolation is used between consecutive pairs to search for an intersection with the array I-V curve. In this manner, loads of any arbitrary shape can be modeled, as long as their I-V curve can be satisfactorily represented by a series of connected line segments.

By examining I-V curves for the various motor/load combinations, it will be shown that the separately excited permanent magnet motor with a centrifugal (fan or pump) load can be designed to achieve a good direct-coupled effectiveness. Direct-coupled effectiveness is defined as the ratio of direct-coupled electric output to the theoretical maximum power point output. It is frequently calculated on a monthly or yearly basis. The series motor is also a satisfactory choice for direct-coupled applications. but the shunt motor is not. These conclusions are reinforced in Chapter 6, where the estimated annual electric output and pumped water volume are shown for various motor/load types of comparable rating.

To make a consistent comparison among the three motor types, hypothetical motors have been designed, each with the same approximate nameplate rating, based on a similar study by Appelbaum [37]. A detailed description of the motor/load combinations used to develop the long-term performance model is provided in Appendix A. The following simplifying assumptions apply to the motor design [38]:

1. The magnetic flux is linearly dependent on field current, and the hysteresis losses are neglected That is, the iron in the field inductor is not magnetized beyond its saturation point, and the magnetization response to current is the same for each magnetization cycle. Hysteresis losses are approximately accounted for by the assumed form of the mechanical torque losses below.
2. The armature and field inductances are constant. This is a common assumption based on uniform winding geometry, negligible temperature effects, as well as the first simplifying assumption.

3. The armature reaction is negligible. This assumes the motor operates at or below its rated current, and is also a consequence of the first assumption above.
4. The mechanical torque losses consist of static and viscous friction of the form $T_{LOSS} = C_{STAT.} + (C_{VISC.}$ x speed).

3.3.1 Series DC motor

Figure 33 is a simplified equivalent circuit for a series motor.

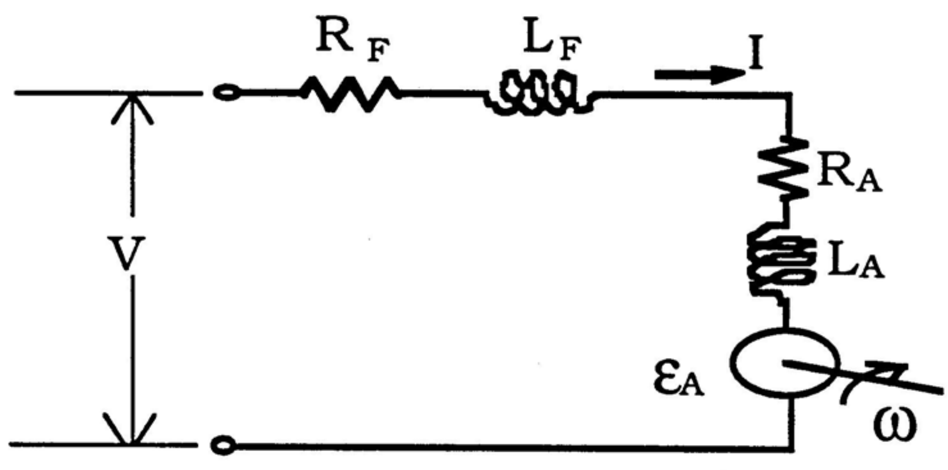


Figure 33. Series Motor Equivalent Circuit

where: R_F = field resistance, Ω

L_F = field inductance, H

R_A = armature resistance,

L_A = armature inductance, H

ε_A = armature electromotive force (EMF), V

ω = motor speed, radians/sec.

At steady state, the I-V equation describing this circuit is [38]:

$$V = \varepsilon_A(I,\omega) + I(R_A + R_F) \quad (3.1)$$

The EMF is equal to:

$$\varepsilon_A(I,\omega) = M_{AF} I\omega \tag{3.2}$$

MAF = mutual inductance between the armature and field, H

The gross motor torque, T, which is the sum of the frictional torque loss plus the load torque, is related to the motor current by:

$$T = T_{LOAD} + T_{LOSS} = M_{AF} I^2 \tag{3.3}$$

To link the mechanical (speed, torque) characteristics of the load to the electrical characteristics of the motor, an explicit expression of the load torque in terms of speed is needed. Often, such an expression is unavailable. Instead, discrete load points where the torque is calculated as a function of speed must be used, which means that I-V points must be calculated one at a time. Section 3.3.4 details three cases: The first is a ventilator fan, where a continuous speed-torque relationship is given [37]; the second case is for a centrifugal water pump, where the torque must be calculated from a manufacturer's performance curve [39]; and the third case is for a positive displacement water pump connected to a permanent magnet motor. For this case, the I-V characteristic is supplied directly by the manufacturer, so no torque-speed conversions are needed [40].

Eqn. 3.3 can be rearranged to solve for I directly in terms of T and M_{AF} (Eqn. 3.4). Equation 3.4 is then used to eliminate I from Eqn. 3.1. The result, Eqn. 3.5, is an explicit expression for the motor voltage in terms of motor speed, torque, and the known motor constant M_{AF} and (R_A + R_F).

$$I = \left[\frac{T(\omega)}{M_{AF}}\right]^{1/2} \tag{3.4}$$

$$V = \left[\frac{T(\omega)}{M_{AF}}\right]^{1/2} \left[M_{AF}\omega + (R_A + R_F)\right] \tag{3.5}$$

At any motor speed, the current and voltage are known. The motor speed ranges from zero up to a maximum rated speed. The motor current is related to the motor torque by Eqn. 3.4. Therefore, by solving for the minimum starting torque from the relationship T(ω) evaluated at ω = 0. Eqn. 3.4 can be used to find the minimum starting current. Depending on the area of the array connected in parallel, knowing the minimum starting current helps determine the minimum_irradiance needed to start the motor. The minimum starting voltage is calculated using Eqn. 3.5. The maximum rated current and voltage are found in a similar way, except that the maximum rated speed is used to evaluate the torque.

Figure 34 shows the I-V characteristic of a nominal 1 1/4 hp, 1800 maximum RPM series DC motor, connected to a centrifugal ventilator load. Also shown are I-V curves for an array of 7 series and 7 parallel 30 W Solarex modules at three irradiance levels. The I-V curve for the motor runs from zero to 1800 RPM, moving from left to right. This system will operate at low levels of solar radiation, but it will not effectively use the array peak power capacity at low irradiance or at very high irradiance. Although the motor output continues to increase with higher speeds, the direct-coupling becomes less effective as the irradiance approaches 1000 W/m^2.

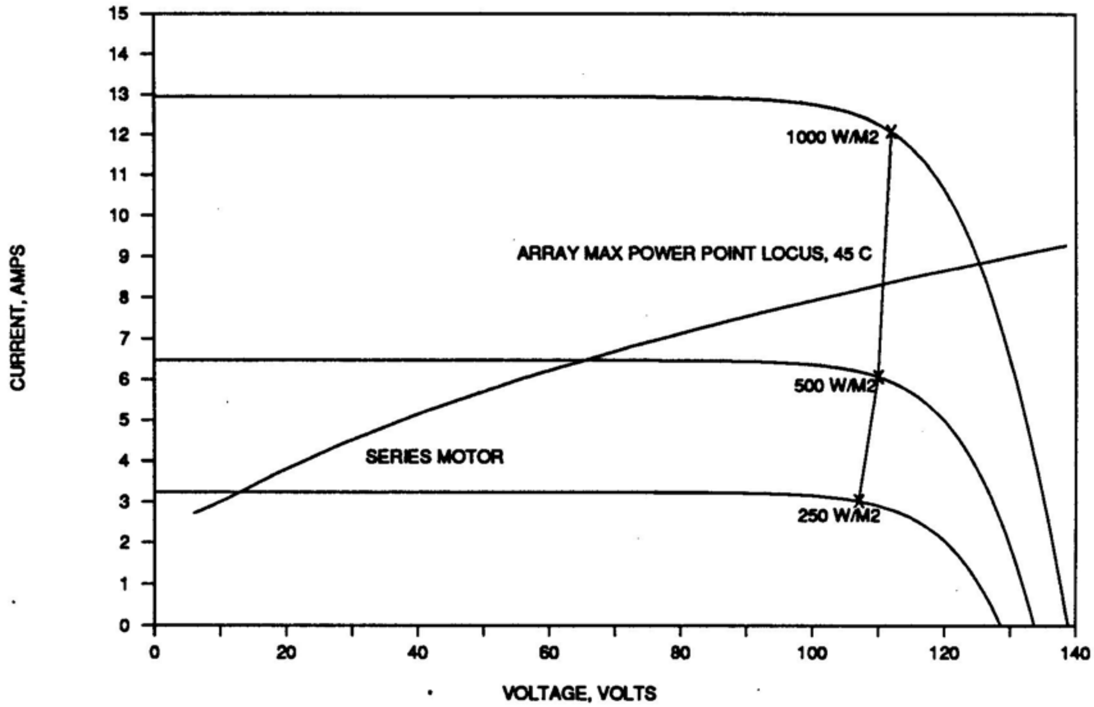


Figure 34. Series Motor I-V Curve

3.3.2 Shunt DC motor

Figure 35 is a simplified equivalent circuit for a shunt motor.

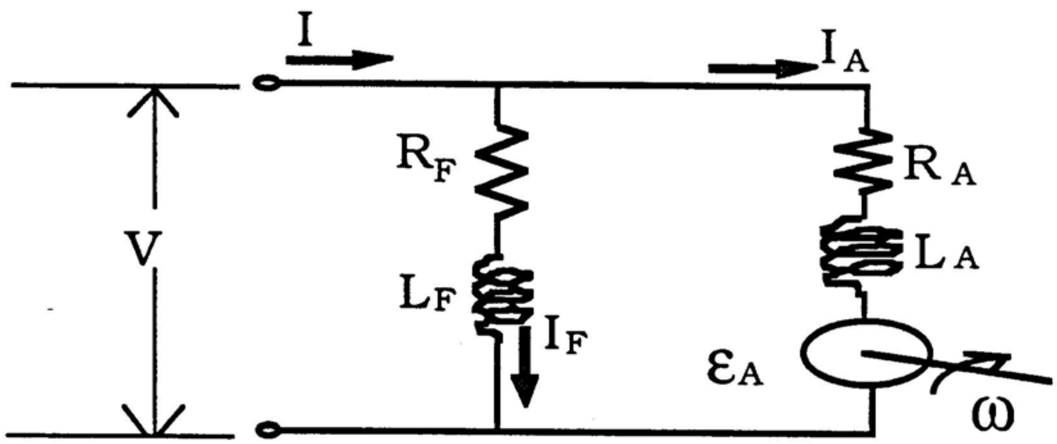


Figure 35. Shunt Motor Equivalent Circuit

where all terms are as defined for the series motor. plus:

I_F = field current, A

I_A = armature current, A

At steady state, the I-V equation describing this circuit is [38]:

$$V = \varepsilon_A(I,\omega) + I_A R_A \tag{3.6}$$

$$V = I_F R_F \tag{3.7}$$

The EMF is equal to:

$$\varepsilon_A(I_F,\omega) = M_{AF} I_F \omega \tag{3.8}$$

The terminal current, I, is equal to:

$$I = I_A + I_F \tag{3.9}$$

The gross motor torque, T, which is equal to the shaft loss torque plus the load torque, is related to the two motor currents by:

$$T = T_{LOAD} + T_{LOSS} = M_{AF} I_A I_F \tag{3.10}$$

Equation 3.11 is an expression for voltage in terms of torque, speed, and motor constants. It is obtained by substituting Eqns. 3.8, 3.7, and 3.10 into Eqn. 3.6 to eliminate I_A and I_F Once the voltage is known, I is calculated by reverse substituting Eqns. 3.10, 3.7, and the voltage, V, into Eqn. 3.9.

$$V = \left[\frac{R_A R_F T(\omega)}{M_{AF}\left(1 - \frac{M_{AF}\omega}{R_F}\right)} \right]^{1/2} \tag{3.11}$$

Figure 36 shows the I-V characteristic of a nominal 1 1/4 hp (930 W), 1800 maximum RPM shunt DC motor superimposed on the curves shown in Figure 34. The shunt motor is connected to the same centrifugal ventilator load as used in the series motor example.

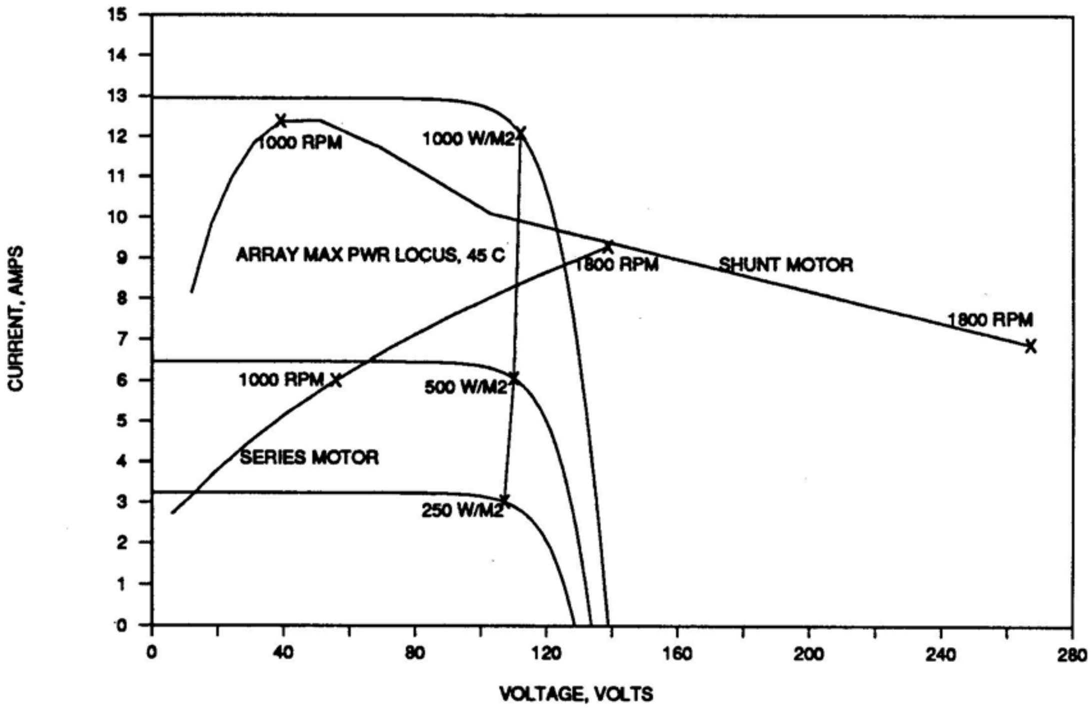


Figure 36. Shunt Motor I-V Curve

Compared to the series motor, the shunt motor has a higher starting torque, and is less efficient, because at the same speed (for example, 1000 or 1800 RPM), it requires more input power to serve the same load. The shunt motor does not follow the maximum power line of the array as well as the series motor (at least not with the 7 series x 7 parallel configuration). It may be prone to unstable operation, because under some circumstances, the load I-V curve may intersect the array I-V curve more than once.

When starting, all direct-coupled DC motors follow a straight (pure resistance) line from the origin to the minimum starting point I-V pair. From that point, the speed, current, and voltage increase steadily along the I-V curve for the series and permanent magnet motors, but not for the shunt motor. In the case of the shunt motor, after it first reaches a steady state intersection with the array I-V curve, other operating states at the same irradiance are possible. This may happen if there are other intersections with the array I-V curve and if the motor is accelerated to a higher speed by an external source. Because of its inherently poor starting torque, poor efficiency, and unstable operation, the shunt motor is not recommended for direct-coupled systems. The same conclusion is cited in other direct-coupled system studies [37,41,42].

3.3.3 Separately excited (permanent magnet) DC motor

Figure 37 is a simplified equivalent circuit for a separately excited (permanent magnet) motor.

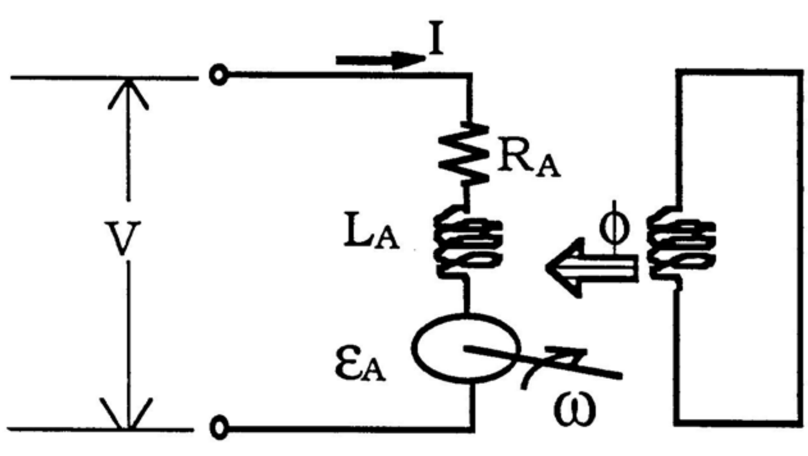


Figure 37. Permanent Magnet Motor Equivalent Circuit

where the elements shown are as defined for the series motor, plus:

φ = permanent magnet flux, Wb

At steady state, the I-V equation describing this circuit is [38]:

$$V = \varepsilon_A(\omega) + IR_A \tag{3.12}$$

The EMF is equal to:

$$\varepsilon_A(\omega) = k\phi\omega \tag{3.13}$$

where k is a dimensionless flux coefficient.

The gross motor torque, T, which is equal to the shaft loss torque plus the load torque, is related to the motor current by:

$$T = T_{LOAD} + T_{LOSS} = k\phi I \tag{3.14}$$

The motor current at any speed is calculated by substituting Eqn. 3.13 into Eqn. 3.12, using the value of I calculated with Eqn 3.14.

Figure 38 shows the I-V characteristic of a nominal 1 1/4 hp (930 W), maximum RPM permanent magnet DC motor and the series motor curves shown in Figure 34. The permanent magnet motor is connected to the same centrifugal ventilator load as used in the series motor example.

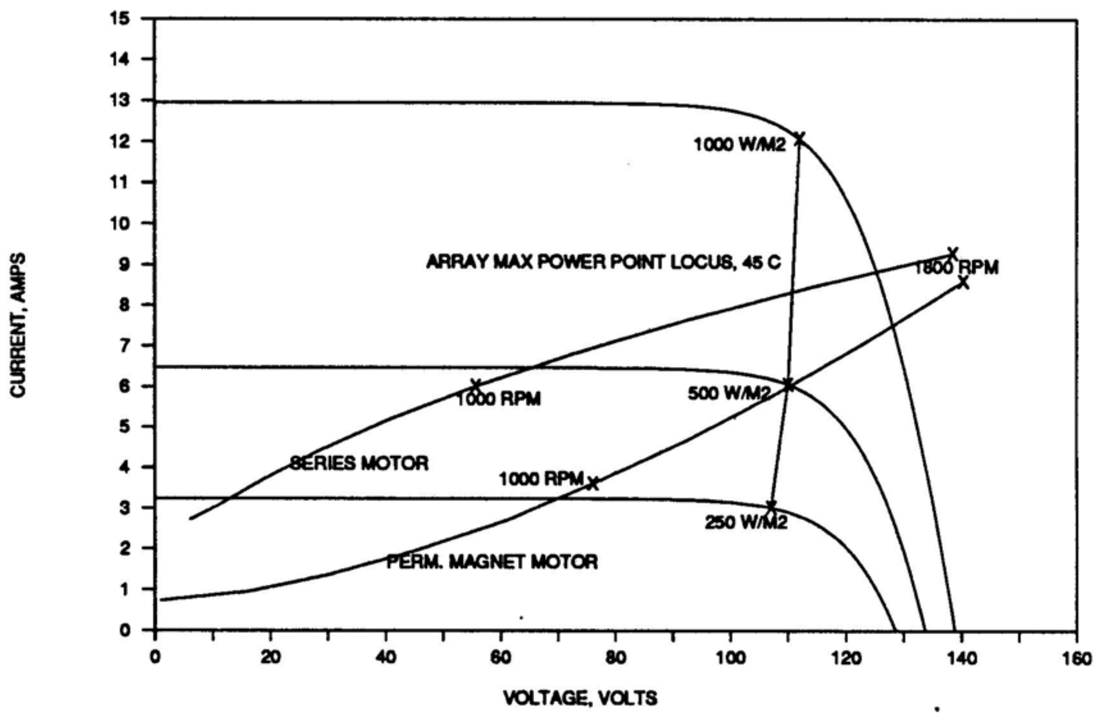


Figure 38. Permanent Magnet Motor/Ventilator Load I-V Curve

Compared to the series motor, the permanent magnet motor has a lower starting torque, and is more efficient, because at the same speed (for example, 1000 or 1800 RPM), it requires less input power to serve the same load power needs. This permanent magnet motor follows the contour of the array maximum power line better than the series motor, but depending on the daily distribution of solar radiation, may not do more work than the series motor, because the series motor I-V curve is nearer to the array maximum power line at high irradiance levels.

One of the motor/load combinations used to test the long-term performance model is a pem1anent magnet motor connected to a positive displacement water pump. Figure

39 illustrates the I-V curve for this combination. The size of the motor and load are much smaller than the previous examples (roughly 40 W, compared to 1000 W for the previous examples), so a direct comparison is not possible, but the shape of the I-V curve is indicative of the different speed-torque characteristics imposed by the positive displacement pump.

The I-V curve is flatter than the curve for the centrifugal load (the general shape of the I-V curve remains the same whether the centrifugal load is a ventilator or water pump). The flatter curve means that the relative starting current, as a fraction of the current at maximum speed is higher for the positive displacement pump load. The flatter curve does not follow the maximum power point line as well as the same type of motor connected to a centrifugal load. The actual shape of any pumping load is dependent on the magnitude of the pumping load and the pump's characteristic performance, as measured by variables such as flowrate, head, efficiency, and pumped fluid specific weight.

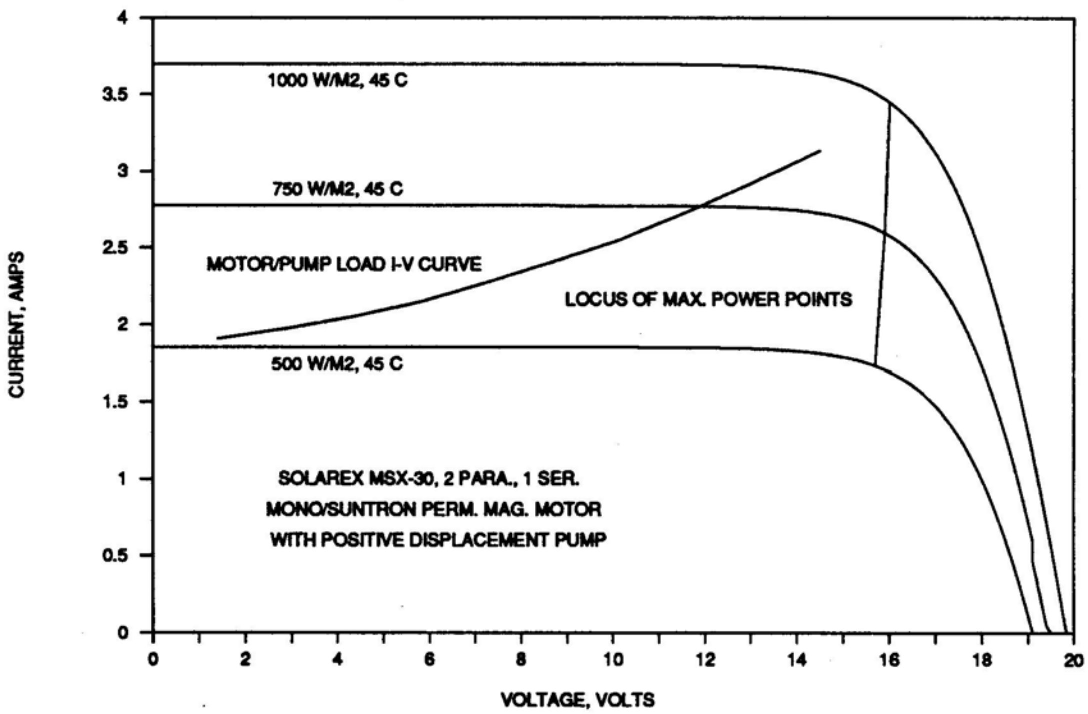


Figure 39. Permanent Magnet Motor/Positive Displacement Pump I-V Curve

3.3.4 Mechanical load torque-speed relationships

As pan of the testing for the long-term performance model, simulations were run for six different motor/load combinations. The six combinations, which include three DC motor types and three load types, are listed below. The horsepower ratings are nominal and vary considerably depending on the operating point. Detailed descriptions of the motor/load characteristics are provided in Appendix A.

1. 1 1/4 hp series motor/1 hp centrifugal fan load
2. 1 1/4 hp shunt motor/I hp centrifugal fan load
3. 1 1/4 hp permanent magnet motor/l hp centrifugal fan load

4. 7 hp series motor/S hp centrifugal water pump load
5. 7 hp permanent magnet motor/5 hp centrifugal water pump load
6. 1/20 hp permanent magnet motor w/positive displacement water pump load

I-V equations for the three motor types were derived in Sections 3.3.1 thru 3.3.3. The relationship of the load speed and load torque must be determined first, in order to use the motor I-V equations. In this section, speed-torque relationships are developed for the centrifugal fan and centrifugal water pump loads. The speed-torque relationship for the positive displacement pump is not shown, because the data for that motor/pump system were supplied in I-V coordinates by the manufacturer [40], and are shown in Figure 39 and listed in Appendix A.

3.3.4.1 Centrifugal Fan Load

The data for this load are taken from a similar study done by J. Appelbaum [37]. The total torque developed by the motor is the sum of the shaft loss torque and the load torque. The loss torque includes a static and viscous component. The first component is constant and represents the starting torque needed to overcome shaft static friction. The second component is assumed to vary linearly with rotational speed.

$$T_{LOSS} = C_{STATIC} + \left(C_{VISCOUS} \times \omega\right) \tag{3.15}$$

C_{STATIC} = 0.2 N•m, $C_{VISCOUS}$ = 0.002387 N•m/rad/sec

The load torque is:

$$T_{LOAD} = 0.3 + 0.00039\omega^{1.8} \tag{3.16}$$

The total torque, as a function of speed, is:

$$T = 0.5 + 0.002387\omega + 0.00039\omega^{1.8} \text{ Nm} \qquad (3.17)$$

3.3.4.2 Centrifugal Water Pump Load

The data for this load are taken from Figure 40, a centrifugal water pump performance curve (reproduced courtesy of Jadco Mfg./Solarjack Solar Pumping Products [39]). The performance curve links four variables: speed, flowrate, head, and efficiency. Selecting any two of these variables fixes values for the other two. Equation 3.18 is a fundamental fluid power relationship which relates the total torque to the pump performance variables.

$$\text{SHAFT POWER, Watts} = T_{TOTAL}\,\omega = \frac{QSWH}{\eta_P} + \text{SHAFT LOSS POWER} \qquad (3.18)$$

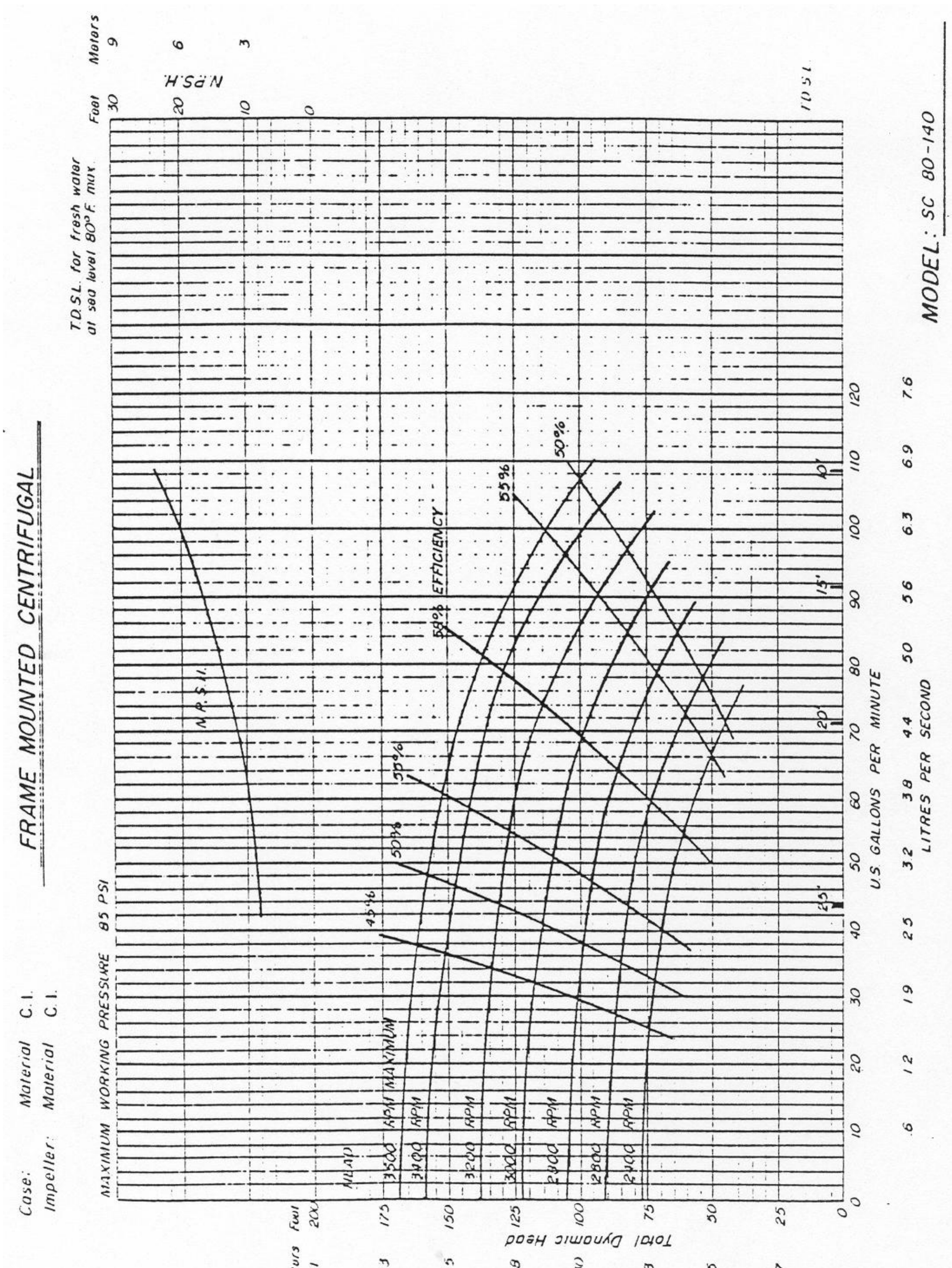
where Q = volumetric flowrate, m^3/sec.

SW = specific weight of water, approx. 9807 N/ m^3

H = total dynamic pressure head, m

η_P = pump efficiency

The total dynamic head is calculated based on site specific characteristics. The sample case in this thesis assumes a constant total dynamic head of 100 feet (30.48 m), and that shaft loss power is negligible. From Figure 40 at a head of 100 ft., if a variety of speeds are chosen over the operating range of the pump. the flowrate and efficiency are known at each speed. The torque can be calculated from Eqn. 3.18 for each chosen speed.


FRAME MOUNTED CENTRIFUGAL
Case: Material C.I.
Impeller: Material C.I.
MAXIMUM WORKING PRESSURE 85 PSI
T.D.S.L. for fresh water at sea level 80°F. max.
N.P.S.H.
Feet Meters
N.P.S.I.
EFFICIENCY
45%
50%
55%
58%
55%
50%
3500 RPM MAXIMUM
3400 RPM
3200 RPM
3000 RPM
2800 RPM
2800 RPM
2400 RPM
HEAD
Total Dynamic Head
US GALLONS PER MINUTE
LITRES PER SECOND
MODEL: SC 80-140

3.4 Summary

In this chapter, the I-V relationships of three different electrical load types were described. The three types are fixed voltage, resistive, and DC motors. Figures illustrating how each of the electrical load types compares to the I-V characteristics of a PV array were presented. The DC motors were further grouped into three categories (series, shunt, and separately excited permanent magnet), and the simulated annual effectiveness for each category was compared. The effect of the type of mechanical load connected to the DC motor was discussed. Finally, six motor/mechanical load combinations used to test the long-term performance model were outlined.