

Chapter 4. WEATHER-DEPENDENT VARIABLES

To decrease the number of calculations needed to estimate annual PV system output, a reduced, or simplified, set of weather data can be generated to represent long-term average weather patterns. This chapter describes a method by which simplified weather data is used to estimate hourly-average plane of array irradiance and cell temperature, using long-term monthly averages of the clearness index, X_T , and ambient temperature, $\overline{T_A}$.

The method presented here is based on two key assumptions. One is that, for any location, day to day variations in the weather variables which affect solar energy systems are small enough so that a month may be characterized by a small number of "typical day" groups, instead of 28 to 31 individual days [43,44]. The other is that, on a monthly-average basis, the hourly variation of weather variables within each day is similar, and approximately symmetric about solar noon [45].

To assess whether the above assumptions are valid, estimated monthly or annual PV system output using a reduced set of weather data are compared to reference simulations which use actual hourly weather data (To be consistent, the long-term data are derived from the same hourly data source). The hourly, or reference, data used is Typical Meteorological Year (TMY) data.

The simplified long-term performance model described in Chapter 5 uses reduced weather data. As a benchmark for evaluating the accuracy of this model, TMY data (for

three U.S. locations) are used in over 800 annual direct-coupled system simulations. In Chapter 6, results from the TMY -based simulations are compared to results from the simplified model, in order to determine how many "typical days" per month are needed to satisfactorily predict monthly and annual PV system performance.

4.1 A Method to Calculate Hourly I-V Model Inputs

In order to determine the I-V characteristics of the PV array for each time interval (an hourly interval is used), the array I-V model requires two inputs: plane of array irradiance (W/m^2), and cell temperature (K). The sequence of steps by which the reduced weather data are generated and then used to calculate these two quantities is summarized in this section. Each step is detailed in the subsequent sections.

1. For each month at the location of interest, estimates of two input variables, X_T , the monthly-average clearness index, and $\overline{T_A}$, the monthly-average ambient temperature, are needed.
2. For each month, a correlation developed by Bendt, Collares-Pereira, and Rabl [44] is used to generate a continuous daily clearness index (K_T) distribution as a function of the monthly-average clearness index and the cumulative fraction of days per month. Other distribution forms could as well be used, for example, those of Hollands and Huget [52] or Liu and Jordan [54].
3. The area beneath the K_T vs. fractional time curve is approximated by dividing it into several rectangular segments of equal width, each of which represents a "typical day." The height, K_T , of each segment is calculated from Bendt's

continuous distribution function, evaluated at the fractional time corresponding to the midpoint of each segment (Figures 47 - 52 show two types of approximations for three locations).

4. For each of the chosen "typical day" segments, the diffuse fraction of total radiation is calculated as a function of K_r for that segment, using a correlation from Collares-Pereira and Rabl [46].
5. The daylength is assumed to be the same for each day within a given month. Therefore, for the location and collector slope in question, the sunset hour angle may be calculated for the representative day suggested by Klein [55]. The hourly fraction of daily total and daily diffuse radiation is then calculated as a function of the hour angle and sunset hour angle, using correlations from Collares-Pereira and Rabl [46] and Liu and Jordan [45].
6. The hourly ambient temperature is calculated as a function of K_r and T_A using a correlation developed by Erbs [47].
7. Using quantities calculated in the previous steps, the average plane of array irradiance for each hour is calculated with an isotropic sky model equation originally developed by Liu and Jordan [48] and an auxiliary equation from Duffie and Beckman [43].
8. Using the hourly ambient temperature calculated in step 6 and the irradiance calculated in step 7, the hourly cell temperature is calculated with Equation 2.89 from Section 2.6.

4.2 Weather Input Variables

Some sources, such as Duffie and Beckman [43], provide the monthly average clearness index directly, while other sources (Cinquemani et al. [20], Lunde [49], National Climatic Data Center [50], UW-Madison Solar Energy Laboratory Report No. 21 [SI]) may provide only the monthly average (or monthly-average daily) global horizontal radiation. The clearness index can be calculated from the measured horizontal surface radiation by dividing by the extraterrestrial radiation calculated over the same period. The following equation is used to calculate the integrated daily extraterrestrial radiation on a horizontal surface. H_o in KJ/m²-day (from Duffie and Beckman [43]):

$$H_o = \frac{24 \times 3600 G_{SC}}{\pi} \left[1 + 0.033 \cos\left(\frac{360N}{365}\right) \right] \times \left[\cos \phi \cos \delta \sin \omega_s + \frac{2\pi\omega_s}{360} \sin \phi \sin \delta \right] \quad (4.1)$$

where: G_{SC} = Solar constant, 1353 W/m²

N = day of year

ϕ = latitude, degrees

$$\delta = \text{declination, degrees} = 23.45 \sin\left(360 \frac{284 + N}{365}\right)$$

$$\delta = \text{declination, degrees} = 23.45 \sin\left(360 \frac{284 + N}{365}\right) \quad (4.2)$$

$$\omega_s = \text{sunset angle, degrees} = \cos^{-1}(-\tan \phi \tan \delta) \quad (4.3)$$

The monthly average daily extraterrestrial radiation may be calculated using N and 5 corresponding to the average day of the month (the average day for each month is listed on page 12 of Duffie and Beckman [43]).

Monthly average ambient temperature data is published by many sources, including those listed above for the radiation data. Figure 41 shows the monthly average clearness index, KT , for three locations used to evaluate the simplified long-term performance model. Figure 42 shows the monthly average ambient temperature for these locations.

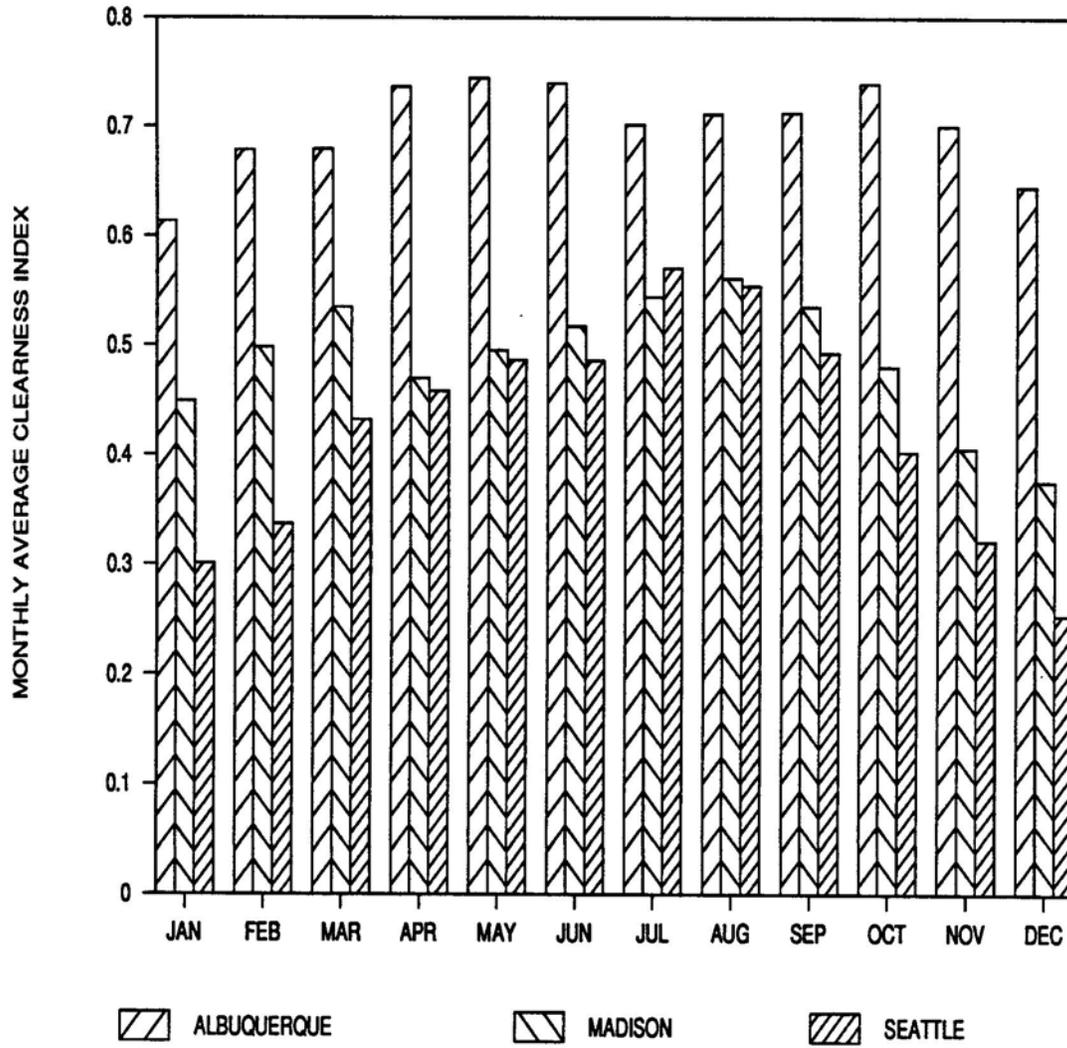


Figure 41. Monthly Average Clearness Indices for 3 Locations

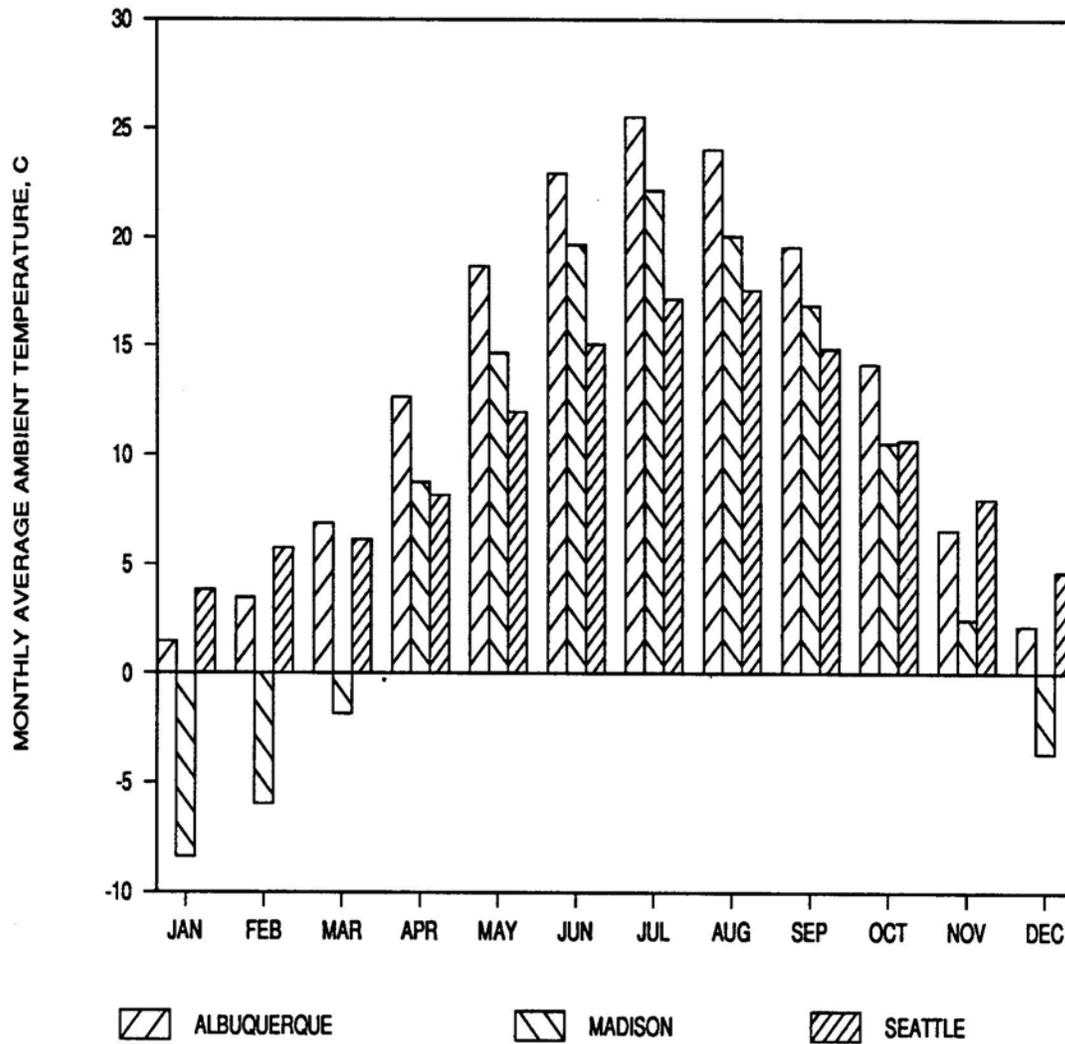


Figure 42. Monthly Average Ambient Temperature for 3 Locations

4.3 Bendt et al. monthly clearness index correlation

Bendt, Collares-Pereira, and Rabl's 1981 correlation [44] is a continuous function describing the long-term distribution of K_T 's within a month, based on $\overline{K_T}$ for that month. The Bendt curve may be shown as a plot of K_T vs. the cumulative fraction of days

within a month. An actual cumulative distribution for one month can only consist of 28 to 31 individual daily K_T 's in ascending order; therefore, values of K_T read from the Bendt curve do not correspond to a particular day, but simply an event of unspecified duration. Rather, the curve should be interpreted to mean that each position on the cumulative fraction scale represents the long-term fraction of days per month that will have a daily K_T less than or equal to K_T evaluated at that position on the cumulative fraction scale. The usefulness of the continuous distribution will be demonstrated in the next section.

The Bendt correlation is shown as Eqn. 4.4, along with auxiliary Eqns. 4.5 - 4.7. Auxiliary Eqn. 4.5 is from a 1983 paper by Hollands and Huget [52] which evaluated applications for the Bendt correlation. Equations 4.6 and 4.7 are from a 1985 M.S. thesis by M. E. Herzog [53]:

$$K_T = \frac{1}{\lambda} \ln[(1 - F) \exp(\lambda K_{MIN}) + F \exp(\lambda K_{MAX})] \quad (4.4)$$

where: $K_T = 0.05$ (assumed to be the worst possible clearness index over any interval)

$$K_{MAX} = 0.6313 + 0.267 \overline{K_T} - 11.9 (\overline{K_T} - 0.75)^8 \quad (4.5)$$

$$\lambda = -1.498 + \frac{1.184 \xi - 27.182 \exp(-1.5 \xi)}{K_{MAX} - K_{MIN}} \quad (4.6)$$

$$\xi = \frac{K_{MAX} - K_{MIN}}{K_{MAX} - K_T} \quad (4.7)$$

$$F = \text{cumulative fraction of days per month } (0 < F < 1) \quad (4.8)$$

Figures 43 - 46 show how the Bendt correlation compares to actual (discrete) clearness index distributions based on TMY data for Albuquerque, Madison, Seattle, and Miami. For each location, the best and worst months are shown, with $\overline{K_T}$ for each listed beside the name of the month. For each month, the areas beneath the Bendt and TMY curves (or, the integral of K_T as F goes from 0 to 1) are the same, and are equal to $\overline{K_T}$.

Albuquerque weather is uniformly clear - both the Bendt and TMY plots are fairly flat with a concentration of very clear days in the range $0.7 < K_T < 0.8$. Many days are alike, even for the worst month. Seattle shows the widest annual variation in monthly average clearness index, from 0.25 in December to 0.57 in July. The Seattle December distribution shows nearly the opposite pattern of Albuquerque; there are many similar poor days ($K_T \sim 0.1 - 0.2$) and only a few mediocre ($K_T \sim 0.5$) days. Madison exhibits less of a month to month variation than Seattle, but during the poorest month, December, there is a wider range and lesser concentration of "day-types" - the gradient from very poor ($K_T \sim 0.1$) to clear ($K_T \sim 0.6$) is more linear. The summer distributions are nearly the same for Seattle and Madison, with a concentration of similar, very clear days. For Miami, the Bendt correlation is not as accurate as for the other locations shown. Both months are characterized by a concentration of mediocre days. The TMY data deviates from the correlation most at the ends of the cumulative time fraction scale.

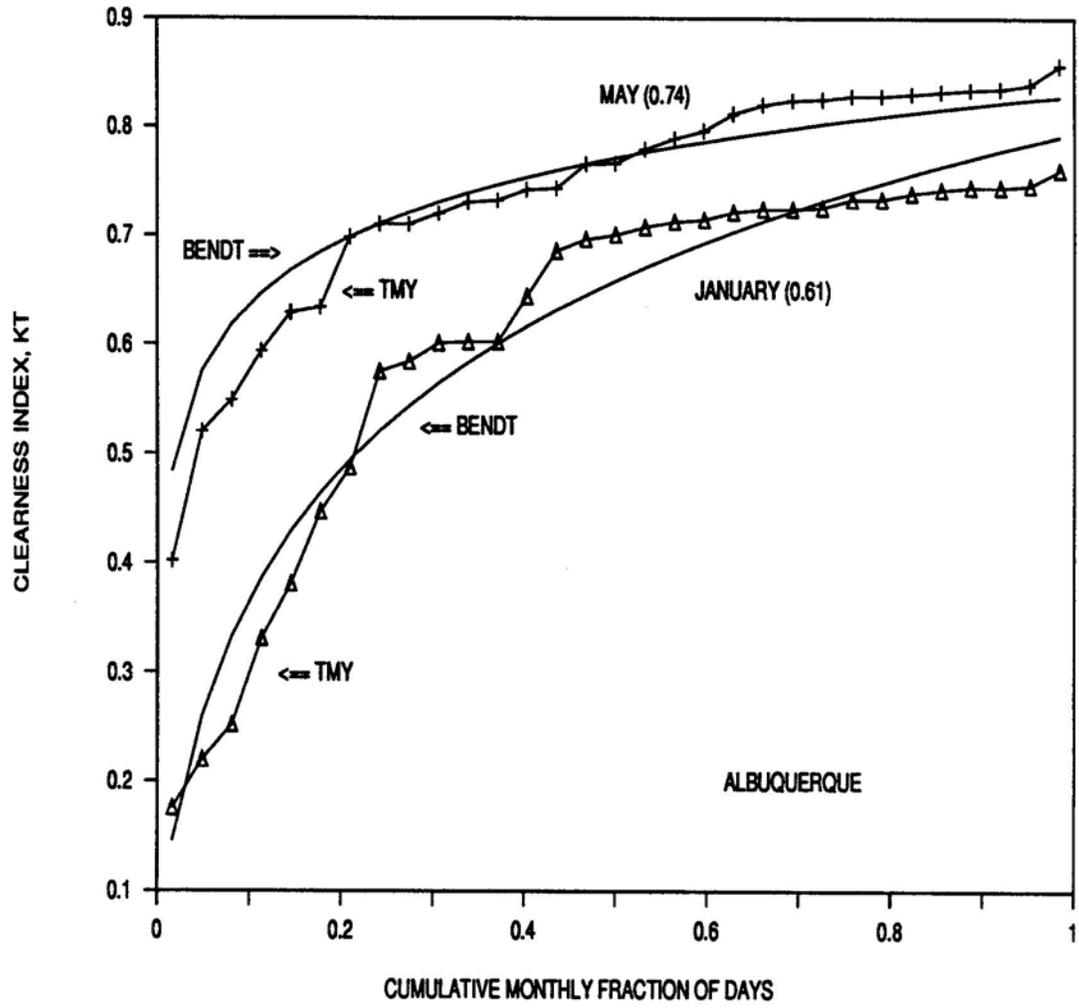


Figure 43. Bendt and TMY K_T Distributions, Albuquerque

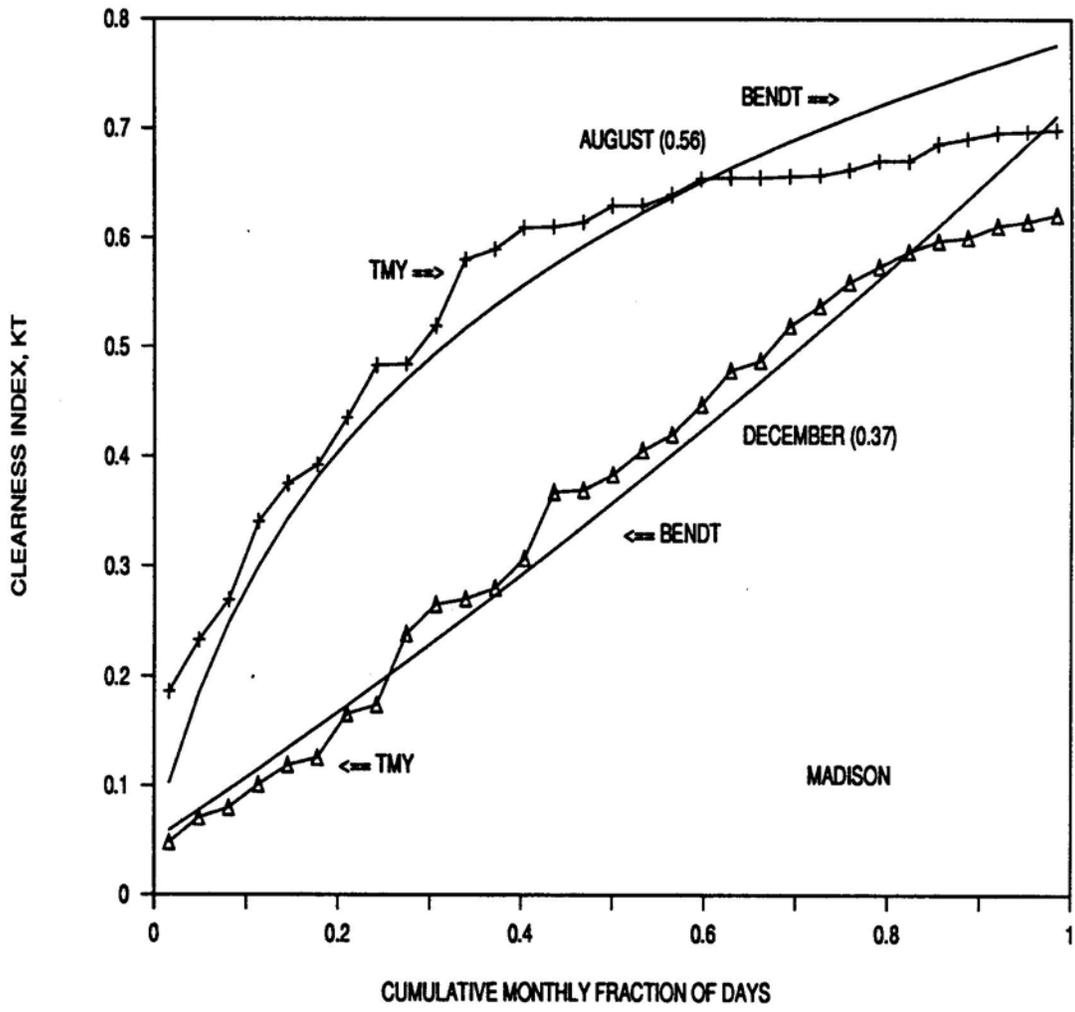


Figure 44. Bendt and TMY K_T Distributions, Madison

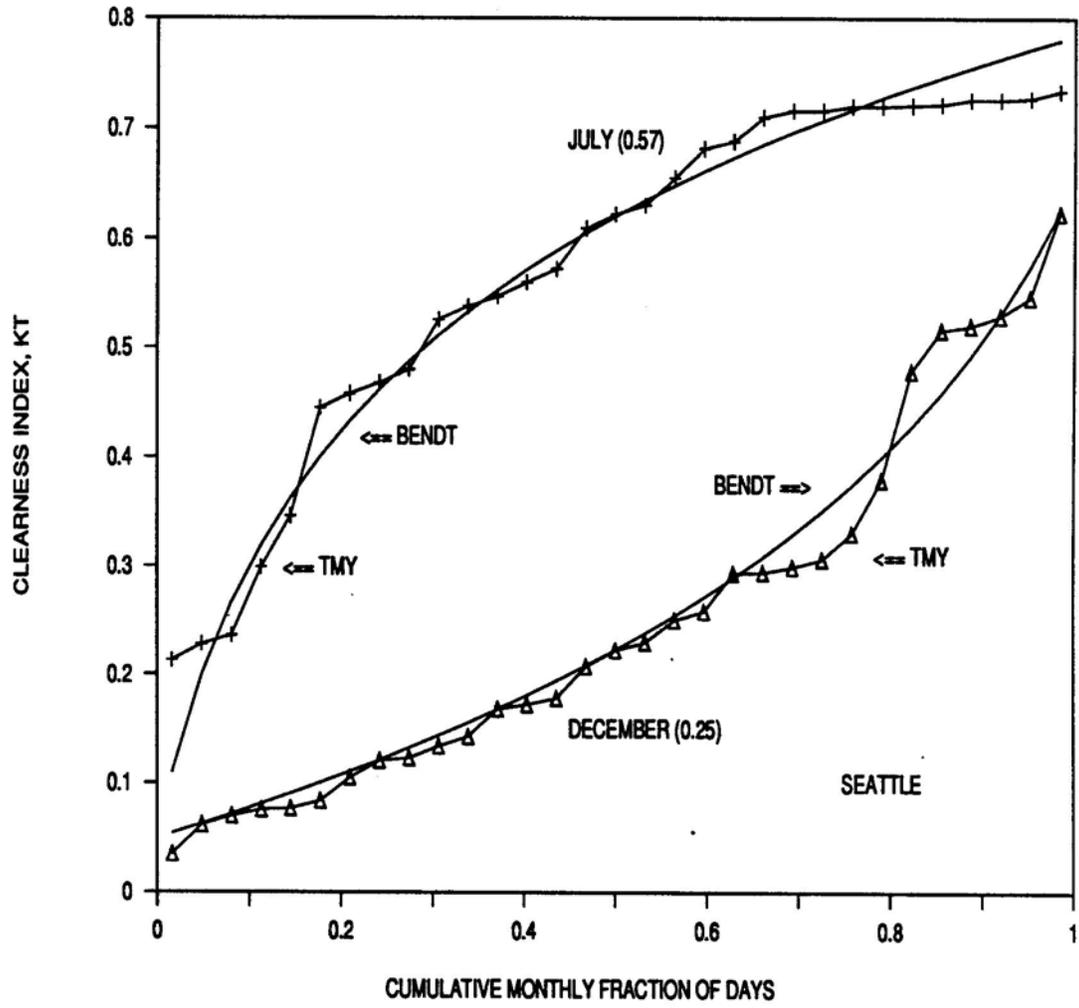


Figure 45. Bendt and TMY K_T Distributions, Seattle

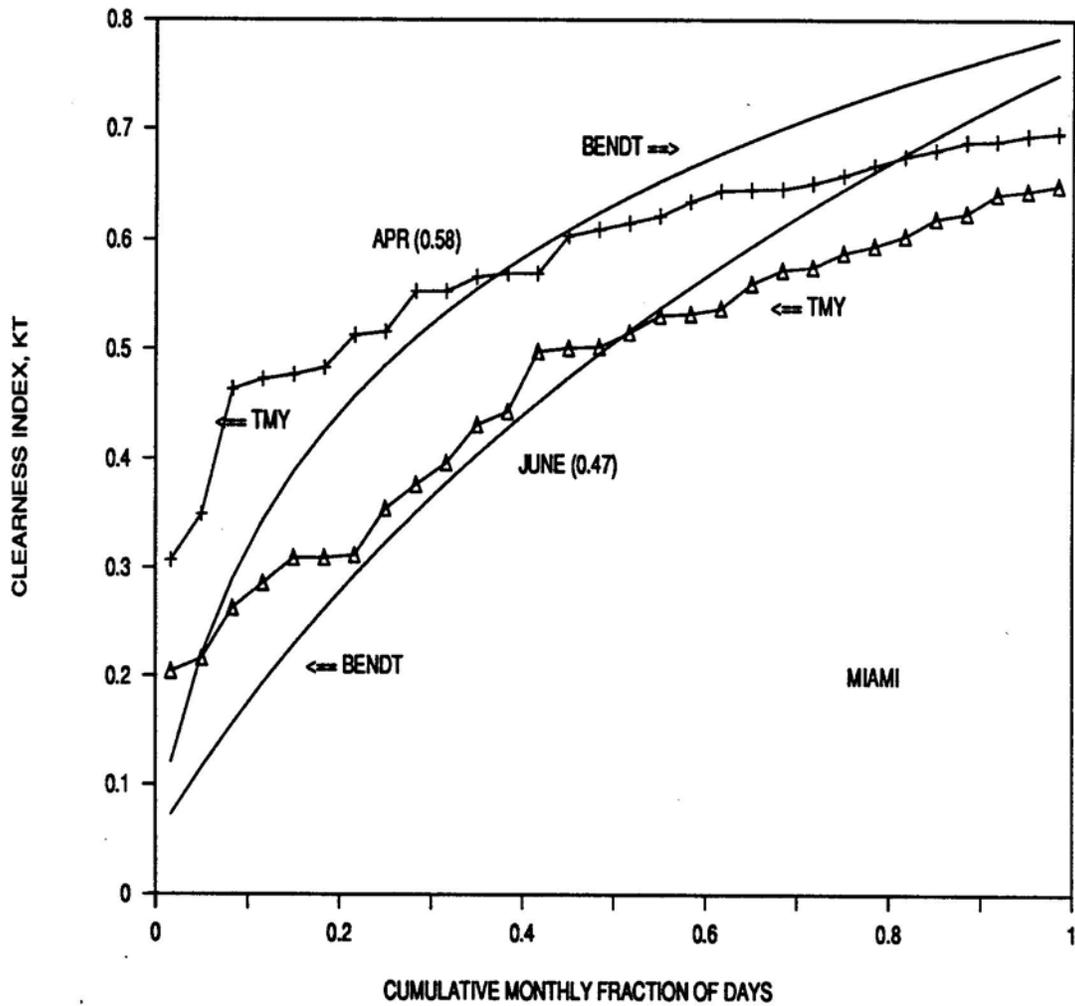


Figure 46. Bendt and TMY K_T Distributions, Miami

The cases shown represent the weather extremes for each location. For each of the cases, the Bendt distribution shows a good correspondence with the measured daily K_T distribution, although the correlation does not always match well with the TMY data at very low and very high cumulative fractions, especially in Miami. The TMY data may not (and probably does not) exactly represent the long-term behavior.

4.4 Approximating the clearness index distribution

The K_T vs. fractional time curve may be approximated by a series of rectangular segments. The number of segments needed to accurately trace the K_T curve depends on $\overline{K_T}$. The height of each segment follows the contour of the Bendt distribution function and is equal to K_T evaluated at the midpoint of each segment. Each rectangular segment represents a type of day, so that an entire month can be approximated by several segments, or "typical day" types. Within each "typical day" type, each day is assumed to have the same K_T .

If a 30 day month is divided into, for example, 5 segments (each segment is chosen to be of equal width, but this is not necessary), each segment will consist of 6 identical days. For performance calculations, only one day from each segment is considered. The output from that day is multiplied by the number of days per segment when calculating monthly totals. The number of segments need not be an integer fraction of the number of days per month.

Figures 47 - 52 show how the \sim curve may be approximated by either 5 or 20 segments, for three representative months and locations: May, Albuquerque; December, Madison; and December, Seattle. The percentage difference in integrated area beneath the Bendt curve and its rectangular approximation is listed on each figure. These examples were chosen to show how the accuracy of the rectangular approximation depends on the monthly average clearness index. For months with very high (Albuquerque in May) or very low (Seattle in December) $\overline{K_T}$ the area computed with the

5 segment approximation is within 1 % of the area beneath the Bendt curve. For a location with an intermediate $\overline{K_T}$, such as Madison, the K_T curve is very linear and the same 5 segment approximation results in a much better estimate - the integrated area difference is only 0.05%. When a 20 segment approximation is used, the integrated area difference is less than or equal to 0.15% for each case.

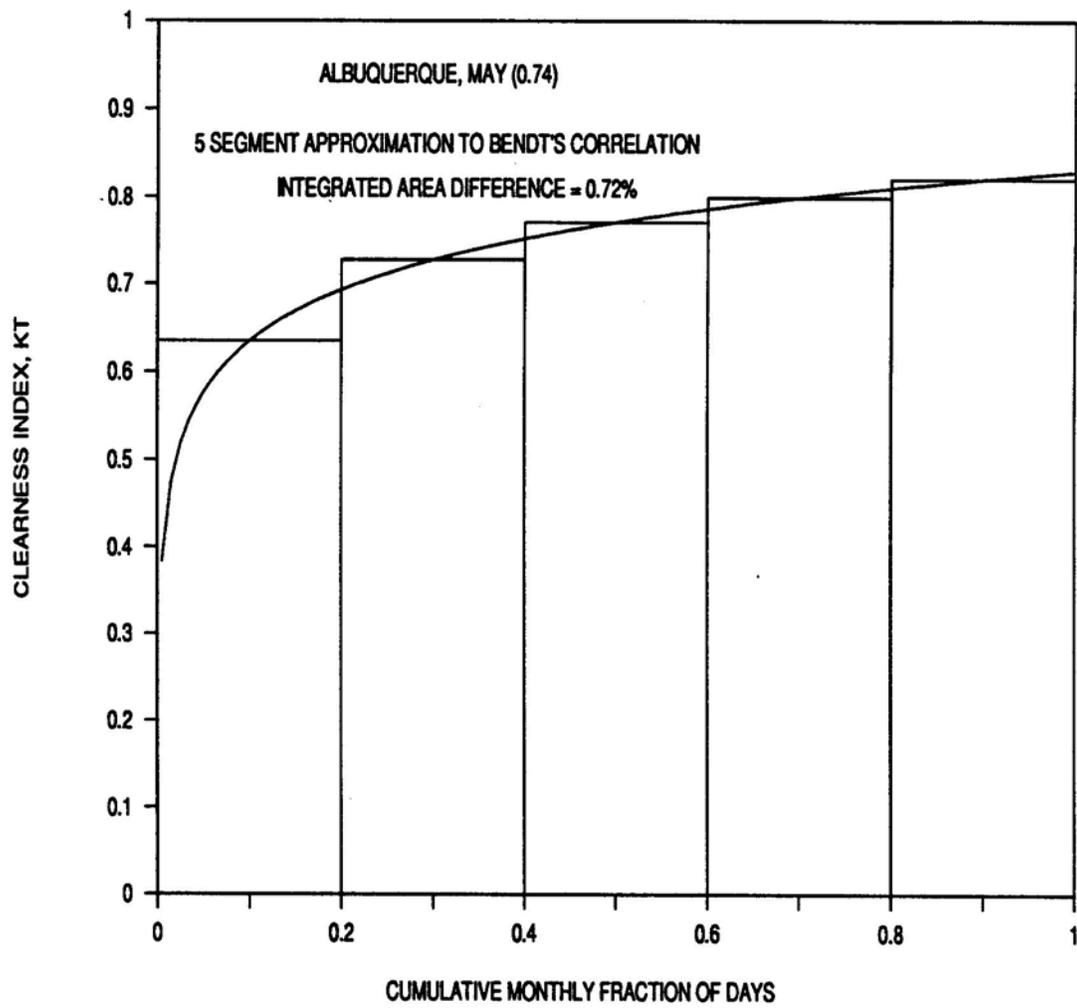


Figure 47. 5 Segment Approximation to the Bendt K_T Distribution, Albuquerque

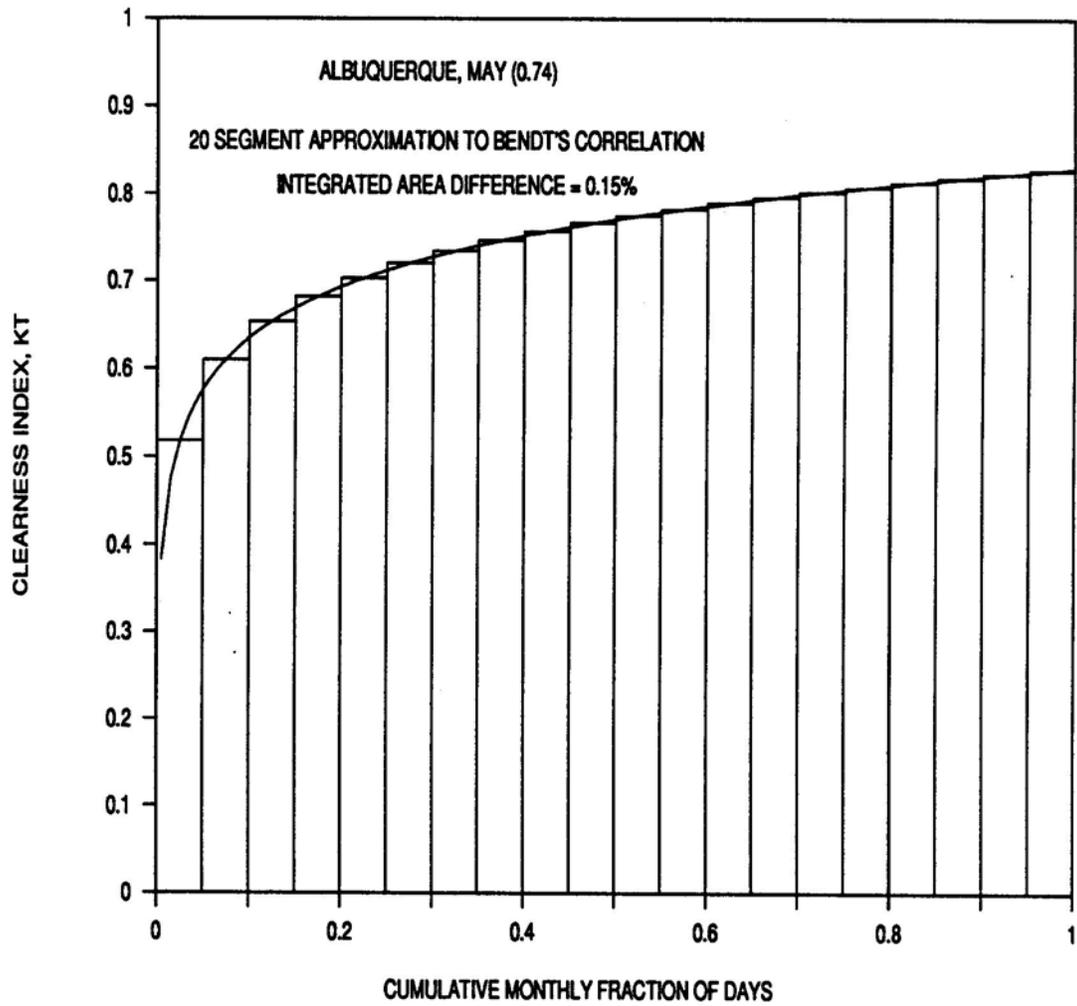


Figure 48. 20 Segment Approximation to the Bendt K_T Distribution, Albuquerque

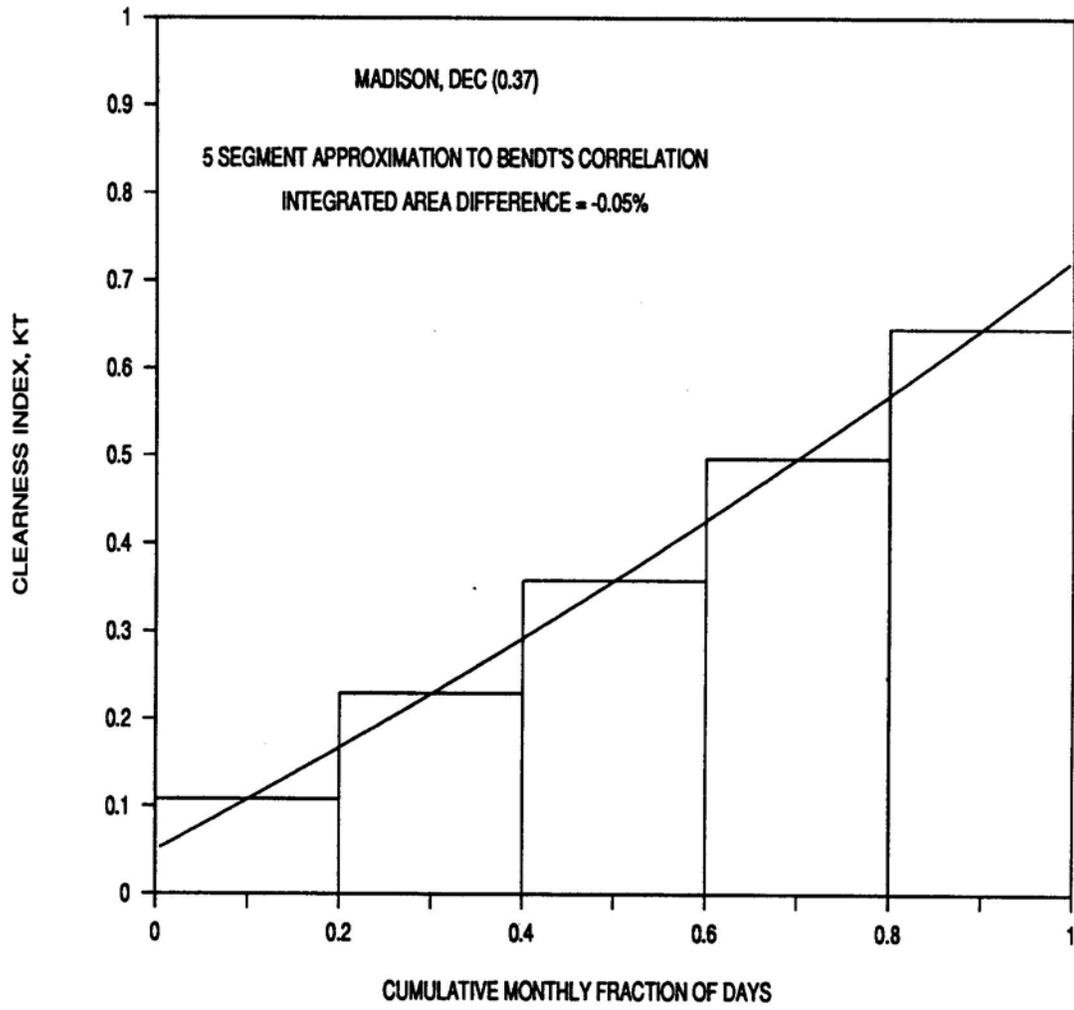


Figure 49. 5 Segment Approximation to the Bendt K_r Distribution, Madison

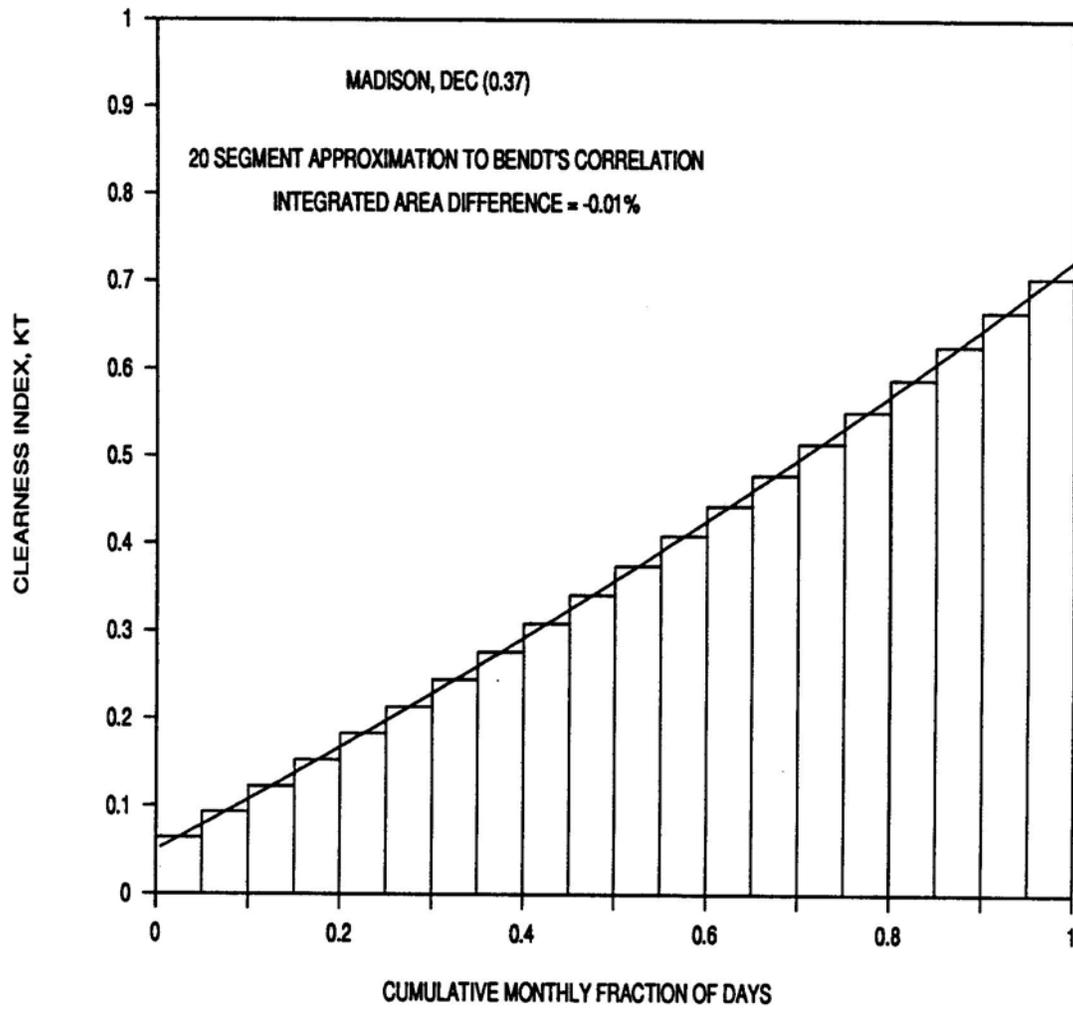


Figure 50. 20 Segment Approximation to the Bendt Kr Distribution. Madison

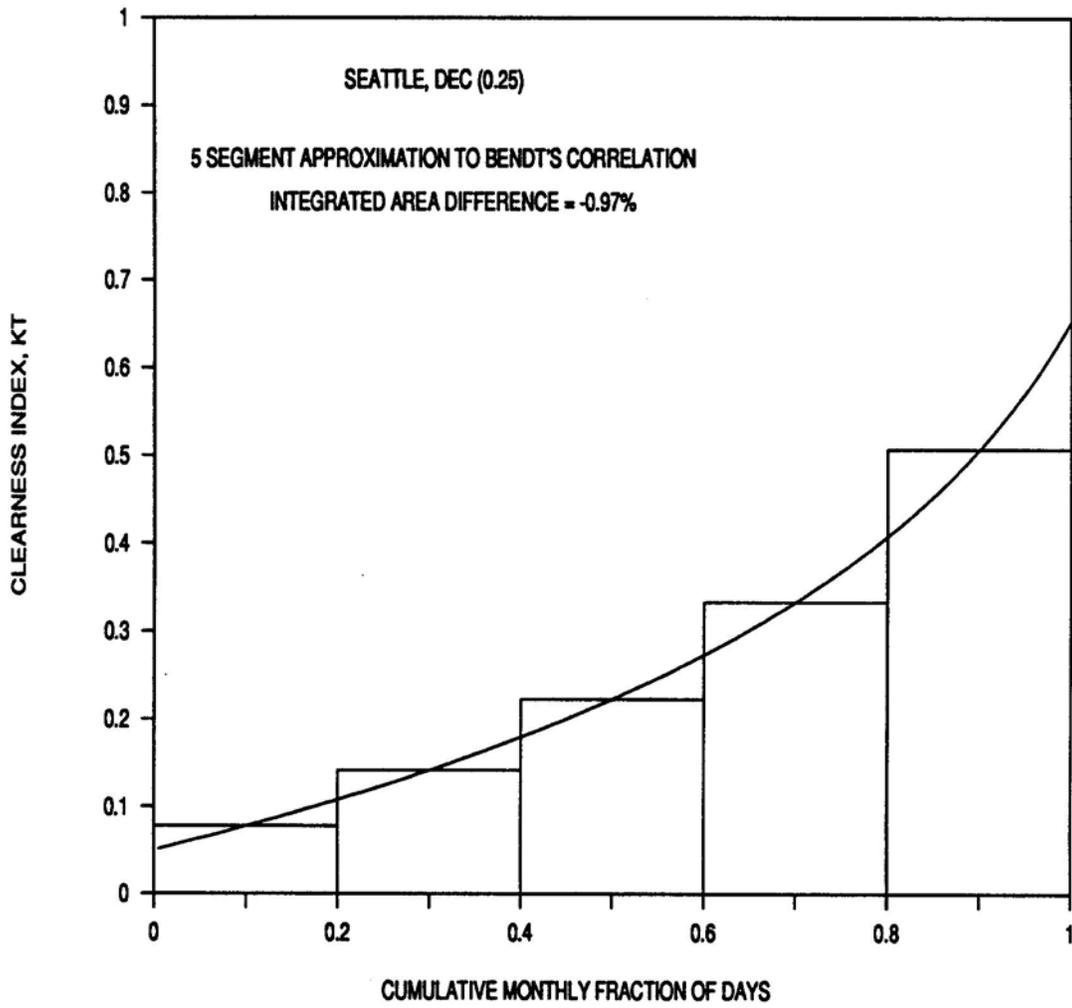


Figure 51. 5 Segment Approximation to the Bendt K_T Distribution, Seattle

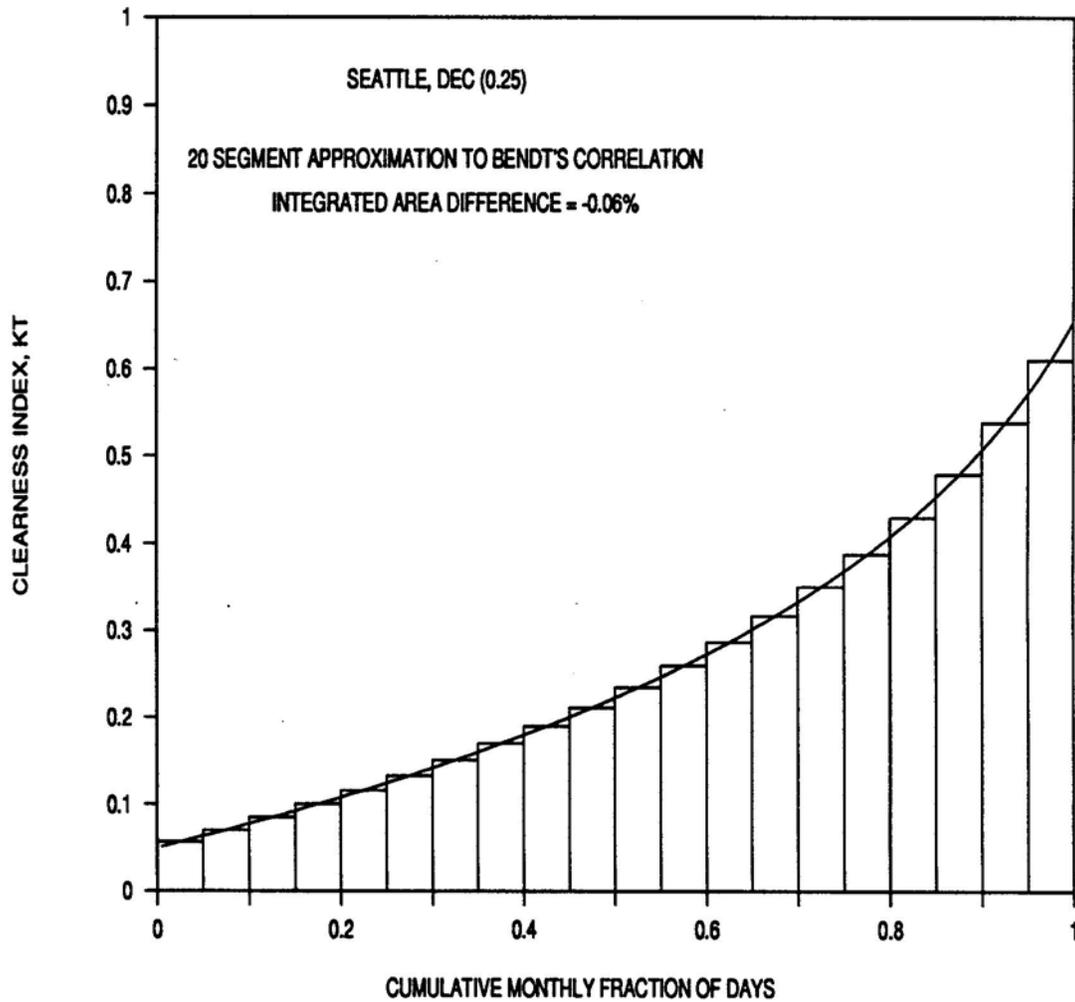


Figure 52. 20 Segment Approximation to the Bendt K_T Distribution, Seattle

As shown in Section 4.1.2, the Bendt correlation does not replicate the exact pattern of individual days. Depending on the location and month, some days may deviate significantly from the correlation curve (e.g., Miami), even though the monthly-average clearness index (or area beneath the curve) is the same for the actual month and the correlation. Therefore, using a large number of segments (15 or more) to approximate the

Bendt curve will improve the integrated area estimate but may not result in a more accurate representation of the long-term weather pattern.

In simulations designed to test the accuracy of the new simplified direct-coupled performance model (described in Chapter 5), each case is replicated four times with the number of segments used to approximate the monthly Bendt distribution being 3, 5, 10, or 20 segments. In Chapter 6, the results from these simulations are compared to simulations using hourly TMY data to see, in general, how few segments can be chosen to speed calculations, yet still produce results similar to the detailed hourly simulations.

4.5 Daily diffuse fraction

Once K_T has been estimated for each "typical day" segment, it is used as input for another correlation designed to estimate the diffuse fraction of that "day's" total radiation. The correlation used is from Collares-Pereira and Rabl [46]. H_D/H is the daily diffuse fraction.

$$K_D / H = 0.99 \quad \text{for } K_T \leq 0.17 \quad (4.9)$$

$$K_D / H = 1.188 - 2.272(K_T) + 9.473(DK_T^2) - 21.865(K_T^3) + 14.648(K_T^4) \quad \text{for } 0.17 < K_T \leq 0.75 \quad (4.10)$$

$$K_D / H = 0.632 - 0.54(K_T) \quad \text{for } 0.75 < K_T < 0.80 \quad (4.11)$$

$$K_D / H = 0.2 \quad \text{for } K_T \geq 0.80 \quad (4.12)$$

H, the total horizontal surface radiation per "typical day," is calculated as the product of K_T and H_0 , the mean daily extraterrestrial radiation calculated using Equation 4.1.

4.6 Hourly fraction of daily total and diffuse radiation

If the radiation pattern within each day is assumed to be symmetric about solar noon, then existing correlations can be used to calculate the hourly fraction of total radiation for each type of "typical day" and also the hourly fraction of diffuse radiation for each type of "typical day." The order of individual days is not known and does not matter if there is no storage, so each "typical day" within a month is assumed to have the same daylength and extraterrestrial radiation, evaluated at the average day of the month.

The following correlation from Collares-Pereira and Rabl [46] is used to calculate the hourly fraction, r_T , of the daily total radiation, H , as a function of the hour angle, ω , and the sunset angle, ω_s . ω is the angle, in degrees, from the local meridian, at 15° per hour, where morning angles are negative and afternoon angles positive. ω_s is calculated using Eqn. 4.3 and is the angular degree equivalent of one-half of the daylength.

$$r_T = \frac{\pi}{24} (a + b \cos \omega) \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \left(\frac{2\pi\omega_s}{360} \right) \cos \omega_s} \quad (4.13)$$

where:

$$a = 0.409 + 0.5016 \sin(\omega_s - 60) \quad (4.14)$$

$$b = 0.6609 - 0.4767 \sin(\omega_s - 60) \quad (4.15)$$

The following correlation from Liu and Jordan [45] is used to calculate the hourly fraction, r_D , of the total daily diffuse radiation, H_D .

$$r_D = \frac{\pi}{24} \left[\frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \left(\frac{2\pi\omega_s}{360} \right) \cos \omega_s} \right] \quad (4.16)$$

The hourly total radiation, I , is equal to the hourly fraction, r_T , multiplied by the daily total radiation, H . The hourly diffuse radiation, I_D , is equal to the hourly diffuse fraction, r_D , multiplied by the daily diffuse radiation, H_D . The hourly beam radiation, I_B , is equal to the difference between the hourly total and hourly diffuse radiation.

4.7 Hourly ambient temperature

The following correlation from Erbs [47] is used to estimate the hourly ambient temperature as a function of the hour of day (HR), K_T and $\overline{T_A}$:

$$\begin{aligned} T_A = \overline{T_A} + (25.8K_T - 5.21)(0.4632\cos(HRX - 3.805) + 0.0984\cos(2*HRX - 0.36) \\ + 0.0168\cos(3*HRX - 0.822) + 0.0138\cos(4*HRX - 3.513)) \end{aligned} \quad (4.17)$$

$$HRX = \frac{(HR - 1)\pi}{12} \quad (4.18)$$

4.8 Plane of array irradiance

The total radiation on a tilted surface per hour is calculated using the following equations from Duffie and Beckman [43] (Equation 4.19 was originally derived by Liu and Jordan [48]). R_B is the dimensionless ratio of beam radiation on a tilted surface to

that on a horizontal surface. β is the array slope, in degrees. ρ is the ground reflectance.

I_T , I_D , and I_B are usually expressed in $\text{KJ/m}^2 \cdot \text{hr}$.

$$I_T = I_B R_B + I_D \left(\frac{1 + \cos \beta}{2} \right) + I \rho \left(\frac{1 - \cos \beta}{2} \right) \quad (4.19)$$

$$R_B = \frac{\cos(\phi - \beta) \cos(\delta) \cos(\omega) + \sin(\phi - \beta) \sin(\delta)}{\cos(\phi) \cos(\delta) \cos(\omega) + \sin(\phi) \sin(\delta)} \quad (4.20)$$

The above equation is valid for south facing surfaces in the northern hemisphere.

In the southern hemisphere the equation becomes:

$$R_B = \frac{\cos(\phi + \beta) \cos(\delta) \cos(\omega) + \sin(\phi + \beta) \sin(\delta)}{\cos(\phi) \cos(\delta) \cos(\omega) + \sin(\phi) \sin(\delta)} \quad (4.21)$$

ϕ is the latitude and δ is the declination. The plane of array irradiance (as defined earlier and denoted by the symbol Φ) has units of W/m^2 , and is obtained by dividing the hourly integrated energy on the tilted surface, I_T , by $3.6 \text{ KJ/W} \cdot \text{hr}$.

4.9 PV Cell Temperature

The cell temperature, T_C , is calculated using Equation 2.89 from Section 2.6. The variables needed are the ambient temperature and the plane of array irradiance. The ambient temperature is calculated using equations from Section 4.7. The irradiance is calculated using equations from Section 4.8. Once the irradiance and cell temperature are known, the a1TaY I-V characteristics are fixed.

4.10 Summary

This chapter described an eight step method by which two long-term monthly average weather statistics, the clearness index and the ambient temperature, are used to calculate a reduced set of hourly estimates of the plane of array irradiance and PV system cell temperature. The irradiance and cell temperature are used in the array I-V equation to fix the hourly array I-V characteristic.