APPENDIX A

Motor/Pump Data

This section details the construction of I-V coordinate pairs for the following motor/pump loads:

1. 11/4 hp series motor/l hp centrifugal fan load

2. 1 1/4 hp shunt motor/l hp centrifugal fan load

3. 1 1/4 hp pennanent magnet motor/1 hp centrifugal fan load

4. 7 hp series motor/5 hp centrifugal water pump load

5. 7 hp permanent magnet motor/5 hp centrifugal water pump load

6. 1/20 hp permanent magnet motor w/positive displacement water pump load

Constants and equations specific to the first three motors (all of which are connected to the same fan load) are shown below. These are used to calculate the load torque, motor current and motor voltage for each speed listed in the table.

Ventilator Centrifugal Fan Load [37]

T = 0.5 + 0.002387 ω + 0.00039$\omega^{1.8}$ Nm

Series Motor:

$$I = \left[\frac{T(\omega)}{M_{AF}}\right]^{1/2}$$

$$V = \left[\frac{T(\omega)}{M_{AF}}\right]^{1/2}\left[M_{AF}\omega + (R_A + R_F)\right]$$

Shunt Motor:

$$V = \left[\frac{R_A R_F T(\omega)}{M_{AF}(1 - M_{AF}\omega / R_F)}\right]^{1/2}$$

$I = [T/(M_{AF}I_F)]+[V/R_F]$

Permanent Magnet Motor:

$I = T/k\phi$

$V = k\phi\ \omega + I\ R_A$

	Series	Shunt	Perm.Mag.
R_A, ohms	1.5	1.5	1.5
R_F, ohms	0.7	100	n/a
M_{AF}, henry	0.0675	0.518	n/a
ω_{RATED},rad/sec	188.5	188.5	188.5
$k\phi$, V sec	n/a	n/a	0.676

SPEED		TORQUE	SERIES		PERM.MAG		SHUNT	
RPM	rad/s	N•m	I	V	I	V	I	V
0	0.0	0.500	2.722	5.99	0.740	1.11	8.142	12.03
200	20.9	0.643	3.087	11.15	0.951	15.59	8.734	14.45
400	41.9	0.924	3.700	18.60	1.367	30.37	9.836	18.49
600	62.8	1.323	4.427	28.51	1.957	45.41	10.954	23.83
800	83.8	1.829	5.205	40.89	2.706	60.69	11.849	3059
1000	104.7	2.437	6.009	55.69	3.605	76.20	12.372	39.27
1200	125.7	3.142	6.823	72.88	4.648	91.92	12.392	51.06
1400	146.6	3.941	7.641	92.43	5.830	107.85	11.735	68.88
1600	167.6	4.831	8.460	114.29	7.147	123.99	10.092	102.92
1800	188.5	5.810	9.277	138.45	8.594	140.31	6.870	267.03

Constants for the fourth and fifth motors (both are connected to the same centrifugal water pump load) are shown below, followed by a list of the I-V pairs used as inputs for simulations. The equations are the same as for the series and permanent magnet motors listed above. The pump data are read from Fig. 40 in section 3.4.4.2 [39].

	Series	Perm.Mag.
R_A, ohms	1.44	1.44
R_F, ohms	0.67	n/a
M_{AF}, henry	0.0275	0.518
ω_{RATED},rad/sec	366.5	366.5
$k\phi$, V sec	n/a	0.59

SPEED RPM (rad/s)	FLOWRATE GPM (m3/hr)	EFFIC. %	TORQUE N•m	SERIES I	SERIES V	PERM.MAG. I	PERM.MAG. V
2775 (290.6)	29.33 (6.66)	45.0	4.229	12.40	125.3	7.17	181.8
2800 (293.2)	32.00 (7.27)	47.5	4.332	12.55	127.7	7.34	183.6
2860 (299.5)	48.00 (10.90)	55.0	5.495	14.14	146.3	9.31	190.1
2900 (303.7)	58.00 (13.17)	56.5	6.374	15.22	159.3	10.80	194.7
3000 (314.2)	69.00 (15.67)	58.0	7.140	16.11	173.2	12.10	202.8
3100 (324.6)	78.00 (17.71)	56.9	7.962	17.02	187.8	13.50	211.0
3200 (335.1)	86.00 (19.53)	55.8	8.672	17.76	201.1	14.70	218.9
3300 (345.6)	93.00 (21.12)	55.0	9.226	18.32	212.7	15.64	226.4
3400 (356.0)	99.00 (22.48)	53.0	9.892	18.97	225.7	16.77	234.2
3500 (366.5)	107.0 (24.30)	50.0	11.01	20.01	243.9	18.66	243.1

The I-V pairs for the sixth motor/pump combination were supplied directly by the manufacturer [40] and are listed below.

I	V
1.905	1.45
1.972	2.89
2.048	4.34
2.146	5.78
2.269	7.23
2.410	8.68
2.545	10.12
2.731	11.57
2.919	13.01
3.135	14.46

APPENDIX B

Operating Point and Maximum Power Point Calculations

The maximum power point is calculated before the operating point. The advantage of doing this is that once I_{MAX} and V_{MAX} are known for each hour, it is easier to establish initial guesses when solving for the direct-coupled operating point with Newton's method, or to establish boundaries when solving for the operating point with a bisection method.

The maximum power point is found by writing the array I-V Eqn. 2.24 at the maximum power point, i.e., use I_{MP} for I and V_{MP} for V. (In Chapter 3, it is stated that the array I-V equation used is Eqn. 2.67, a simplified form of Eqn. 2.24. However, in the FORTRAN programs, array Eqn. 2.24 is used. The difference is insignificant.) Then, the derivative of the product of (Eqn. 2.24 x V) is set to zero and I_{MP} and V_{MP} are used in this expression as well. V_{MP} can be solved for explicitly by rearranging the original expression and then this is substituted into the derivative expression for every V_{MP} term. This leaves one equation with one unknown, I_{MP}, which is rearranged so that all terms are on one side and then solved implicitly using Newton's method. An initial guess for I_{MP} is given by:

$$I_{MAX,GUESS} \approx \frac{\Phi}{\Phi_{REF}} \cdot NP \cdot (I_{MAX,REF} + \mu_{ISC} \cdot (T_C - T_{C,REF})) \qquad \text{(B.1)}$$

The objective function is given by Eqn. B.2, where each value of I_{MAX} is a guess:

$$OBJ = 0 = I_{MAX} + \frac{(I_{MAX} - I_L - I_O) \cdot (\ln(1 + (I_L - I_{MAX})/I_O) - I_{MAX} qR_S / \gamma k T_C)}{1 + (I_L - I_{MAX} + I_O) \cdot qR_S / \gamma k T_C} \qquad \text{(B.2)}$$

Its derivative with respect to k is given by:

$$OBJ' = 2 + \frac{(\ln(1 + (I_L - I_{MAX})/I_O) - qR_S / \gamma k T_C)}{(1 + (I_L - I_{MAX} + I_O) \cdot (qR_S / \gamma k T_C))^2} \qquad \text{(B.3)}$$

Each new guess for I_{MAX} is calculated by subtracting the ratio of (Eqn. B.2/Eqn. B.3) from the current guess. The process is repeated until successive guesses differ by less than 5E-O5 amps. With I_{MAX} known, V_{MAX} is calculated by rearranging Eqn. 2.24 to solve explicitly for V, substituting I_{MAX} for I.

For fixed voltage loads, the operating point is found by substituting the known load voltage, V, directly into array I-V Eqn. 2.24 for V, if V is less than the open circuit voltage (otherwise. there is no solution - the power is zero). Newton's method is used to solve for I. An initial guess for I is established by using a simplified 3 parameter I-V equation explicit in I. The other parameters are known. The tolerance is the same as for the maximum power point solution.

$$I_{GUESS} \approx I_L - I_O \cdot (\exp(qV / \gamma kT_C) - 1.0) \tag{B.4}$$

The objective function, Eqn. B.2, and its derivative, Eqn. B.3, are given by:

$$OBJ = 0 = -I_{GUESS} + I_L - I_O \cdot (\exp(q(V + I_{GUESS} R_S) / \gamma kT_C) - 1.0) \tag{B.5}$$

$$OBJ' = -1 - I_O \cdot (qR_S / \gamma kT_C) \cdot \exp(q(V + I_{GUESS} R_S) / \gamma kT_C) \tag{B.6}$$

For resistive loads, the quantity (I x R_S) is substituted into Eqn. 2.24 for V. Newton's method is used to solve for I. The tolerance is the same as for the maximum power point solution. A guess for I is established by first checking the load resistance relative to the optimum load resistance, $R_{OPT} = V_{MAX}/I_{MAX}$. If the load resistance R_L is less than the optimum, then the initial guess is scaled between I_{MAX} and I_{SC} using Eqn. B.7. If the load resistance is greater than the optimum, then the initial guess is scaled between 0 and I_{MAX} using Eqn. B.8.

$$I_{GUESS} \approx I_{MAX} + (I_L - I_{MAX}) \cdot \frac{R_L - (V_{MAX} / I_{MAX})}{-V_{MAX} / I_{MAX}} \tag{B.7}$$

$$I_{GUESS} \approx \frac{V_{MAX}}{R_L} \tag{B.8}$$

The objective function, Eqn. B.9, and its derivative, Eqn. B.l0, are given by:

$$OBJ = 0 = -I_{GUESS} + I_L - I_O \cdot (\exp(qI_{GUESS} (R_L + R_S) / \gamma kT_C) - 1.0) \tag{B.9}$$

$$OBJ' = -1 - I_O \cdot (q(R_L + R_S) / \gamma kT_C \cdot \exp(qI_{GUESS} (R_L + R_S) / \gamma kT_C) \tag{B.10}$$

For motor loads, the load I-V curve is approximated by a series of straight line segments. The search for an operating point begins with the leftmost I-V coordinate pair (i.e., that of smallest I and V). To screen out most of the possible non-solutions, if the first load point voltage is greater than the open circuit voltage, then there is no solution, or if the first load point current is greater than the short circuit current, there is no solution. A [mal check for non-solutions is done by solving for I*, the current on the array I-V curve evaluated at the voltage of the first load point. If the current at the first load point is greater than I*, there is no solution.

A straight line equation is calculated between the first two points. If the intersection of this equation with the array I-V curve occurs within the bounds of the first two load points, that intersection determines the operating point. Otherwise, the program advances to the next load I-V pair, calculates a new straight line equation, and repeats the search. If no suitable intersection is found after checking each line segment, the power is zero.

The slope, *SL,* of a load I-V line segment is equal to the ratio of $(I_{NEXT} - I_{PRESENT})/(V_{NEXT} - V_{PRESENT})$. The V-intercept, C, is equal to $V_{PRESENT} - (I_{PRESENT}/SL)$. The I-intercept is equal to -C x *SL.*

The next step is to determine whether Newton's method can be used to solve for I, or whether a slower but more stable bisection method is necessary. If the slope is < 0 and the V-intercept is > V_{OC}, then the bisection method is used. If the slope is < 0 and the I-intercept is greater than the short circuit current and the V-intercept is > V_{OC}, there is no solution. Otherwise, Newton's method is used. Regardless of the search method, the same tolerance is used as for the previous load types.

If the bisection method is used, the upper bound estimate for I is equal to $I_{PRESENT}$. The lower bound estimate is equal to *SL* x (V_{OC} - C). The objective function is given by Eqn. B.11, where in this case, I_{TMP} represents either the present high, low, or midpoint estimate for I:

$$OBJ = 0 = I_L + I_O - I_{TMP} - I_O \cdot \exp(q(C + I_{TMP} \cdot (R_S + 1/SL))/\gamma k T_C) \quad \text{(B.11)}$$

If Newton's method is used, different initial guesses are calculated, depending on the slope and intercepts of the load I-V line relative to the array I-V curve. For all of the possible combinations of initial guesses, the reader is referred to the section of the FORTRAN code which handles this task (starting 2 lines after numbered statement #250 on the DCPVSIMP and DCPVDET programs). The objective equation and its derivative are given by Eqns. B.12 and B.13.

$$OBJ = 0 = I_L + I_O - I_{GUESS} - I_O \cdot \exp(q(C + I_{GUESS}(R_S 1/SL))/\gamma k T_C) \quad \text{(B.12)}$$

$$OBJ' = -1 - I_O \cdot \exp\left(\frac{q(C + I_{GUESS} \cdot (R_S + 1/SL))}{(\gamma k T_C)}\right) \cdot (R_S + 1/SL) \cdot (q/\gamma k T_C) \quad \text{(B.13)}$$

For pumping applications, the hourly volume of water pumped is calculated by interpolating between the flowrates corresponding to the two load points of the line segment where the operating point occurs.