4 HEAT AND MASS TRANSFER IN ROTARY REGENERATORS

In this chapter several existing models that describe the mechanism of combined heat and mass transfer in rotary regenerators will be introduced, and the utility of these models for simulating the regenerator operation and estimating the amount of energy that can be recovered will be discussed. None of the existing models is suitable for computing the regenerator performances at all operating conditions encountered in space-conditioning systems, and therefore a new computationally simple model is developed for the appropriate range of parameters.

The purpose of this new model, which is described in Section 4.4, is to predict the outlet states of the two air streams that are passed through the matrix (Figure 1.1) with good accuracy for a specified range of operating conditions. The model must also have a relatively simple form in order to allow simulations over a longer period of time without excessive computational effort.

All the models, the existing ones as well as the new one, allow calculation of the properties of the two outlet air states as a function of the inlet states, the matrix design and properties and the ratio of matrix rotation speed to air flowrate.

4.1 Mathematical Formulation of Combined Heat and Mass Transfer

Existing mathematical models used to describe the heat and mass transfer mechanisms inside the enthalpy exchanger matrix are based on the following assumptions:

1. The state properties of both inlet air streams are spatially uniform and constant in time at the inlet face of the regenerator.

2. The thermodynamic properties of the air and the matrix are not affected by the pressure drop in axial direction of the matrix which is small compared to the total pressure.
3. There is no mixing or carry-over between the two air streams
4. The fluid and matrix states are considered to be uniform in radial direction
5. Angular and axial heat conduction and vapor diffusion due to temperature and concentration gradients, respectively, are neglected.
6. The matrix is considered to be a homogeneous solid with constant matrix characteristics (Figure 1.1)
7. The convective heat and mass transfer coefficients between the air streams and the matrix are constant throughout the system.
8. The rotary regenerator operates adiabatically.

The heat and mass transfer model is based on mass and energy conservation, and with the assumptions listed above the governing equations can be written in terms of dimensionless coordinates as stated by Van den Bulck [36]:

1. Mass conservation:

$$\frac{\partial w_f}{\partial z} + \frac{M_f}{M_m}\frac{\partial w_f}{\partial \tau} + \frac{\partial W_m}{\partial \tau} = 0 \tag{4.1}$$

2. Energy conservation:

$$\frac{\partial i_f}{\partial z} + \frac{M_f}{M_m}\frac{\partial i_f}{\partial \tau} + \frac{\partial I_m}{\partial \tau} = 0 \tag{4.2}$$

3. Mass transfer rate:

$$\frac{\partial W_m}{\partial \tau} = NTU_w \left(w_f - w_m \right) \tag{4.3}$$

4. Thermal energy transfer rate:

$$\frac{\partial I_m}{\partial \tau} = NTU_t \frac{\partial i_f}{\partial T_f} \left(T_f - T_m \right) + NTU_w \left(w_f - w_m \right) i_w \tag{4.4}$$

The coordinate system that leads to Equations (4.1) to (4.4) is in accordance with Figure 1.1, where the angular coordinate system is rotating with the matrix. Therefore the angular coordinate Θ is actually a time coordinate. The dimensionless parameters used in the equations above are defined as:

$z = \frac{x}{L}$ dimensionless length in flow direction

$\tau = \frac{\Theta}{T_j \Gamma_j}$, $0 \le \tau \le \frac{1}{\Gamma_j}$ dimensionless time

$NTU_w = \frac{h_w A}{\dot{m}_f}$ number of transfer units - mass transfer

$NTU_t = \frac{h\ A}{\dot{m}_f c_{p,f}}$ number of transfer units - heat transfer

T_j is the rotation time for one period, which is one half of the time needed for one complete matrix rotation if the two air flow rates are equal, as it is the case in this study.

Γ_j is the capacitance rate ratio of one period and is defined as the ratio of the "matrix flowrate" to the air flow rate of the period j:

$$\Gamma_j = \frac{M_m}{T \cdot \dot{m}_{f,j}}$$

However, an analytical solution for this set of coupled partial differential equations is not possible. Numerical solutions to this problem are available but they require significant computational effort. For this reason the model is restricted to estimations of instantaneous energy recoveries and not suited for the transient simulations that have to be executed in this study.

4.2 Equilibrium Theory

A solution to the combined heat and mass transfer problem has been developed by Van den Bulck et al. [36] based on the equilibrium theory for regenerative heat and mass exchangers. The equilibrium theory assumes infinite overall transfer coefficients for heat and mass exchange, resulting in a complete thermodynamic equilibrium between the matrix and the air at all times and positions. This assumption leads to a reduced set of only two differential equations.

1. Mass conservation:

$$\frac{\partial w_f}{\partial z} + \Gamma_j \beta_j \frac{\partial W_m}{\partial \Phi} = 0 \qquad (4.5)$$

2. Energy conservation:

$$\frac{\partial i_f}{\partial z} + \Gamma_j \beta_j \frac{\partial I_m}{\partial \Phi} = 0 \qquad (4.6)$$

where β_j is the time fraction of period j: $\beta_j = \frac{T_j}{T}$

and Φ is the dimensionless time coordinate: $\Phi = \frac{\Theta}{T}$

The two conservation equations (4.5) and (4.6) are still coupled due to the thermodynamic state property relations:

$$i_f = i_f\,(T_f,\ w_f) \tag{4.7}$$

$$I_m = I_m\,(T_f,\ W_m) \tag{4.8}$$

$$W_m = W_m\,(T_f,\ w_f) \tag{4.9}$$

but they can be expressed as a pair of uncoupled kinematic wave equations:

$$\lambda_1 \frac{\partial F_1}{\partial z} + \frac{\partial F_1}{\partial \tau} = 0 \tag{4.10}$$

$$\lambda_2 \frac{\partial F_2}{\partial z} + \frac{\partial F_2}{\partial \tau} = 0 \tag{4.11}$$

F_1 and F_2 are called combined potentials of heat and mass transfer and they are similar to lines of constant enthalpy and constant relative humidity in a psychrometric chart, respectively. The F-potentials are defined by the differential equation:

$$dF_i = a_1 a_3\, dT_f + (a_3 - a_5 \lambda_i) a_2\, dw_f \tag{4.12}$$

λ_1 and λ_2 are the dimensionless wave speeds at that the potentials move in flow direction through the matrix. They can be computed as the roots of the following quadratic equation, referred to as the equation of the direction of the characteristics:

$$a_2 a_5 \lambda^2 + (a_1 a_4 - a_2 a_3 - a_5)\lambda + a_3 = 0 \tag{4.13}$$

The coefficients a_i are partial derivatives of the state properties and are defined in the following way (Van den Bulck [36]):

$$a_1 = \left(\frac{\partial W_m}{\partial T_m} \right)_{w_f} \tag{4.14}$$

$$a_2 = \left(\frac{\partial W_m}{\partial w_m} \right)_{T_f} \tag{4.15}$$

$$a_3 = \left(\frac{\partial i_f}{\partial T_f} \right)_{w_f} \tag{4.16}$$

$$a_4 = \left(\frac{\partial i_f}{\partial w_f} \right)_{T_f} - \left(\frac{\partial I_m}{\partial W_m} \right)_{T_f} \tag{4.17}$$

$$a_5 = \left(\frac{\partial I_m}{\partial T_m} \right)_{W_m} \tag{4.18}$$

The two wave speeds are always positive and, for the investigated air-water system, the wave speed λ_1 of the constant enthalpy potential F_1 is always greater than the speed of the F_2-potential. Generally, λ_1 exceeds λ_2 by the factor 100 to 1000.

However, it can be shown (see Klein [21]) that Equation 4.12 is not a total differential, which means that the result of this equation depends on the path of integration. The expression can be transformed into a total differential if an integration factor $G = G(T_f, w_f)$ is found. Due to the difficulties in evaluating the integration factor G, this set of differential equation is also not used to calculate the enthalpy exchanger outlet states in this study, but the definitions of the F-potentials and their wave speeds λ_i will be used in the following models.

4.3 Simplified Solution for the Case of Maximum Enthalpy Exchange

Klein et al. [21] studied the performance of enthalpy exchangers and stated that, in order to achieve maximum enthalpy exchange, the regenerator has to be operated at conditions such that neither of the two transfer waves reaches the outlet of the matrix. After comparing the theory of equilibrium exchange systems with the numerical solutions of the coupled equations for finite transfer coefficients, he claimed that the ratio of the dimensionless capacitance rate ratio to the fastest dimensionless wave speed Γ/λ_1 has to be greater than 1.5 in order to operate the enthalpy exchanger at a point where the enthalpy exchange effectiveness is determined only by the number of transfer units.

If the matrix rotation speed is high enough, the mass and heat transfer effectivenesses that are defined in terms of humidity and temperature differences:

$$\varepsilon_w = \frac{w_{f,j,out} - w_{f,j,in}}{w_{f,(3-j),in} - w_{f,j,in}} \tag{4.19}$$

$$\varepsilon_T = \frac{T_{f,j,out} - T_{f,j,in}}{T_{f,(3-j),in} - T_{f,j,in}} \tag{4.20}$$

can be calculated in terms of the number of transfer units only. For a counterflow exchanger and equal capacitance rates the effectivenesses become (see Incropera and DeWitt [20]):

$$\varepsilon_w = \frac{NTU_{o,w}}{1 + NTU_{o,w}} \tag{4.21}$$

$$\varepsilon_T = \frac{NTU_{o,T}}{1 + NTU_{o,T}} \tag{4.22}$$

$$\text{where} \qquad NTU_{o,w} = \frac{NTU_w}{2} \tag{4.23}$$

$$\text{and} \qquad NTU_{o,T} = Le \cdot NTU_{o,w} \tag{4.24}$$

Once the number of transfer units for the heat and mass exchange are known for a specified enthalpy exchanger matrix that is rotated at a high enough speed, it is a simple matter to calculate the humidities and temperatures of the two outlet states for fixed inlet conditions using Equations 4.19 and 4.20.

Generally it is desirable to operate an enthalpy exchanger at a rotation speed that results in the maximum possible enthalpy transfer between the two air streams, since such an operation allows the maximum possible energy recovery from the exhaust air stream, thus the greatest energy savings. For this reason Klein's model for the case of maximum enthalpy exchange is appropriate for simulating the performance of regenerators, and it is used in this study to describe the enthalpy exchanger operation in the cooling mode, where temperatures are well above freezing and intermediate to low relative humidities are typically observed.

However, if the enthalpy exchanger is operating in the heating mode, two situations can occur that result in a necessity to lower the matrix rotation speed. The first situation is that relatively mild outdoor temperatures, just slightly below the point where no heating is needed at all, can result in a supply air outlet temperature that is too warm and has to be cooled before it can be ventilated into the heating zone. In this case the enthalpy exchanger and the additional cooling system would be working against each other.

The regenerator outlet temperature can be controlled by lowering the rotation speed which decreases the regenerator effectiveness, thereby avoiding the need for additional cooling.

The second situation occurs when the outdoor temperature is very cold and the indoor humidity high. In this case, the exhaust air stream might be cooled below the saturation temperature and condensate can form on the matrix. For temperatures below freezing, the condensate will freeze and eventually block the matrix. Even without freezing, the condensation should be avoided because the liquid phase could do harm to the water based polymer coating and because it is undesirable that excess water leaks out of the ventilation system at an unpredictable location. The problem of condensation is even more serious for sensible heat exchangers, and in both cases the problem can be solved if the temperature effectiveness is decreased by lowering the matrix rotation speed.

However, for a decreased effectiveness Klein's model for the case of maximum enthalpy exchange is obviously not valid anymore and a new model that provides estimates of the enthalpy exchanger outlet states as a function of the rotation speed has to be developed.

4.4 Simplified Model for Intermediate and High Rotation Speeds

The original set of partial differential equations (Equations 4.1 to 4.4) was solved numerically for many different operating conditions with the computer program MOSHMX [25], and a function of three independent variables was fit to the results for the regenerator effectivenesses. Only two of the three regenerator effectivenesses for temperature, humidity and enthalpy are independent, and it was chosen to model the effectivenesses for temperature and enthalpy since they can be described by simpler equations than the humidity effectiveness which is negative for a dehumidifying operation at very slow rotation speeds.

The independent variables in the functions for the effectivenesses are the supply inlet temperature (outdoor temperature) T_{si}, the number of transfer units between air and matrix NTU and the ratio of "matrix flow rate" to air flow rate Γ which is proportional to the rotation speed for a constant air mass flow rate. The effect of the temperature was investigated for a range from -15°C to +15°C, the NTUs were varied between 1 and 20 and the dimensionless rotation speed ranged from $\Gamma = 0$ to $\Gamma = 10$.

The exhaust inlet temperature (indoor temperature) was kept constant at 23°C. This assumption is reasonable since the objective of an air-conditioning or heating system is to maintain a room or building at constant comfortable conditions, and therefore the indoor temperature will always be close to the chosen value.

The Lewis number of the matrix material is assumed to be equal to one throughout this study. This assumption represents the ideal case and this fact has to be taken into account when the results of the simulation are discussed. If the Lewis-number is actually greater than one, the amount of energy that can be recovered by an enthalpy exchanger will be over predicted.

For high rotation speeds this model has to predict the same values that are obtained by the model for the case of maximum enthalpy exchange, and therefore an equation of the following form was chosen for both temperature and enthalpy effectiveness:

$$\varepsilon = \frac{NTU}{NTU+2}\left(1-\exp[f(\Gamma)]\right) \tag{4.25}$$

The parameter NTU used in this equation represents the number of transfer units between air and matrix, whereas the parameter NTU_O in Klein's model (Equations 4.21 to 4.24) is the overall number of transfer units between the two air streams. Since NTU is equal to two times NTU_O,

$$\frac{NTU}{NTU+2} = \frac{NTU_o}{NTU_o+1} \tag{4.26}$$

and therefore this model produces the same output for high rotation speeds as Klein's model.

The exact functions that were found for temperature and enthalpy effectivenesses are listed below.

Temperature Effectiveness:

$$\varepsilon_T = \frac{NTU}{NTU+2}\left(1-\exp\left[a_T\Gamma^2 + b_T\Gamma\right]\right) \tag{4.27}$$

$$a_T = a_{T_1} + \frac{a_{T_2}}{NTU^{a_{T_3}}}$$

$$a_{T_1} = 0.002259 - 1.376\times10^{-3}\cdot T_{amb} - 6.91\times10^{-5}\cdot T_{amb}^2$$

$$a_{T_2} = 0.09084 - 3.263\times10^{-4}\cdot T_{amb} + 7.4\times10^{-5}\cdot T_{amb}^2$$

$$a_{T_3} = 0.7388 - 0.01994\cdot T_{amb} - 3.829\times10^{-4}\cdot T_{amb}^2$$

$$b_T = b_{T_1} + \frac{b_{T_2}}{NTU^{b_{T_3}}}$$

$$b_{T_1} = -1.007 + 0.0093\cdot T_{amb} + 2.778\times10^{-4}\cdot T_{amb}^2$$

$$b_{T_2} = -1.533 + 0.02287\cdot T_{amb} - 2.356\times10^{-4}\cdot T_{amb}^2$$

$$b_{T_3} = 1.111 - 2.667\times10^{-3}\cdot T_{amb} + 1.378\times10^{-4}\cdot T_{amb}^2$$

Enthalpy Effectiveness:

$$\varepsilon_i = \frac{NTU}{NTU+2}\left(1-\exp\left[a_i\Gamma^3 + b_i\Gamma^2 + c_i\Gamma\right]\right) \tag{4.28}$$

$$a_i = a_{i_1} + a_{i_2}NTU + a_{i_3}NTU^2$$

$$a_{i_1} = \begin{cases} 3.381\times10^{-3} - 9.679\times10^{-4}\cdot T_{amb} & for\ \ T_{amb} \le 0°C \\ 3.381\times10^{-3} - 4.127\times10^{-5}\cdot T_{amb} & for\ \ T_{amb} > 0°C \end{cases}$$

$$a_{i_2} = 5.088\times10^{-2} + 4.89\times10^{-5}\cdot T_{amb}$$

$$a_{i_3} = -5.298\times10^{-6} - 7.652\times10^{-7}\cdot T_{amb}$$

$$b_i = b_{i_1} + b_{i_2}NTU + b_{i_3}NTU^2$$

$$b_{i_1} = 6.237\times10^{-2} + 8.827\times10^{-3}\cdot T_{amb} - 6.042\times10^{-4}\cdot T_{amb}^2$$

$$b_{i_2} = -0.02123 + 1.323\times10^{-4}\cdot T_{amb}$$

$$b_{i_3} = 4.908\times10^{-4} + 6.46\times10^{-6}\cdot T_{amb}$$

$$c_i = c_{i_1} + \frac{c_{i_2}}{NTU^{0.8}}$$

$$c_{i_1} = -0.4087 + 0.00253 \cdot T_{amb} + 3.34 \times 10^{-4} \cdot T_{amb}^2$$

$$c_{i_2} = -1.449 + 0.02337 \cdot T_{amb} - 5.578 \times 10^{-4} \cdot T_{amb}^2$$

In order to be able to assess the accuracy of the developed model, the results of the curve fit were compared with the numerical results obtained with MOSHMX [25] for operating conditions different than the ones used for the curve fitting. Figure 4.1 shows the temperature effectiveness as a function of various *NTUs* and capacitance rate ratios Γ for an intermediate supply air inlet temperature of 0°C.

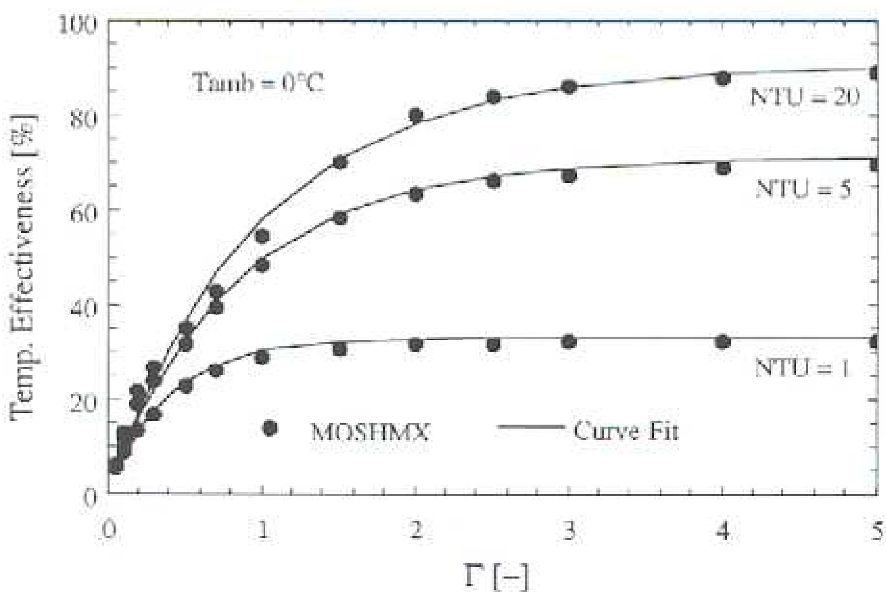

Figure 4.1: Temperature Effectiveness - MOSHMX Output and Curve Fit

Figure 4.2 shows the actual outlet states of the two air streams that are passed through an enthalpy exchanger with $NTU = 5$ for an outdoor temperature of 0°C, an indoor temperature of 23°C and typical humidities. The outlet state of the exhaust air stream that is calculated with the new model as a function of the matrix rotation speed is also shown in the graph.

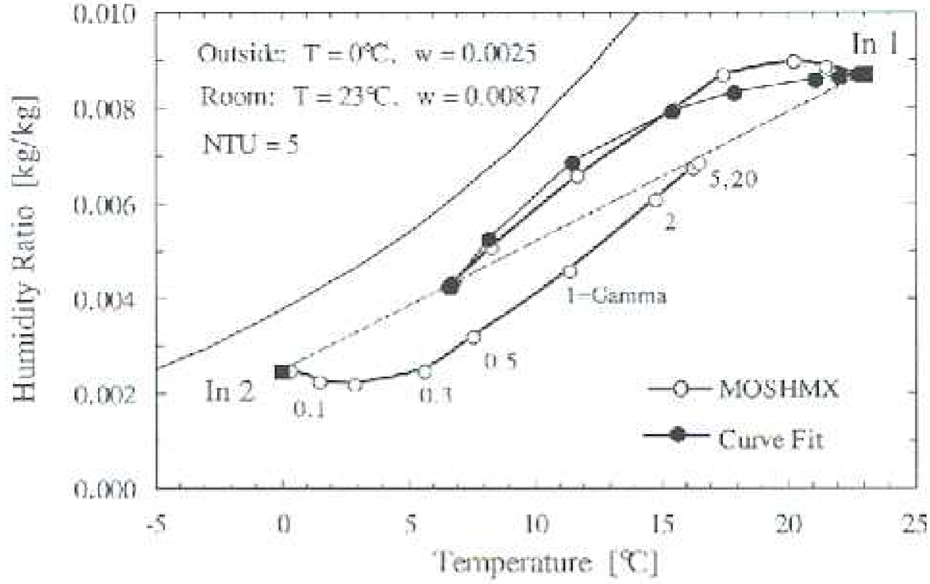

Figure 4.2: EX Outlet States for Various Rotation Speeds - MOSHMX Output and Curve Fit

The two figures show that the new model represents the actual performance of an enthalpy exchanger reasonably well in the range of parameters appropriate for the enthalpy exchanger operation in a space-conditioning system. For fast rotation speeds, the model predicts exactly the same values that are obtained by both the numerical solution of the original differential equations and Klein's model for the case of maximum enthalpy exchange. If the regenerator is operated at intermediate rotation speeds, i.e. $\Gamma > 0.4$, the outlet states calculated by the new model will also be close to the numerical solution. It can be seen that the temperature transfer exceeds the humidity transfer for these operating conditions. However, at very low matrix rotation speeds ($\Gamma < 0.4$) the model differs from the actual behavior of the regenerator (determined by MOSHMX [25]) and the dehumidifying section is not well reproduced. Therefore this model should not be used to simulate the performance of rotary dehumidifiers. It should also be mentioned, that this model is only valid for the range of parameters specified previously in this section and the matrix material investigated in this study. For a different material the form of Equations 4.27 and 4.28 is most likely the same, but the parameters a, b and c will be different.