High energy density plasma simulations using ultracold neutral plasmas

Jimfest 2018, Madison, Wisconsin, April 20, 2018
Plasma physics has an impressive history at UW

- Pegasus Toroidal Experiment
- Helically Symmetric Experiment
- Plasma theory and computation

Madison Symmetric Taurus

...as well as...

- Plasma-aided manufacturing
- Magnetic self-organization
- Fusion technology institute
- Advanced fuels project
- Fluctuation diagnostics development
- Plasma-surface interactions
- Rotating wall machine
- Shock tube laboratory
Z-pinch
Z-pincho

Tens of Millions of Amps
Tens of Millions of Volts
350 TW emitted x-ray power
National Ignition Facility
192 Laser Beams: 4 million Joules
1400 kg
19 meters
260,000 J
1400 kg
19 meters
260,000 J
Focused to a tiny spot
Making a tiny H-bomb
National ignition campaign

- All-out effort in 2009-2012 to achieve burning plasma at NIF
National ignition campaign

- All-out effort in 2009-2012 to achieve burning plasma at NIF
- Fusion has been achieved
- Gain and complete burn of the fuel has not
  - Instabilities
  - Diagnostics

“Ignition on the NIF is a grand challenge undertaking”
What can laser-cooled atoms tell you about all this?
A hierarchy of length scales

Debye length

\[ \lambda_D = \sqrt{\frac{e_0 k_B T_e}{n e^2}} \]
A hierarchy of length scales

Debye length
\[ \lambda_D = \sqrt{\frac{e_0 k_B T_e}{n e^2}} \]

Wigner-Seitz radius
\[ a_{WS} = \left( \frac{3}{4\pi n} \right)^{1/3} \]
A hierarchy of length scales

Debye length

$$\lambda_D = \sqrt{\frac{e_0 k_B T_e}{ne^2}}$$

Wigner-Seitz radius

$$a_{WS} = \left(\frac{3}{4\pi n}\right)^{1/3}$$

Closest approach

$$b = \frac{e^2}{4\pi \varepsilon_0} \frac{1}{k_B T}$$
A hierarchy of length scales

Debye length

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Wigner-Seitz radius

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Closest approach

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debye length</td>
<td>20000 nm</td>
</tr>
<tr>
<td>Wigner-Seitz radius</td>
<td>300 nm</td>
</tr>
<tr>
<td>Closest approach</td>
<td>0.1 nm</td>
</tr>
<tr>
<td>Electron temperature ( T_e )</td>
<td>10 eV</td>
</tr>
<tr>
<td>Electron density ( n )</td>
<td>( 10^{12} \text{ cm}^{-3} )</td>
</tr>
</tbody>
</table>
Ratios of characteristic lengths and energies

**Energy**

Potential Energy \( = \frac{e^2}{4\pi \varepsilon_0 a_{\text{ws}}} \)

Kinetic Energy \( = k_B T \)

**Length**

\( b = \frac{e^2}{4\pi \varepsilon_0} \frac{1}{k_B T} \)

\( a_{\text{ws}} = \left( \frac{3}{4\pi n} \right)^{1/3} \)

\[
\frac{\text{Potential Energy}}{\text{Kinetic Energy}} = \frac{e^2}{4\pi \varepsilon_0 a_{\text{ws}}} \frac{1}{k_B T} = \frac{b}{a_{\text{ws}}} = \frac{1}{3} \left( \frac{a_{\text{ws}}}{\lambda_D} \right)^2 \equiv \Gamma
\]
Thermodynamic properties scale with \[ \Gamma \propto \frac{n^{1/3}}{T} \]
Rutherford scattering

\[ b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 m v_0^2} \cot(\theta / 2) \]
Rutherford scattering

- We have the idea of "cross section"
  - An effective size of the scatterer
  - Allows us to write collision rates as
    \[ \gamma_{ee} = n\sigma v \]
  - Allows us to think about things like mean free path, scattering probability, etc.

\[ b = \frac{Z_1 Z_2 e^2}{4\pi \varepsilon_0 m v_0^2} \cot(\theta/2) \]
Rutherford scattering

- The Coulomb cross section diverges

\[
\frac{d\sigma}{d\Omega} = \left( \frac{Z_1 Z_2 e^2}{8\pi \epsilon_0 m v_0} \right)^2 \frac{1}{\sin^2(\theta/2)}
\]

\[
\sigma = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega} \rightarrow \infty
\]

\[
\sigma = 2\pi \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{d\sigma}{d\Omega}
\]

\[
b = \frac{Z_1 Z_2 e^2}{4\pi \epsilon_0 m v_0^2} \cot(\theta/2)
\]
Rutherford scattering

- And there are other particles to worry about

\[ \gamma_{ee} = n \sigma \nu \Lambda \]
Thermodynamic properties scale with \[ \Gamma \propto \frac{n^{1/3}}{T} \]

\[ \gamma_{ee} = n \sigma v \Lambda \]

\[ \Lambda = \frac{1}{2} \ln \left[ 1 + \left( \frac{\lambda_D}{b} \right)^2 \right] \]
Ultracold neutral plasmas

PRL 95, 235001 (2005)

Neutral calcium

\[ \text{4s}^2\text{1S}_0 \ 0.00 \text{ cm}^{-1} \]
\[ \text{4s4p} \ 1\text{P}_1 \ 23652 \text{ cm}^{-1} \]
\[ \text{4s3d} \ 1\text{D}_2 \ 21854 \text{ cm}^{-1} \]
\[ \text{4s3d} \ 3\text{D} \]
\[ \text{4s4p} \ 3\text{P} \]

423 nm

Neutral ytterbium

\[ \text{6s}^2\text{1S}_0 \ 0.00 \text{ cm}^{-1} \]
\[ \text{6s6p} \ 1\text{P}_1 \ 25068 \text{ cm}^{-1} \]
\[ \text{6s5d} \ 3\text{D} \]
\[ \text{6s6p} \ 3\text{P} \]

399 nm
Ultracold neutral plasmas

PRL 95, 235001 (2005)

Ionization limit 49306 cm$^{-1}$

4s$^2$ $^1$S$^0$ 0.00 cm$^{-1}$

4s$^2$3d $^1$D$_2$ 21854 cm$^{-1}$

4s$^2$3d $^3$D

4s$^2$3d $^1$P$_1$ 23652 cm$^{-1}$

4s$^2$3d $^3$P

4s$^2$ $^3$P

4s$^2$ $^1$S$^0$ 0.00 cm$^{-1}$

Ionization limit 50443 cm$^{-1}$

6s$^2$ $^1$S$^0$ 0.00 cm$^{-1}$

6s6p $^3$P$_1$ 25068 cm$^{-1}$

6s6p $^3$P

6s6p $^1$P$_1$ 25068 cm$^{-1}$

6s6p $^3$D

6s6p $^3$D

6s5d $^3$D

Neutral calcium

Neutral ytterbium
Our laser-cooling lab
10 million atoms, 1 mm diameter trap
Laser-induced fluorescence

- Trap neutral atoms in the MOT
- Resonantly photo-ionize the atoms to create a plasma
- Measure ion fluorescence vs. time
- Use CW laser beam
- Detune from resonance
Laser-induced fluorescence

- Trap neutral atoms in the MOT
- Resonantly photo-ionize the atoms to create a plasma
- Measure ion fluorescence vs. time
- Use CW laser beam
- Detune from resonance
- Analyze horizontal cut in data to find $T_i(t)$
The rms ion velocity

\[ v_{i,\text{rms}} = \sqrt{\frac{k_B}{m_i} \left[ T_i(0) + T_e(0) \frac{t^2}{t^2 + \tau^2} \right]} \]

AC Stark broadening
Disorder-induced heating
Plasma expansion

M. Lyon, S. D. Bergeson, and M. S. Murillo
The rms ion velocity

\[ v_{i,\text{rms}} = \sqrt{\frac{k_B}{m_i} \left[ T_i(0) + T_e(0) \frac{t^2}{t^2 + \tau^2} \right]} \]

- Atoms are initially randomly located in space
- Essentially zero atom-atom interaction
- Photoionization impulsively “hardens” the potential energy landscape
- Ions move to minimize their potential energy
We are measuring energy transfer in laser cooled atoms

• Collisional energy transfer between two different ions
• Trap Yb and Ca atoms in a MOT
• Photo-ionize at resonance
• Measure how each species influences the other
We are measuring energy transfer in laser cooled atoms

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**Diagram:**
- Yb\(^+\) plasma
- Yb\(^+\) fluorescence
- Ca\(^+\) plasma
- Ca\(^+\) fluorescence
Yb⁺ plasma expansion
$\text{Yb}^+$ plasma expansion with $\text{Ca}^+$ at 3 $\mu$s
We are measuring energy transfer in laser cooled atoms

• Collisional energy transfer between two different ions
• Trap Yb and Ca atoms in a MOT
• Photo-ionize at resonance
• Measure how each species influences the other
We are measuring this energy transfer in laser cooled atoms

- Ca+ plasma alone is described “perfectly” by the Vlasov equation
- Adding Yb+ ions significantly changes the Ca+ velocity distribution
1D1V PIC calculation

- Start from the Navier-Stokes equation, assume iso-thermal electrons
- Acceleration
  \[ a_i(r, t) = -\frac{k_B T_e(t)}{m_i} \frac{\nabla n_t(r, t)}{n_t(r, t)}, \]
- Ion Friction
  \[ F_{12} = -\mu n_2 \pi \left(\frac{e^2}{2\pi \varepsilon_0 \mu}\right)^2 \frac{(v_1 - \langle v_2 \rangle)}{|v_1 - \langle v_2 \rangle|^3} \Lambda, \]
The Coulomb logarithm in the strongly coupled regime

• The Rutherford scattering cross-section gives:

\[ \Lambda = \frac{1}{2} \ln \left[ 1 + \left( \frac{\lambda_D}{b} \right)^2 \right] \]

• In terms of the strong-coupling parameter and the scaled screening length, this can be written as

\[ \Lambda = \frac{1}{2} \ln \left( 1 + \frac{1 + \kappa \Gamma}{\kappa^2 \Gamma^2 + 3 \Gamma^3} \right) = 0.04 \]
Comparing the measured and computed Ca$^+$ velocity distributions
Summary

• Ultracold neutral plasmas probe an interesting region of Gamma-kappa-Lambda space

• Streaming, stopping power, thermal relaxation, etc., are all available to us and can be measured with high precision.

• I learned many of these tools in Jim’s lab
  • Experimental design
  • Computer/equipment interface
  • Laser-induced fluorescence
  • Dye laser/amplifier design
  • Data processing
  • Persistence and patience