The economic analysis of the UTC system is performed using the $P_1$, $P_2$ method of life cycle savings [Duffie and Beckman, 1991]. The optimal collector area is found by maximizing the life cycle savings.

### 5.1 $P_1$, $P_2$ Method of Life Cycle Savings

The total life cycle savings of a UTC system can be calculated by the $P_1$, $P_2$ method [Duffie and Beckman, 1991].

$$LCS = P_1 \, C_F \, F \, Q_{trad} - P_2 \, (C_E + C_A \, A) \tag{5.1.1}$$

$C_F$ is the cost of fuel, $C_E$ is the fixed equipment first cost, and $C_A$ is the equipment first cost per unit area. The auxiliary heating unit must be able to meet the entire heating load of the building, so there is no equipment cost savings on the traditional unit.

$P_1$ and $P_2$ are determined from economic parameters (e.g. interest rate, inflation rate, period of economic analysis). $P_1$ is the ratio of life cycle fuel savings to first year fuel savings. Typically, for a period of economic analysis of $N$ years, $N/2 < P_1 < N$. $P_2$ is the ratio of life cycle expenditures to initial investment. Typically, $0.5 < P_2 < 1.0$. This method simplifies the economic analysis by concentrating all of the economic parameters into the two parameters $P_1$ and $P_2$.

$F$ is the solar fraction, defined as the fraction of the traditional load that is met by the solar energy system [Duffie and Beckman, 1991].
The solar fraction is a function of the collector area, as seen for a sample UTC system in Figure 5.1.1. $Q_{\text{save}}$ cannot exceed $Q_{\text{trad}}$, as discussed in Section 2.5. Therefore, the solar fraction cannot exceed 1.0. As is the case in Figure 5.1.1, the solar fraction is often not close to 1.0 because there are cold periods during the year for which the useful energy from the UTC system cannot come close to meeting the heating load of the building.

\[ F = \frac{Q_{\text{save}}}{Q_{\text{trad}}} \]

Figure 5.1.1. Solar fraction and approach velocity as a function of collector area.

The approach velocity at $\gamma_{\text{min}}$ as a function of collector area, as discussed in Section 2.8, is also shown for the sample UTC system in Figure 5.1.1. As the collector area increases above 900 m$^2$, the decreasing approach velocity causes the efficiency of the UTC system to decrease [Kutscher, 1992]. The slope of the solar fraction curve decreases due to the decreasing efficiency of the UTC system. In Figure 5.1.1, the maximum collector area is 2250 m$^2$ because the minimum approach velocity should not be below 0.02 m/s. The maximum area is determined as discussed in Section 2.6.
A UTC system is a good investment if the life cycle savings, calculated from Equation 5.1.1, is above zero. However, only the ratio $P_1/P_2$ is needed to determine whether a UTC system is a good investment. Equation 5.1.3 yields the minimum ratio $P_1/P_2$ necessary for the life cycle savings to exceed zero.

$$\frac{P_1}{P_2} = \frac{(C_E + C_A A)}{(C_F F Q_{trad})} \quad (5.1.3)$$

The ratio $P_1/P_2$ depends on economic parameters. Typically for a period of economic analysis of $N$ years, $N/2 < P_1/P_2 < 2N$. If the ratio $P_1/P_2$ is greater than the ratio calculated from Equation 5.1.3, then a UTC system is a good investment.

For those readers not familiar with the $P_1$, $P_2$ method of life cycle savings, Equation 5.1.3 is also the simple payback period, defined as the amount of time to earn back the first cost of a system in fuel savings.

### 5.2 Collector Area Optimization

For an economic analysis of a UTC system on a specific building, the life cycle savings of the UTC system is a function of collector area, as shown in Figure 5.2.1. As the collector area becomes large, the life cycle savings decreases because the energy savings approaches a limit, but there is no limit on the collector first cost. The y-intercept in Figure 5.2.1 is $(-P_2 C_E)$. The life cycle savings curve is fairly linear up to 900 m$^2$ because the solar fraction curve is fairly linear in Figure 5.1.1 for a constant approach velocity.

The maximum life cycle savings occurs where the slope in Figure 5.2.1 is zero.

$$0 = \frac{d(LCS)}{dA}$$

$$= P_1 C_F Q_{trad} \left(\frac{dF}{dA}\right) - P_2 C_A \quad (5.2.1)$$

Rearranging Equation 5.2.1 yields Equation 5.2.2.

$$\frac{dF}{dA} = \frac{(P_2 C_A)}{(P_1 C_F Q_{trad})} \quad (5.2.2)$$

d$F$/d$A$ is the slope in Figure 5.1.1. For an economic analysis of a UTC system on a specific building, $C_A$, $C_F$, and $Q_{trad}$ are known. For a given ratio $P_1/P_2$, $dF/dA$ is calculated, and the
optimal collector area is found from Figure 5.1.1. The magnitudes of \( P_1 \) and \( P_2 \) affect the magnitude of the life cycle savings, but only their ratio affects the optimal collector area which maximizes the life cycle savings.

![Graph showing life cycle savings for \( P_1/P_2 = 4.0 \).]

Figure 5.2.1. Life cycle savings for \( P_1/P_2 = 4.0 \). In this case, a UTC system is not a good investment.
If the ratio $P_1/P_2$ is increased, the slope $dF/dA$ at the economic optimum decreases, and the optimal collector area increases. As seen in Figure 5.2.3, the optimal collector area may be above the maximum collector area. The maximum area is used in this case.

Figure 5.2.2. Life cycle savings for $P_1/P_2 = 3.0$.

Figure 5.2.3. Life cycle savings for $P_1/P_2 = 6.0$. 

To summarize, a procedure for optimizing the collector area is outlined. Economic parameters are estimated to find the ratio $P_1/P_2$. If this ratio is greater than the ratio calculated from Equation 5.1.3, the life cycle savings is positive, and a UTC system is a good investment. The slope $dF/dA$ is calculated for the optimal collector area from Equation 5.2.2. Using the UTC system model, a plot of solar fraction as a function of the collector area is generated. The optimal collector area is found from the slope of this plot. If the optimal area is above the maximum area, the maximum area is used. Optimizing the collector area in this way maximizes the life cycle savings of the UTC system.