The idea of internal time steps has been alluded to briefly in previous chapters. This chapter provides a complete explanation of why the internal time step is needed, how it is computed, and how its size affects the accuracy of the solution.

4.1 The Need for Internal Time Steps

The size of the TRNSYS time step is set by the user. An experienced TRNSYS user knows that when a simulation is having difficulty converging on a solution, one of the first components to examine for problems is the tank. The old tank model was typically the most sensitive TRNSYS component to time step size. To ensure the old tank was giving correct results, users would choose time steps on the order of 10-30 minutes, whereas most simulations are more conveniently run at one-hour time steps.

The reason for the problem is that typical tank phenomena happen more quickly than a typical TRNSYS time step. A conventional household water heater may contain two 4.5 kW electric heaters. When the electric heaters are on at full power, it takes much less than one hour for the tank to heat up to its set point. When a tank requires auxiliary heat, there are two ways in TRNSYS to prevent the temperature from exceeding the set point. One method is to use a
fraction of the power for the entire TRNSYS time step. This method of control is referred to as energy rate control. The other method is to use full power for a fraction of the TRNSYS time step and determine when the temperature reaches the set point. This method of control is referred to as temperature level control and requires smaller, internal time steps within the tank component subroutine. The difference between internal time step heaters and energy rate control heaters are discussed in section 4.2. Section 4.1 provides justification for the use of internal time steps, which is part of the new tank model.

4.1.1 Critical Euler Time Step

In section 3.6, results to the solution methods were presented at three different time steps, and the time steps were expressed as some fraction of the critical Euler time step. Without regard to the solution method used, the larger the time step, the more inaccurate the solution. Consider the typical solar storage tank shown in figure 4.1.1.

\[ \Delta t_{\text{crit}} = -\left( \frac{-1}{\frac{0.08333}{30 \text{ kg}}} \right) = 360 \text{s} = 6 \text{ min} \]

\[ n\dot{V} = 300 \text{ kg/hr} = 0.08333 \text{ kg/s} \]

Conduction, losses, etc. can be ignored because their effects are very small compared to the axial flow contribution. Using equation 3.6.2, the critical Euler time step is:

\[ V_{\text{tank}} = 300 \text{ L} \]

\[ N = 10 \text{ Nodes} \]

\[ \rho = 1000 \text{ kg/m}^3 \]

\[ \dot{m} = 300 \text{ kg/hr} \]

\[ \dot{m} = 0.08333 \text{ kg/s} \]
If the user decides that largest time step the model should use is $1/6$ the critical value, the time step size in this example would be 1 minute. The tank in figure 4.1.1 was put into TRNSYS and run with both the new and old models. Figure 4.1.2 shows the results of the simulation when a TRNSYS time step of 1 minute is used.

![Graph showing model comparison](image)

**Figure 4.1.2** Model comparison using 1/6 critical time step

The old model uses the DIFFEQ technique and the new model uses the Crank-Nicolson technique. (Refer to chapter 3). When the models are run at 1 minute time steps, the only noticeable discrepancies occur due to the difference in solution techniques, which are still almost too small to see in the figure. However, it would not be practical to perform an annual simulation using 1 minute time steps. If this same simulation is run using 1 hour time steps, (a reasonable
time step size) the differences are discernible:

![Graph showing temperature over time for New Model and Old Model.](image)

Figure 4.1.3 Model comparison using 1 hour time steps

It is important to note that the models output the time-averaged values at the end of a time step. For example, the temperature shown at hour 1 would not be the tank’s actual temperature at 1 hour, but rather the integrated-mean temperature for the hour. However, as the time step size decreases, the more closely the time-averaged output resembles the real-time output. Figure 4.1.4 depicts this difference graphically by comparing 1 hour and 5 minute time step simulations.
Figure 4.1.4 is an example simulation and not the same simulation shown in figures 4.1.2 and 4.1.3.

Referring back to figures 4.1.2 and 4.1.3, take as an example the temperatures between hours 2 and 3. Figure 4.1.2, with 1 minute time steps, shows the real-time temperature profile of the node. As can be seen in figure 4.1.2, the temperature over the hour is very close to 15°C, hence, the time-averaged temperature should also be 15°C. However, from figure 4.1.3, one can easily see the problems incurred when internal time steps are not used. The old model reports a time-averaged temperature of about 17.5°C, whereas it should have been about 15°C. Internal time steps prevent the errors shown in figure 4.1.3 by not allowing the model to use time steps that exceed the critical Euler time step.
4.1.2 Internal Heat Exchanger

One other justification for internal time steps occurs when the tank contains an internal heat exchanger. The heat transferred to the tank from an internal heat exchanger is calculated by:

\[ q_{hx} = U A_{hx} \cdot \text{lmtd} \]  

(4.1.2)

Where

\[ \text{lmtd} = \frac{T_{in} - T_{out}}{\ln\left(\frac{T_{in} - T_i}{T_{out} - T_i}\right)} \]  

(4.1.3)

\( T_i \) is the temperature of the node containing the heat exchanger. Figure 4.1.5 shows qualitatively how \( q_{hx} \) varies with time as the tank temperature approaches the heat exchanger fluid inlet temperature.

Both \( U A_{hx} \) and \( \text{lmtd} \) change as the tank temperature approaches the heat exchanger fluid temperature. Because \( q_{hx} \) changes with time, assuming a constant value will lead to incorrect results. (The old model assumes all parameters and inputs are constant for the entire TRNSYS
time step). The value of $q_{hx}$ must be updated within the tank subroutine, which requires internal time steps. Although it would require internal time steps of infinitesimal size to obtain the exact solution, smaller internal time steps reduce the error incurred by assuming a constant value for $q_{hx}$.

4.2 Temperature Level Control Versus Energy Rate Control Auxiliary Heat

The old model uses energy rate control for the auxiliary heaters, i.e., auxiliary heat is added by turning the heaters on for the full TRNSYS time step at some fraction of maximum power. The new model turns the heaters on at full power for some fraction of the time step. The difference between the two is small, but it does affect the solution accuracy.

The advantage to energy rate control is that the auxiliary heaters can heat the tank to the set point (without exceeding it) and use the entire TRNSYS time step to do so. This eliminates the need for internal time steps, which allows the model to run faster. The fraction of the heater's maximum power is governed by how much energy is required and how long the TRNSYS time step is. The fractional power is determined using a two-step process. During the first time through, $A$ and $d$ (see chapter 3) contain all parameters that affect the energy balance on the node except for auxiliary heat. The first time through gives a prediction for the temperature at the end of the TRNSYS time step. Knowing that, the model then knows how much auxiliary power would have been needed to make the temperature end up at the set point, which will be some fraction of the maximum power. During subsequent iterations, this fractional power from the auxiliary is placed in the energy balance with the other terms. [28] Figure 4.2.1 shows a tank node temperature profile when energy rate control is used.
The tank shown in the figure has a set point of 55 °C and a deadband of 5 °C. (Note: for clarification purposes, the temperatures shown in figures 4.2.1 - 4.2.3 are the temperature profiles at the time they occur, rather than the time-step-averaged profiles that were described in section 4.1.1). For example, between the hours 2 and 3 the model predicted the temperature would be less than 50 °C. During that time step, the power needed to bring the tank to the set point was computed and then added into the energy balance equation. The correct amount of energy was added to make the temperature end up at the set point.

However, actual auxiliary heaters in a tank do not use energy rate control. Electric and gas heaters are usually on at full power until the set point is reached. In reality, auxiliary heaters are turned on when the tank cools below the set point temperature less the deadband, not when the TRNSYS time step ends. If a such a "real" temperature profile was superimposed on the
graph in figure 4.2.1, the inaccuracies of energy rate control become evident. The "real" profile was obtained by using very small time steps.

The gray line in figure 4.2.2 follows the "real" tank temperature as heaters cycle on and off. As can be seen in the figure, the "real" profile is not governed by the size of the time steps being used in the simulation.

When smaller, internal time steps are used, the model can allow the heaters to be on at full power for a fraction of the TRNSYS time step. This allows the model to follow the "real" temperature profile internally. When the internal time has reached the end of a TRNSYS time step, the model outputs the temperature. Figure 4.2.3 shows the temperature profile when internal time steps are used.
It is clear that using small, internal time steps provides a more accurate solution than using energy rate control. Internal time steps (temperature level control) is the method used by the new tank model. Internal time steps allow the simulation to more closely model reality. However, with the use of internal time steps come some additional programming considerations. Section 4.3 describes how the model chooses the size of the internal time step, and section 4.4 shows how time step size affects solution accuracy.

4.3 Choosing the Time Step Size

The model takes the largest time steps possible to make the simulation run as quickly as possible. However, the internal time step size is limited by three possibilities and can not exceed the smallest of:
• Time until the end of the TRNSYS time step
• Fraction of the critical Euler time step entered by user
• Time until an auxiliary heater must turn on or off

The first item listed is calculated as:

$$\Delta t = \text{time at end of TRNSYS timestep} - \text{internal time}$$  \hspace{1cm} (4.3.1)

The other two items on the list require slightly more complex calculations. They are described in the sections below.

4.3.1 Critical Euler Time Calculation

Recall equation 3.6.2 for the critical time step:

$$\Delta t_{\text{crit}} = \text{minimum} \left( -\frac{1}{b_i} \right)_{i=1,n}$$  \hspace{1cm} (3.6.2)

Where $b_i$ is the constant coefficient on $T_i$. (see chapter 3). If the tank does not have any internal heat exchangers, computing the critical time step is quite simple. However, when an internal heat exchanger is present, the calculations become more complicated. Recall from section 4.1 the equation for the log-mean temperature difference of the internal heat exchanger:

$$\text{lmtd} = \frac{T_{\text{in}} - T_{\text{out}}}{\ln \left( \frac{T_{\text{in}} - T_i}{T_{\text{out}} - T_i} \right)}$$  \hspace{1cm} (4.1.3)

Because it is not algebraically possible to isolate $T_i$ in equation 4.1.3, the energy from the internal heat exchanger must be modeled as direct energy input ($q_{h_x}$ is part of the $d_i$ constants in equation 3.1.3). When $q_{h_x}$ is not included in the $b_i$ constants, the effects of the heat exchanger
are not included in the critical time step calculation.

To include the effects of the heat exchanger, assume the energy balance equation (equation 3.1.1) was written in the form of equation 4.3.2

\[
M_i C_p \left( \frac{dT_i}{dt} \right) = \frac{(k + \Delta k) A_c}{\Delta x_{i+1 \rightarrow i}} (T_{i+1} - T_i)
\]

\[
+ \frac{(k + \Delta k) A_c}{\Delta x_{i-1 \rightarrow i}} (T_{i-1} - T_i)
\]

\[
+ (U + \Delta U) A_s (T_{env} - T_i) + U_{flue} (T_{flue} - T_i)
\]

\[
+ \gamma_1 \dot{n}_{down} C_p (T_{i-1}) - \gamma_2 \dot{n}_{up} C_p (T_i)
\]

\[
- \gamma_3 \dot{n}_{down} C_p (T_i) + \gamma_4 \dot{n}_{up} C_p (T_{i+1}) + \gamma_5 Q_{aux1}
\]

\[
+ \gamma_6 Q_{aux2} + U_{hx} \left( lmtd(T_{in} - T_i)(T_{in} - T_i) \right)
\]

\[
+ n_{in} C_p T_{1in} - n_{1out} C_p T_i + n_{2in} C_p T_{2in}
\]

\[
- n_{2out} C_p T_i
\]

Equation 4.3.2 is algebraically equivalent to equation 3.1.1, but note that the heat exchanger term has been re-written. The heat exchanger term in equation 4.3.2 can be re-arranged as shown in equation 4.3.3.

\[
\left( \frac{dT_i}{dt} \right) = \ldots + \left( \frac{U_{hx} lmtd}{M_i C_p (T_{in} - T_i)} \right) T_{in} - \left( \frac{U_{hx} lmtd}{M_i C_p (T_{in} - T_i)} \right) T_i \ldots
\]

By assuming the ratio of the \( lmtd \) to \( (T_{in} - T_i) \) is constant, the positive term on the right-hand side of equation 4.3.3 is a constant, and is placed in the \( d \) array with the other constant terms (see section 3.1). The negative term in equation 4.3.3 is a constant coefficient on \( T_i \), and can be placed in with the other \( b_i \) terms in the matrix \( A \).
However, the bracketed terms in equation 4.3.3 are not really constant. The UA value of the heat exchanger does vary slightly with time due to changing fluid properties. The ratio of the \( l mtd \) to \( (T_{in} - T_i) \) changes slightly also. Rather than using the terms in equation 4.3.3 as part of the general energy balance, they are used only for time step calculations.

It was discussed briefly in section 4.1.2 how internal heat exchanger parameters need to be updated several times within the TRNSYS time step. The negative term in equation 4.3.3 is computed within the heat exchanger subroutine. The heat exchanger subroutine returns this correction value every TRNSYS time step.

\[
h_{x_{corr}} = \left( -\frac{UA_{hx} \ l mtd}{M_i \ C_p (T_{in} - T_i)} \right) \tag{4.3.4}
\]

Finally, the correct time step (fraction of the critical Euler) can be computed using equation 4.3.5:

\[
\Delta t = \min\left( \frac{-1}{b_i + h_{x_{corr}}} \right)_{i=1,n} \times \frac{1}{\text{Crit. Fraction}} \tag{4.3.5}
\]

As the tank temperature approaches the heat exchanger fluid temperature, the value of \( h_{x_{corr}} \) decreases (becomes more negative), thereby increasing the critical time step. Therefore, adjusting the time step size once every TRNSYS time step, rather than once every internal time step, causes no detrimental effects on solution accuracy.

### 4.3.2 Auxiliary Heater Time Calculation

For the model to turn auxiliary heaters on and off as occur in reality, the temperature profile must be followed internally. Figure 4.3.2 shows a portion of the "real" profile used in figures 4.2.2 and 4.2.3:
In the figure, it can be seen that the auxiliary heater needs to come on at about 2.8 hours and then turn off at 3.3 hours. The TRNSYS time periods of 0-1, 1-2, and 4-5 hours are of little interest here because there is no need to compute an internal time step. The hours 2-3 and 3-4 require the auxiliary heaters to change their on/off status. The following explanation will use the hour 2-3 in figure 4.3.1 as an example.

The time until the heater must shut off (labeled $\Delta t_{\text{internal}}$ in figure 4.3.2) is roughly calculated using a straight line approximation as given by equation 4.3.6.
Equation 4.3.6 assumes $\Delta t_{\text{TRNSYS}}$ is the largest possible time step without auxiliary heater considerations.

Actual temperature profiles are slightly curved. Depending on the shape of the curved profile, the straight-line approximation will either underestimate or overestimate the proper time for $\Delta t_{\text{internal}}$.

If $\Delta t_{\text{internal}}$ is overestimated, $T_{\text{new},i}$ will still exceed $T_{\text{switch}}$, so the model will recognize that the time step is too large. The model will then re-calculate the time step until a value of $\Delta t_{\text{internal}}$ is found such that $T_{\text{new},i} - T_{\text{switch}}$ is within a specified margin of error. The time step
then becomes $\Delta t_{\text{internal}}$.

\[ \Delta t = \Delta t_{\text{internal}} \]  \hspace{1cm} (4.3.7)

However, if $\Delta t_{\text{internal}}$ underestimates the time, a different calculation must be performed. Figure 4.3.3 shows how the straight-line approximation can underestimate the correct time.

![Figure 4.3.3 Straight line approximation underestimating correct time](image)

If $\Delta t_{\text{internal}}$ is used, the time when the tank reaches $T_{\text{switch}}$ will be underestimated. The problem is that $\Delta t_{\text{internal}}$ did not cause an overshoot (or undershoot, as the case may be) of the temperature past the switch temperature, so $\Delta t_{\text{internal}}$ is used as the internal time step. On the
next internal time step, the straight line approximation will underestimate the time again. As this process continues, $\Delta t_{\text{internal}}$ will asymptotically approach 0, causing the model to take several thousand internal time steps to reach $T_{\text{switch}}$.

To ensure that the straight line approximation does not underestimate the correct internal time step, the model computes the tangent to the curve at $\Delta t_{\text{internal}}$ as shown in figure 4.3.3. The tangent is computed by evaluating the temperature at $\Delta t_{\text{internal}}$ and $\Delta t_{\text{internal}} + 1$ second. The tangent line is used to compute $\Delta t^2$ as shown in the figure. Adding $\Delta t^2$ to $\Delta t_{\text{internal}}$ provides an overestimated time step that will not cause problems.

### 4.4 Effect of Time Step Size on Solution Accuracy

The Crank-Nicolson solution method is unconditionally stable and any time step size can be used. However, stability does not guarantee accuracy. Table 3.6.1 presented solution results for the various methods at three different time step sizes. Table 4.4.1 presents the results of the same simulation shown in table 3.6.1 at more time step variations, and only for the Crank-Nicolson solution method. Table 4.4.1 is provided to the user to assist in choosing the critical time step fraction for either an accurate or quickly-running tank model.
The model allows the user to choose the fraction of the critical Euler value. By choosing a small time step size, an accurate solution will be obtained, but at the expense of longer computation time. Conversely, choosing a large time step size will result in a faster running simulation that is not as accurate. Figure 4.1.1 shows how time step size affects solution accuracy in an example simulation.

<table>
<thead>
<tr>
<th>Fraction of critical Euler</th>
<th>Solution obtained</th>
<th>Iterations / time step</th>
<th>Total number of steps required</th>
<th>%error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/10^7</td>
<td>8.2133</td>
<td>1.0</td>
<td>10,000,000</td>
<td>0.000</td>
</tr>
<tr>
<td>1/100</td>
<td>8.2134</td>
<td>3.0</td>
<td>300</td>
<td>0.002</td>
</tr>
<tr>
<td>1/50</td>
<td>8.2139</td>
<td>3.0</td>
<td>150</td>
<td>0.007</td>
</tr>
<tr>
<td>1/20</td>
<td>8.2172</td>
<td>4.0</td>
<td>80</td>
<td>0.048</td>
</tr>
<tr>
<td>1/10</td>
<td>8.2291</td>
<td>4.8</td>
<td>48</td>
<td>0.192</td>
</tr>
<tr>
<td>1/6</td>
<td>8.2574</td>
<td>5.5</td>
<td>33</td>
<td>0.536</td>
</tr>
<tr>
<td>1/4</td>
<td>8.3136</td>
<td>6.5</td>
<td>26</td>
<td>1.221</td>
</tr>
<tr>
<td>1/3</td>
<td>8.3939</td>
<td>7.7</td>
<td>23</td>
<td>2.199</td>
</tr>
<tr>
<td>1/2</td>
<td>8.6366</td>
<td>10.0</td>
<td>20</td>
<td>5.154</td>
</tr>
<tr>
<td>1</td>
<td>10.3999</td>
<td>21.0</td>
<td>21</td>
<td>26.622</td>
</tr>
</tbody>
</table>
Although a simulation would never use two times the critical value as a time step size, it is presented in the figure to illustrate the errors incurred with large time steps. Even using a time step equal to the critical value can produce errors in excess of 2°C for the 300 liter tank shown in the figure.