Effectiveness of Montessori Materials

The Effectiveness of Montessori Materials in Multiplication Understanding as Evidenced by Teacher Made Assessments

A Master’s Paper
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Effectiveness of Montessori Materials

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Abstract

Recent assessment results have indicated a need for strengthening multiplication understanding in students at the researcher’s school. Montessori materials have been developed to practice, implement, and master academic concepts—including math skills. Research supports the notion that a Montessori environment provides students with a strong understanding of math skills, as the learning begins with the concrete before addressing the abstract. The combination of a supportive environment, proper Montessori materials, and teacher training in the Montessori Method supported student success in multiplication mastery. It was hypothesized that students would develop mastery of multiplication with the assistance of one of these materials, the Checkerboard, which serves as a concrete representation of the problems presented.

Observations of daily lessons at a small suburban Montessori public charter school involving the checkerboard were analyzed to track student growth and knowledge. Twenty students at the fourth and fifth grade level were given lessons on multiplication using the Checkerboard as a manipulative for solving problems. Data was collected in the form of interest surveys, competence surveys from both students and parents, and both pre and post assessment scores. Developmentally appropriate materials extended improvements in the reasoning skills of all children in a classroom, including those who independently solve math problems given the Montessori environment. Post surveys and assessments showed growth in understanding, confidence and math vocabulary.

Keywords: Montessori, multiplication checkerboard, manipulatives, math
Literature Review

Introduction

Mathematic manipulatives can be found in many classrooms; however, the ones in a Montessori setting are unique. These materials are intended to teach children concrete concepts before moving to the abstract notion and have proven to be effective in this endeavor. “Children who attend Montessori programs in early childhood demonstrate high levels of mathematics achievement” (Laski, Jor’dan, Daoust, & Murray, 2015, p. 2). The research suggests that “the benefits of the Montessori approach to mathematics learning in early childhood may, at least in part, be due to its effective use of manipulatives” (Laski et al., 2015, p. 1).

“Manipulatives are concrete materials (e.g., blocks, tiles) used to demonstrate a mathematical concept or to support the execution of a mathematical procedure” (Laski, 2015 et al., p. 1). Contemporary research supports the need for an environment that includes manipulatives as envisioned by Maria Montessori. “The benefits of the Montessori approach to mathematics learning in early childhood may, at least in part, be due to its effective use of manipulatives” (Laski et al., 2015, p. 1)

The physical space can also greatly impact how the children feel, grow, and learn. When children feel ownership over a space, they also feel a sense of responsibility to maintain it, and do so with surprising consistency. Some of the crucial factors needed for optimal learning are “an orderly, prepared environment, a humble, observing teacher, and freedom for the child” (Standing, 1962). This demonstration of ownership is a representation of the importance of environment and atmosphere, and its effect on the children.
Cosmic education is the theory that if children are given an understanding of the connection of each thing they encounter and the cosmic universe, they will develop a better understanding of purpose, relationships, and meaning. This belief is another contributing factor to the effectiveness of the Montessori Method. The purpose of cosmic education is to help children feel a sense of connection, purpose and responsibility in the world. “Using the manipulative helps establish a basic understanding of the math concept that in turn promotes deeper insights into how the material relates to the concept that in turn leads to better understanding and the concept and so on” (Laski et al., 2015, p. 2). By teaching children about the bigger picture we are giving them a view of how things all effect each other and the way that things are inter-related.

All included research acknowledges the interrelatedness of the aforementioned components of preparation of the environment, cosmic curriculum connections, space ownership, and use of manipulatives. When these elements are properly utilized, students are successful in their learning experiences. It is also crucial that families of Montessori students are educated on the framework and basis, and have a clear understanding of the curriculum and pedagogy to support the child. The combination of these working together will honor the vision of Maria Montessori.

**Montessori Environment**

The environment in which Montessori education takes place is as important as any physical object in that room. In order for the room to be productive, conducive to learning, and pique children’s interest and curiosity, it must be prepared with the child in mind. Maria Montessori referred to the Prepared Environment as the New Third Factor in Education
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(Standing, 1962). In her work she describes the environment as being as important as the teacher and the child. “Montessori classrooms are usually multi-age (serving students of a 2- or 3-year age range and therefore a range in grade levels), allowing the opportunity for children to act as followers, peers, and leaders with different classmates” (Donabella & Rule, 2008, p. 5). The setting was important as Montessori recognized the shortcomings of pairing students only with those in narrow proximity to their own age. “The multi-year time frame and the consistency...provides extensive opportunities for children to abstract the mathematical concepts represented by the Montessori math manipulatives and to gradually develop more sophisticated knowledge over an extended period of time” (Laski, 2008, p. 2). The length of exposure to the mathematic materials serves to lay a strong foundation of knowledge. Students are provided with both the opportunity to learn from and teach their peers, as well as the time to expand in both endeavors.

Research findings indicate that a “close adherence to the Montessori approach seems to promote better math learning as children who attend high-fidelity Montessori programs are more likely to have higher standardized math scores than those who attend lower fidelity Montessori or traditional early childhood programs” (Murray, 2008, p. 2). This approach was built upon many theories and findings of its creator, which also indicated the need for a peaceful environment to thrive. By honoring the findings of Maria Montessori in this regard, we are also following the child and their inner need for silence. In the article Getting it Together: The Fundamental Exercise, A Montessori Lesson Plan, there is emphasis on the importance of preparation for teachers everywhere. “One part of the prepared environment is the individual activities that are designed to address the children’s needs and sensitive periods. Because it is
important to think through every aspect of an activity before offering it to the children, a Montessori teacher usually writes an activity plan that includes these basic elements” (Woods & Turner, 1999, p. 24).

Silence is an often overlooked need of children, especially today with the constant noise that surrounds them. When children are surrounded by silence, they are able to connect with their thoughts and feelings, and this centers their focus and creativity. As technological advances have increased the noise engulfing children, the need has only become greater. “It is only in solitude and silence that our life is really present, that we are truly responsive to the heartbeat of the universe, and free to contemplate the miracle of existence” (Haskins, 2011, p. 34). By striving to create a calm and peaceful environment, the teacher will seek an atmosphere that offers the least distraction, maximum exposure, broad experiences, and limitless activities. “When done well, it is an indication of the teacher’s understanding of the fundamental components of a successful program, as well as her [sic] ability and willingness to carry out a core element of Maria Montessori’s vision for facilitating the unveiling of a child’s potential” (Haskins, 2012, p. 36). Research stresses “the theory that the Montessori Method is an effective learning curriculum and environment for students when the ideals of its creator are honored within Montessori schools” (Haskins, 2012, p. 36).

The relationship between the environment and the child’s development is supported by Dr. Montessori as she stated that “the first aim…is to render the growing child independent of the adult…it is a place where he can do things for himself—live his own life—without the immediate help of adults” (Standing, 1962, p. 267). This emphasizes the importance of giving students the opportunity to have experiences, not just hear about them. Every physical object in
the room has a reason for being there and is available for the child to experiment with and to figure out for themselves, or with guidance if requested. “What Montessori has done is this: realizing the peculiarly absorbent nature of the child’s mind, she has prepared for him a special environment and then, placing the child within it, has given him freedom to live in it, absorbing what he finds there” (Standing, 1962, p. 265). As the children discover the environment, the teacher may then guide them to the representations of materials and manipulatives. “Teachers may need to be outright methodical in pointing out the connections between the representations that students’ construct and the more abstract concepts” (McNeil & Uttal, 2009, p. 139). Within the lessons the students are introduced to the concepts behind the physical materials used. It is through this progression that the students make connections between the concrete and abstract; “the sequence in which the Montessori materials are introduced is structured to move children to increasingly abstract representations over time” (Laski et al., 2015, p. 4). The foundations for these concepts begin early in the educational journey, and progress as the children move through the program.

**Montessori Materials**

In math and other curricular areas, manipulatives are used to help students learn large concepts gradually. “Montessori methods and materials are especially powerful for teaching mathematics because they form a coherent curriculum, progressing from concrete representations of concepts to levels of increasing abstraction, culminating in paper and pencil algorithms” (Donabella & Rule, 2008, p. 4). Dr. Montessori discussed her methods of finding the right materials for the classroom setting as a learning sequence progresses. “The reactions of children to various objects…the frequency with which they used them…built up reliable criteria for the
elimination, modification, and acceptance of apparatus to be used in our schools… Everything about these various objects were all determined by experience” (Montessori, 1967, p. 99). This description of the development of material sequence indicates that these objects were not chosen haphazardly, but with great precision, care, purpose, and reason. There is a justifiable reason for every piece of a well-prepared environment. “Materials are designed to lead the child from the beginning simple, concrete representations of numbers and mathematical concepts to the more complex, abstract mathematical ideas” (Donabella & Rule, 2008, p. 4). It is through the Montessori training that teachers learn how to do this in a way that is developmentally appropriate for their students.

The observations of Maria Montessori guided her in selecting appropriate manipulatives for the learning environment. “Montessori watched children in the classroom and thought about their developmental needs; she developed materials that she thought would suit those needs, and then she watched the children with the materials and revised and refined them until she thought she had a material that would meet one or more specific needs” (Rosanova, 2003, p. 9). The materials in a Montessori classroom are each thought out, practiced, and researched before being incorporated into the curriculum. “The simplicity of Montessori materials is that though they are superficially less interesting or appealing than more broadly used manipulatives, they are designed in ways that are more likely to focus children’s attention on the attributes that represent the mathematical concept and increase learning” (Laski et al., 2015, p. 5). They have withstood the test of time as they continue to hold interest as well as help children learn and progress to the next skill. “Tremendous thought and experimentation went into the development of these materials and their use” (Lillard, 2008, p. 21). Through her own observations and experiences
Maria Montessori was able to find manipulatives that were, and are, productive, helpful, and engaging for students of all ages. “All of our work in Montessori begins with a philosophical conviction; the idea that meaning is both possible and worthwhile” (Rosanova, 2003, p. 9).

Montessori materials are initially the most obvious sign of uniqueness and present a difference from traditional schools. “The curriculum is much more thorough and carefully articulated than what is generally available in American schools” (Rosanova, 2003, p. 8). Upon entering a Montessori class without background knowledge the materials are foreign and confusing, yet to a student who has used them since a young age, they are natural stepping stones to learning and understanding. It is through these carefully chosen materials that Montessori students master concrete concepts before moving toward the abstract.

The Checkerboard serves as an abstract representation of facts the students traditionally are asked to memorize without understanding. “The Checkerboard for Multiplication serves as a bridge between the concrete methods…and the abstract algorithm used in paper and pencil computation” (Donabella & Rule, 2008, p. 11). When this is used as an aid in introducing students to multiplication they are better able to explain what the multiplicand and multiplier represent. “Concrete, dynamic, hands-on materials: The beads allow the student to see what multiplication means. Seeing the problem concretely promotes deep understanding. The board builds the background for more abstract work” (Donabella & Rule, 2008, p. 21). When students begin work with the Checkerboard they are familiar with the coloring pattern and beads as these have been used extensively in their previous learning. “The Checkerboard for Multiplication has three components: a place mat-sized board with green, blue, and red alternating squares, numeral tiles, and a set of bead bars” (Donabella & Rule, 2008, p. 6). Students have used bead bars from
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A young age in counting, grouping and other mathematical activities and have been guided by the colors representing the hierarchy of numbers with bead frames and stamps, so the transition is seamless. “Steps of multiplication on the board are analogous to the steps used in paper and pencil work” (Donabella & Rule, 2008. p. 21). It is when the students are able to articulate the meaning of the steps and make connections to its purpose that they move toward abstraction.

“The Checkerboard for Multiplication combines the abstract concept of place value squares on a board with the concrete use of bead bars to show quantities within each place” (Donabella & Rule, 2008. p. 11). While students are expected to work with place value extensively, few understand what this represents, even after completing their schooling. “Preservice and in-service teachers introduced to this technique often remark that this is the first time they understand the reason for the zero in the ones place of the second partial product” (Donabella & Rule, 2008. p. 21). Students who use the Checkerboard for multiplication are able to explain the steps of multiplication as well as understand why they are successful.

**Cosmic Education**

Cosmic education is the idea that if children are given an understanding of the connection between each thing they encounter and the cosmic universe, they will develop a better understanding of purpose, relationships, and meaning. By teaching children about the bigger picture, we are giving them a view of how things all affect each other and the way that things are inter-related. This concept can be taught through many areas of the curriculum as the concept is so broad and can be integrated into basically any part of education. “The *Timeline of Mathematical Ideas*, emphasizing contributions of different cultures, is one of the Great Lessons
presented annually in all classrooms in different ways. Connections are regularly made between mathematics, art, other academic subjects, and everyday life” (Donabella & Rule, 2008, p. 5).

The main focus of the article *On Becoming a Cosmic Educator, Spotlight: Cosmic Education* is on how the Cosmic Education came to be and why. “The Montessori approach provides children with multiple opportunities to make connections between a physical representation and the underlying mathematical concept through incorporating the same physical representation in multiple materials” (Laski, 2008, p. 2). Embracing this appeals to their imagination, it is when we understand children that we can see how they think. “Cosmic education is more a pedagogy than a curriculum. Many Montessori teachers equate it with Five Great Lessons: The Fable of Creation, Timeline of Life, Evolution of Humans, History of Writing, and History of Mathematics...The challenge is to provoke students' imaginative participation in the stories, to stimulate them to create with us images and feelings that represent the possible” (Maier, 2002, p. 42). These lessons show the importance of math within our whole world, which amazes many children.

The purpose of cosmic education is to help children feel a sense of connection, purpose, and responsibility in the world. The manipulatives in the classroom bridge this with a sense of understanding many things in a child's world. “The solid foundation of hands-on work with simple, appealing materials affords students a deep understanding of place value, number, and operation concepts” (Donabella & Rule, 2008, p. 4). With the Checkerboard students are able to understand all components of multiplication, which allows them to use it on a deeper level. “Montessori math manipulatives are basic representations of mathematical entities that do not resemble real objects or possess irrelevant perceptual features” (Laski et al., 2015, p. 5). By
understanding the relationship among all things, children are more apt to have appreciation for
all things in life, as well as life itself. “If the idea of the universe be presented to the child in the
right way, it will do more for him than just arouse his interest, for it will create in him admiration
and wonder, a feeling loftier than any interest and more satisfying” (Montessori, 1967, p. 6).
The Montessori Method does this with math as well as the other curricular areas, which make
this educational approach both unique and tailored to children’s interests and natural curiosities.

While cosmic education may appear to be a grandiose idea, it is the core of the
Montessori Method and is incorporated throughout all curriculum areas. “Drawing linkages
between concrete and abstract representations of mathematical concepts may do far more to
advance students’ understanding than working on either in isolation” (McNeil & Uttal, 2009,
p. 139). The combination of ideals and theories held by Maria Montessori create an atmosphere
that nurtures the children. “The goal of nurturing peace in ourselves, in our children, is not
central, incorporated into every act of teaching, every nuance, every contact with a child’s
engaging mind” (Torrence, 2002, p. 11). When a child is at peace they are open to learning and
expanding their minds. By meeting this need first the teacher is able to present new ideas and
concepts to children.

Public Misconceptions

As Montessori education is not the most often used approach in the United States, there is
still much confusion over what it entails. The article Public Knowledge of Montessori Education
demonstrates popular beliefs that the Montessori Method is either too confined and rigid, or too
lax and unstructured. Nevertheless, “Nine in 10 survey participants recognized that Montessori
children are allowed to work together in small groups and that the primary goals of Montessori
education include teaching children to cooperate with one another and developing children’s sense of community at school” (Murray, 2012, p. 20). While a small percentage of people are familiar with the terminology, most have misconceptions or falsehoods that they believe to represent what Montessori schools encompass. Through blind surveys the authors were able to identify the biggest mistaken assumptions people make regarding Montessori classrooms and the trends among specific populations therein. “The American public generally recognize the name ‘Montessori’ because so many schools across the country and around the world use the Montessori name” (Murray, 2012, p. 18). The problem with this is that while some institutions have strict adherences to the Montessori Method, others use the name and loosely incorporate the concepts as intended. While also noting that “for those who had heard the term “Montessori” their average score on a series of Montessori knowledge questions was only 64% correct” (Murray, 2012, p. 18). When Montessori schools are offered at the public level more widely, the need to educate families on what this type of education represents is crucial.

Throughout the article “Public Montessori Elementary Schools: A Delicate Balance,” the authors analyzed “the degree to which schools reported they were living up to the ideals of establishing truly Montessori environments within public schools” (Murray & Peyton, 2008, p. 26). By having teachers that are properly trained, materials that meet the classroom needs, and professional support to address concerns, public Montessori schools can expect continued success in the future. “Better understanding of the reasons behind the unique structure of the schedule on a Montessori classroom will help people see that the differences from other educational environments are based on a comprehensive philosophy of child development”
(Murray, 2012, p. 21). The need to educate families is an important component to gathering support within the community to better aid the education of the child.

**Following the Child**

The article *Montessori, Individual Work and Individuality in the Elementary School Classroom* examines the history of the educational movement in both Europe and the United States. It compares practices, beliefs, and teaching methods from the late 1800s through the mid-1900s. The comparison between the Montessori Method and whole-class teaching is most evident when focusing on individualism. “Lessons are given to individuals or small groups of children who are ready for the concept. Each child may therefore progress at the child’s own pace, not having to ‘catch up’ with or ‘wait’ for others” (Donabella & Rule, 2008, p. 5). While Montessori education follows the child, the child in many traditional settings is expected to follow the lead of others and not stand out.

In *Why Montessori? Answers from a Parent's Perspective* the theory that the Montessori Method is an effective learning curriculum and environment for students is investigated. The parent compares and contrasts traditional schooling with the structure and beliefs of Montessori schools. The teacher must be very observant in learning how the students absorb information, what drives and interests them, and how they solve problems independently. “The usual source of conceptual development proceeds in three phases: from (a) acting on concrete objects, to (b) forming images of the concrete constructions, to (c) adopting symbolic notations” (McNeil & Uttal, 2009, p. 138). By taking the time to understand how the children learn initiates better ways to reach them effectively and efficiently. While being observant, the ideal teacher must
present an aura of peace and acceptance so that the child does not feel threatened or judged, but free to be themselves.

To have a clear understanding of both the absorbent mind and sensitive periods is also necessary to be an effective educator in a Montessori setting. When the Montessori Method is used correctly, children thrive, they find their confidence, they explore, they feel safe to try new things, and they believe in themselves. “Hands-on materials guide the student to ever-increasing levels of abstraction until the student is able to manipulate the quantities mentally or with paper and pencil using the traditional algorithms” (Donabella & Rule, 2008, p. 11). The teacher in the classroom must also have the training to know their role in the child’s educational path. While some educators see themselves as the giver of knowledge, those trained in the Montessori Method are more humble and see themselves as actual guides. They recognize that they are there to help children find resources, guide their curiosity in productive ways, but most importantly, to observe the child.

In the Montessori school setting there is freedom for the child; freedom within limits. When a child feels a sense of choice, they take pride and ownership in their work, their learning, and their behavior. Instead of being told what to do, how to do it, and how to act, they are given the choices and will choose the one that best suits their personality, interests, and needs. In using the Checkerboard for multiplication that students are able to progress independently as “the board arrangement guides the student through the problem” (Donabella & Rule, 2008, p. 21). This allows freedom for exploration and practice for the child.

If student choices are not beneficial and disturb the goal indefinitely, then the guide will discuss this with the child and offer other choices, not redirect them to their choices. “Armed
with the ability to think thoroughly, know their strengths and weaknesses, and strategically solve problems, Montessori children may be less stressed in times of trial, and may even enjoy working through the challenges they face” (Karna, 2013, p. 25). To truly use any theory as it was intended, it must be properly researched, practiced with fidelity, and self-assessed regularly. It is the responsibility of the guide to know the materials and their intended purpose. “Concepts are introduced concretely in many different ways with related materials. Concepts build from the simple (here, multiplication facts) to the complex (three-digit multipliers)” (Donabella & Rule, 2008, p. 11). The teachers must make the manipulatives accessible and explain them to children so that the children have the tools as intended. They also must provide an environment conducive to learning, growing, exploring, and independence. “Through the Montessori approach, he develops the love of learning, and love of the world that he lives in, learns to love and care for himself, and understands how to share responsibility for the world’s progress” (Karna, 2013, p. 28).

Conclusion of Literature Review

The research indicates that Montessori manipulatives do support understanding. While my initial focus was solely on the Checkerboard for Multiplication manipulative, the range broadened when the underlying concepts aforementioned were considered. It is not one material, process, or ideal that is successful in and of itself, but the collaborative theories, practices, and procedures that support each other to guide student success. The background of research, design, and implementation all are imperative to the execution of presentation and the understanding of this from the teacher’s perspective is crucial to proper practice. When an educator is thoroughly engrossed in the history of the Montessori Method and all that it encompasses, they are ready to
begin their work with children and learn from them. “When done well, it is an indication of the teacher’s understanding of the fundamental components of a successful program, as well as her ability and willingness to carry out a core element of Maria Montessori’s vision for facilitating the unveiling of a child’s potential” (Haskins, 2012, p. 36).

**Purpose**

The purpose of this study was to determine the effect of the checkerboard work (a Montessori manipulative) on multiplication mastery. As the researchers, I explored how a concrete representation assisted students in internalizing concepts and procedures. The following questions will help guide the study:

1. Does the Checkerboard support multiplication understanding?
2. Does concrete application of multiplication strengthen abstract understanding?
3. Are students able to transfer word problems to the Checkerboard?
4. Do children have a deeper understanding of multiplication when they are introduced to it with the Checkerboard?

**Method**

**Participants**

Students involved in the study are typical fourth-, fifth-, and sixth-graders in a mainstreamed setting, enrolled in a suburban, public Montessori elementary school. The classroom is comprised of predominantly white, mixed gender, 9- to 11-year-olds.
By working with this particular group of 4th, 5th and 6th grade students, I aimed to increase their multiplication understanding as they utilized Montessori materials. Student names were stored separately from numerical and qualitative data. No names or identifying characteristics were included in the analysis of the data. The students and their parents were provided with a letter explaining the study and given the opportunity to opt out of participation, without penalty, if they chose. No recruitment occurred outside of the researcher’s classroom.

**Procedure**

The study was conducted in a classroom at the public Montessori charter school. The researcher was the teacher. Students were given direct instruction in the area of multiplication with the use of the Checkerboard for support. The researcher collected data based on work completion and parent and student surveys. Data were then recorded in both numerical and narrative form. No previously collected data was included in the study.

1. Parents and students were asked to complete the survey about multiplication skills and understanding to gather baseline data.

2. Students participated in direct instruction as part of their regular school day.

3. Students completed work from their lessons as part of their regular routine.

4. Parents and students then completed the survey a second time to look for growth.

The researcher used the current classroom grade book (record keeping) to track work completion. See attachments for the parent/student survey. It was not anticipated that this study would present any risk greater than what a student might experience in a typical school day.

The general findings of the study were shared with the students and their parents. The results of the study could be beneficial to students and families by giving them an increased
understanding of the student’s multiplication understanding, which has been correlated with improved academic success. The results will also benefit the researcher in planning for and implementing future direct instruction in multiplication and related content areas in math. It is possible that the findings of the study could be used in a professional developmental setting or academic conferences.

The information obtained from subjects will be used to analyze the correlation between direct teaching in multiplication with the Checkerboard manipulative and student understanding. Student information will in no way be tied to the findings. Explanations and consent forms for both parents/guardians and students were distributed prior to the action research beginning. The forms were addressed to the students in class and addressed to the parents/guardians.

At the onset of this research students were given an assessment (see Appendix). This was used with all students and they were given oral instructions to use the rubric as a guide to determine what is required. We initially went through the rubric collaboratively allowing students an opportunity to seek clarification in any areas that were not understood. Students showed varying degrees of apprehension regarding this assessment but were assured that they will be given the tools to solve these problems. Coming from different teachers prior to the current school year also determined varied exposure to both multiplication and the Checkerboard specifically. Approximately one-third of the focus group had used the Checkerboard previously, and two-thirds had seen it but not personally used it. The students were split into groups to accommodate their comfort level with the Checkerboard as well as the guidance needed initially.

The first group (referred to as Group A) began with lessons introducing the Checkerboard and its components. The presentations began with the basics such as vocabulary, names and
procedures for using the material. While some students in Group A had not practiced multiplication at all there was much work to be done. This group received daily check-ins for progress the first 2 weeks to ensure that they were comfortable with the tasks. Having students from multiple classrooms in Group A, each student was able to ask questions without feeling singled out. This also provided an opportunity for students to share their experience and help each other.

While Group B had experience with either/both the Checkerboard and the execution of multiplication they still had lessons on incorporating the two together. Students were able to have discussions on how to apply assigned problems to the Checkerboard. These conversations were the basis of applying abstract problems to concrete materials. Through each step the students were expected to explain how the numbers applied to the materials to ensure understanding was evident.

Both groups were given weekly lessons on Wednesdays that incorporated the Checkerboard. They also were given practice follow up to complete with a partner or small group throughout the weekdays that math was not taught. The repetition of daily practice and exposure strengthened the connection to understanding. The Checkerboard also strengthens memory as studies show the link between movement and memory. When the brain is able to associate a physical movement with a task or concept it is better able to recall that at a later date. “Several studies have found that use of representations can lead to improvement and development of students’ mathematical abilities and understanding” (van Garderen, 2012, p. 25).

Math is typically taught as an abstract concept where students are asked to memorize facts, procedures and formulas without having a clear understanding what it represents or the
Effectiveness of Montessori Materials

purpose of doing so. Having a representation that students are able to manipulate makes the process more concrete and relatable for students. When one understands something they are more likely so feel confident, to further explore and to question anything that they are struggling with.

Materials

○ Checkerboards
○ Students
○ Bead bars
○ Assessment
○ Rubric
○ Time to Collaborate with peers
○ Surveys

Standards (State standards addressed)
From the Common Core State Standards:
● Operations and Algebraic Thinking 4.oa
  ○ Use the four operations with whole numbers to solve problems.
    1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
● Use place value understanding and properties of operations to perform multi-digit arithmetic.
  ○ Number and Operations in Base Ten 4.nBt
    5. Multiply a whole number of up to four digits by a one-digit number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (Wisconsin Department of Public Instruction, 2011)
Effectiveness of Montessori Materials

Procedure

The teacher gave a pre-assessment of multiplication problems to students at the onset of the research. These scores were recorded and then the participating students and their parents were given a survey that ranked their skill level, confidence level, interest, and concerns with multiplication. The surveys also specifically regarded the Checkerboard that is used in a Montessori classroom to support such application as this was the basis for research. The findings were used to drive instruction in both multiplication and the steps of using the Checkerboard. Students were split into groups based upon their previous exposure to the Checkerboard and lessons were tailored to meet their needs. Throughout the set time frame students had weekly lessons in using the Checkerboard as well as daily practice time with peers to work on problems collaboratively. At the end of the research time student growth was analyzed by the teacher, which showed growth in multiplication when using the Checkerboard.

Findings and Discussion

At the end of the research time frame the same assessment and surveys were given to track growth. The assessment scores had increased, showing gains in multiplication mastery. The surveys from students and parents also showed growth in the areas of confidence, interest, and skill. Upon completion of the research timeline, student scores in successful multiplication execution and confidence rose, which supports the theory that Montessori materials indeed are effective in the understanding of multiplication. The survey results show that confidence in both students and parents increased as well. These results can additionally be interpreted by the data collection methods of: observations of student work, vocabulary used, student interviews, student surveys, parent surveys, pre and post assessment results and student work.
Results

Figure 1: Student Using Checkerboard

Question 1: Does the Checkerboard support multiplication understanding?

The post assessment scores show a growth in multiplication understanding, which is a result of the work done using the Checkerboard. The 6 weeks of lessons laid groundwork for practicing, exploring, and manipulating the numbers in a concrete fashion. A statement from the student survey stated: *The Checkerboard helps me with multiplication.* The initial response for this statement averaged low, with less than half of the participants agreeing. This result indicated that students did not view this as a valuable tool. This could be from lack of exposure or understanding, both of which were not able to be pinpointed initially. The explanation of why certain tools were chosen relates to this question in that students must have an understanding to be able to explain. Initially, 42% of the respondents chose Level 3 (*I showed one way that my answer was correct*). The next most selected rating of Level 2 (*I didn’t show that my answer is*
Correct) with 36% choosing this answer. This score is from all respondents that participated in the research with the Checkerboard practice. The same question was presented after the lesson and exposure to the math manipulative.

![Pie chart showing explanation levels](image)

**Figure 2: Pre-Assessment Explanation of Tools**

When the results from the same question were examined post-intervention, it was found that not 63% of respondents chose Level 3 on the rubric (*I showed one way that my answer was correct*) and the next highest selection was 31% indicating Level 4 (*I gave a detailed explanation on why my answer was correct*). This data show a growth in student perspective on how and why they chose the tools that they used.
Question 2: Does concrete application of multiplication strengthen abstract understanding? As the practice was all completed with concrete materials, students had to use this knowledge to answer the abstract questions. The answers to this demonstrates the connections that they made. *I have tools to solve multiplication problems that I do not know* was on the student survey and initially the general consensus was about 30% of students felt strongly that this was applicable to them. This was revisited in another format on the rubric as when asked to *Tell why your answer is correct*; initially 42% of students chose Level 3 (*I showed one way that my answer was correct*).
Figure 4: Pre-Assessment Explanation of Answer

Upon the conclusion of the lessons given the same survey was given and the results of the same question this time was significantly higher. The majority (52%) of respondents now indicated that they could show one way that their answer was correct. Students that were able to show more than one way that their work was correct rose 10% as well.
Figure 5: Post-Assessment Explanation of Answer

Question 3: Are students able to transfer word problems to the Checkerboard?

The results show that students are able to transfer word problems to the Checkerboard successfully. During the lessons students were given the Checkerboard for support and word problems and asked to transfer the information to the Checkerboard.
Figure 6: Layout of Materials

When asked to explain rules of patterns used upon solving multiplication problems, initially only 15% of those surveyed ranked themselves at Level 4 (I gave a detailed explanation on the rules or patterns that I used).
Figure 7: Pre-Assessment Explanation of Rules/Patterns

When this was revisited with the follow-up survey, it was found that a greater amount of participants were comfortable explaining the process they used. Those who could rank themselves at a Level 4 (*I gave a detailed explanation on the rules or patterns that I used*) now had risen to 32%, which is a significant gain.
Figure 8: Post-Assessment Explanation of Rules/Patterns

Questions 4: Do children have a deeper understanding of multiplication when they are introduced to multiplication with the Checkerboard? Results support a deeper understanding when they are introduced to multiplication with the Checkerboard as evidenced by both the student and parent surveys. When asked the survey question: *Montessori materials help me with multiplication*, the majority responded positively both before and after the intervention. When asked to show work in multiple ways, pre-intervention indicated that 63% ranked themselves Level 3 or higher.
Figure 9: Pre-Assessment Showing Work

This was again examined post intervention and the Level 3 or higher rose to an astonishing 94%.
Figure 10: Post-Assessment Showing Work

When reviewing the surveys upon completion of the study, it was evident that confidence in both students and parents has risen considerably. In all areas regarding multiplication understanding scores had increased, tremendously in some areas.

Figure 11: Students Working on Multiplication with Checkerboard

Analysis and Conclusion

Upon the conclusion of this research project, it is my finding that the Montessori materials do indeed aid in the mastery of multiplication understanding. The pre and post assessments were identical, which allowed for scores to show the growth. Beyond the scores of the assessments, it was also determined that confidence, ability, and vocabulary have improved as well. These things all aid in the process of multiplication and math application in general.
The additional components of the Montessori environment, cosmic education, addressing public misconceptions, and following the child were all taken into account throughout this process as well. I feel that these contribute heavily to the success of multiplication mastery in a Montessori setting. Limitations of this study include time available, pool of participants, and possibly other variables such as the practice effect of using the same problems on the assessment tool as well as other learning experiences that could contribute to growth.

Figure 12: Student Work

Future Action Plan

In the future I will begin the school year determining what exposure students have had to materials. Assuming that students have gone through the curriculum in previous years and are at the same spot can hinder progress throughout the school year. Not only have my students had different teachers, they also have had different experiences in all subject areas. Determining where each child is at will allow me to better plan what we will need to cover, as well as how in depth. This process will save a lot of time and frustration when lesson planning. The
Checkerboard is only one of many examples of Montessori math materials that are used to support math in the classroom. In order to adequately prepare for scope and sequence, the teacher must determine for which ones the students need an introduction. By exacting student knowledge, a more precise and accurate plan of student goals can be planned. It is with an understanding of where a child is at, and a goal of where we would like to guide them, that we are able to plan for the journey of success. Both of these pieces are crucial to helping a child receive what they need based upon their unique knowledge, background, and interest. One of the greatest freedoms of a Montessori environment is the guideline to follow the child. The qualities of a Montessori classroom that contribute to the development of the will are consistent with these same sources. “Under proper conditions, the will is a force which impels activities beneficial to life….A will in agreement with what the individual is doing finds the path open for its conscious development. Our children choose their work spontaneously, and by repeating the work they have chosen, they develop an awareness of their actions” (Montessori, 1949, p. 253). The Montessori classroom supports this by allowing children to experience repetition when deemed appropriate. They are given the freedom to practice skills until they are satisfied with their mastery of it. By the choices they have, they can work at their own speed. ”The exercise which develops life consists in the repetition not in the mere grasp of the idea. When a child has attained this stage of repeating an exercise, he is on the way to self-development, and the external sign of this condition is self-discipline” (Montessori, 2003, p. 358). Having the Montessori training, I feel prepared to meet the needs of my students. This research also taught me that importance of gauging student knowledge early into the year to avoid confusion and re-teaching later on. As students are not grouped strictly by grade level in a Montessori classroom,
it is the expectation that the teacher will plot a course appropriate to their background knowledge, interests, and expectations. This is even more crucial in this type of environment as Montessori teachers do not follow a daily guide of what to teach and when. It is by knowing our students as individuals as well as school expectations, that we can pave the most appropriate path to guide them.

The conclusion of this research is also the beginning of further studies in this regard. The data used will drive future planning in mathematics, however there are many other areas in which I would like to review as well. Montessori manipulative are relatively unchanged from their inception almost one hundred years ago and I would like to investigate their relevance, appropriateness and efficiency. By focusing on a specific material, as was done for this research, I plan to research these concepts. There are so many aspects of Montessori education that I want to learn more about and I plan to begin with the concrete materials that are helping these students with their abstract understanding.

Beginning my journey towards a Montessori degree was initially an unremarkable event for me. Having gone through multiple educational programs I felt that this was just another process of writing papers, reading texts, and showing my knowledge. I can now admit that I was arrogant in this way of thinking and honestly believed that this would be a short journey that would simply provide another certification. Although I was warned, I was completely unprepared for the ways in which this adventure would change me- as an educator, as a person, and as a part of the human race. The concepts of cosmic education, following the child, the absorbent mind and the prepared environment are more than tools to teach, they are guides for understanding ourselves and those around us. I had to undergo a transformation of unlearning
what I thought I knew in order to relearn how to view children, the environment, and myself. It is with hard earned humility that I finally seceded the notion that I knew better, and accepted that I knew very little. It was through that process that I truly began to learn: how to observe, how to listen, how to respect the environment and all within it, and how to let children teach me what they need as opposed to me trying to teach them. I simply want to learn more now and have found the educational program that supports what I want for myself and my students. By continuing my studies in the Montessori program I believe that I will finally fulfill my purpose for my path of life.
References


Appendix A: Parent Letter

Tuesday, November 12, 2016

Dear Families,

As part of my Montessori training and my continuing learning about best teaching practices, I will be conducting an action research project in our classroom this year. There have been many studies examining the relationship between using Montessori materials and academic success. As you know, I have already been using the materials in teaching of math, as the teacher’s before me have done. This specific study focuses on the checkerboard in the understanding of multiplication.

1. Purpose:
   The purpose of this action research study is to examine the effects of using the checkerboard for multiplication skills. This will be demonstrated in the following ways:
   ● To learn the level of multiplication skills in which the students are already successful, as well as those that will most likely improve using explicit teaching.
   ● To gain knowledge as to which activities will help improve the multiplication understanding of all children in the classroom, including those who independently solve multiplication problems given the Montessori materials.

2. Procedure:
   Students and parents will complete a survey about the student’s ability to understand, execute, verbalize, and demonstrate multiplication understanding. Over the next several weeks, students will receive lessons about multiplication, word problems, and the checkerboard as they complete their normal school routine. After the lessons have been given, students and parents will complete the same survey a second time.

3. Time required:
   Parent participation will involve two 15- to 20-minute sessions to complete the survey. Student participation will include attending school on a regular basis. Two 15- to 20-minute sessions will be included in the school day to complete the surveys.

4. Risks:
   It is not anticipated that this study will present any risk to participants greater than what might be experienced in a daily routine, other than the inconvenience of the time taken to complete the surveys.

5. Your rights as a subject:
   (i) The information gathered will be confidential. Results will be reported in a university presentation and perhaps a professional development setting or academic conference, but in any presentation no names or other identifying information will
be used. Whether your child participates or not will in no way affect his/her standing with the teacher or school. Data or summarized results will not be released in any way that could identify your child.

(ii) If you do not want to not answer a particular item or if you want to withdraw from the study at any time, you may do so without penalty. The information collected from you up to that point would be destroyed if you so desire.

(iii) At the end of the session, you have the right to a complete explanation (“debriefing”) of what this experiment was all about. If you have questions afterward, please ask me or contact:
Michael Miller  
Dept. of Education, Wyman Building, UW-RF  
715/425-3206  
Michael.miller@uwrf.edu

Also, once the study is completed, you may request a summary of the results.

This research project has been approved by the UW-River Falls Institutional Review Board for the Protection of Human Subjects, protocol # _______.

I am requesting your permission to use samples of your child’s work as evidence in my research. If you are willing to allow your child’s participation, please sign this form and return it to me as soon as possible.

If you have any questions or concerns, please feel free to call me (715-425-7645) or email me (katie.purington@rfsd.k12.wi.us). You may also contact my supervisor, Michael Miller, by phone (715-425-3206) or email (Michael.miller@uwrf.edu).

Sincerely,

Katie Purington  
Upper Elementary Teacher

I have read the above information and willingly consent to have my child participate in this study.

Parent Name (please print) ____________________________

Parent Signature _________________________________

Date ________________________________
Appendix B: Student Letter

Dear Students,

As part of my Montessori training and my continuing learning about best teaching practices, I will be conducting an action research project in our classroom this year. There have been many studies examining the relationship between using Montessori materials and academic success. As you know, I have already been using the materials in teaching of math, as the teacher’s before me have done. This specific study focuses on the checkerboard in the understanding of multiplication.

1. Purpose:
   The purpose of this action research study is to examine the effects of using the checkerboard for multiplication skills. This will be demonstrated in the following ways:
   ● To learn the level of multiplication skills in which the students are already successful as well as those that will most likely improve using explicit teaching.
   ● To gain knowledge as to which activities will help improve the multiplication understanding of all children in the classroom including those who independently solve multiplication problems given the Montessori materials.

2. Procedure:
   You and your parents will complete a survey about your ability to understand, execute, verbalize, and demonstrate multiplication understanding. Over the next several weeks, you will receive lessons about multiplication, word problems, and the checkerboard as they complete their normal school routine. After the lessons have been given, students and parents will complete the same survey a second time.

3. Time required:
   You will attend school on a regular basis. Two 15- to 20-minute sessions will be included in the school day to complete your surveys.

4. Risks:
   It is not anticipated that this study will present any risk to you greater than what might be experienced in a typical day of school, other than the inconvenience of the time taken to complete the surveys.

5. Your rights as a subject:
   (i) The information gathered will be confidential. Results will be reported in a university presentation and perhaps a published article, but in any presentation no names or other identifying information will be used. Whether you participate or not
will in no way affect your standing with your teacher or school. Data or
summarized results will not be released in any way that could identify you.

(ii) If you do not want to not answer a particular item, or if you want to withdraw from
the study at any time, you may do so. The information collected from you up to
that point would be destroyed if you so desire.

(iii) At the end of the session, you have the right to a complete explanation of what this
experiment was all about. If you have questions afterward, please ask me or
contact:
Michael Miller
Dept. of Education, Wyman Building, UWRF
715-425-3206
Michael.miller@uwrf.edu

Also, once the study is completed, you may request a summary of the results.

This research project has been approved by the UW-River Falls Institutional Review Board for
the Protection of Human Subjects, protocol # ________.

I am requesting your permission to use samples of your work as evidence in my research. If you
are willing to participate, please sign this form and return it to me as soon as possible.

If you have any questions or concerns, please feel free to call me (715-425-7645) or email me
(katie.purington@rfsd.k12.wi.us). You may also contact my supervisor, Michael Miller, by
phone (715-425-3206) or email (Michael.miller@uwrf.edu).

Sincerely,

Katie Purington
Upper Elementary Teacher

I have read the above information and willingly consent to participate in this study.

Student Name (please print) _______________________

Student Signature ________________________________

Date ________________________________
Appendix C: Student Survey

Name: __________________________                      Date: _______________

Multiplication Questionnaire
( Student Version)

Multiplication skills include knowing the memorization facts, understanding the steps, being familiar with the vocabulary, and executing the process.

Scoring Key

Answer how well each statement describes you when you don't use special aids or tricks you have developed to get around difficulties you might have. Score each answer as follows:

0 - doesn't describe me at all

1 - describes me somewhat

2 - describes me pretty well

3 - describes me very well

<table>
<thead>
<tr>
<th>Statement</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have trouble with multiplication.</td>
<td></td>
</tr>
<tr>
<td>I don’t understand multiplication.</td>
<td></td>
</tr>
<tr>
<td>I have trouble solving multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>I don't do multiplication correctly.</td>
<td></td>
</tr>
<tr>
<td>It is hard for me to do multiple problems in a row.</td>
<td></td>
</tr>
<tr>
<td>I like math.</td>
<td></td>
</tr>
<tr>
<td>I practice math every day.</td>
<td></td>
</tr>
<tr>
<td>Learning math is fun for me.</td>
<td></td>
</tr>
</tbody>
</table>

Score
<p>| I use different ways to solve problems.                        |
| I understand the inverse of multiplication.                  |
| I am good at math in general.                                |
| I am good at multiplication.                                 |
| I have some of my multiplication facts memorized.            |
| I have most of my multiplication facts memorized.            |
| I have all of my multiplication facts memorized.             |
| I have tools to solve multiplication problems that I don’t know. |
| I know how to use the checkerboard.                          |
| I can describe what a multiplier is.                         |
| I can describe what a multiplicand is.                       |
| I can describe what the product is.                          |
| I know who to get help from if I am stuck.                   |
| My parents are good at math.                                 |
| My parents help me with math.                                |
| I will get better at math.                                   |
| The checkerboard helps me with multiplication.               |
| Montessori materials help me with multiplication.           |</p>
<table>
<thead>
<tr>
<th>Statement</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>I use materials to solve multiplication.</td>
<td></td>
</tr>
<tr>
<td>I use the checkerboard to solve multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>I don’t use any materials to solve multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>I use paper and pencil to solve multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>I will use materials if I get stuck.</td>
<td></td>
</tr>
<tr>
<td>I am comfortable with the materials.</td>
<td></td>
</tr>
<tr>
<td>The checkerboard has helped me learn multiplication.</td>
<td></td>
</tr>
<tr>
<td>I can use the checkerboard to solve word problems with multiplication.</td>
<td></td>
</tr>
<tr>
<td>I don’t understand the checkerboard.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix D: Parent Survey

Name: __________________________                      Date: _______________

Multiplication Questionnaire
(Parent Version)

Multiplication skills include knowing the memorization facts, understanding the steps, being familiar with the vocabulary, and executing the process.

Scoring Key

Answer how well each statement describes you when you don't use special aids or tricks you have developed to get around difficulties you might have. Score each answer as follows:

| 0 - doesn't describe me at all |
| 1 - describes me somewhat |
| 2 - describes me pretty well |
| 3 - describes me very well |

| Score |
| My student has trouble with multiplication. |
| My student doesn’t understand multiplication. |
| My student has trouble solving multiplication problems. |
| My student doesn’t do multiplication correctly. |
| It is hard for my student to do multiple problems in a row. |
| My student likes math. |
| My student practices math every day. |
| Learning math is fun for my student. |
Effectiveness of Montessori Materials

<table>
<thead>
<tr>
<th>Statement</th>
<th>Score</th>
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<tbody>
<tr>
<td>My student uses different ways to solve problems.</td>
<td></td>
</tr>
<tr>
<td>My student understands the inverse of multiplication.</td>
<td></td>
</tr>
<tr>
<td>My student is good at math in general.</td>
<td></td>
</tr>
<tr>
<td>My student is good at multiplication.</td>
<td></td>
</tr>
<tr>
<td>My student has some multiplication facts memorized.</td>
<td></td>
</tr>
<tr>
<td>My student has most multiplication facts memorized.</td>
<td></td>
</tr>
<tr>
<td>My student has all of the multiplication facts memorized.</td>
<td></td>
</tr>
<tr>
<td>My student has tools to solve multiplication problems that they don’t know.</td>
<td></td>
</tr>
<tr>
<td>My student knows how to use the checkerboard.</td>
<td></td>
</tr>
<tr>
<td>My student can describe what a multiplier is.</td>
<td></td>
</tr>
<tr>
<td>My student can describe what a multiplicand is.</td>
<td></td>
</tr>
<tr>
<td>My student can describe what the product is.</td>
<td></td>
</tr>
<tr>
<td>My student know who to get help from if they are stuck.</td>
<td></td>
</tr>
<tr>
<td>I personally am good at math.</td>
<td></td>
</tr>
<tr>
<td>I personally will help my student with math.</td>
<td></td>
</tr>
<tr>
<td>My student will get better at math.</td>
<td></td>
</tr>
<tr>
<td>The checkerboard helps my student with multiplication.</td>
<td></td>
</tr>
<tr>
<td>Montessori materials help my student with multiplication.</td>
<td></td>
</tr>
<tr>
<td>My student uses materials to solve multiplication.</td>
<td></td>
</tr>
<tr>
<td>Statement</td>
<td>Score</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>My student uses the checkerboard to solve multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>My student doesn’t use any materials to solve multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>My student uses paper and pencil to solve multiplication problems.</td>
<td></td>
</tr>
<tr>
<td>My student will use materials if they get stuck.</td>
<td></td>
</tr>
<tr>
<td>My student is comfortable with the materials.</td>
<td></td>
</tr>
<tr>
<td>The checkerboard has helped my student learn multiplication.</td>
<td></td>
</tr>
<tr>
<td>My student can use the checkerboard to solve word problems with multiplication.</td>
<td></td>
</tr>
<tr>
<td>My student doesn’t understand the checkerboard.</td>
<td></td>
</tr>
</tbody>
</table>

Comments:
## Appendix E: Rubric Used

<table>
<thead>
<tr>
<th>Name:</th>
<th>Show work in multiple ways (MP1 and MP4)</th>
<th>Tell why I chose certain tools (MP3 and MP5)</th>
<th>Tell why your answer is correct (MP1, MP3, and MP6)</th>
<th>Tell about rules or patterns that you used (MP7 and MP8)</th>
<th>Persevering on Problems (MP1)</th>
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<tbody>
<tr>
<td>I didn’t show my work</td>
<td>I didn’t explain why I chose my tools</td>
<td>My answer isn’t correct</td>
<td>I didn’t use rules or patterns</td>
<td>The problem didn’t make sense and I gave up.</td>
<td>The problem didn’t make sense and I gave up.</td>
</tr>
<tr>
<td>I showed my work one way</td>
<td>I gave explanations on why I chose certain tools</td>
<td>I didn’t show that my answer is correct</td>
<td>I used a rule or pattern but didn’t use it correctly</td>
<td>I tried approaching this problem from one direction</td>
<td>I tried approaching this problem from one direction</td>
</tr>
<tr>
<td>I showed my work in two ways</td>
<td>I gave two explanations on why I chose certain tools</td>
<td>I showed one way that my answer was correct</td>
<td>I used a rule or pattern but didn’t explain</td>
<td>I tried approaching this problem from multiple directions</td>
<td></td>
</tr>
<tr>
<td>I showed my work in three or more ways</td>
<td>I gave a detailed explanation on why I chose my tools</td>
<td>I gave a detailed explanation on why my answer was correct</td>
<td>I gave a detailed explanation on the rules or patterns that I used</td>
<td>I tried approaching this problem from multiple directions</td>
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- **Show work in multiple ways (MP1 and MP4):**
  - I didn’t show my work
  - I showed my work one way
  - I showed my work in two ways
  - I showed my work in three or more ways

- **Tell why I chose certain tools (MP3 and MP5):**
  - I didn’t explain why I chose my tools
  - I gave explanations on why I chose certain tools
  - I gave two explanations on why I chose certain tools
  - I gave a detailed explanation on why I chose my tools

- **Tell why your answer is correct (MP1, MP3, and MP6):**
  - My answer isn’t correct
  - I didn’t show that my answer is correct
  - I showed one way that my answer was correct
  - I gave a detailed explanation on why my answer was correct

- **Tell about rules or patterns that you used (MP7 and MP8):**
  - I didn’t use rules or patterns
  - I used a rule or pattern but didn’t use it correctly
  - I used a rule or pattern but didn’t explain
  - I gave a detailed explanation on the rules or patterns that I used

- **Persevering on Problems (MP1):**
  - The problem didn’t make sense and I gave up.
  - I tried approaching this problem from one direction
  - I tried approaching this problem from multiple directions
Appendix F: Assessment

Name: __________________________                      Date: _______________

Directions: Complete the questions below. Use the paper provided to show all of your work. This includes describing patterns or rules that you found and used to solve the problem. You are welcome to use materials to solve all problems, but be sure to include the name of the material and a description of how you used it. Finally prove your answer by using the inverse operation.

1. Matthew can mow a 200 ft. lawn in 8 hours. How long will it take him to mow a 400 ft. lawn?

2. Annie passed out 347 balloons at the fair. She came home with 14 balloons. How many balloons did Annie start with?

3. Felix bought 36 lollipops for his nephews. He wants each of his nine nephews to have the same amount of lollipops. How many will each nephew get?

4. Sally planted 16 rows of carrots. Each row has 27 carrots in it. How many carrots did Sally plant?

5. Mandy earns money by delivering groceries. She earned $4 on Monday, $7 on Tuesday, $5 on Wednesday, $4 on Thursday, and $5 on Friday. What is the average amount of money Mandy earned per day?

6. Calvin paints pictures and sells them at art shows. He charges $56.25 for a large painting. He charges $25.80 for a small painting. Last month he sold six large paintings and three small paintings. How much did he make in all? Show your work and label your answer.

7. Brayden and Gavin were playing touch football against Cole and Freddy. Touchdowns were worth 7 points. Brayden and Gavin scored 7 touchdowns. Cole and Freddy's team scored 9 touchdowns. How many more points did Cole and Freddy have than Brayden and Gavin? Show your work and label your answer.

8. Kenneth practiced his trumpet each day from Monday through Friday. He practiced for 10 minutes on Monday and 25 minutes on each of the other days. How many minutes did he practice in all?