New Methods of Constructing 4-Dimensional Tops

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Introduction

String Theory predicts that the universe has several extra dimensions, which have the structure of Calabi-Yau varieties; the universes defined by these varieties are conjectured to occur in physically indistinguishable pairs. The mathematical field of mirror symmetry seeks to understand the geometric correspondences between paired Calabi-Yau varieties. The polar duality transformation takes a polytope with integer lattice points to its polar dual.

Let $\Delta$ be a lattice polytope which contains $\tilde{0}$. The polar polytope $\Delta^\circ$ is the polytope given by:

$$\{(m_1, \ldots, m_k) : (n_1, \ldots, n_k) \cdot (m_1, \ldots, m_k) \geq -1 \ \forall \ (n_1, \ldots, n_k) \in \Delta\}$$

A lattice polytope is defined to be reflective if its polar dual is also a lattice polytope.

**From Reflexive Polytopes to Dual Tops**

Bouchard and Skarke classified families of 3D tops by relating them to 2D reflexive polytopes. To generate these tops, a reflexive 2D polytope was chosen as a base and points were defined beneath it to form a dual top. We require that the dual tops be convex, so we must constrain the points beneath the polytope accordingly. By generating dual tops in this fashion, we can extend into higher dimensions. The classification of reflexive polytopes yields a finite number of equivalence classes. In contrast, the classification of tops yields infinite families of equivalence classes. We construct 4D tops as follows:

**Constructing a 4-Dimensional Top**

- Choose reflexive polytope base for dual top
- Fix a lattice triangulation of this polytope
- Construct a 4-dimensional top with triangulated boundary. We choose the fourth coordinate for each lattice point of base while satisfying the convexity condition (Four of these are fixed up to overall change of coordinates)
- Our construction shows that we can use lattice triangulations to construct and create tops with a given reflexive polytope as its base.

In [Detal14] we construct infinite families of tops and explore their implications.

**Dynkin Diagrams**

Dynkin diagrams are used in various areas such as Lie theory and the classification of semisimple Lie algebras as well as finite reflection groups.

**Exceptional Tops**

The Standard Simplex is the simplest case. The Bouchard and Skarke classification of 3D tops with the dual top boundary polytope shown in Figure 1 includes two infinite families, $A_n$ and $C_n$, plus an exceptional top corresponding to the Dynkin diagram $E_6$.

From this result we learned that the classification would be more difficult than with the standard simplex. Our next step is to expand the methods used by Bouchard and Skarke to 4-dimensions.

**References**


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