Definition. A time scale $\mathbb{T}$ is a non-empty, closed subset of $\mathbb{R}$.

Definition. Let $\mathbb{T}$ be a time scale and $t \in \mathbb{T}$. The forward jump operator $\sigma : \mathbb{T} \rightarrow \mathbb{T}$ and backward jump operator $\rho : \mathbb{T} \rightarrow \mathbb{T}$ are defined by

$$\sigma(t) = \inf \{ s \in \mathbb{T} : s > t \} \quad \text{and} \quad \rho(t) = \sup \{ s \in \mathbb{T} : s < t \}.$$ 

Notation. $f^\sigma(t) = f(\sigma(t))$.

Definition. The graininess function $\mu : \mathbb{T} \rightarrow [0, \infty]$ and the backwards graininess function $\nu : \mathbb{T} \rightarrow [0, \infty]$ are defined by

$$\mu(t) = \sigma(t) - t \quad \text{and} \quad \nu(t) = t - \rho(t).$$

Example. If $\mathbb{T} = \mathbb{R}$, $\mu(t) = 1$. This makes sense due to the density of $\mathbb{R}$, that is, adjacent values in $\mathbb{R}$ are arbitrarily close to each other.

Definition. Let $f : \mathbb{T} \rightarrow \mathbb{R}$ and $t \in \mathbb{T}$, where sup $\mathbb{T} = \infty$. Then the $\Delta$-derivative of $f$ is defined by

$$f^\Delta(t) = \frac{f(t) - f(\rho(t))}{\mu(t)} \quad \text{and} \quad \sigma(t) = t \quad \text{provided the limit exists}. \quad \text{If the $\Delta$-derivative of $f$ exists, we say $f$ is $\Delta$-differentiable}.$$ 

Definition. Let $f : \mathbb{T} \rightarrow \mathbb{R}$ and $t \in \mathbb{T}$, where inf $\mathbb{T} = -\infty$. Then the $\nabla$-derivative of $f$ is defined by

$$f^\nabla(t) = \frac{f(t) - f(\sigma(t))}{\nu(t)} \quad \text{and} \quad \rho(t) = t \quad \text{provided the limit exists}. \quad \text{If the $\nabla$-derivative of $f$ exists, we say $f$ is $\nabla$-differentiable}.$$ 

2. Second-Order Dynamic Equations

There is an intimate relationship between the second-order dynamic equations shown below.

Delta-Sigma: \( (p(t)x^\Delta)^\Delta + q(t)x = 0 \)

Delta: \( (p(t)x^\Delta)^\Delta + q(t)x = 0 \)

Nabla-Hil: \( (p(t)x^\nabla)^\nabla + q(t)x = 0 \)

Nabla: \( (p(t)x^\nabla)^\nabla + q(t)x = 0 \)

and the dynamic Riccati equations. We can use the Riccati substitutions to solve for the four Riccati equations. The Riccati substitutions are:

$$z = \frac{p(t)x^\Delta}{x^\nabla} \quad \text{and} \quad w = \frac{p(t)x^\nabla}{x^\Delta}.$$ 

where $x$ is a nonzero solution of the second-order equation.