A Study of Integrating Vocabulary Instruction of Linear Equations in a Middle School Classroom

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Abstract

Over the last decade, a recent shift in education requires students to do more in K-12 than numerical calculations; instead, students must utilize higher levels of thinking to analyze, process, and use problem-solving strategies to complete multi-step problems. This study looked at incorporating direct vocabulary instruction and word problem solving strategies in a small classroom of students declared as academically at-risk. The class was a group of 10 students that consistently performed below grade-level on standardized mathematics assessments. These students spent two weeks focusing on vocabulary and word problems involving linear equations. While the results are not conclusive, over half of the students in the class experienced growth either with quality of responses on assignments or on standardized assessments. The results have allowed me to consider planning more time for vocabulary instruction and teaching students how to break down multi-step words problems using reading strategies in mathematics.
Introduction/Overview

Mathematics involves more than numerical calculations and rote-memorization. Recent shifts in education require students to analyze, process, and use problem-solving strategies to answer more complex mathematical questions. Despite asking more out of students’ ability in math classes, the level of performance and college-level course readiness remains lower than other countries (Leinwand, Brahler, & Huinker, 2014). The push to close the achievement gap continues, yet students that perform below grade-level struggle to improve their performance. This study implemented a curriculum in a small class of ten specifically made for students with low performance scores on standardized tests. The curriculum used exposed students to literacy strategies and problem-solving strategies to support their learning and performance on more complex math problems.

The design and implementation of this study originated from a simple observation in the classroom; students were capable of completing problems that involved numerical calculation and rote-memorization, yet problems that involved application and several steps were a struggle. Through research of studies looking at literacy and problem-solving, I was able to put together a curriculum that would help the students process mathematics on a literal level. The content that was specifically chosen was linear equations in an applied setting. Students had the opportunity to identify and continuously connect with vocabulary specific to linear equations, and practice different problem-solving strategies with word problems involving linear equations. The algebra strand of the Minnesota State Mathematics Standards focuses primarily on linear equations, hence the focus of the study (Minnesota has not adopted the Common Core State Standards). The study took place in February 2015 at John Glenn Middle School in Maplewood, MN.
The results of this project may indicate that there is an educational benefit to incorporating more vocabulary instruction and strategies to solve word problems. Students involved in the project showed growth in understanding vocabulary specific to linear equations, which was evident through a pre- and post-project survey given to students, continuous journaling on the researcher’s behalf throughout the project, and quality of student responses on the class activities.
Literature Review

A recent shift in mindset of K-12 mathematics focuses less the direct numerical calculation and directs the attention of educators to involve more problem-solving and literacy in the content area. Many educators today have experienced situations wherein students are capable of performing at grade level when assessed on problems that involve little to no application, yet when a student is required to apply a concept to a real-world situation, it’s as if the student didn’t know anything of the concept. Several researchers address this common issue experienced by educators. Fortunately, from said research, many instructional strategies have been developed to address these concerns and issues.

According to Burns (2014), “all instruction must foster students’ ability to think, reason, and solve problems” (p. 66). In order to nurture our students’ ability to do this, teaching strategies must focus on all elements of the content area, including literacy and problem-solving. These strategies include seeing mathematics as a separate language than as simply arithmetic. The language of mathematics involves decoding words and symbols that have different definitions than what students are exposed to in their home lives (Kenney, 2005). To aid in decoding and understanding this language, teaching reading, writing, and applying those skills to problem-solving can help students feel more successful with applied problems.

Reading in mathematics primarily focuses on word problems as applied math in K-12. Commonly overlooked as less challenging than reading in a different content area, a problem involving a small number of sentences requires more reading skills than what is usually perceived. As a student reads a text problem, they must process a lot of information that contains little to no redundancy, and contains words as well as numeric and non-numeric symbols (Metsisto, 2005). These words and symbols may have several definitions, some involving
mathematics and some not. Lack of attention on seeing mathematics as a language can be
detrimental to the success of students, especially students who are classified as “at-risk”
aacademically. When students who struggle with math read a problem, they “must be able to
parse the word problem in meaningful chunks while converting the information to formal
mathematical notions” (Kuihara & Witzel, 2014, p. 235). To address the needs of this particular
group of students, focusing on reading strategies and showing students different question
structures can aid their success.

A study conducted by Metsisto (2005) found that using a general strategic reading outline
in a classroom enhances the way students learn to read in math. Through modeling “the process
by reading the problem out loud and paraphrasing the authors’ words” and using “context clues
to figure out word meanings” students see how to approach a mathematical problem involving
reading (Metsisto, 2005, p. 16). Reinforcement and scaffolding are key strategies to help students
access and build off of their prior knowledge when addressing word problems. It is, however,
important to be aware of one’s approach to teaching reading strategies in the classroom. Metsisto
reminds educators to avoid simplifying statements and instead approach problems using
questioning, such as “What information do you have that might help you answer this question?”
and “Does the fact that this is a ‘follow-up’ help us decipher the question?” (Metsisto, 2005, p.
17). Utilizing guiding questions rather than explicit ones support the students’ ability to read and
process their thinking in math.

Compared to other core content areas, reading in mathematics requires students to
observe the text in different structures. Barton, Heidema, & Jordan (2002) state that math texts
require students to read and interpret a problem in unfamiliar ways, including horizontally (both
left to right and right to left), vertically (both top to bottom and bottom to top), and diagonally (a
combination of horizontally and vertically, i.e. a graph). By incorporating strategies “to help activate prior knowledge, master vocabulary, and make sense of unfamiliar text”, teachers can provide students methods to comprehend text in math (Barton, Heidema, & Jordan, 2002, p. 25). By using questions to access and understand students’ prior knowledge, teachers can build a better understanding of the knowledge and misconceptions students bring in the classroom.

Some people believe that vocabulary in mathematics instruction can never receive too much exposure. In several of the mentioned studies, authors claim that acquisition and use of mathematical vocabulary requires time and focused instruction, including processing time for students when learning said vocabulary. Utilizing graphic organizers for vocabulary instruction and semantic feature analysis charts provide various avenues for students to understand and build connections in math terminology (Metsisto, 2005). In addition to the graphic organizers, word walls (display of vocabulary words used within a classroom) provide students a reference and “interactive” display of particular language used within a unit. Cronsberry (2014) claims that word walls build vocabulary, reinforce understanding of content terminology, and encourage student independence when reading and writing by providing a visual cue that is easily accessible in the classroom. Consequently, allowing time and repeated opportunities for math language understanding and acquisition enhances reading in the content area.

Reading and vocabulary act as the foundation of math literacy, followed by understanding textual structures and utilizing strategies to develop a conclusion to a problem. As previously mentioned, text in mathematics may appear in unconventional ways, including horizontally, vertically, diagonally, and using graphics (i.e. tables, graphs, charts). Meyer (2014) incorporated predicting, clarifying, questioning, and summarizing to support students’ ability to comprehend and solve mathematical word problems. Though the strategies took more
instructional time (leaving less time to dive deeper in to content), this study found that students produced more accurate answers than those who did not receive the instruction previously stated. Applying problem-solving skills allows students to analyze the full context of a question, rather than simply looking at the math on a basic level.

Kiuhara & Witzel (2014) echo the same reasoning by providing approaches that help students “synthesize novel information, use language to articulate math-reasoning processes, and applying these higher-level skills when solving real-world problems” (p. 235). Directly teaching students to observe common underlying structures of problems (i.e. change problems, compare problems) provides students the opportunity to witness the semantics used in particular structures, thus better understanding how to find the solution. Using the knowledge of semantics and problem structures, students may use a reading strategy called SQRQCQ (Survey, question, read, question, compute, question) to break a problem into smaller pieces (Metsisto, 2005). Using a structure to analyze and solve real-world problems allows students to process and understand the context of the question and access information to find the solution.

In addition to teaching and utilizing word problem-solving strategies, teachers can incorporate writing to access the understanding of students and bring pupils to a higher-level of cognitive processes. Tuttle (2005) states that writing math explanations provide students an outlet to show what a solution is about and why the solution works. In order to allow students opportunities to feel successful with writing in mathematics, teachers must provide questions that are appropriate for the group and will allow students to feel confident about their abilities to find and explain the solution. “Providing a structured guide for their writing can be particularly helpful for students” (Tuttle, 2005, p. 38) and give students a chance to fully explain their solution. Tuttle’s study suggests that by incorporating reading, vocabulary, word problem-
solving strategies, and writing, students effectively develop mathematics literacy and understanding.
Justification for Development of Project

I have taught 8th grade intervention math classes along with mainstream math classes at two different middle schools over the course of three years at the time of this study. During that time I noticed students were typically able to complete pure computational problems, but struggled when applying the concepts to all types of word problems. As a reflective practitioner, I was constantly trying new strategies in the classroom to see if anything was helpful. During this past year, I began teaching an English-Language-Learners (ELL) inclusion class. This course was unique compared to the other courses I taught because it was co-taught with an ELL teacher. During this time, I observed that language must be taught explicitly, giving students the opportunity to constantly speak and use the words whenever possible. I felt this mindset would also benefit the intervention math class I was teaching. This lead to the development of this project.

As I began my research on math literacy, I became convinced that mathematics is a language that I have not taught optimally in my classroom. The constant coding and decoding of symbols, charts, words (with multiple meanings), and numbers takes more explicit instruction than what I previously felt it needed. An example of this includes seeing a statement with a mathematical representation of “6 > -1”. The students must read and comprehend the statement in the English language as “six is greater than negative one”. Through teaching an ELL-inclusion course, I was able to understand that the mental processes to comprehend all the language in a mathematics class went beyond a simple acquisition; students need constant interaction with the vocabulary through speaking, structuring the word in the English language, and developing the ability to apply their understanding of the language without the aid of an instructor via language interaction.
Using a combination of several of the studies outlined above and my own personal take on the strategies researched, I studied the following question: “Does spending instructional time teaching and using vocabulary, instructing and practicing word problem-solving strategies, and including writing with solutions increase students’ ability to comprehend and correctly answer word problems with linear equations?”

The Minnesota State Standard I focused on in particular was MN 8.2.4, which outlines understanding and representing linear situations verbally, graphically, and symbolically. Having taught 8th grade mathematics for two years, this benchmark was one that was quite relevant and a major focus algebraically for the students of this age group, as it is stated to be up to 30% of the Minnesota Comprehensive Assessment in Mathematics (MCA-III).
Design of the Curriculum Project

The design of my curriculum was tailored to the group of students in my 8th grade intervention math course. These individuals were placed in a second math class (Math Strategies Plus 8) because of low performance on state standardized tests and teacher recommendation from the year prior. I decided my project should last approximately two weeks, or ten instructional days with some flexibility. Based on the pacing calendar of their mainstream 8th grade mathematics course, I incorporated my project in late February of 2015, soon after they learned linear equations, but not so long after that I would not be able to utilize prior knowledge.

To help my group of students build a better understanding of vocabulary and to use strategies to solve word problems, I decided to incorporate two elements in each lesson: a vocabulary element and a word problem solving strategy. Thus, students would be able to read word problems involving linear equations and, in time, be able to recognize and understand the use of specific vocabulary words. The word problem solving strategies taught included Survey, Question, Read (SQR), Visualization (draw a picture), and Paraphrasing (putting the problem in their own words). To elaborate the structure of my project design, each day’s design and purpose are discussed in detail in Appendix A. Following is a brief listing of the outline of the unit plan.

- **Day 1: Baseline Data and Vocabulary Recognition**
  - Objective: Student will assess their levels of understanding of linear equations by rating their confidence level to solve word problems. Students will also recall, list, and organize their knowledge of vocabulary of linear equations to create a word wall for the unit.

- **Day 2: Vocabulary Review and Slope Break-Down**
Objective: Students will recap and review the vocabulary from the word wall and focus on understanding application of slope in word problems.

- Day 3: Vocabulary Usage and Application of Slope
  - Objective: Students will continue to review and develop an ability to use the vocabulary from the word wall, and also discuss applications of slope.

- Day 4: Connecting Vocabulary to Word Problems
  - Objective: Students will connect vocabulary to word problems involving linear equations.

- Day 5: Identify Different Types of Word Problems
  - Objective: Students will learn to identify problems involving mathematics and identify problems involving different math concepts.

- Day 6: Problem-Solving Strategies
  - Objective: Students will learn of and how to apply word problem-solving strategies for linear equation word problems.

- Day 7: Review Word Problem Solving Strategies
  - Objective: Students will review and apply word problem-solving strategies previously learned. Students will also learn another word problem-solving strategy.

- Day 8: Writing Solutions
  - Objective: Students will use the strategies taught in class to write up a solution to a problem using math vocabulary and breaking down the process.

- Day 9: Continue Writing Solutions
Objective: Students will assess responses from problems used the day before to
discuss what creates a quality solution to a math word problem.

- Day 10: Using Word Problem-Solving and Writing Skills to Solve Word Problems
  
  Objective: This cumulative day will provide students the opportunity to use all of
  the skills learned over the unit to solve word problems involving linear equations.

As the instructor of the course, I took sole responsibility for implementing the project to
the group of ten students. The data collected and utilized throughout the project included 1)
student responses to questions; 2) student surveys; and 3) general notes taken by me following
the lesson of the day (I was fortunate to have my prep hour following this class, so was able to
journal items quickly). Though not used to entirely measure the impact of this project, the scores
of this cohort of students on the MCA-III Mathematics test from Spring 2014 (these students
were 7th graders during this year, in Math Strategies Plus 7 with a different instructor) to Spring
2015 (8th graders in my Math Strategies Plus 8 course) were also used to determine any potential
growth.
Results

To begin my analysis of the project, I looked at individual student responses to each hand-out over the course of the project. I specifically looked for completion and quality of responses. Questions I asked myself to analyze the effectiveness of the project included: “Did the student change responses to questions involving linear equations?”, “How did the student incorporate aspects of the project in their responses to the handouts?”, and “Were students able to take ownership of the strategies and use them to solve linear equation word problems?”

Overall, there was a positive response to those questions from the majority of the class, as will be shown with 1) student responses from worksheets, 2) student surveys, 3) my personal journaling throughout the process, and 4) MCA-III Math scores.

My observations on the change of student responses focused primarily on the student survey given before and after the project called “Linear Equations Word Problems” (Appendix B). Though each student showed growth in different ways, I noticed that overall there was a better understanding of what a question was asking of the student and an increased vocabulary recognition and application of word problem-solving strategies. For example, question #2 on the student survey said: “A server earns $10 per hour at a restaurant. During one four-hour shift, the server earned $25 in tips. How much did the server earn for the shift? *In your own words, what is question #2 asking you to do?*” Examples of changes in responses are illustrated in the following table.

<table>
<thead>
<tr>
<th>Before Project Response</th>
<th>After Project Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>“I’m not sure how to do this”</td>
<td>“First I will do y = mx + b. I will put 10 in m. I will put 30 in b.”</td>
</tr>
<tr>
<td>“How much 8 goes in 25 and how much tips it is.”</td>
<td>“Asking me how much did the server earn for the shift.”</td>
</tr>
<tr>
<td>“I think cross multiply. If not, I don’t know.”</td>
<td>“It is asking me how much did the server earn for that shift.”</td>
</tr>
</tbody>
</table>
“I don’t know.”

“A server gets $10 per hour at their job. When he works 4 hours he earns $25 in tips, so how much did he earn for that shift.”

It is quite clear the change in the selected students’ ability to read and comprehend a word problem improved from before the project was incorporated in the classroom to after.

Observing and taking note of how the group of students applied the word problem-solving strategies and constant involvement with vocabulary helped me measure the effectiveness of that aspect of the project. Students were exposed to three different forms of strategies to help process and solve a linear equation word problem. Each student was asked to use each strategy on a set of given questions to practice applying the methods. Following the focused instruction, students were then given the opportunity to use whichever method they preferred when asked to answer questions. The table below shows a few responses from the latter activity, which allowed students to use their choice of strategy. The question students were required to answer was “A video streaming website charges a $10 yearly membership and $0.50 per video. Part A: Write an equation that represents the total cost, $T$, to upload $v$ videos. Part B: What would it cost you to upload 25 videos?”

<table>
<thead>
<tr>
<th>Student Example #1:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What problem-solving strategy are you going to use?</strong> SQR</td>
<td></td>
</tr>
<tr>
<td><strong>What is the answer to the problem?</strong></td>
<td></td>
</tr>
<tr>
<td>a. $y = mx + b$, $y = 0.50x + 10$</td>
<td></td>
</tr>
<tr>
<td>b. $y = 0.50(25) + 10 = 22.5$</td>
<td></td>
</tr>
<tr>
<td><strong>Explain how you got your answer using words.</strong> I used key word[s] to solve my problem.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Example #2:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What problem-solving strategy are you going to use?</strong> (student left this question blank, but had “$10 yearly membership” and “$0.50 per video” underlined)</td>
<td></td>
</tr>
</tbody>
</table>
What is the answer to the problem?
   a. \( y = 0.50x + 10 \)
   b. \( y = 0.50(25) + 10, \ y = 12.5 + 10 \)

Explain how you got your answer using words. *Broke the problem down step by step and solved it.*

**Student Example #3:**

What problem strategy are you going to use? *SQR*

What is the answer to the problem?
\( y = 0.50x + 10 \) (student has arrows pointing to 0.50 saying “slope, per” and arrows pointing at 10 saying “charges, y-intercept”)

Explain how you got your answer using words.
*How I got my answer \( y = 0.50x + 10 \) is because how I put it is \$0.50 right next to the problem said ‘per’ so I said to myself that per means slope so then I thought that was my slope. How I got ‘y’ basically the same thing. 10 was the y-intercept right next to it was charges. Charges means y-intercept.*

As I looked at the responses of students, I noticed many were capable of using a strategy to create their equations (noting that they used \( y \) and \( x \) instead of \( T \) and \( v \) as they were asked to).

However, some struggled to follow through with the second part of the question, whether they did not calculate the total or they started the problem but did not finish (like Student Examples #2 and #3). Their explanations gave me deeper insight as to how they created their equations and of their abilities to express their understanding using words rather than numbers or equations.

The majority of students volunteered to use the SQR method, while a small number chose to use visualization; no students chose paraphrasing. As a result, students utilized vocabulary words more often than not; as stated in my project design, the SQR method requires the reader the underline or highlight key words to help find the solution. Since part of the project focused on key vocabulary words seen in linear equation word problems, students were able to connect the vocabulary to the word problems, and apply the concepts to create equations. I speculate that
they chose this method because it was similar to reading strategies they learn in their Language Arts classes; it did not require them to create something on their own—they used information from the question, rather than drawing a picture or paraphrasing from their understanding.

To get a deeper understanding of the students’ processes when solving word problems, I developed a lesson where students learned to write solutions (see “Writing Solutions Guided Practice Day 2” in Appendix B). Below are examples of two students’ responses to the written solution to the word problem: “A boat rental company charges $50 to rent a boat and $8 per hour of use. Part A—Write an equation that represents the total cost, T, to rent the boat for h hours. Part B—If the total cost to use the boat was $118, how many hours was it rented for?”

<table>
<thead>
<tr>
<th>Student Example #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain step-by-step and in detail everything you do to find the answer.</td>
</tr>
<tr>
<td>First I write an equation ( y = 8x + 50 ). Then I need to use 118 in the problem. Next I put 118 in for ( y ), ( 118 = 8x + 50 ). After that I solved the problem. Finally I got my answer.</td>
</tr>
<tr>
<td>My answer is:</td>
</tr>
<tr>
<td>68</td>
</tr>
<tr>
<td>My answer makes sense because. . .</td>
</tr>
<tr>
<td>I did it step by step and solved it.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Example #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain step-by-step and in detail everything you do to find the answer.</td>
</tr>
<tr>
<td>First I write my equation ( y = 8x + 50 ). Then I need to use 118 in ( m ) equation. Next I put 118 in my ( y )-value. After that I solve my equation: ( 118 = 8x + 50 ). Finally I get my answer.</td>
</tr>
<tr>
<td>My answer is:</td>
</tr>
<tr>
<td>My answer is 68</td>
</tr>
<tr>
<td>My answer makes sense because. . .</td>
</tr>
<tr>
<td>My answer makes sense because I used my equation and numbers.</td>
</tr>
</tbody>
</table>

As I assessed the written responses, it was obvious to me that students excelled at knowing key words that pointed to slope and \( y \)-intercept. However, it was also quite clear to me
that their justification for steps they took lacked true reasoning; students said they did a step because they felt that’s what should happen next, rather than understanding why a value went where it did (i.e. why you must substitute 118 for y). Also, from journaling notes, I commented that students did not really use any of the word-problem solving strategies to find their solutions. This could mean they either felt confident with their ability to answer the question, or they did not use it since it was not required as part of the worksheet. Some students also were not able to completely answer the question—their final answers were not correct.

A tool I used to measure the retention and application of the strategies used in the project included the MCA-III Mathematics test results. Overall, of the 10 students involved in the project, six students increased their performance on the standardize test, two students had scores that went down, one student had a score that did not change, and one student was not in attendance during the test (so information for that student is not applicable in this specific situation). Of the six students whose scores improved, four of those students moved from either “Does Not Meet Standards” to “Partially Meets Standards”, or “Partially Meet Standards” to “Meets Standards”. I cannot conclude that my project was the only reason that these students had successful scores, but my hope is that because of the vocabulary instruction, and from breaking down the structure of linear equation word problems, students were able to apply the strategies in an assessment setting.

With all gains, there may exist losses, and with every project implemented within a classroom, there are successes and failures. During this specific project, I experienced a huge struggle with “Slope Exploration” (Appendix B). When I assigned this task to students, I assumed students were able to bring in the concepts of slope and constant rate of change as prior knowledge. An excerpt from my journaling provides insight to how I noticed I expected too
much on the second day: “Activity—WAY too challenging; had to provide a lot of support; vocabulary concepts too abstract; need to remind students how to find slope; perhaps change it to more basic and start with some type of warm-up or reminder?” Unfortunately, I spent most of the class period answering questions about how to find slope and what certain words meant (i.e. constant rate of change, steepest, rate). We decided as a group that the assignment was beyond the students’ ability to complete on an independent level, so I asked the students to stop and respond to a prompt asking what they liked about the day’s activities and what they needed next. Responses for what they enjoyed about the day’s activities included figuring out slope, with one student feeling that it wasn’t hard, and some indicated they didn’t like it. Students requested as a “next step” to spend time getting help, knowing how to use a table, having more information, while some did not know or want any next steps. Following this, I continued with my original plan, knowing that the next few lessons did not include prompts as did this day, and that I would change these problems for the future.

Overall, the implementation of focused vocabulary instruction and the application of word problem-solving strategies seemed to improved students’ ability to respond to linear equation word problems. This is specific to the window of the project; there was not much for results to measure how well the students retained the strategies and applied them on their own after the project. I am hopeful, however, that if strategies such as the ones I utilized in this project were used more often in the classroom, students and teachers would experience higher rates of success for word problems.
Reflection

Overall, this project opened my eyes to the complexities that exist in mathematical language. I often took mathematics for granted because it is a content area that simply makes sense to me. For example, a simple statement that reads “eight miles per hour” immediately registers as a constant rate of change to me, yet for some students (in particular the at-risk students I worked with) there was little to no understanding of what the phrase means. Instead of attempting to quickly explain the idea, and then move on to the next step of the solution, I see this now as an opportunity to discuss why it is a constant rate of change, and the vocabulary words that signal this concept to the reader.

After seeing the influence of vocabulary instruction on higher rates of success, I am compelled to take more time to discuss content-specific words and provide students ample opportunities to interact with the language. I cannot conclude that my project influenced the growth of the students on the MCA-III, but I am comfortable stating there is a possibility that those students felt more familiar with linear equations words than before the project. This gives me encouragement to incorporate language interaction into my teaching, including sentence frames, Pictionary, and more methods that I may find through personal research or through the ELL teachers in my building.

Looking at my results, I noticed that what I read during my literature review held true on some level. By providing my students with ample opportunities to work with linear equation vocabulary and finding ways to express their understanding in writing, I was able to witness what felt like improved performance on creating equations from word problems and true understanding of the thought processes of students with their solutions. In particular, I mentioned previously that showing different question structures can aid in the success of students: analyzing
questions to notice words that meant slope or y-intercept truly helped the students understand how to create their equations. I see this as an opportunity for me to help students break down word problems and notice patterns to help them be successful.

My career in mathematics contributed greatly to this project. The coursework through the Master’s program at the University of Wisconsin-River Falls helped shape the motivation I had to create a project that would possibly shape deeper mathematical understanding in at-risk students. Throughout the program, I experienced learning that inspired me to find different, more effective ways to instruct my students. For example, one of my instructors developed her class to be somewhat inquiry based; the experience I had developing my own conjectures helped me understand the power of exploration and creating conjectures. Granted, I did not completely follow the exact same structure as the mentioned instructor, however, it gave me a mindset to find ways to allow students to take ownership of concepts, with guidance from the teacher.

If I were to incorporate a project similar to this in the future, I would spend more time discussing slope, specifically building a deeper conceptual understanding of constant rate of change. Students were able to say slope also meant constant rate of change, yet I do not feel confident they truly comprehend constant (i.e. something that always has the same rate, something that never changes, a self-referential so as to have a better connection with the word). I would also find ways to allow students to share their solutions and give each other feedback on their solutions. For example, after completing an activity where students were asked to write an equation using a word problem-solving strategy, I would provide students an activity to give one another a chance to present and get feedback on solutions and justifications. This would allow other students to see more than one route to finding a solution, and another opportunity to interact with the vocabulary.
In addition to adding more time for slope and sharing of solutions, I would focus on teaching students how to justify their answers. Though it is not a Minnesota State Standard, I feel students would benefit from learning more about reasoning and communication in their mathematics experience. I often encounter students who struggle to articulate their reasoning for completing a specific step, and so allowing more instruction and time for justification would support the learning in the classroom.

Since completing this project, I have focused most of my research for classroom instruction ideas on vocabulary and effective questioning. I have found that vocabulary instruction is priceless for the learning experience in my classroom. When students feel more comfortable with content-specific words, they are more prone to know how to approach a problem on their own. Effective questioning includes strategies to require students to dig deeper within their thinking, which was inspired by my observation of the inability of my students to justify their solutions.

Altogether, this project taught me not to be afraid to try new things in my classroom as long as it has potential to benefit the students. I can access resources to find other research similar to whatever project I am considering trying in my classroom, and adjust it to fit my style and my students. Language acquisition takes time, practice, and patience; teaching mathematics and the language of mathematics is no different.
Bibliography


## Appendix A: Detailed Project Outline

### Day 1: Baseline Data and Vocabulary Recognition

**Objective:** Students will assess their levels of understanding of linear equations by rating their confidence level to solve word problems. Students will also recall, list, and organize their knowledge of vocabulary of linear equations.

**Overview of the Lesson:** The main purposes of the lesson is to first get a baseline reading on the students’ abilities to solve complex word problems and second to develop a word wall that will be used throughout the unit. At the beginning the class, students will respond to several questions focused on word problems with linear equations. Students must respond to survey-type questions such as “Do you know what the question is asking of you?”, “What is your confidence level of answering this question?” etc. On a blank sheet of paper, students will be given a minute to write as many words as they can that they feel are connected to linear equations. Once time is up, the class will develop a cumulative list of vocabulary words that they may encounter when dealing with word problems involving linear equations. The teacher will also have a few words already on a list to discuss and place in the cumulative list. When words are written, a short discussion will occur to meaning of word and relevance of word (should we keep it on our list? Why/why not?).

### Day 2: Vocabulary Review and Slope Break-Down

**Objective:** Students will recap and review the vocabulary from the word wall and focus on understanding applications of slope in real-world situations.

**Overview of the lesson:** The lesson begins with a review of the words from the word wall. Sentence frames will be provided for students, such as “An example of a line with a positive slope is _____, because _____ is a positive number.” This will provide students with a way of using the vocabulary in a sentence and also reinforce concepts of the words from the wall. Following vocabulary review, the lesson will focus on slope. The main idea of slope is a constant rate of change—yet students struggle to see that. To help reinforce the idea of a constant rate of change, several lines will be given to students. Using prompts provided by the teacher, students will analyze the rate of change in several types of lines (linear, non-linear). The main idea is for students to walk away from this lesson with the ability to articulate that a straight line has a constant rate of change, and that is what we call slope. To assess their understanding for the day’s lesson, students will be asked to explain how slope and straight lines are connected. Another possible structure could be cooperative groups where students solve problems and give feedback to others.

### Day 3: Vocabulary Usage and Applications of Slope

**Objective:** Students will continue to review and develop an ability to use the vocabulary from the word wall, and also discuss applications of slope.
Overview of the lesson: To begin reviewing the vocabulary, the class will play a game called “Fact or Fib”. Each student will be given two cards that read “fact” and “fib”. The teacher will make a statement about a word from the word wall, and students must respond with whether the statement is a fact or fib. Based on feedback from students, the teacher may step back and allow the class to start making statements for the words.

To focus on applications of slope, students will first work in small groups and review slope (constant rate of change). In their groups, students will brainstorm ideas of things that move at a constant rate. A cumulative list will be created and discussed. The list will then be divided into the following categories: objects or actions. For example, a monthly bill would be placed under actions because it is an action that is done monthly, whereas a car moving would be placed in the object category. The goal is to help students realize slope can be an object actually moving at a constant rate, but it can also be an action that is completed at a constant rate, such as paying a monthly bill.

Day 4: Connecting Vocabulary to Word Problems

Objective: Students will connect vocabulary to word problems involving linear equations.

Overview of the lesson: To get students started on using the vocabulary, the class begins with an activity called “Connect” where students are given words and asked to create a sentence that either compares or contrasts the words. Following the review activity, the class will spend time reading word questions and finding vocabulary words they recognize. The main goal is to get students to find key words that mean slope and key words that signify y-intercept. A graphic organizer will be used to help students build vocabulary connections with slope and y-intercept.

Day 5: Identifying Different Types of Word Problems

Objective: Students will learn to identify problems involving mathematics and identify problems involving different concepts in mathematics, specifically linear equations.

Overview of the lesson: Class begins with a game of Pictionary. Words from the word wall and also random words will be used. The teacher will start the game, but if students feel comfortable, they will also be invited to draw pictures on the board. This will help reinforce vocabulary and also give students an opportunity to recognize a visual representation of a vocabulary word. Following the activity, students will learn how to identify math word problems. To address whether or not students acknowledge mathematics in a word problem, first the teacher will model two types of problems: one involving mathematics and one that doesn’t. Students will see how to find key vocabulary words that help identify the problem. Using white boards, students will be shown a word problem and be asked to identify the type of problem—they will also be given a hard copy to mark the text. After identifying math/non-math problems, class will continue by learning how to identify different types of mathematics, specifically linear equations, proportions, geometry, and solving equations. Using the same structure, the teacher will model, then provide students the opportunity to practice.
### Day 6: Word Problem-Solving Strategies

Objective: Students will learn of and apply two problem-solving strategies for word problems that involve linear equations.

Overview of the lesson: The beginning of class will require students to answer a few questions about vocabulary words. Questions may include “What does _____ look like?” or “How would you explain __________ to another person?”. Using a cold call strategy, student responses will be collected out loud at random. Following the review, students will be introduced to a problem solving strategy called “Survey, Question, Read”. The teacher will model how to use the strategy in a word problem, then give students an opportunity to use the strategy on a select number of questions. Students need not answer the question—the goal is to simply get comfortable using the strategy while reading a word problem. The second strategy that will be modeled and practiced is the “Paraphrasing Strategy”.

### Day 7: Review Word Problem Solving Strategies

Objective: Students will review and apply problem solving strategies previously learned. Students will also learn another problem solving strategy.

Overview of the lesson: The group will collectively review the problem-solving strategies previously learned. The class will discuss how to use the strategies. The strategy that students will learn is called “Visualization”, where students create a visualization of the problem by drawing a diagram or a pictorial representation of the problem. The teacher will model the strategy and give students the opportunity to practice the strategy. At the end of the hour, students will be asked to provide feedback on problem solving strategies taught in the course.

### Day 8: Writing Solutions

Objective: Students will use the strategies taught in class to write up a solution to a problem using math vocabulary and breaking down the process.

Overview of the lesson: The lesson’s focus will primarily be on writing out solutions to a math word problem involving linear equations. To help students develop writing skills in math, the teacher will provide a model for students about how to write a solution to a problem. Students will be given a problem (the same problem for the whole class) and have questions to prompt responses using vocabulary. There will also be emphasis on using problem solving strategies learned in previous lessons.

### Day 9: Continue Writing Solutions

Objective: Students will assess responses from problems used the day before to discuss what creates a quality solution to a math word problem.

Overview of the lesson: A few short problems will be listed on the board. Students will be given a few minutes to write out solutions to the problems, then be placed in groups to read
responses and provide feedback. The teacher will emphasize two major items: does the solution make sense, is it possible to understand how to solve without a math background? The class will then discuss a few examples from the day prior—the teacher will show examples on the board and lead the class to assess the quality of the response. Responses will be typed so writing will not be recognizable. Following this exercise, students will work in cooperative groups to write solutions to problems. The cooperative groups will focus on: a student writes down the first step, the sheet gets passed to the next student, that student reads and provides feedback, and then writes the second step. This continues until the solution is complete.

**Day 10: Using Problem Solving and Writing Skills to Solve Word Problems**

Objective: This cumulative day will provide students the opportunity to use all of the skills learned over the unit to solve word problems involving linear equations.

Overview of the lesson: The class begins with a vocabulary activity where students give hints to another student to guess the word on their back (called “Vobackulary”). Following the activity, students will be given the same word problems from the beginning of the unit and complete the same survey questions like before. After the survey is complete, students will simply provide the teacher feedback on what they enjoyed about the unit and what they would change if possible. This may lead in to expanding the unit if students desire more practice with the strategies.
Appendix B: Worksheets from Project Outline

Student Survey
Linear Equations Word Problems

Read each problem and answer the survey question to the best of your ability. You do NOT need to solve the math problem.

1) A cell phone company charges $0.10 per text and a flat fee of $40 for 150 minutes a month. Write an equation that represents the total cost, c, for number of text messages sent, t.

What is your confidence level to answering question #1?
Very Confident-------Somewhat Confident------A little Confident------Not Confident

2) A server earns $8 per hour at a restaurant. During one four-hour shift, the server earned $25 in tips. How much did the server earn for that shift?

In your own words, what is question #2 asking you to do?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

3) Jason put a $125.00 down payment on a motorbike and is making monthly payments of $70.00. Write an equation that represents the number of months it will take for Jason to pay off his bike.

Do you know what question #3 is asking of you? Yes--------No

4) Shannon graphed the line y = 2x + 1. Sean then graphed a line with a slope steeper than Shannon’s. Give an example of what Sean’s equation could possibly be.

Are there any math terms in the problem you don’t know? Yes--------No
If yes, what is/are the word(s)?
________________________________________________________________________
________________________________________________________________________

5) The equation y = 0.1x – 2.5 represents Barb’s earnings each day at her sales job, selling x magazines. What does the slope represent?

What would be your answer for problem #5?
________________________________________________________________________

Explain your answer.
________________________________________________________________________
________________________________________________________________________

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Slope Exploration Worksheet

(adopted from (Briars, Cook, & Lynn, 2013))

Slope Exploration

Use the following graph to answer questions #1-3.

1) Find the slope of each line.
   Line A: ___________   Line B: ___________   Line C: ___________

2) Which line is the steepest? Explain your answer.
   ___________________________________________________________
   ___________________________________________________________
   ___________________________________________________________

3) Which line would model a student spending their money on a monthly basis? Explain your reasoning.
   ___________________________________________________________
   ___________________________________________________________
   ___________________________________________________________

4) Another way to define slope is constant rate of change. What is something you know moves at a constant rate of change? Explain your answer.
   ___________________________________________________________
   ___________________________________________________________
   ___________________________________________________________

(continued on next page)
5) The community center is draining a pool for cleaning. Picture the water being pumped from the swimming pool, and answer the questions below.

a. Do you think the water drains at a constant rate of change? Explain.

b. What does it mean to drain at a constant rate?

c. Complete the table showing the volume of water in the pool at different times.

<table>
<thead>
<tr>
<th>Time Elapsed (hours)</th>
<th>Volume of water in pool (gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>750</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

d. Find the rate of change. Include units.

e. What does the rate mean in the context of the pool?

f. Draw a sketch of the graph of the pool draining data. Label the axes.
g. Explain why a negative slope makes sense for this situation.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

h. What kind of situation would be described by a positive rate of change for the pool situation?

________________________________________________________________________
________________________________________________________________________

i. Draw a sketch of the graph if the rate of change in the pool context was positive. Label the axes.

6) You and your friends are going to a movie. It costs $6 per ticket plus a flat fee of $5 to reserve your seats in the theater.
   a. Use the information to fill in the table below.

<table>
<thead>
<tr>
<th># of people</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

   b. What is the rate of change? Explain.

   (continued on next page)
c. Graph the data from the table. Label your axes.

d. If the cost per ticket changed to $10 per person, how would that affect the line on the graph? Explain.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

e. If the cost per ticket changed to $2 per person, how would that affect the line on the graph? Explain.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

7) How does rate of change affect the data in a graph or table?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Real-World Problems Used in Word Problem Break-Down

(Note: these questions were adopted from worksheets and assessments given in the 8th grade Linear Algebra class)

You are going to a concert and need to save money. You have $30 right now and you save $4 per week. The initial fee to go to a party is $50 for room rental. It then costs $6 per person.
You have $20 and spend $3 a day for lunch.
You have $25 right now and plan to save $5 per week.

A babysitting company charges $5 an hour after an initial $25 fee.
A snow-shoveling service offers a basic care package for $10 plus $15 per hour of shoveling.
A dog-sitting company charges an initial fee of $40 for watching your dog plus $20 a day.

A cell phone plan costs $40 per month after a $60 initiation fee.

A lawn service offers a basic care package for $50 plus $10 per hour of planting.

You are saving for a new phone. You have $25 right now and you save $5 per week.

A video streaming website charges a $10 yearly membership and $0.50 per video.

A camera shop charges $4 for an enlargement of a photo. Enlargements can be delivered for $1.50 per order.

You are saving money to buy a new iPhone. You have saved $35 so far and plan to save $5 per week.

A printer can print 15 pages each minute.

You hike 5 miles before taking a break. After your break, you continue to hike at an average speed of 3.5 miles per hour.

A boat rental company charges $50 to rent a boat and $8 per hour of use.
A bicycle rental store charges $12 to rent a bicycle and $4 per hour of use.

Amanda earns $6 per hour of babysitting. She charges an extra $3 for travelling.

A child starts a savings account with $100. He later adds money to the account at a constant rate of $25 each month.

Alma plants 3 rows of tomatoes with n plants in each row. She also plants 1 row of carrots with 5 plants in the row.

You are going to a concert and need to save money. You have $30 right now and you save $4 per week. The initial fee to go to a party is $50 for room rental. It then costs $6 per person.
You have $20 and spend $3 a day for lunch.
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A babysitting company charges $5 an hour after an initial $25 fee.
A snow-shoveling service offers a basic care package for $10 plus $15 per hour of shoveling.
A dog-sitting company charges an initial fee of $40 for watching your dog plus $20 a day.

A cell phone plan costs $40 per month after a $60 initiation fee.

A lawn service offers a basic care package for $50 plus $10 per hour of planting.

You are saving for a new phone. You have $25 right now and you save $5 per week.

A video streaming website charges a $10 yearly membership and $0.50 per video.

A printer can print 15 pages each minute.

A boat rental company charges $50 to rent a boat and $8 per hour of use.
A bicycle rental store charges $12 to rent a bicycle and $4 per hour of use.

Amanda earns $6 per hour of babysitting. She charges an extra $3 for travelling.

A child starts a savings account with $100. He later adds money to the account at a constant rate of $25 each month.
Problem-Solving Strategy #1: SQR
1. The initial fee to go to a party is $50 for room rental. It then costs $6 per person.
   a. Write an equation to represent total cost, \( T \), for \( p \) people to attend.
   b. How much would it cost for 10 people to attend the party?

2. A cell phone plan costs $40 per month after a $60 initiation fee.
   a. Write an equation to represent total cost, \( T \), for \( m \) months.
   b. What would be the cost of your bill after 5 months?

3. A dog-sitting company charges an initial fee of $40 for watching your dog plus $20 a day.
   a. Write an equation that represents total cost, \( T \), for \( d \) days.
   b. If your bill cost $120, how many days was your dog at the dog-sitting company?
Paraphrasing Guided Practice

Problem-Solving Practice: Paraphrasing

1. Amanda earns $6 per hour of babysitting. She charges an extra $3 for travelling.
   a. Write an equation that represents the total amount, \( T \), Amanda makes for babysitting \( h \) hours.
   b. If Amanda works 10 hours, how much money does she make?

   Read the question.

   Underline or highlight any important information/numbers.

   Restate the problem in your own words.

2. A boat rental company charges $50 to rent a boat and $8 per hour of use.
   a. Write an equation that represents the total cost, \( T \), to rent the boat for \( h \) hours.
   b. If the total cost to use the boat was $118, how many hours was it rented for?

   Read the question.

   Underline or highlight any important information/numbers.

   Restate the problem in your own words.

3. A video streaming website charges a $10 yearly membership and $0.50 per video.
   a. Write an equation that represents the total cost, \( T \), to upload \( v \) videos.
   b. What would it cost you to upload 25 videos?

   Read the question.

   Underline or highlight any important information/numbers.

   Restate the problem in your own words.
Visualization Guided Practice
Problem-Solving Strategy Practice: Visualization

1. You are saving money to buy a new iPhone. You have saved $35 so far and plan to save $5 per week. Write an equation to represent your total savings, T, after w weeks.

   Read the question.

   Underline words that represent images.

   Draw a picture to represent the problem.

2. A snow-shoveling service offers a basic care package for $10 plus $15 per hour of shoveling. Write an equation to represent your total cost, T, of shoveling h hours.

   Read the question.

   Underline words that represent images.

   Draw a picture to represent the problem.

3. Alma plants 3 rows of tomatoes with n plants in each row. She also plants 1 row of carrots with 5 plants in the row. Write an equation that represents the total number of vegetables, T, of n plants.

   Read the question.

   Underline words that represent images.

   Draw a picture to represent the problem.
**Writing Solutions Guided Practice Day 1**

1. A video streaming website charges a $10 yearly membership and $0.50 per video.
   a. Write an equation that represents the total cost, \( T \), to upload \( v \) videos.
   b. What would it cost you to upload 25 videos?

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<td>What problem-solving strategy are you going to use?</td>
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<tr>
<td>What is the answer to the problem?</td>
</tr>
<tr>
<td>Explain how you got your answer using words.</td>
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2. The initial fee to go to a party is $50 for room rental. It then costs $6 per person.
   a. Write an equation to represent total cost, \( T \), for \( p \) people to attend.
   b. How much would it cost for 10 people to attend the party?

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Writing Solutions Guided Practice Day 2

(Note—structure of this worksheet adopted from (Tuttle, 2005))

1. A dog-sitting company charges an initial fee of $40 for watching your dog plus $20 a day.
   a. Write an equation that represents total cost, T, for d days.
   b. If your bill cost $120, how many days was your dog at the dog-sitting company?

Read the problem.

Write your solution.

-Paragraph One:
  -What is the problem about?

-What are you supposed to find?

-Paragraph Two:
  -Explain step-by-step and in detail everything you do to find the answer. (First I... Then I... Next I... After that I... Finally I...)

-Paragraph Three:
  -My answer is. .

-My answer makes sense because. .
2. A boat rental company charges $50 to rent a boat and $8 per hour of use.
a. Write an equation that represents the total cost, \( T \), to rent the boat for \( h \) hours.
b. If the total cost to use the boat was $118, how many hours was it rented for?

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<td><strong>Write your solution.</strong></td>
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