Manipulatives in the Math Classroom

Advisors signature

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Abstract

The use of manipulatives have been a big debate with math teachers. Are they helpful? Do they distract students? What makes a good manipulative? What are the manipulatives that are distracting? Can the use of manipulatives help middle school students bridge their learning from concrete to abstract? In a middle school math classroom setting, teaching concepts were used in different ways to help understand if manipulatives would help students understand the math concepts better than students did the year before. Test scores were compared from the previous year when manipulatives were not used to the current year when manipulatives (including Montessori manipulatives) were used. A comparison was also made between manipulatives to determine which ones were more effective. Students were also able to choose a pencil and paper method if that that is how they considered that they learned best. Results were compared for a the same two month period for the current year with the past year. These results will help guide my teaching in the future. So far tests scores, when compared to last year, have increased. This leads a math teacher to believe that manipulatives can affect a students ability to retain and use information more effectively on a test with real world problems as opposed to no manipulatives being used.

Introduction

Manipulatives have been a part of my teaching career since the beginning. The big debate has been whether they will help or hinder a student from learning a new concept. In mathematics, a manipulative is an object, which is designed so that a learner can perceive some mathematical concept by manipulating it, hence its name. The use of manipulatives provides a way for children to learn concepts in a developmentally appropriate, hands-on way. In my research I wanted to find out if Montessori manipulatives would help students connect their learning of math concepts from previous grades, especially if they came from a Montessori school. Through my research I wanted to find out how manipulatives could be used to enhance understanding of a math concept and help students move from concrete to abstract.

Literature Review

Pros of Using Manipulatives

Manipulatives bring a lot of different perspective to a lesson that pen and paper may not be able to teach a student. They can help a students explore both visually and tactilely. Students get a hands on experience that helps them connect their hand movement to what they are learning (McNeil, 2009). This is most helpful because McNeil explains that "children do not come in to the world with the capacity for abstract thought." They can, however, grasp these abstract thoughts through their interactions with concrete objects during their lessons. Specifically, she argues that manipulatives provide students with additional resources that can help students draw on the practical, real-world knowledge; lastly, they induce physical action which enhances memory and understanding(Mcneil, 2009). It has also been stated that it is easier for children to understand a mathematics concept with the support of concrete materials then when something is presented with out it (Uttal, 2009). One point that Ella Shoval stated was that manipulatives or movement had the greatest affect on lower achievers than other students (Shoval, 2011).

Cons of Using Manipulatives

In my research I also found the contrast to be true when it comes to using manipulatives for a lesson if an object is not used correctly. The teacher has to be very careful how to use the object so that it is useful. Teachers may sometimes use them just for the sake of adding variety to their classroom instead of using them to engage a student and enhance the new concept or help the student go from concrete to abstract. You can not assume that because a student is interacting with a manipulative that the child will immediately learn the math concept(McNeil, 2009) .

One argument states that, "Manipulative might lead students to focus on having fun at the expense of deep learning. Second, manipulatives might make learning more difficult because they require dual representation" (McNeil, 2009, p. 309).A teacher needs to be able to answer these certain questions about the objects that they may use to enhance the lessons. For example, Does it effectively build students' conceptual understanding of mathematical equivalence and help students prepare for writing and solving equations, or does it divert students' attention away from the symbolic notation of mathematics to something else (McNeil, 2009)?

Some findings found that low-income children need to balance their math curriculum with more emphasis on higher order mathematics content and less on math manipulatives and math games. Teachers Should focus and provide more learning opportunity on learning higher order content, such as telling time, using measurement tools, accurately estimating quantities and knowing the value of coins and cash (Wang , 2009).

What is a manipulative?

Teachers need to be very careful when selecting an object to teach a new concept or enrich a concept already taught. The manipulative needs to be represented as an object and as a symbol of a mathematical concept or procedure (McNeil, 2009). Manipulative such as; card sorts, dice, cards, three dimensional objects and versatiles. The world is different today and we do not know what modern tools Maria Montessori would have used in the classroom. A computer manipulative might help facilitate the development of mathematical thinking, which she may have thought was an appropriate way of teaching mathematics, depending on the use. Uttal states that, “Computer manipulatives provide the ideal level of guidance and direction for students.” He also says that “Work with concrete materials in ways that allow them to make the proper connections and to see the mathematical concepts inherent in the materials” (Uttal, 2009, p. 139).

Calculators were another manipulative brought up in researching helpful tools for mathematics lessons. Manipulatives need to be something that will “stimulate or inspire as many students as possible to realize their potential, not only master skills but to learn important mathematical ideas- with – out feelings of frustration and despair” (Margaritis, 2003). Margaritas felt that calculators do not hinder basic skills and can stimulate and inspire uninspired students or students that feel unsafe in math class. They give them a safety net. They provide the students with a tool that fosters learning and thinking by providing immediate feedback that will support their reasoning and enabling them to visualize their actions and learn from their mistakes; Calculators also help students see their entire operation and sequence of events of a problem. Students can make vital connections, and practice critical skills that they will be able to use in the real world. Calculators are not a solution to a problem, but a very important tool that allows students to explore concepts and ideas that otherwise would be difficult to grasp. (Margaritas, 2003).

What is not a manipulative?

Teachers need to be very careful when choosing an object to enhance their math lesson. “If children are very familiar with a particular object as a toy, it may be difficult for them to view the object as a mathematics symbol” (McNeil, 2009). Manipulatives are not using pen and paper, taking notes, being present at a lesson.   Another obstacle is if a student has seen an object used in teaching one concept and is told to use it in another way. This can cause confusion and thus have the opposite effect of that student making a connection to the math concept (McNeil, 2009). Another misconception is that educators believe that children are learning simply if they are busy working on something (Uttal, 2009).

Montessori on Manipulatives

Maria Montessori had a lot to say about using manipulatives in a math classroom. She believed that true mathematical understanding came as a result of taking the time to connect the hands to the mind and the abstract and theoretical to the practical and meaningful (Morrison, 2011). This argument is formed because the research states that children's thinking is inherently concrete. Using manipulatives helps to bridge the gap for students to learn the concept in a more real-world or abstract way (Uttal, 2009).

Mathematics is a critical part of academic preparation of the middle school child, teachers need to resist hurrying children through the study of mathematics (Morrison, 2011).

Some of the materials Maria Montessori wanted in her math classroom was, concentric figures, wooden cylinders, red rods, geometry cabinet, binomial cube, whiteboards and markers among some other objects. All of the materials has students moving, tracing and counting to learn. "Montessori teachers sometimes discuss the importance of children making up their own problems, rather than being assigned problems by a teacher, thus giving the child more ownership of the work as opposed to the idea that work is for the teacher" (Lillard, 2008, p.28).

Abstract Vs Concrete Learners:

There are two types of learners; abstract learners and concrete learners. A lot of the middle school students are still stuck in a concrete way of thinking, which means they are practical and they appreciate being taught by lectures. Abstract thinkers are more imaginative, emotional and holistic. They like to experience things and create things. When teaching math it is important for students to be able to reach a more abstract way of thinking. They need to be able to take the concepts being taught on a concrete level and bridge them in to an abstract way so there is a connection beyond the practice and it can be used in the real life examples on an MCA test.

Research Design

Purpose:

The purpose of my Action Research topic, was to see if students can learn a topic more abstractly using Manipulatives that they may or may not be familiar with from their elementary classrooms.

Question:

How can the use of Manipulatives enhance a student retaining a new math concept, over using standard junior high teaching methods, like pen and paper?

Sub-Questions:

1) Do Manipulatives help a student learn a math concept more abstractly?

2) Does it matter if the manipulatives are from a Montessori elementary classroom?

4) Does it matter if the students is an abstract or concrete learner?

5) What other methods help students grasp new concepts and help the student move from concrete to abstract?

Participants

I used my students from my prealgebra math class that I taught last year. There are 8 students I taught in that class, on whom I will be concentrating most of my findings.

Setting:

The study will take place inside my classroom at a Midwestern urban Montessori middle school.

Materials:

I used manipulatives from the Holt math materials to teach concepts. I also recreated Montessori manipulatives to use in my classroom. I created card sorts in keeping with Montessori methods. I used versatiles in my lessons, which are squares that have numbers on them and you have to move them in the right spots with the right answers, if you get it right it will create the design desired. I gave a test to determine abstract vs. Concrete learners. The students will be able to pick worksheets to practice as well. The students will use Ipads to do buzzmath lessons, which are standard based lessons that students do on the computer. They involve moving things around on the Ipad surface. Objects I used were: dice, cards, 3 dimensional shapes, nets, pennies and spinners.

Data Collection

I collected data from January to April. I used careful *observation* to collect the majority of my data. I compared test scores from last year, when I did not have these manipulatives to this year when I did use manipulatives and noted the change. I do have a lot of students that are the same from last year so it will be a fair comparison. I found that last year a lot of the students were struggling with connecting the math problems to the real world so my plan was to create my formative assessments so that they ranged conceptually from concrete math questions to more abstract to see if students this year would make better connections.

The MCA test that is given at the end of April was good indicator to look at for improvement with my students. The questions are very abstract. I compared the results to last year's test results. I also surveyed my students and asked them how they felt they learned a concept best. I think a lot of them had a good idea of what helps them learn. I compared their own self assessments to their test scores to see if they were accurate about what helps them learn and if it was correlating to their math outcomes.

Student	Type of learner	2014 MCA	2015 MCA	Versatiles	Ipad interactive	worksheet	Puzzle	objects	Money	Montessori Elementary
Laquan (1)	Concrete	624	620	5	10	20	15	5	5	yes
Newjai (2)	Concrete	631	640	20	25	10	20	15	5	yes
Chueshi (3)	Concrete	622	628	25	20	5	25	20	10	yes
Larry (4)	Concrete	630	640	22	25	10	20	17	7	no
Victoria (5)	Abstract	643	650	15	20	15	10	10	0	no
Mattiyoss (6)	Abstract	633	630	0	20	25	10	5	0	yes
Nayelli (7)	Abstract	625	630	25	10	10	24	10	10	no
Chambrey (8)	Concrete	629	642	5	5	30	10	5	5	no

Table 1.

This is the breakdown of the 8 students I had last year and this year. I kept track of what they used on their own during shelf work along with our lessons. I also wrote down what they self assessed themselves for their type of learner. The number under their name is what I used for in the table of data instead of their names.

I used a survey for my students at the beginning of the year and towards the end of my collection of data in April. The tables of their results are on the next page as well. You can find the survey in appendix A.

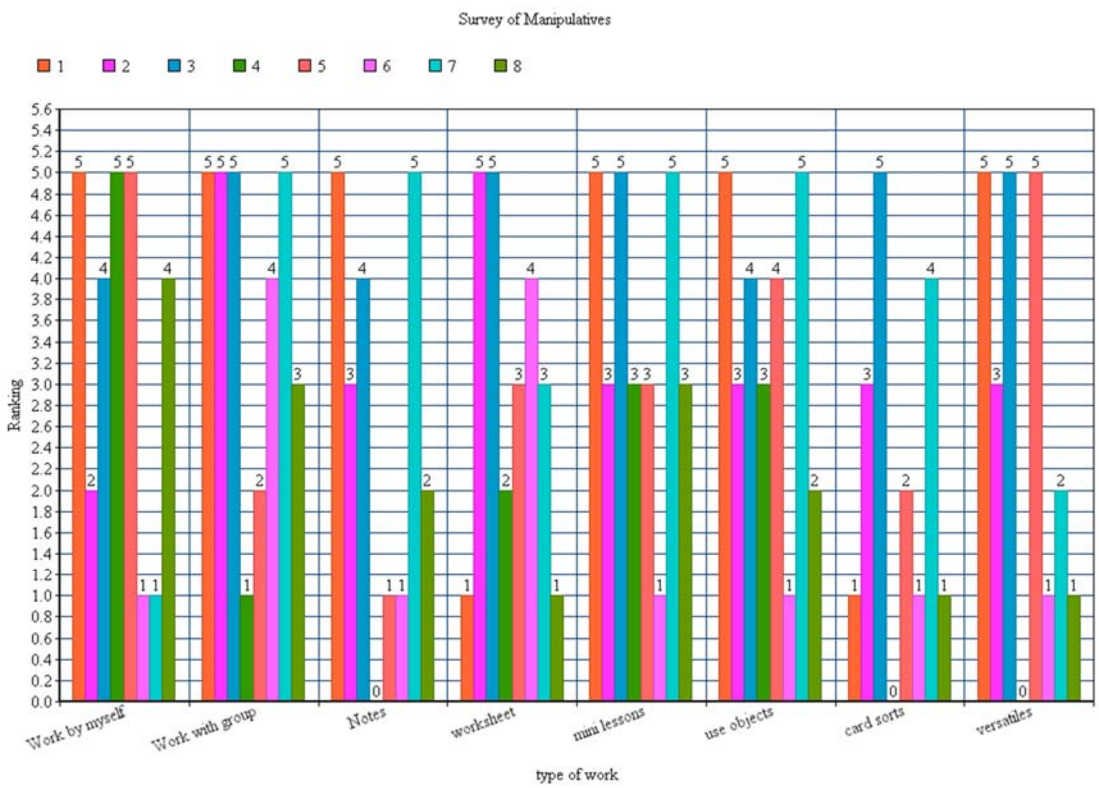


Figure 1.

At the beginning of the year this is how my students felt they learned best. The highest was working with a group, working by themselves and coming to mini lessons. I wanted to make sure I incorporated those things but also add some manipulatives to their learning. Some of them I could see needed a push to start using these objects. The students who struggled to use them were labeled as a concrete learner. See appendix A for survey.

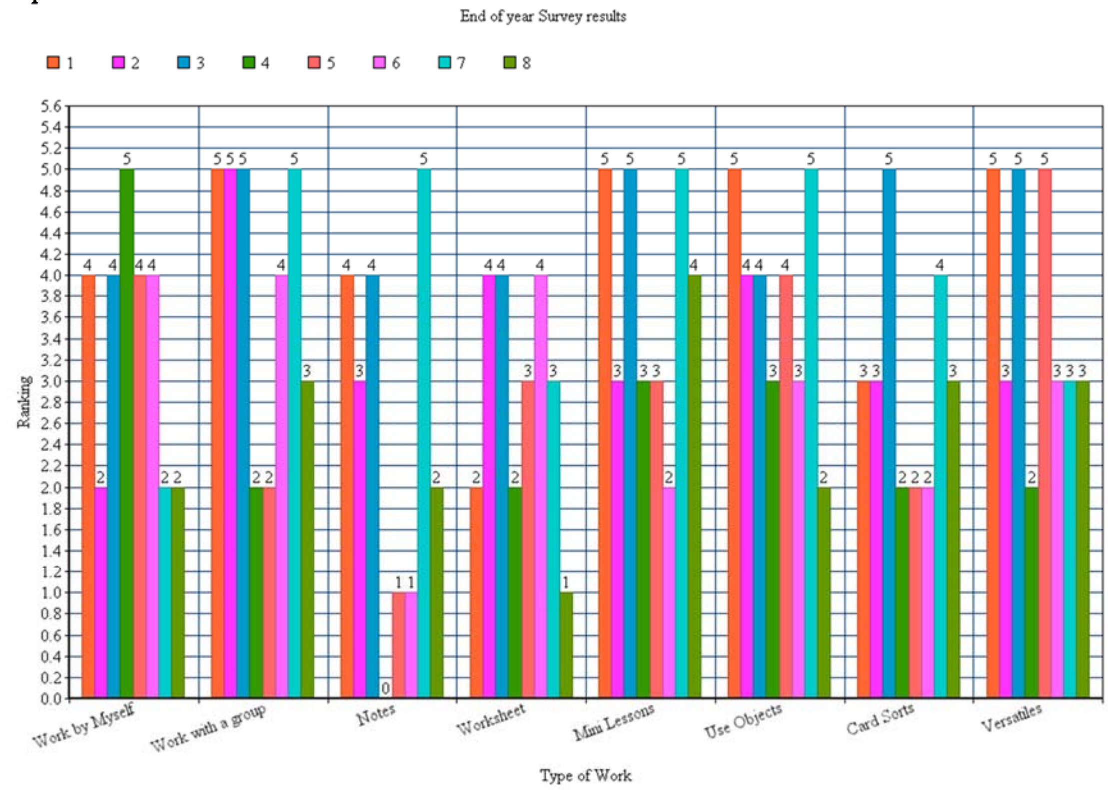


Figure 2.

When comparing the ranking from the beginning of the year with the same 8 students I noticed that on average work by myself went down, work in groups was the same or went up for some students. I think work with groups increased for some because I was very deliberate in teaching the students how to use their groups better. They learned how to ask them questions and get to answers instead of copying them or always going to the teacher. I also gave them manipulatives that had controls built in to them so they could find out if they were right or wrong with in their groups instead of always checking in with the teacher. Taking notes was still unpopular. Mini lessons stayed the same with in this group of students. The students liked the versatiles, card sorts and objects. I was more intentional with the uses of these objects as well. I also made some students try them at least once instead of it just being an option. I'm glad to see that they did start to enjoy them and choose them more on their own. This does not include activities that were hands on during the lessons. I will talk more about that in my journal.

I analyzed the test data from last year to the same tests given this year to students I taught. Overall there was an increase in test scores for which I used manipulatives. Not every individual students went up but on average the class as a whole increased their test scores from the previous year. I found that the individual students that did not increase their score had a smaller number of times of using the hands on manipulatives then students who did use more manipulatives to practice with. There was a positive correlation between test scores and number of uses with manipulatives. It is important to understand our scale in math for grading.

1= n (not passing), 1.5= D, 2 = c, 2.5 = B, 3 = A, 3.5- 4 = A + (above the standard)

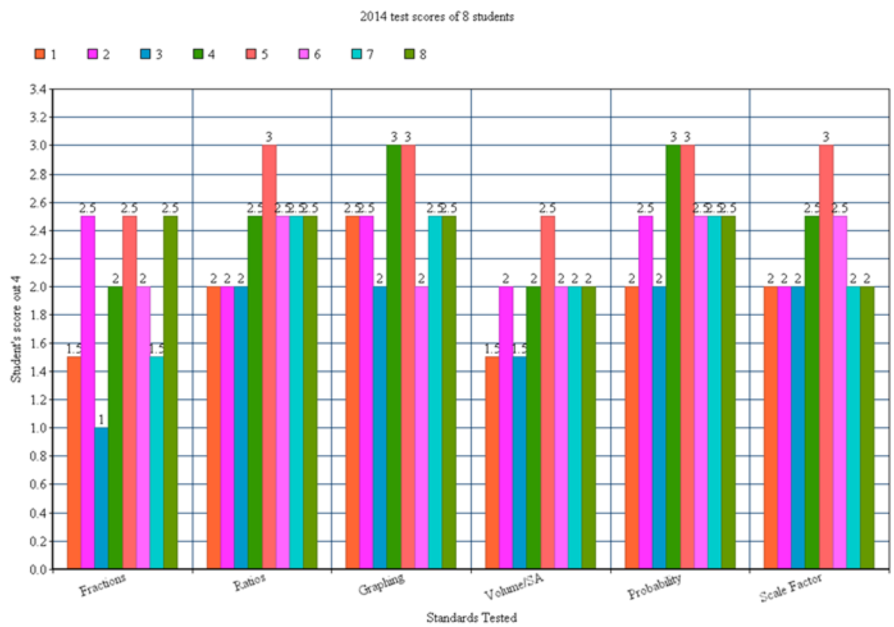


Figure 3.

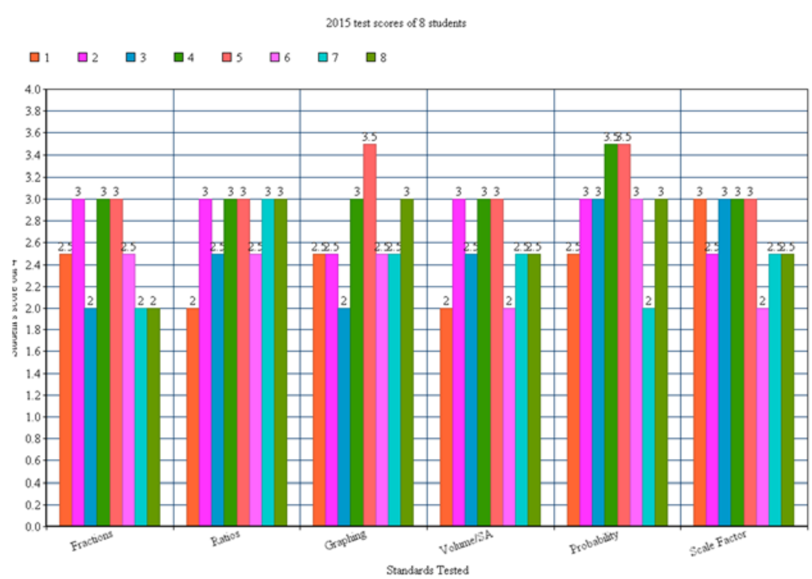


Figure 4.

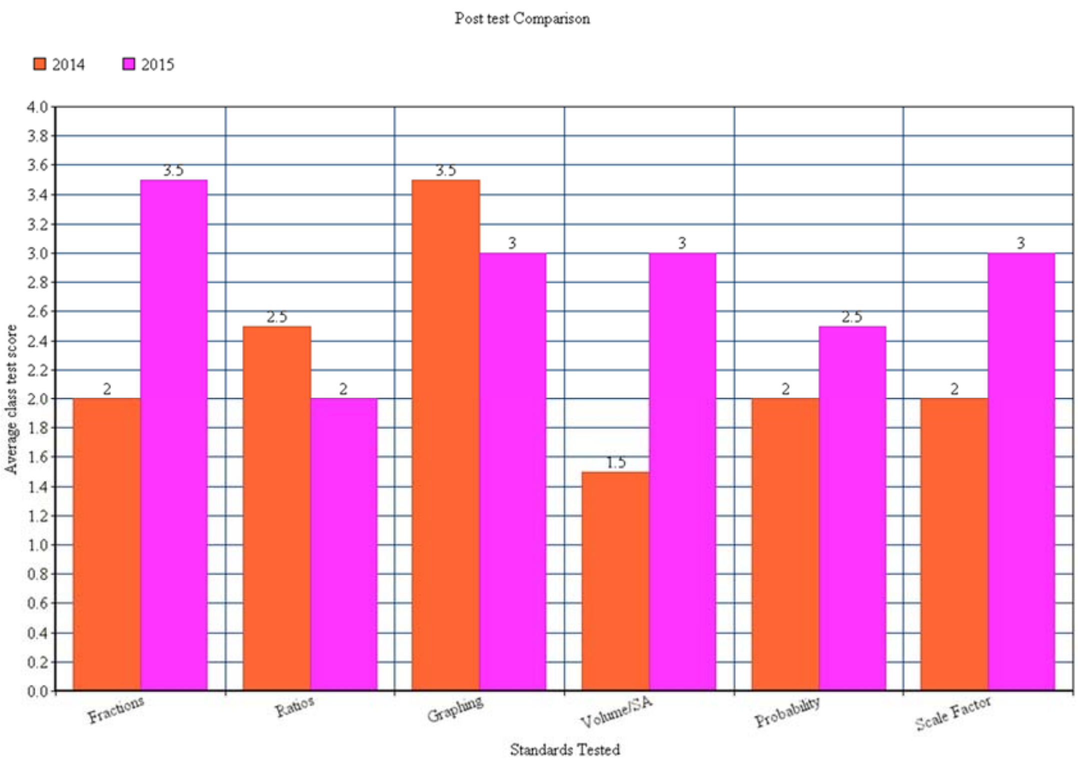


Figure 5.

All students increased their test scores overall. It was a range between 4% to 42% increase.

Data Analysis

To analyze my data, I will look at my notes from my daily observations and my charts where I record daily observations such as how often I saw students on task, how often students chose to work on something hands on or the practice sheet, and the correlation to their test scores.

Jounral Entries:

Fraction unit (1/5 – 1/16)

This is the first unit I observed the classroom and the students I had last year to see what types of materials they would grab to help them in their learning. The extra materials I had out for their individual use was; money, dice, cards and a card sort. Students were very shy about pulling the money off of the shelf. They thought it was too much of an elementary tool to use. Some students grabbed it for play the first day and I had to put a stop to that. Once I showed a couple students, who I know struggle with fractions, how to use the money to help, I think they really started to wrap their brain around the real world problems instead of thinking only in naked math problem ideas, which are problems with just numbers and no words. Overall in my class students increased their test scores by 1.5 points, which in our scale correlates to a grade and a half. The students that used the money the most, improved their scores, Nayalli used it 10 our of the 10 activities time and increased by a half of a grade, Larry used it 7 out of 10 times and went up a grade from B to A and Chueshi used it 10 out of 10 times and he also jumped a grade level from a D to a C. See appendix J for test.

Ratio unit

During this unit I struggled to find good manipulatives that would help students understand ratios a little bit more abstractly. We started the unit off with a candy jar and 2 different candies that they would have to compare in the jar. This brought us to finding equivalent ratios and proportions. I also did an "orange juice" task to show them the ratio of putting more cool aid in some mixes and less in others to see the difference. They had 1 hands on card sort to work through, some puzzle sheets and a versatile to manipulate. After their first post test on this unit on average the class went down by 0.5 or a half of a letter grade. I think that I was thinking too hard about using too many things and forgot about teaching some of the basics that just need to be practiced more, like the procedures to finding the proportions. It brought me back to making sure I am not just using manipulatives to use them, but that they have a real purpose to them. See appendix K for test.

Graphing 1/19- 1/30

Graphing unit consists of having the students understand how to graph coordinate planes, how to find points and what is on a coordinate graph. For this unit I used dice to have students do an activity with rolling two dice and graphing the coordinate; I had them get up and move from their own origin marked on the floor and then they moved their bodies left, right, up or down based on the coordinate directions. Other activities included; puzzle sheets to solve riddles with graphing, and students did a card sort to place the name correctly on the coordinate grid. The activity that I thought was the most helpful was the activity where students had to stand up and actually move to the correct point on the graph. It helped them understand the movement better, and gave me a good idea of what students were struggling to understand where the points are supposed to be on a graph. I had more students understand the different parts of the coordinate grid this year with the use of the card sort then I had in previous years. This was an activity that I had all of my students doing and using. On average the class still passed this standard but went from a 3.5 last year to a 3 this year. I struggled with this dip even though they were passing because I wasn't sure what activity could help the students understand graphing in a more abstract way. See appendix L for test.

Volume and surface area 2/17 – 2/27

In this unit I brought out a lot of 3 dimensional shapes, I had students doing activities with filling up shapes and measuring, making nets, which are flat versions of 3 dimensional shapes that they had to construct, building different shapes and measuring the dimensions. I had students walking around the building pointing out shapes that they had seen. Students, during these lessons, really got in to the versatiles and I think that is because students struggle a lot with the formulas and versatilies are a good way to check yourself. This year I focused more on how the formulas come about and not just on plugging numbers into the formula. I was very excited about the students and their test scores for this unit since I felt this is the unit I have changed the most in my teaching. I think I was always worried about pulling out the messy manipulatives, like filling up water or rice, but these kids need to actually do things to understand what they are learning. The students average went from a 1.5 to a passing grade of 3. The biggest jump I had from last year to this year. See appendix M for test.

Probability 3/2 – 3/14:

This unit had the least amount of change from last year to this year. I think it is hard to teach probability without dice, cards, spinners or pennies. The students always like the activity they are doing for the day but some find it hard to connect it to the bigger abstract problems. I struggle with finding a way to make that more apparent connection. Probability is a big unit that will come up for them later in high school. It's always fun to teach but hard to reach the concrete learners even with all the manipulatives being used. The test scores stayed at an average of a B from both years. See appendix N for test.

Scale factor 3/10 – 3/28:

For the unit of scale factor I had the students doing a lot of hands on measuring. They had to measure shadows and the object to figure out the height of tall things like trees, or bushes that they might be able to find with out knowing the scale factor. We also talked about blue prints and how to expand a picture or make a picture smaller. They had to use scale factor with a drawing they created by enlarging the picture. I found that the students were very engaged during these lessons. I asked a few students what they liked about the unit and their responses were that it seemed like something they could use in the real world or for a job. Their test scores went from a 2 to a 3 so from a C to an A average. This unit made me realize the importance of connecting it to their real life and not just finding something for them to manipulate. See appendix O for test.

Individual students

Student one improved on all tests by an average of half a letter grade. The only two he stayed the same at were Ratios and Graphing. He struggled at the beginning to use manipulatives but once he was shown how they can help he really grasped on to them and enjoyed them. He did go down on the MCA test and I think is a result of him still struggling to get to the abstract with out the manipulative in front of him at testing. It was fun to see his growth on the benchmark tests in class. See appendix B for graph.

Student two improved on the MCA and the benchmark tests; fractions, rations, volume and scale factor. He went down with probability and stayed the same with our graphing unit. He was a student that enjoyed using manipulatives from the beginning. He had a language barrier with the word problems so introducing him to the hands on materials was helpful for him to make deeper connections. See appendix C for graph.

Student three was one of my lower students to begin with this year. He did improve on the MCA test this year and had big improvements on the benchmark tests, see appendix (,) for tests. The only test he didn't improve on was graphing but he stayed the same as last year. He was another student that needed to be pushed to use manipulatives this year. He really enjoyed using the money for our fraction unit and went up a letter grade for his test. See appendix D.

Student four became a more serious student this year compared to last year. He would always choose the card sorts or hands on activity to learn the concepts. He told me he enjoyed the independent activities and the built in controls with the card sorts. He improved on all tests and stayed proficient with graphins. See appendix E.

Student five was labeled abstract and I believe the closest to that in my classroom. She never felt the need to use the hands on activities. She would do them with her group at times and when I had them be a part of a lesson but she was capable of doing the real world problems with out the use of manipualtives. She was the only student out of the 8 to be proficient on the MCA test. She was also proficient on all the benchmark tests in class. See appendix F.

Student six had a hard time this year taking math seriously. He was absent 20 times this year so was always struggling to keep up with the class. He went down for his MCA score and most of the benchmark tests. The only one he improved was the graphing unit. See appendix G.

Student seven made a lot of gains this year. She was one of my lower students that really grasped and enjoyed using the manipulatives. She love the money unit and understood the connection between fractions and using the money. She improved on most test except for probability. See appendix H.

Student eight made the biggest gain on the MCA test. She went up 14 points and went from not proficient to partially proficient. She embraced the new learning style the most and was a great teacher to other students who weren’t as enthusiastic about using them as she was. It makes me excited to use more of the student teaching student model when I see some students appreciate being taught by their peer. See appendix I for graph.

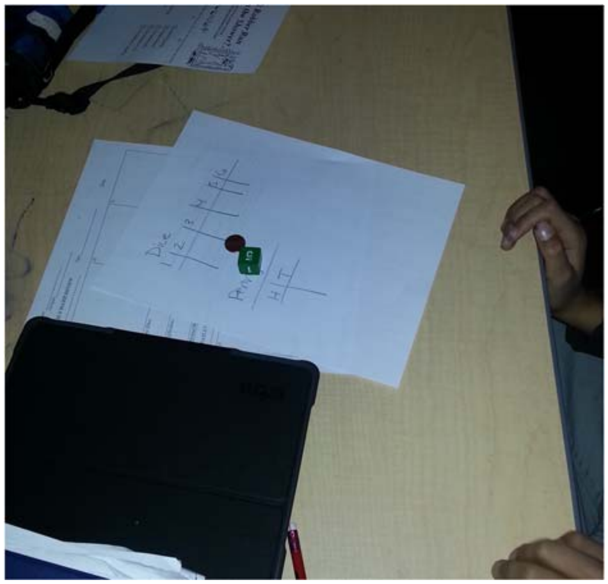


Figure 6.

Using dice and pennies for probability unit. The students like these lessons and using these manipulatives.

These are the manipulatives that did not change as much from last year.

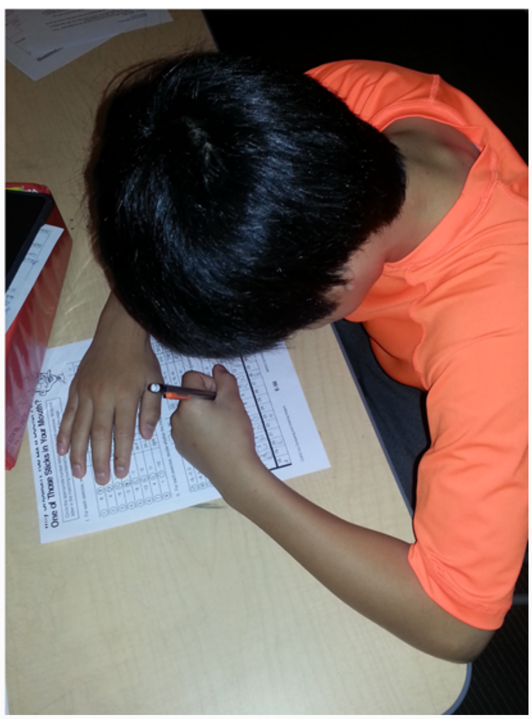


Figure 7.

A student working on a puzzle sheet. Students liked these because they had a built in control and solved riddles when they got the right answers.

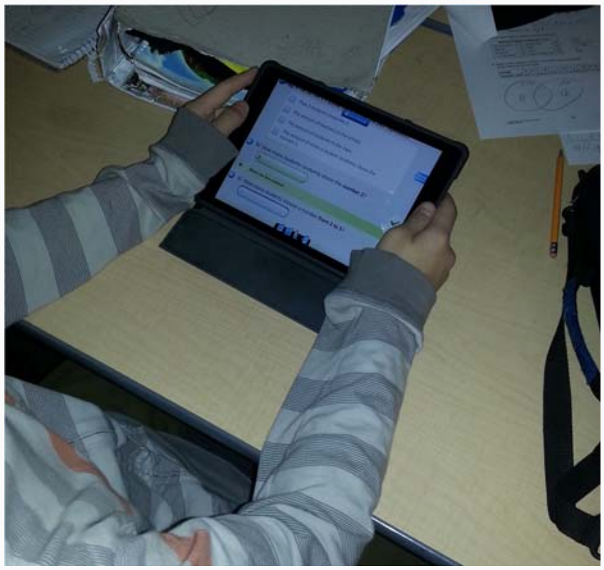


Figure 8.

A student working on his ipad with buzzmath. I felt that this was the most connnected to the MCA test since their test was also on the computer and used the same manipulation. The student's like the variety it brought to the lessons.

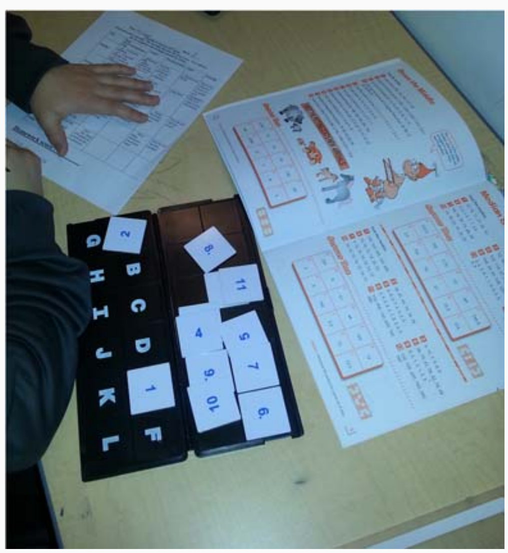


Figure 9.

A student using versatiles. Versatiles were the new manipulative being used this year. The students liked that they were able to self check and work independently

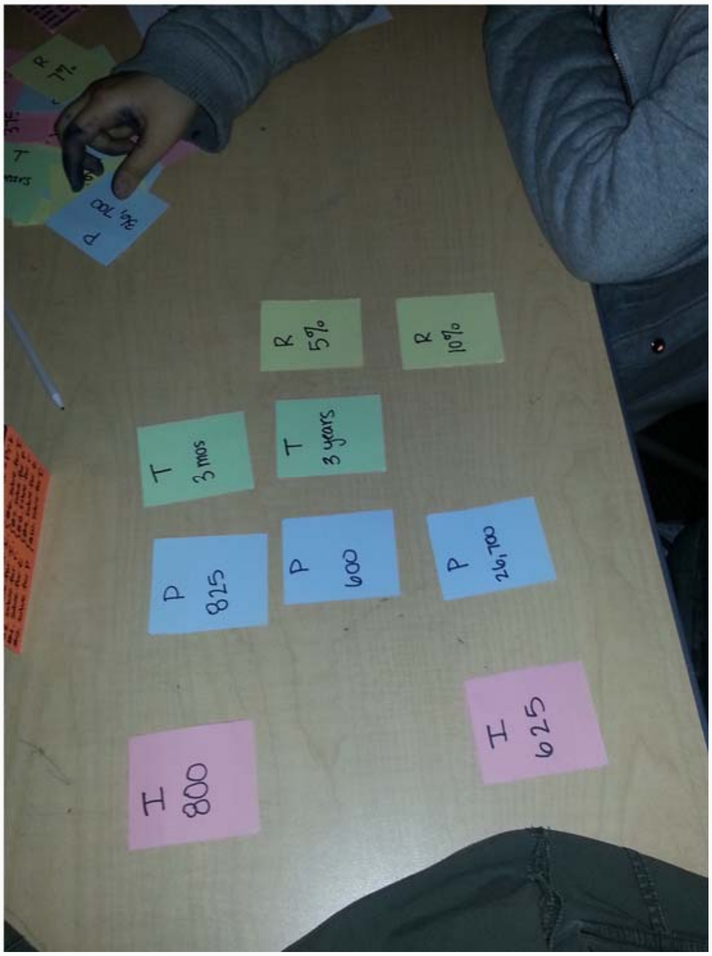


Figure 10.

A student using a card sort. Some of the card sorts involved vocabulary, ordering, or solving for equations. The students liked being able to choose the card sorts over practicing with a worksheet. They also have a built in control, so they could work independently.

Conclusion:

Manipulatives used in the right way can help students gain a more abstract understanding of a math concept. The students that were already labeling themselves as abstract enjoyed using manipulatives to learn the concepts, and the concrete learners needed more of a push into using them but I found that they made bigger gains when pushed outside their learning comfort zone.

It doesn't take finding any object for the student to use in learning the math lesson. The object needs to be carefully picked out or it could have the opposite effect on the student's understanding the concept. This is what I learned when doing my lessons on ratios. I was too concerned about the students using manipulatives and not on making sure they helped students gain a better understanding on ratios. Maria Montessori knew what objects would be the most helpful and after many years after her passing they still hold value in the classroom. There are of course objects that could be added but we should not discount what was already provided for a Montessori teacher. This is what I realized with my 8 students, the students don't need to be from a Montessori elementary classroom in order to gain a better understanding with the Montessori manipulatives, like the card sorts, and versatiles, but those manipulatives were the most popular ones that all students would go for in their shelfwork. When talking to students it was because they had a built in control when working through them so they can do them more independently.

I had 20 concrete learners in my room and out of those 20, 14 improved their MCA score which was 70%. I had 5 abstract learners and out of those 5, 3 improved, which is a 60%. Overall 68% of my students improved with the use of manipulatives being introduced and a part of the lessons compared to last year when I did not use these manipulative and had only 25% of student's improve on the MCA test. When I broke down the data to the 8 students I had taught last year, 3 abstract learners and 2 of them improved for a percent increase of 66%. The other 6 considered themselves concrete learners and 3 out of those 6 showed improvement on the MCA test which was an increase of 50%. Overall in my finding the students that were concrete learners and used more manipulative this year in math class tested higher on their MCA test. The abstract learner didn't seem to be affected either way if they used the manipulative or not. I felt that this is because they are more independent and creative in the way of thinking and don't need the hand manipulation to help get them there.

I am very proud of the MCA results but I am also proud of the standardized tests used for each benchmark. The results I observed for each test, even if the students did not improve helped me to reflect on my teaching process and get to know what my students needs were on a more individual level. I intend to continue to observe and record data for my classroom and make sure I adapt my lessons to accommodate and reach all my students. My limitations this year was only having the eight students from last year to this year. Next year I will have more students from the previous year so my data will be more meaningful with a larger group. I hope that my research can help convince the district to supply us with more of the elementary manipulatives for the math classrooms.

Bibliography

Creech, N., & Bhavnagri, N. (2012). Teaching Elements of Story Through Drama to 1st Graders. *Childhood Education, 78*(4), 219-224.

Lillard, A. (2011). What Belongs in a Montessori Primary Classroom. *Montessori Life, 23*(3), 18-32.

Margaritis, A. (2003). Using Graphing Calculators in the Montessori Middle School Classroom. *Montessori Life, 15*(2), 42-43.

McNeil, Nicole, and LInda Jarvin. "When Theories Don't Add Up: Disentangling the Manipulative Debate." *Theory into Practive* 46.4 (2007): 309-16. Print.

McNeil, N., & Uttal, D. (2009). Rethinking the Use of Concrete Materials in Learning: Perspectives From Development and Education. *Child Development Perspectives, 3*(3), 137-139.

Morrison, M. (2011). Math Tracks: What Pace in Math is Best for the Middle School Child? *Montessori Life, 23*(4), 26-35.

Nguyen, H. (2013). Teach Mathematics to ELLs. *Peer Reviewed, 89*(6), 392-395. Retrieved February 25, 2015, from http://www.tandfonline.com/loi/uced20

Parke, Carol S; Lane, Suzanne. The Journal of Educational Research101.3 (Jan/Feb 2008): 132-146.

Scheiter, K., Gerjets, P., & Catramborie, R. (2006). Making the abstract concrete: Visualizing mathematics solution procedures. *Computers in Human Behavior, 22*(1), 9-25. Retrieved February 1, 2015, from www.sciencedirect.com

Shoval, E., & Shulruf, B. (2011). Who Benefits From Cooperative Learning with Movement Activity? *School Psychology International, 32*(L), 58-72.

Vinson, Beth. Early Childhood Education Journal, 2001, Vol.29(2), pp.89-94 [Peer Reviewed Journal]

Wang, A. (2010). Optimizing Early Mathematics Experience for Children from Low-Income Families: A study on Opportunity to Learn Mathematics. *Early Childhood Education*, *37*(J), 295-302.

Appendix A:

Survey of Manipulative

Rank each statement 1-5, 1 meaning never and 5 meaning always.

___I like to work by myself

___I like to work with a group

___I like writing notes

___ I learn best by doing a worksheet for practice

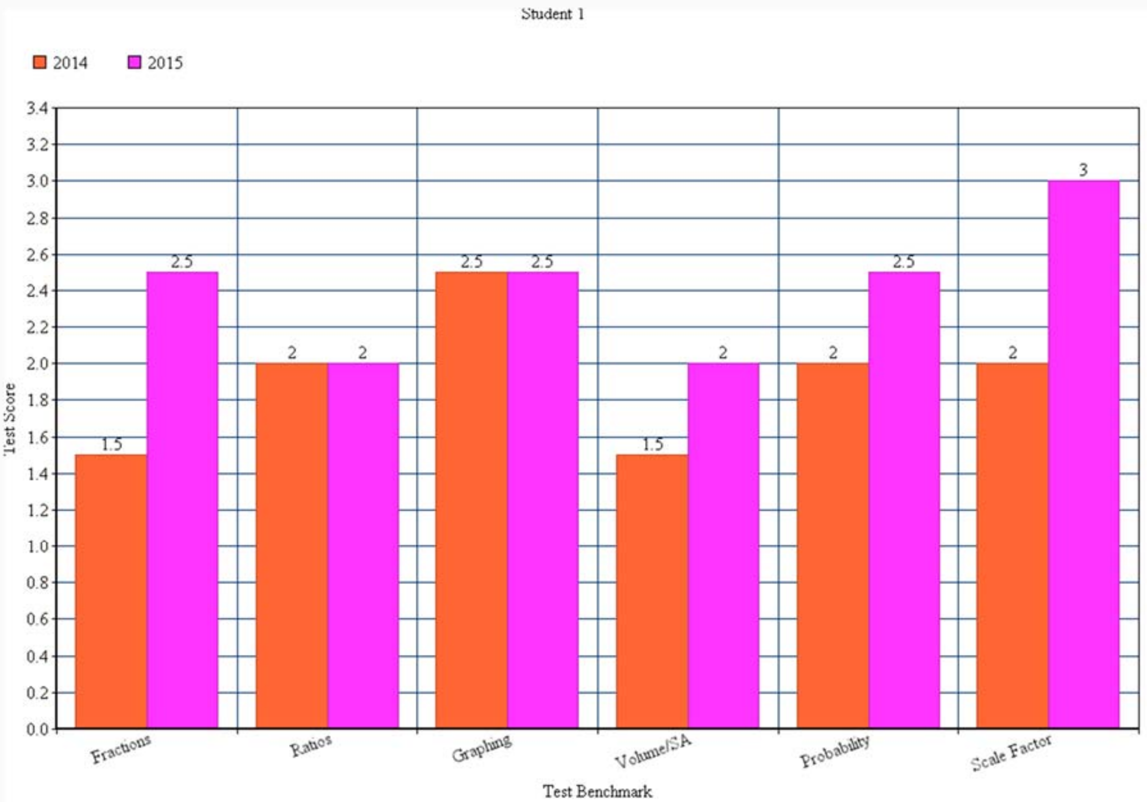
___ I learn best with mini lessons

___ I learn best when I can use objects like dice, cards or other hands on things

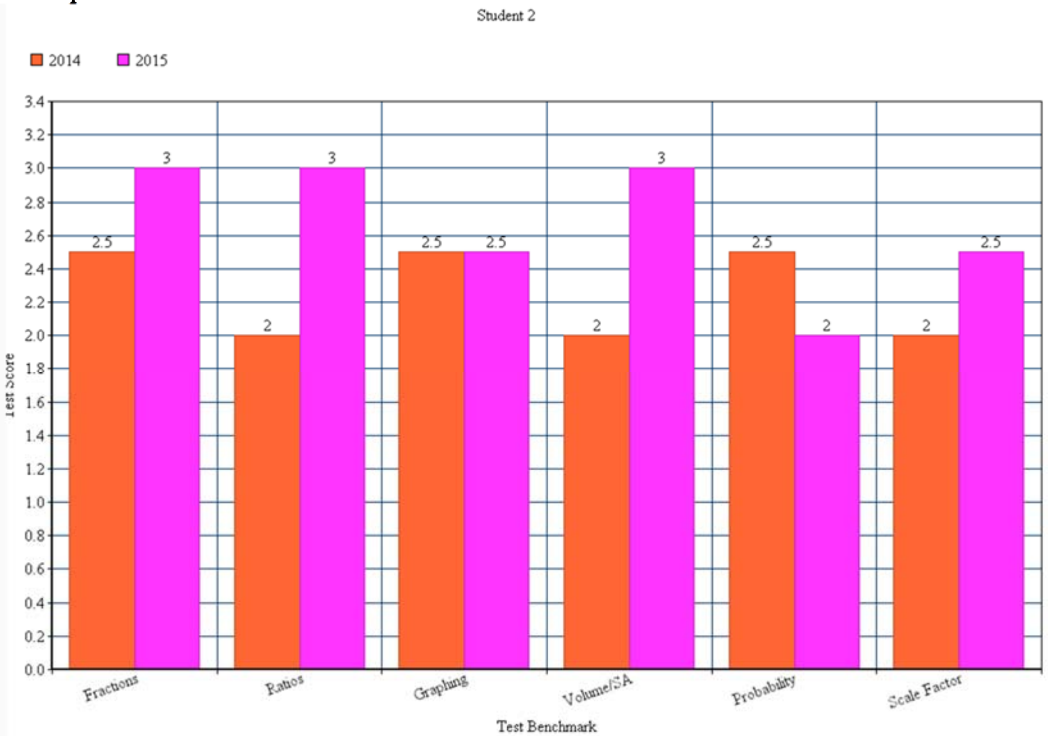
___I choose card sorts to do when they are available

____ I choose versatiles when they are available

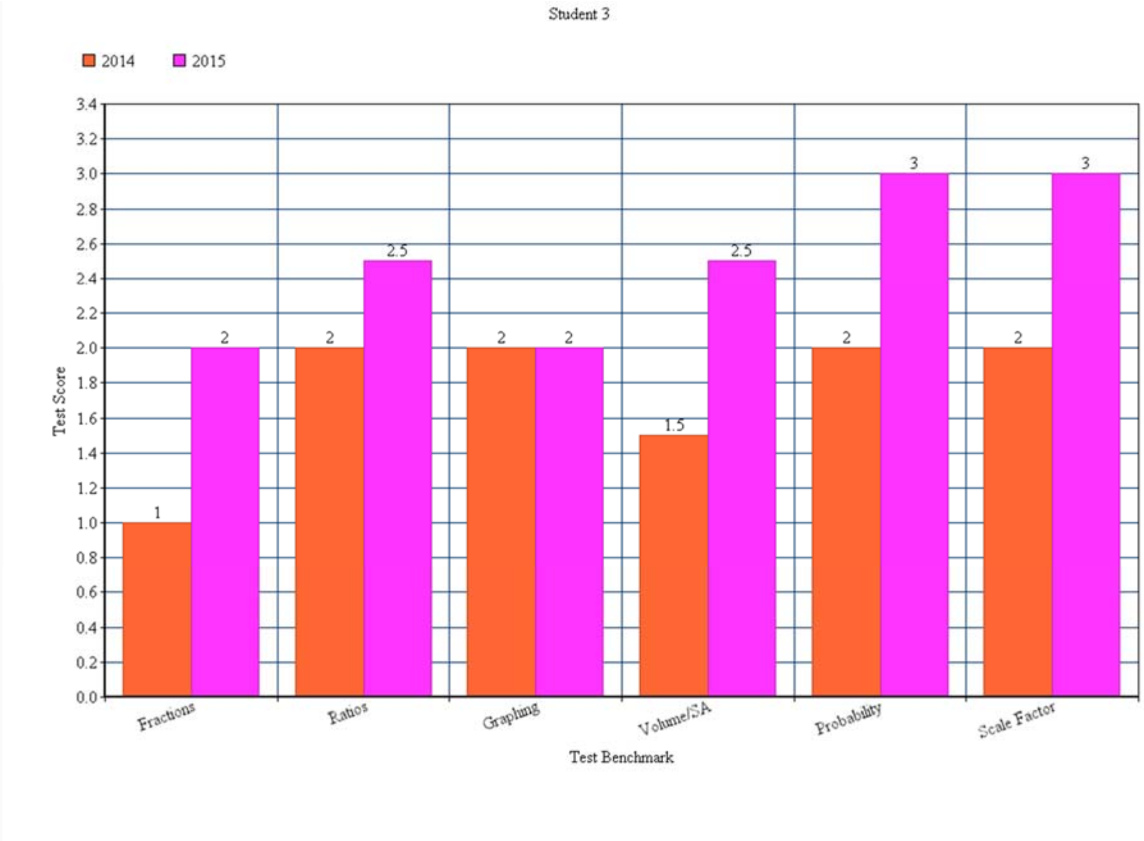
Appendix B



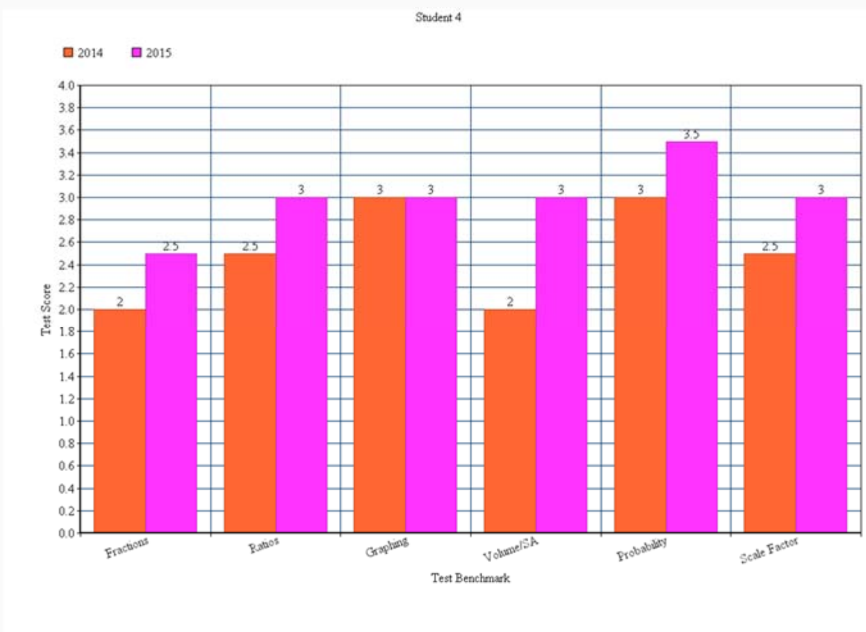
Appendix C


Student 2
2014
2015
Test Score
Test Benchmark
Fractions
Ratios
Graphing
Volume/SA
Probability
Scale Factor
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
2.5
3
2
3
2.5
2.5
2
3
2.5
2
2
2.5

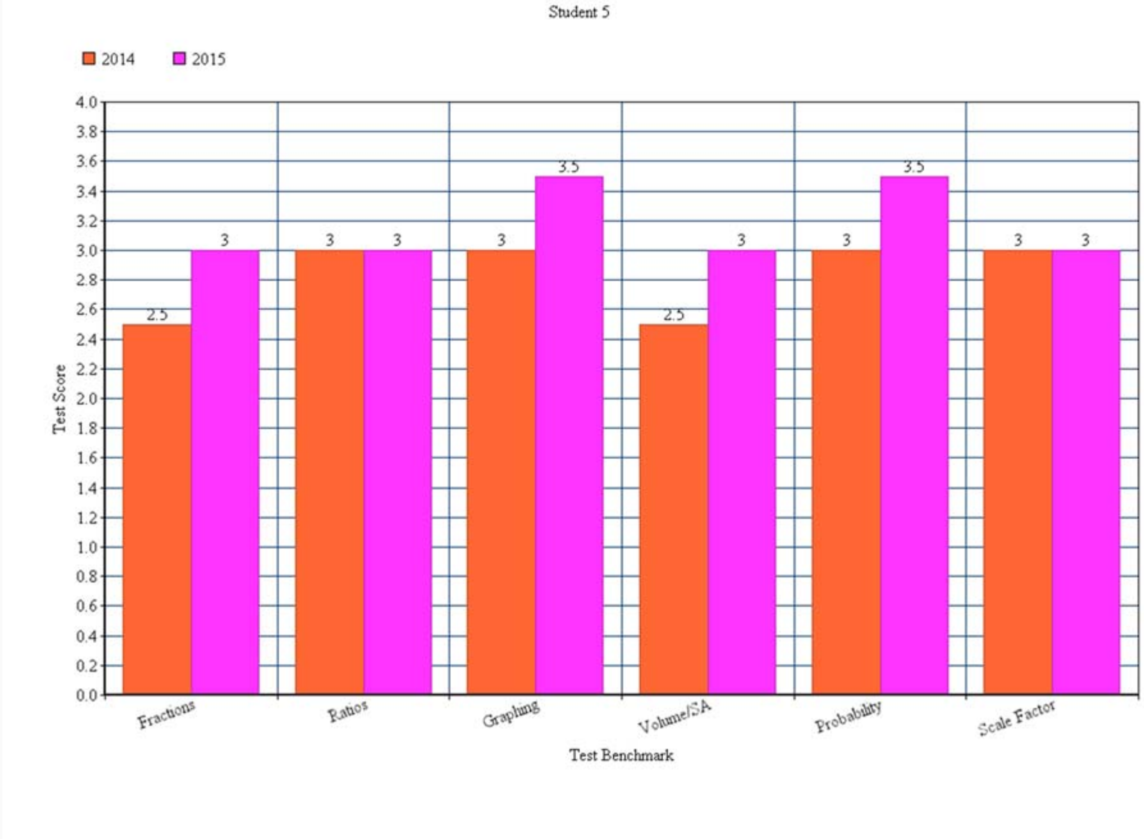
Appendix D.



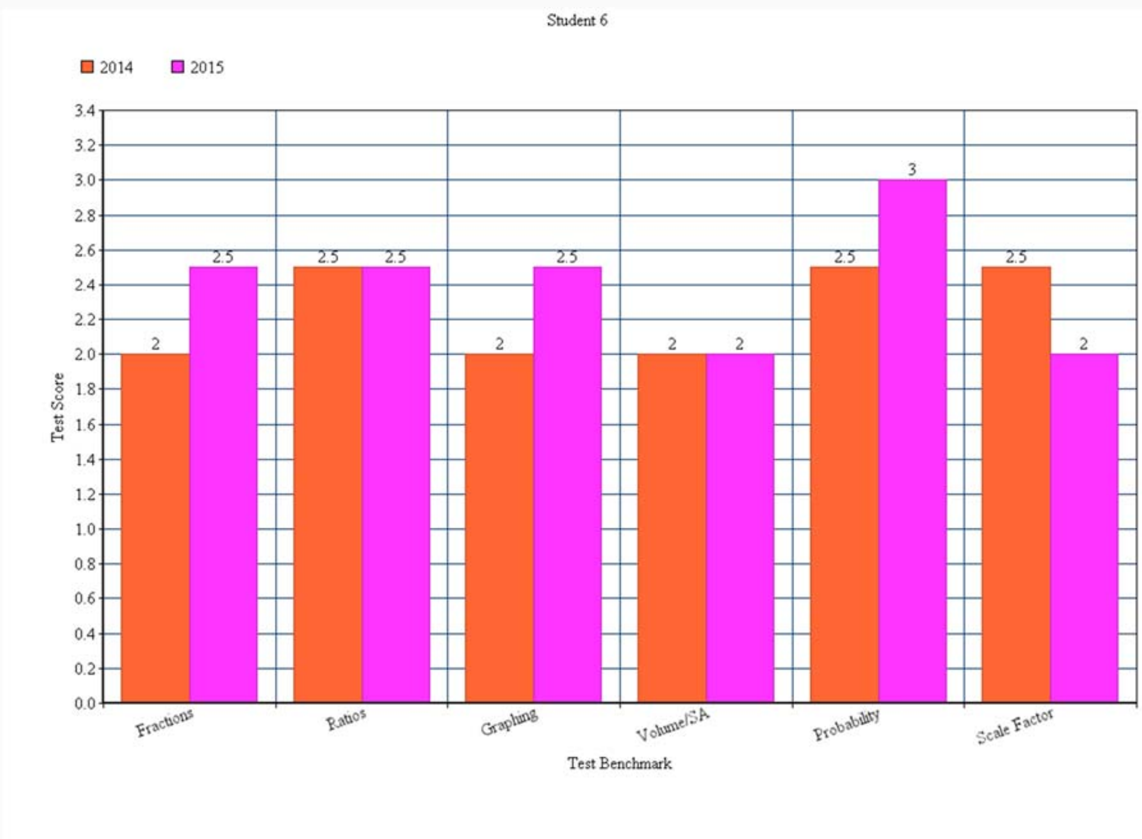
Appendix E.



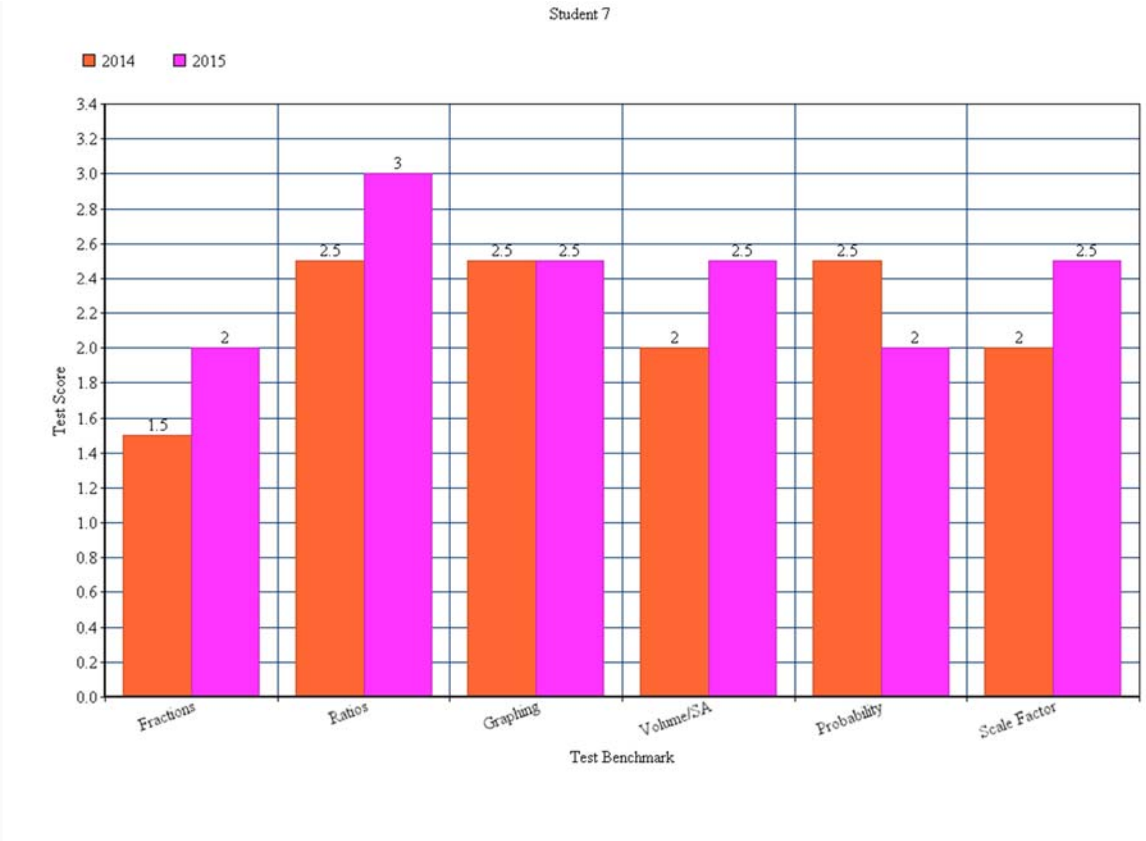
Appendix. F



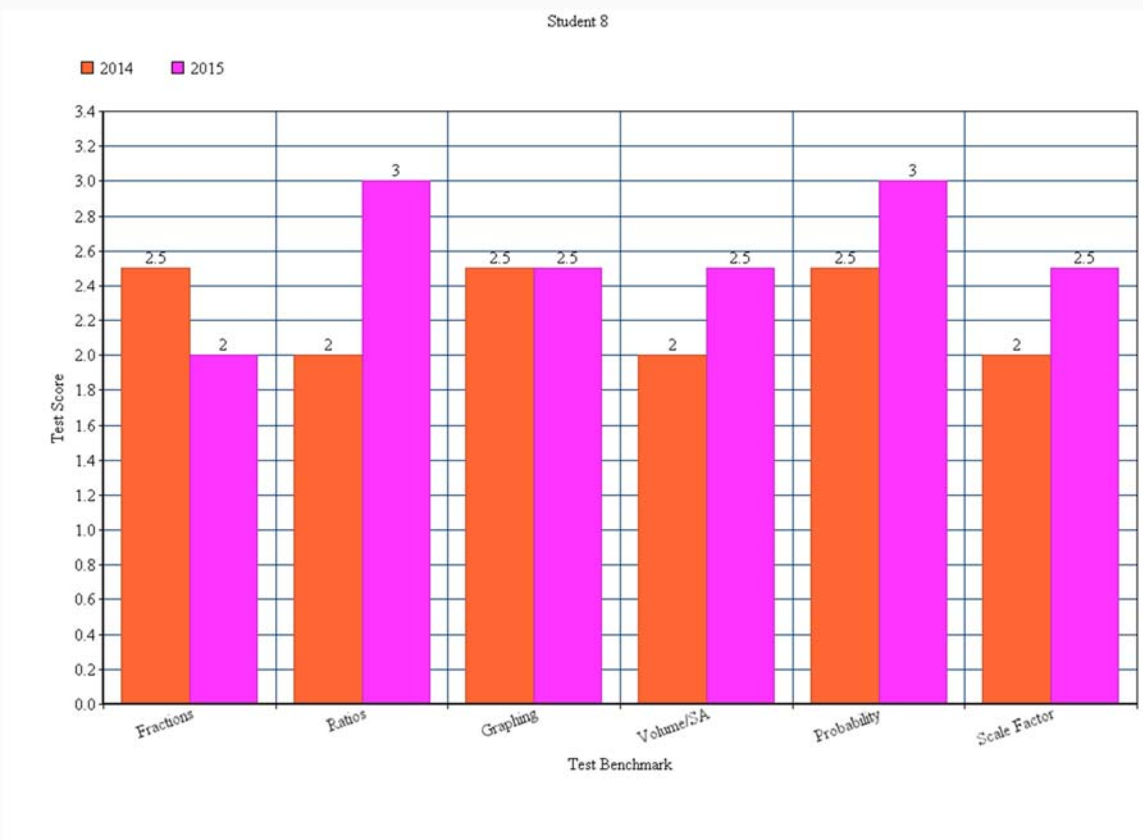
Appendix. G



Appendix. H



Appendix. I



Appendix J

Test given on Fractions:

Learning Target 7.1.2.1e – I can add & subtract with positive and negative fractions.

17.

A. Explain the difference between factors of a number and multiples of a number. Give examples of factors and multiples of 12.

B. What are the factors of 18?

18. Find the common denominator between $\frac{1}{4}+\frac{1}{3}=$

19.

A. $\frac{1}{2}+\frac{1}{2}=$

B. $\frac{7}{9}-\frac{2}{3}=$

C. $\frac{5}{6}+\frac{4}{9}=$

D. $-3\frac{1}{2}+4\frac{1}{3}=$

E. $5\frac{2}{5}-2\frac{3}{10}=$

F. $-2-4\frac{2}{3}=$

20. Mark has baked a birthday cake. His brother took 2/5 of the cake. How much of the cake is remaining for the rest of the family?

Learning Target 7.1.2.1f – I can multiply & divide with positive and negative fractions.

21.

A. 3 • -7 =

B. $(-3)(-16) =$

C. $-2 \bullet 10 =$

D. 15 ÷ -3 =

E. -20 ÷ -4 =

F. $\frac{-42}{6} =$

22. Explain how multiplication is part of dividing fractions.

23.

A. $\frac{1}{4} \times \frac{2}{3} =$

B. $\frac{4}{5} \times \left(-\frac{1}{8}\right) =$

C. $\frac{3}{7} \div \frac{1}{2} =$

D. $\left(-\frac{2}{5}\right) \div \frac{1}{3} =$

24. Maya is drinking her favorite juice. There are $2\frac{3}{4}$ servings remaining in the bottle. Maya pours only $\frac{1}{4}$ of a serving into her glass at a time. How many glasses can Maya have before the bottle is empty?

Appendix K.

Test on Ratios:

Equivalent Ratios

1) Which one of the following is not an equivalent ratio to the others? Circle your answer.

a) $\frac{18}{42}$ b) $\frac{24}{48}$ c) $\frac{15}{35}$ d) $\frac{3}{7}$

2) True or False:
The unit price of 13 donuts for $7.28 is $1.79 per donut.
2)__________

3) Cereal is on sale this week. Is it a better buy to get the 10-ounce box for $2.14 or the 14-ounce box for $2.45? Show your work for credit.

3) ____________________

4) Working as a salesman, James visits 9 cities per week. At this rate, about how many cities will he visit in 2 years? Show your work for credit.

4)____________________

5) Explain how you would find out how many eggs you would need to make 27 dozen cookies if 6 dozen cookies uses 4 eggs.

Application of Proportions

6) Solve the following proportion: $\frac{N}{36} = \frac{108}{9}$

a) N = 3 b) N = 12 c) N = 432 d) N = 324

7) What is the correct proportion to find 22% of 86? Circle your answer.

a) $\frac{22}{86} = \frac{x}{100}$ b) $\frac{22}{x} = \frac{86}{100}$ c) $\frac{100}{x} = \frac{22}{86}$ d) $\frac{22}{100} = \frac{x}{86}$

8) In a garden there is a large statue of a rabbit. It stands 6 ft tall and casts a shadow of 5 ft. If there is a tree in the garden that casts a shadow of 30 ft, how tall is the tree? Show your work for credit.

8)______________

9) The height of a tower on a scale drawing is 26 centimeters. The scale is 2 cm : 21 m. What is the actual height of the tower? Show your work for credit.

9)______________

10) In her catering business, Elena usually estimates that 64 ounces of meat will feed about 16 people. How many POUNDS of meat should she prepare for 96 people?

Appendix L.

Test on Graphing

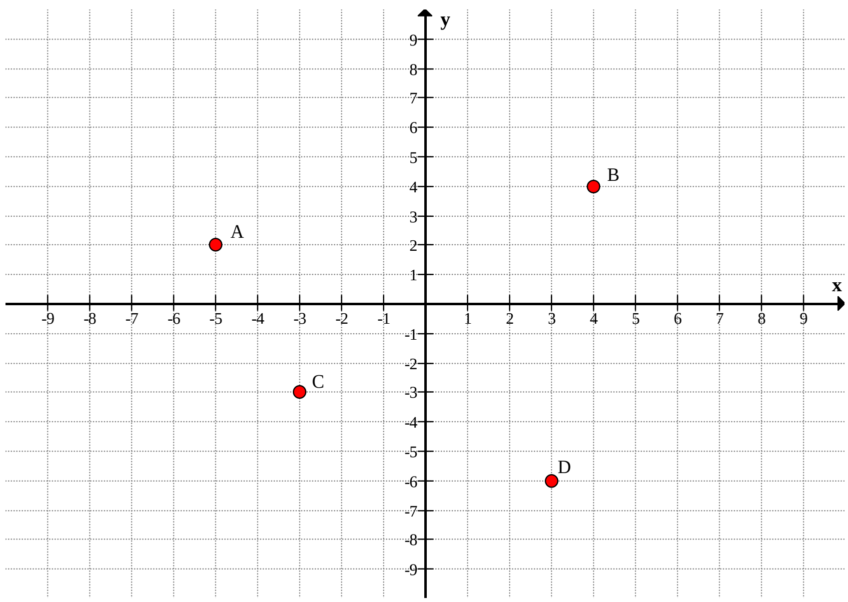
Learning Target 7.1.1.3d - I can plot ordered pairs on a coordinate grid.

1.) For the ordered pair (2,-6), which number is the x-coordinate and which number is the y-coordinate?

x-coordinate = ________ **y-coordinate** = ________

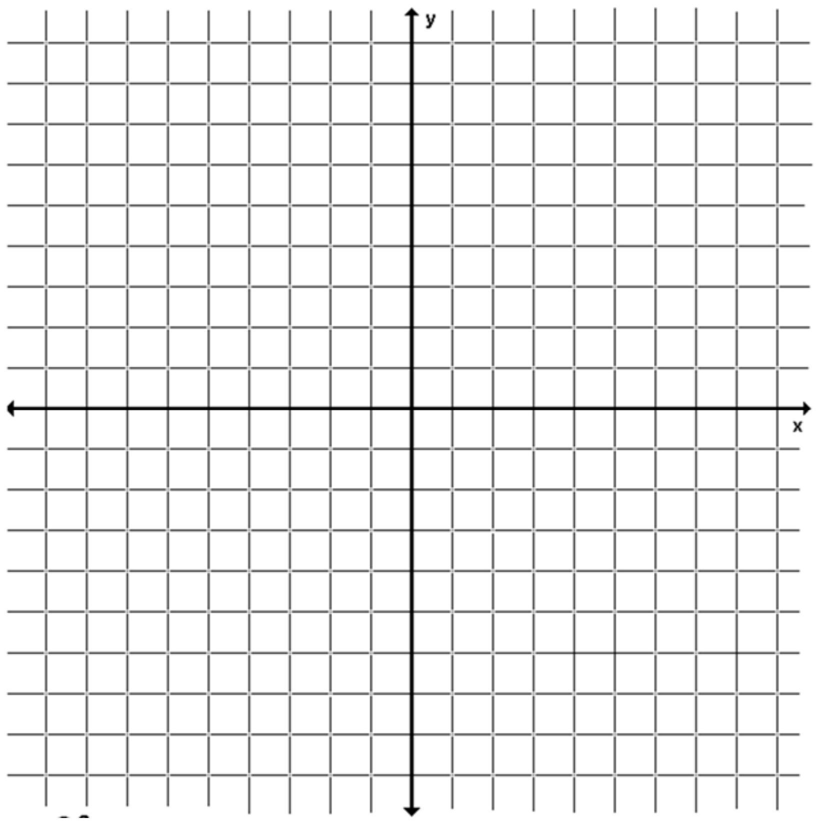
2.) Write the ordered pairs for the following points:

A: ________ B: _________ C: _________
D: _________



3.) The coordinate grid below has an interval of 1. Plot and label the points below on the coordinate plane:

E $(-\frac{1}{4}, 4)$ **F** (-4, 2) **G** (3, 0) **H** (2.5, -3) **I** (0, -3)
J (-4, -2)



4.) Explain how you plot a point on a coordinate plane. Use at least five out of these seven terms:

- origin
- x-axis
- y-axis
- positive
- negative
- interval
- coordinate(s)

Write in full sentences. Include visuals if you like.

Appendix M.

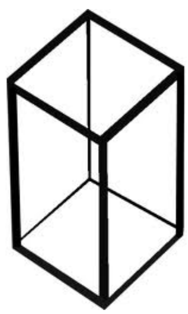
Test on Volume and Surface Area

Learning Target 6.3.1.1a - I can calculate the surface area of prisms and use appropriate units.

Learning Target 6.3.1.1c - I can justify the formula of surface area of a prism.

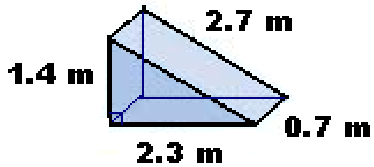
1. A) Decompose the following rectangular prism into a 2-dimensional net.

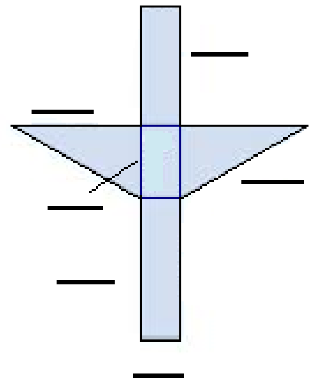
   B) Decompose the following rectangular prism into a different 2-dimensional net.



2. A) Use the triangular prism on the left to help you fill in the measurements on the net.

   B) Explain why this activity helps you determine the surface area of the triangular prism.

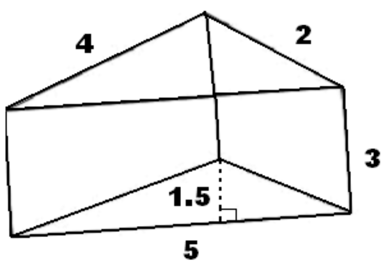




3. A) Find the surface area of the following shape.

   Surface Area = ______

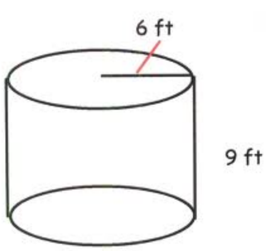
   B) Draw the corresponding net, solve and show all your work.



4. A) Calculate the surface area of the cylinder. (π = 3.14)

   Surface Area = _______

   B) Create a net to show your thinking.



Learning Target 6.3.1.1b - I can calculate the volume of prisms and use appropriate units.

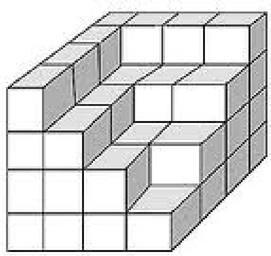
Learning Target 6.3.1.1d - I can justify the formula for the volume of a prism.

5. A) Determine the number of cubes in the rectangular prism when it is filled.

   Volume = ______

   B) How many cubes did you need to finish filling up the rectangular prism? ______

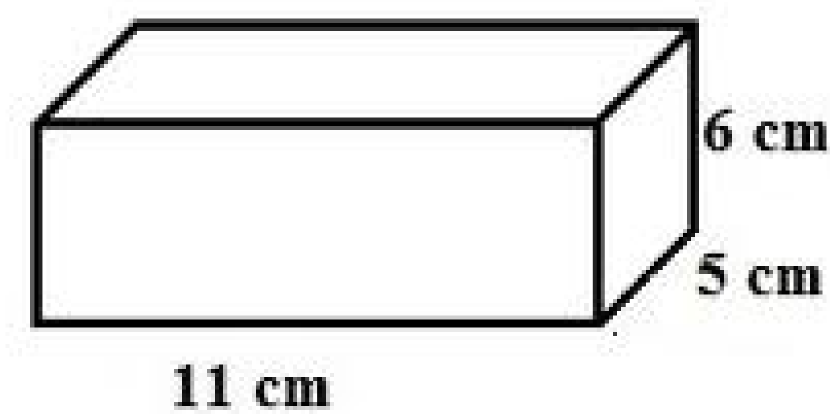
   Show all your work.



6. Janine keeps her jewelry in a jewelry box like the figure below.

   A) Find the volume of Janine's jewelry box. Volume = _______

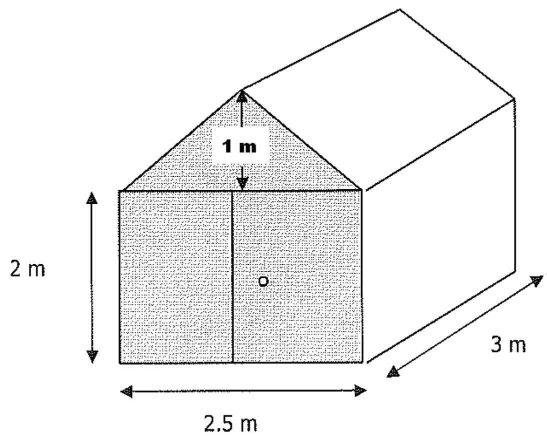
   B) Draw the lines to show the cubes to prove your solution.



7. A) What is the volume of the garage?

   Volume =______

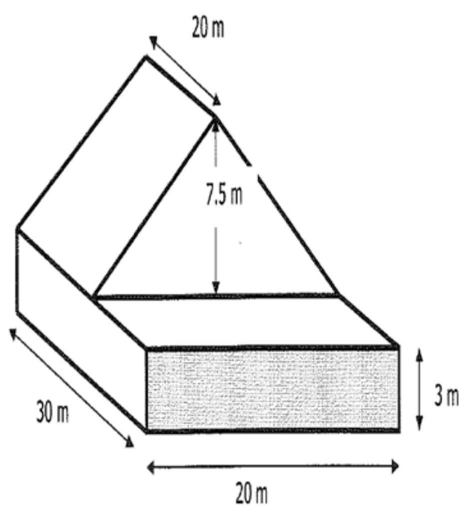
   B) Explain how you solved the problem.



8. A) The diagram below show the outside dimensions of a partially finished factory building.

   What is the volume of the missing roof section? ______

   B) Explain how you solved the problem.



Appendix N.

Test on Probability

Learning Target 7.4.3.2a – I can write probabilities as fractions, decimals, and percents.

1. Convert each fraction into a decimal AND a percent.

A. $\frac{13}{25}$

B. $\frac{3}{5}$

C. $\frac{2}{9}$

2. *Multiple Choice:* A jar contains blue and red marbles. The probability of choosing a red marble at random is 6 out of 24. Which represents the probability that a marble chosen at random is red?

a) 6%

b) 24%

c) 25%

d) 24

3. Sunny McStorm is the local weather forcaster. She is predicting a 60% chance of rain. Express the probability that it will be sunny as a fraction, decimal, and percent.

4. Use the table provided to find each probability. Give each answer as a fraction and a percent.

Outcome	A	B	C	D

Probability	0.2	0.2	0.1	0.5

P(D) =

P(B or C) =

P(not A) =

Learning Target 7.4.3.2b – I can calculate probability using sample spaces.

5. *Multiple Choice:* Which answer gives the sample space for the outcome of rolling a single number cube?

a) {0, 1, 2, 3, 4, 5, 6}

b) {1, 2, 3, 4, 5, 6}

c) {0}, {1}, {2}, {3}, {4}, {5}, {6}

d) the number that is rolled

6. *Multiple Choice*: A sample space consists of 18 separate events that are equally likely. What is the probability of each of those events happening?

a) 0

b) $\frac{1}{18}$

c) 1

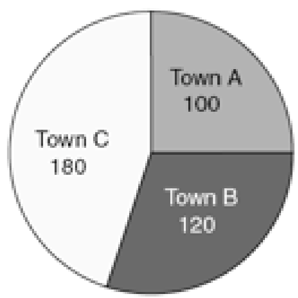
d) 18

7. Explain why the probability of rolling an even number on a fair number cube is not one out of six.

8. A bag of M&M's has 4 red, 7 green, and 5 light brown candies. How many yellow M&M's must be added to make the probability of choosing a red candy 1 out of 5?

Learning Target 7.4.3.2c – I can calculate probability as a fraction of area.

9. *Multiple Choice:* Students from Tri-City Middle School come from towns A, B, and C. The graph below shows the number of students from each town.



A student is chosen at random will most likely be from:

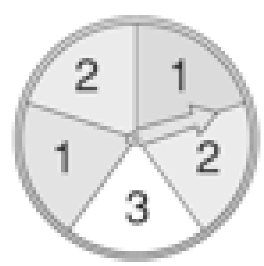
a) Town A

b) Town B

c) Town C

10. Use the spinner below to determine each probability. Give your answer as a fraction.

A. Spinning a 1



B. Spinning a 3

11. *Multiple Choice*: Use the graph below to answer the question.

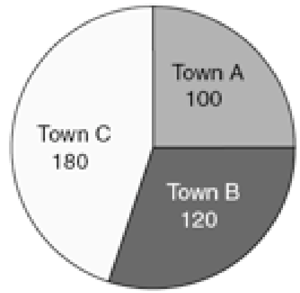
What is the probability that a student picked at random is from Town B?

a) 6%

b) 12 %

c) 30%

d) 36%



12. Draw a spinner, with equally sized sections, on which the probability of spinning a 1 is 50%, spinning a 2 is .25, and spinning a 3 is ¼.

Appendix O.

Test on Scale Factor

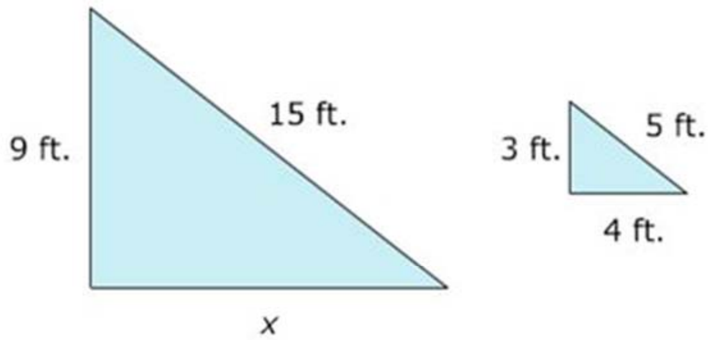
Learning Target 7.3.2.2.a - I can use scale factor to determine side lengths of similar geometric figures.

1. Solve:

   a) $12 \bullet \frac{2}{3}$

2.

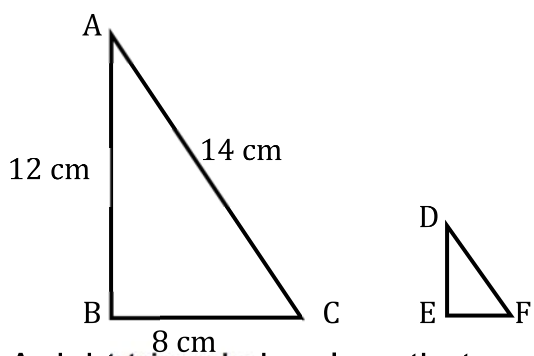
The triangles shown are similar.



What is the value of *x*?

- **A.** 8 ft.
- **B.** 12 ft.
- **C.** 14 ft.

3. Triangles ABC and DEF are similar. Use the scale factor of $\frac{1}{4}$ to find the side lengths of triangle DEF.



4. A right triangle has legs that measure 4 cm and 15 cm. Another right triangle has legs that measure 10 cm and 37.5 cm. Are the triangles similar? Justify your answer by showing your work.

Learning Target 7.3.2.2.b - I can use scale factor to determine side lengths of similar geometric figures.

1. Solve:

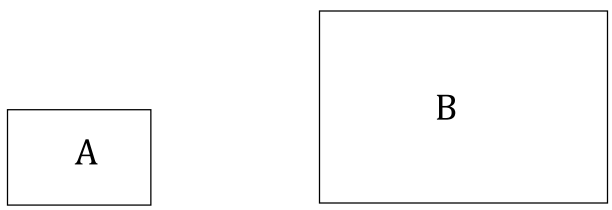
a) 7^2

b) $\left(\frac{2}{3}\right)^2$

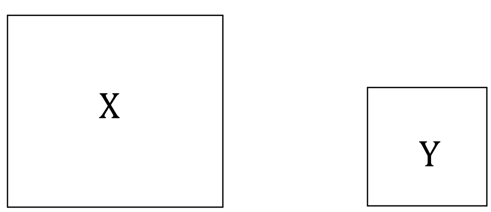
c) $\sqrt{81}$

d) $\frac{\sqrt{16}}{\sqrt{64}}$

2. The two rectangles are similar. The area of rectangle A is 24 square units and rectangle B is 96 square units. Find the area ratio A:B.



3. Square X has an area of 36 square feet and is similar to square Y. Find the area of square Y if the scale factor of corresponding sides is $\frac{1}{2}$.



4. The ratio of corresponding sides between rectangles D and F is $\frac{1}{3}$. If the rectangle D has dimensions of 6 inches by 3 inches, find the area of rectangle F.