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Analysis of Unreplicated Split-Plot Experiments With Multiple Responses

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ABSTRACT

The purpose of this study is to demonstrate an effective strategy for analyzing unreplicated split-plot experiments with multiple responses. Through principal component analysis (PCA) the response variables are reduced to only those that describe different phenomena among the experimental samples. These selected response variables are then analyzed individually using ANOVA and Normal probability plots to identify the factors with the greatest influence on the quality and cost of the product. This approach makes it possible to take both the preferred quality characteristics and the production costs into account when studying a process or product. A case study from a fish food manufacturing company is used to illustrate our ideas.

KEYWORDS: *Split-plot experiment; Principal component analysis; ANOVA; Normal probability plots.*

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I. INTRODUCTION

In industrial experimentation it is important to organize experiments so they are easy to run. Very often there are restrictions on how an experiment can be performed. Some factors might be easy to change whereas others may be more difficult to change. In such cases it might be natural, convenient and efficient to run the experiment in split-plot mode (Cox, 1958; Box and Jones, 1992). The factors which are difficult to change and the factors which are easy to change are commonly referred to as the whole plot factors and subplot factors respectively. It may further be necessary to examine the effects of the experimental factors on several performance characteristics of the product or process. In this paper we propose a method of analysis for unreplicated split-plot experiments with multiple responses.

Using Principal Component Analysis (PCA; Mardia et al., 1980) the different types of responses are reduced to only those that describe different important phenomena of the product or process. Then, using the reduced set of responses, each response is individually analyzed as an unreplicated split-plot experiment. To motivate the need for this type of analysis strategy, consider an example based on the manufacture of fish food.

A fish food manufacturer wanted to study the effect of using different carbohydrates ground up at different levels as raw material for producing fish food pellets. They also wanted to study how various processing variables affected both the pellet quality and the production costs, and to see if the effect of the processing variables depended on the raw material used. It was decided that designed experimentation should be used as the means to study this manufacturing process. While discussing the different experimental strategies the manufacturer conveyed that it was easy to change some of the processing factors but very difficult to change the carbohydrate type and the way in which it was ground up. Thus, a fully randomized experiment was not practical. Instead a split-plot experiment was proposed where the raw material with a given grinding level was first produced. Then, it was processed varying the "easy to change" processing conditions according to a two-level factorial design. So, what could be randomized, was randomized.

Typically, experimental design techniques have been used for optimizing the product towards one or a few response variables. However, the total quality of the pellet could not be expressed by using only one response variable. In this experiment several response variables were measured, as the goal was not only to improve the quality of the pellet but also reduce the production costs. The goals set by the manufacturer were to produce a pellet which sank to the bottom, i. e. sank 100%, had high fat content after absorption and leakage, resulted in minimum dust and breakage and was produced with minimum use of energy (measured as ampere). In addition they wanted an elastic and strong pellet, i. e. low texture gradient and relatively high texture peak.

Determining the optimum processing conditions when several response variables have to be taken into account can be difficult. The optimum processing condition for one response variable can be quite different from the optimum condition for another response variable. It is therefore necessary to identify the most critical response variables that affect the quality of that process or product and find the best compromise between the optimal manufacturing conditions for each of those response variables. However, very often several response variables describe the same phenomena among the samples. In such cases the number of response variables can be reduced by selecting one of the responses to represent a particular phenomena among the samples. Principal component analysis (PCA) is an effective tool for that purpose (Wold & Sjostrom, 1977).

In the next section we describe in detail the fish food experiment which inspired this research. Then we explicitly outline our analysis strategy and give a demonstration of its application. Finally, we discuss the analysis and its apparent level of success.

II. EXAMPLE

For the fish food experiment five factors were chosen for the study; carbohydrate type (*C*), degree of grinding (*G*), temperature (*T*), feeder rate (*F*) and percent of water (*W*). Furthermore, five levels were selected for *C* and two levels were selected for each of the remaining factors.

Initially a 2^{4-1} fractional factorial experiment was constructed for each carbohydrate type. This resulted in 8 runs for each carbohydrate type and a total of 40 different runs. However, as stated earlier, this experiment could not be fully randomized. There were restrictions on randomization regarding carbohydrate type and degree of grinding. Two big batches were to be produced from each carbohydrate type; one which was coarsely ground and another which was finely ground. Then for each carbohydrate type - grinding level combination four randomized experiments were performed regarding factors *T*, *F* and *W*. Thus only 2 batches were required for each carbohydrate type. A fully randomized experiment would have required 8 batches to be made for each carbohydrate type! In Table 1 the restricted randomized experiment is presented, where *C* and *G* are the whole plot factors and *T*, *F* and *W* are the subplot factors.

This experiment ended up being a 2^{3-1}x(5x2) cross-product design with split-plot confounding (Bisgaard, 1992). Factor *G* was confounded with the three factor interaction between *T*, *F* and *W*. This resulted in the following production scheme; at the low level of *G* the −*TFW* fraction was produced and at the high level of *G* the +*TFW* fraction was produced.

Eight response variables were measured to characterize the pellet quality. These were fat % after absorption, fat % after leakage, pellet density, texture-peak, texture-area, texture-gradient, sink % and amount of dust and broken. In addition the current was measured in amperes as an indication of the production costs. The goal for each of the different attributes was presented in the introduction.

Table 1.
2^{3-1}*x(5x2) cross-product design with split-plot confounding G=TFW.*

			Carb. (C)	*1*	*1*	*2*	*2*	*3*	*3*	*4*	*4*	*5*	*5*
			Grind. (G)	−	+	−	+	−	+	−	+	−	+
Temp. (T)	Feeder (F)	Water (W)											
				I=−*TFW*	I=+*TFW*	I=−*TFW*	I=+*TFW*	I=−*TFW*	I=+*TFW*	I=−*TFW*	I=+*TFW*	I=−*TFW*	I=+*TFW*
−	−	−		1		9		17		25		33	
+	−	+		2		10		18		26		34	
−	+	+		3		11		19		27		35	
+	+	−		4		12		20		28		36	
−	−	+			5		13		21		29		37
+	−	−			6		14		22		30		38
−	+	−			7		15		23		31		39
+	+	+			8		16		24		32		40

III. ANALYSIS STRATEGY

Our strategy for analyzing unreplicated split-plot experiments with multiple responses is outlined below in Figure 1.

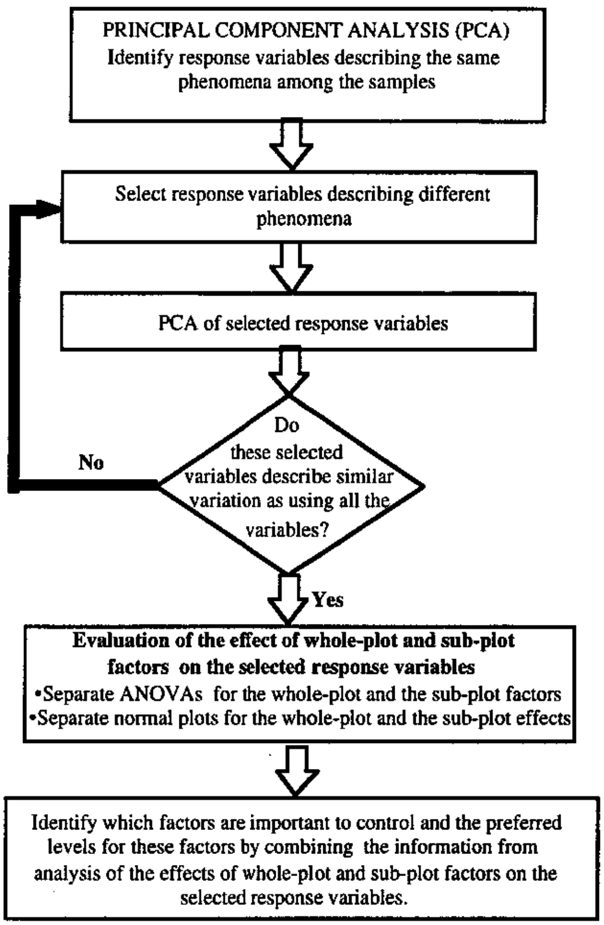

Figure 1. *An illustration of our strategy for analyzing unreplicated split-plot experiments with multiple responses.*

From Figure 1 we see that first PCA is used on the response variables describing the quality of the product or process. PCA examines all the response variables simultaneously. The data are then modeled in terms of a few significant principal components, denoted PC, which express the main phenomena or systematic variability present in the data. Plots of the data based on the PCA model are also useful.

Score plots graphically display the variation among the samples and loading plots express the original response variables contribution to describing the variation among the samples. Score plots and loading plots should be evaluated simultaneously in order to better understand the variation in the data. Furthermore, points that fall close together in a loading plot are of particular interest. This is an indication of response variables that describe the same phenomena or variation among the samples. Thus, fewer of the original response variables may be selected to reflect the main variation present in the data. These selected response variables can then be used to evaluate the effect of whole plot and subplot factors investigated in the study.

Note that typically the scales will vary for the different response variables as in this case. In order to equalize the chance for the different response variables to contribute to the PCA model the response variables should be standardized to equal variance and centered mean prior to PCA. Full cross-validation (Martens and Næs, 1989) should also be performed to validate the PCA model.

Once the response variables are narrowed down to the important few, standard unreplicated experimental analysis techniques for each response variable can be used. Two commonly used tools for judging the significance of effects in unreplicated experiments are the normal plot of effects (Box, Hunter and Hunter, 1978) and the ANOVA where the error variance is estimated by pooling the sums of squares associated with higher order interactions (Box and Jones, 1992). Whereas it has been our experience that some authors and/or instructors advocate the use of either ANOVA or Normal probability plots, we prefer to use them as complementary sources of information. Agreement and discrepancies between the analyses can provide useful information. For example, by comparing the two analyses it can be seen which conclusions are robust to the analysis methods used and thus most likely significant.

It should also be noted that although this strategy was developed to analyze unreplicated split–plot experiments, many of these ideas can easily be adapted to the case where there is replication. In the next sections we will demonstrate our analysis strategy based on the example described in the previous section.

IV. PCA ANALYSIS

PCA was performed on the eight response variables used to describe the pellet quality. This analysis revealed that a PCA model with only 5 PC's describes 95% of the variation among the samples. However, a significant proportion of the variation, 82%, is described by only the first three PC's. Hence, these three PC's can be used to identify variables describing similar variation among the samples.

The main variation among the samples is visualized by the score plots of PC1 versus PC2 and PC1 versus PC3 as shown in Figures 2a and 2b receptively. The loading plots for the same pairs of PC's, Figures 2c and 2d, show the original response variables contribution to describing the variation among the samples. The score plots in Figure 2a and 2b show that the main variation among the pellet samples is related to the water content (*W*) of the pellets. Pellets with high water content are within the area marked with a thick line and pellets with low water content were within the area marked with a thin line. The loading plots in Figure 2c and 2d show the response variables which describe the main variation among the samples as those with high positive or negative loadings at that PC. Thus, we can see that samples with high water content had high pellet density, high sink %, low amount of dust and broken and lower fat after leakage and fat after absorption than pellets with low water content.

To better understand the carbohydrate effect, the pellet samples in Figures 2a and 2b are marked according to the type of carbohydrate used. The pellets made from carbohydrates 1, 2, 3 or 5 appear to be randomly scattered in the score plots. This indicates that there is no significant difference in pellet quality when using these carbohydrates. However, the pellets made from carbohydrate 4, denoted by squares, all have positive scores at PC1. Thus, when using carbohydrate 4 the pellets have lower density, lower sink %, lower texture peak and higher amount of dust and broken.

From Figures 2c and 2d we see that pellet density and sink %, texture-peak and texture-area, and fat after absorption and fat after leakage are situated very near each other in three different clusters. One implication from this is that pellet density and sink % describe the same phenomena among the samples. The same is implied for texture-peak and texture-area, and fat after absorption and fat after leakage. This is visualized for pellet density and

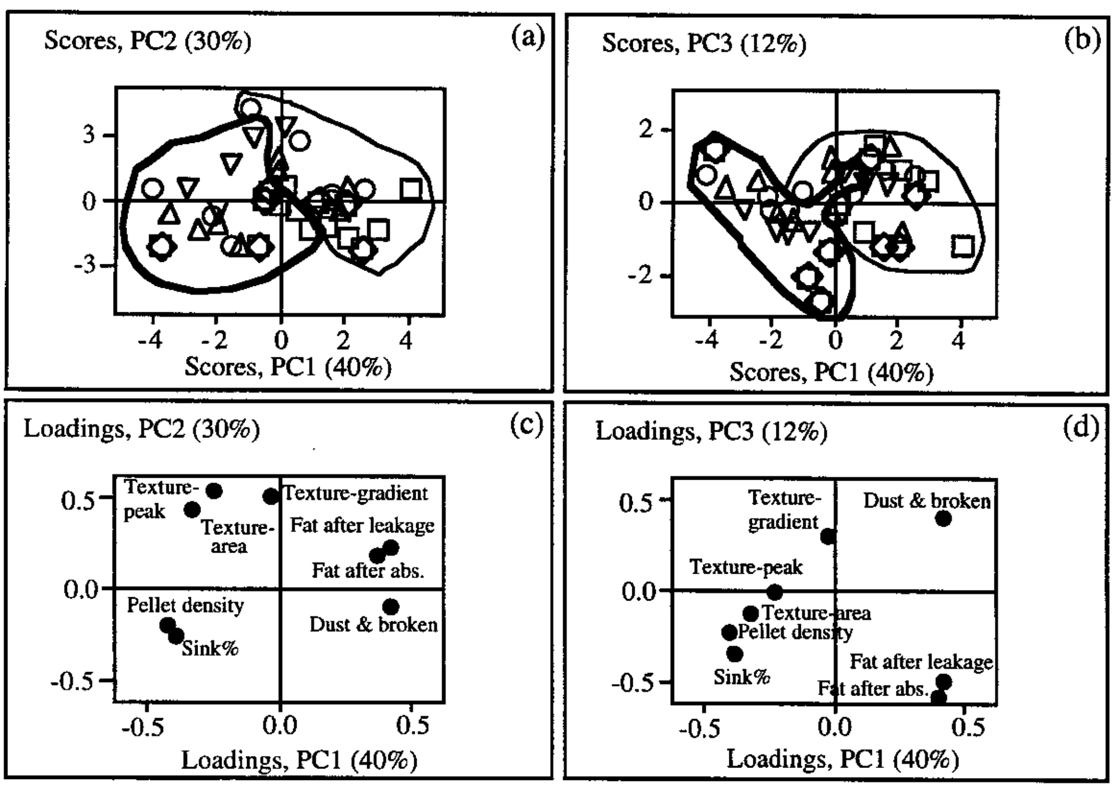

Figure 2. *The Score plots of (a) PC1 vs. PC2, (b) PC1 vs. PC3, and Loading plots of (c) PC1 vs. PC2 and (d) PC1 vs. PC3. The different shapes in (a) and (b) represent different carbohydrates used. Open circles = carbohydrate 1; triangle (up) = carbohydrate 2; diamond = carbohydrate 3; square = carbohydrate 4 and triangle (down) = carbohydrate 5. Samples within the same area have the same moisture level. High water content samples are within the area marked with a thick line, and low water content samples are in the area marked with a thin line.*

sink % in Figure 3. This plot shows that pellets with low density sink badly. A pellet should sink 100%. This is achieved when pellets have density greater than 0.86.

The number of response variables can be reduced by selecting one of the response variables from each cluster in Figures 2c and 2d to represent the different phenomena in the data. Several selection criteria can be used. The variables selected might be those which are easiest to measure, give the most objective measurement or best describe the phenomena. In this case pellet density, fat after absorption and texture-peak were selected to represent the three clusters. Pellet density was selected since this variable gave a more objective measurement than sink %. Texture-peak and fat after absorption were selected since the variation between the samples in these properties is better described than the variation between the samples in fat after leakage and texture-area.

A new PCA confirmed that the selected response variables along with dust and broken and texture-gradient describe approximately the same variation in the data as when the original response variables are used. With this reduced set of responses the three largest PC's describe 75% of the variation in the data. Thus, the five response variables; pellet density, dust and broken, texture-peak, texture-gradient and fat after absorption, can be used to evaluate the factor's effects on the quality of the pellet. Also, the response variable ampere can be used to evaluate the cost of producing the pellet under the different factor conditions.

An alternative approach would have been to analyze the scores from the first three PC's as response variables (Ellekjaer et al., 1995). In this case it would then have been possible to reduce the number of response variables from eight to three. However, it would have been more difficult to discuss the results in relation to the goal for each of the different pellet attributes. The use of the scores as response variables is more suitable when a reference product with the preferred quality characteristics is included in the experiment. In such cases the scores associated with the reference product for the different principal components can be used as the goal.

Another approach to the problem of studying multiple responses is to simply plot the original data in various ways. Scatter plots are often used to study relationships between two variables. In some cases they might also be used for identifying response variables that describe the same phenomena in the data. However, making a large number of scatter plots is laborious and making relative comparisons between the plots is often difficult. For the fish food experiment where there is 8 response variables, it would be necessary to make a total of 8*(8-1)/2=28 scatter plots. Furthermore, according to Chatfield and Collins (1980), scatter plots might reveal bivariate relationships and obvious outliers but might sometimes be rather misleading as they consists of projections onto various planes. The advantage of using PCA to identify response variables describing the same phenomena is that typically only two or three loading plots need to be studied regardless of the number of response variables. In addition, it is possible to verify that the response variables selected

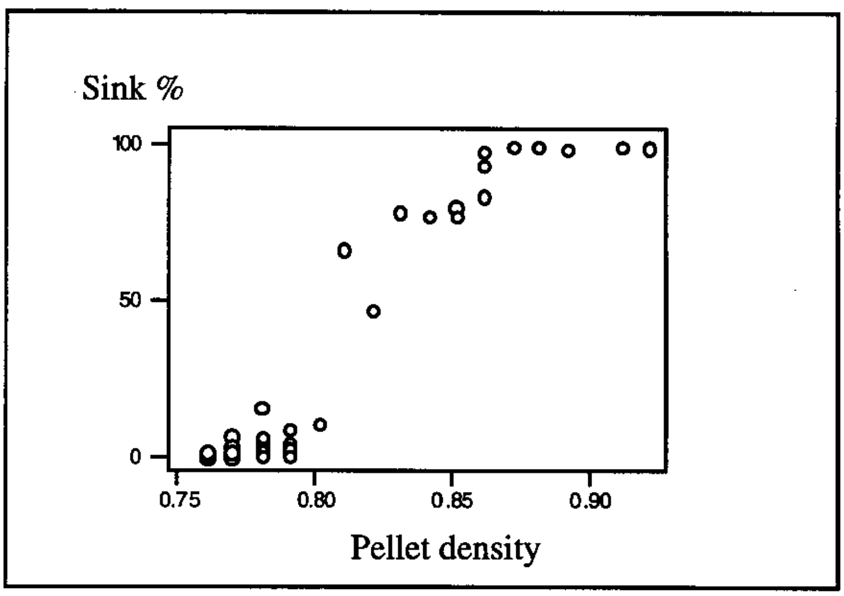

Figure 3. *A scatter plot of the experimental responses Pellet density vs. Sink %.*

describe the main variation in the data, and to get information about the main variation among the samples which would not be easily available from the scatter plots.

V. ANOVA ANALYSIS

One difference between split-plot experiments and fully randomized experiments is that two different error variance estimates are required to analyze the data; one error variance estimate for the effects associated with the whole plot factors and another error variance estimate for the effects associated with the subplot factors. Thus, use of the ANOVA requires two separate analysis; ANOVA of the whole plot effects and ANOVA of the subplot effects. This has to be taken into account when designing and analyzing data from split-plot experiments (Box and Jones, 1992).

When performing the ANOVA of whole plot effects in this experiment an estimate of the whole plot expected Mean Square Error (MSE) can be obtained by assuming the CG interaction is insignificant and using the CG mean square as an estimate of the MSE. Likewise, an estimate of the subplot expected MSE can be obtained by pooling the sum of squares associated with third and higher order subplot interaction effects. However, the analysis may be extremely sensitive to these assumptions. For instance, it may be that the CG interaction is active. However, if it is not assumed inert we have no way to estimate the expected MSE of the whole plot effects.

Using the reduced number of response variables, the mean squares for the factors and their interactions for each response variable were computed and are given below in Table 2. Carbohydrate type (C) and grinding (G) are the whole plot factors and temperature (T), feeder rate (F) and water % (W) are the subplot factors. The effects in bold type are significant at the 5% level. Additional information about the type of effects and their significance may be obtained from normal plots.

VI. NORMAL PROBABILITY PLOT ANALYSIS

Like the ANOVA use of the normal probability plot in a split-plot experiment requires two separate analyses; one consisting of a probability plot of only whole plot effects and another consisting of a probability plot of only subplot effects. However, unlike the ANOVA, use of the normal plot does not require prior assumptions about which effects are inert but does assume that all the effects are independent, have equal variance and are Normally distributed. However, the normal probability plot may still be useful when these assumptions are not completely satisfied.

When computing the carbohydrate (C) effect to be used with the normal plots it was first decomposed

Table 2. Mean Sum of Squares for the effects of the whole plot and subplot factors.

Source	Factors	D. F.	Fat after absorp.	Pellet density	Texture peak	Texture gradient	Dust and broken	Ampere
Whole plot factors	Carbohydrate type (C)	4	10.26	**0.0030**	**6976073**	9136997	77.2	**9.66**
	Degree of grinding (G)	1	0.48	0.0010	15327	4464913	3.1	0.90
	CG=Whole Plot Error	4	5.52	0.0003	442534	3468755	16.0	1.21
Sub plot factors	Temperature (T)	1	17.96	**0.002**	380055	1445520	12.1	2.50
	Feeder rate (F)	1	1.23	**0.004**	**19244626**	**26082250**	**242.0**	**44.10**
	Water % (W)	1	**56.17**	**0.050**	1360503	**9308390**	**2749.0**	**44.10**
	FC	4	5.47	0.000	987991	1418416	2.6	0.29
	TC	4	7.25	0.000	686909	1316458	2.6	0.06
	WC	4	3.87	**0.004**	1875563	3027810	**58.4**	0.29
	$TW = GF$	1	0.00	**0.002**	271096	163328	3.4	0.00
	$FW = GT$	1	6.56	**0.002**	2846756	404010	**161.6**	0.00
	$FT = GW$	1	**32.40**	**0.001**	95551	43296	**50.2**	0.40
	Higher Order Inter.=SubPlot Error	12	5.80	0.0002	691683	1185597	7.3	0.20

into four single degree of freedom effects denoted *Ci5* for i = 1, 2, 3, 4 as shown in Appendix B. These effects can be interpreted as a comparison between the current carbohydrate type (type 5) and the four alternatives (type 1 - type 4).

The whole plot effects were then computed by regressing the data on columns under the heading whole plot effects in Table A1. With this coding the variance-covariance matrix of whole plot effects, denoted Σ_w, is

$$\Sigma_w = \begin{bmatrix} 0.1 & -0.025 & -0.025 & -0.025 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.025 & 0.1 & -0.025 & -0.025 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.025 & -0.025 & 0.1 & -0.025 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.025 & -0.025 & -0.025 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.025 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1 & -0.025 & -0.025 & -0.025 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.025 & 0.1 & -0.025 & -0.025 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.025 & -0.025 & 0.1 & -0.025 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.025 & -0.025 & -0.025 & 0.1 \end{bmatrix} \sigma_w$$

where σ_w is the whole plot expected MSE. From Σ_w we see that the main effects *C15*, *C25*, *C35* and *C45* are correlated with each other, the interaction effects *GC15*, *GC25*, *GC35* and *GC45* are correlated with each other but independent of all the whole plot main effects, and effect *G* is independent of all the whole plot effects. Furthermore, all the whole plot effects have the same variance except factor *G*. Thus, in this case, the assumption of constant variance can easily be satisfied by scaling the completely independent effect *G* by two. Even so, the effects that were correlated remain correlated and hence the assumption of independence of effects will only be approximate. This technique of scaling the completely independent effects to get constant variance can also be applied to the subplot effects. From the variance-covariance matrix of subplot effects it can be shown that only the interactions involving *Ci5* for each *i* are correlated with each other.

The normal plots of whole plot effects and of subplot effects are given below in Figures 4 and 5 respectively.

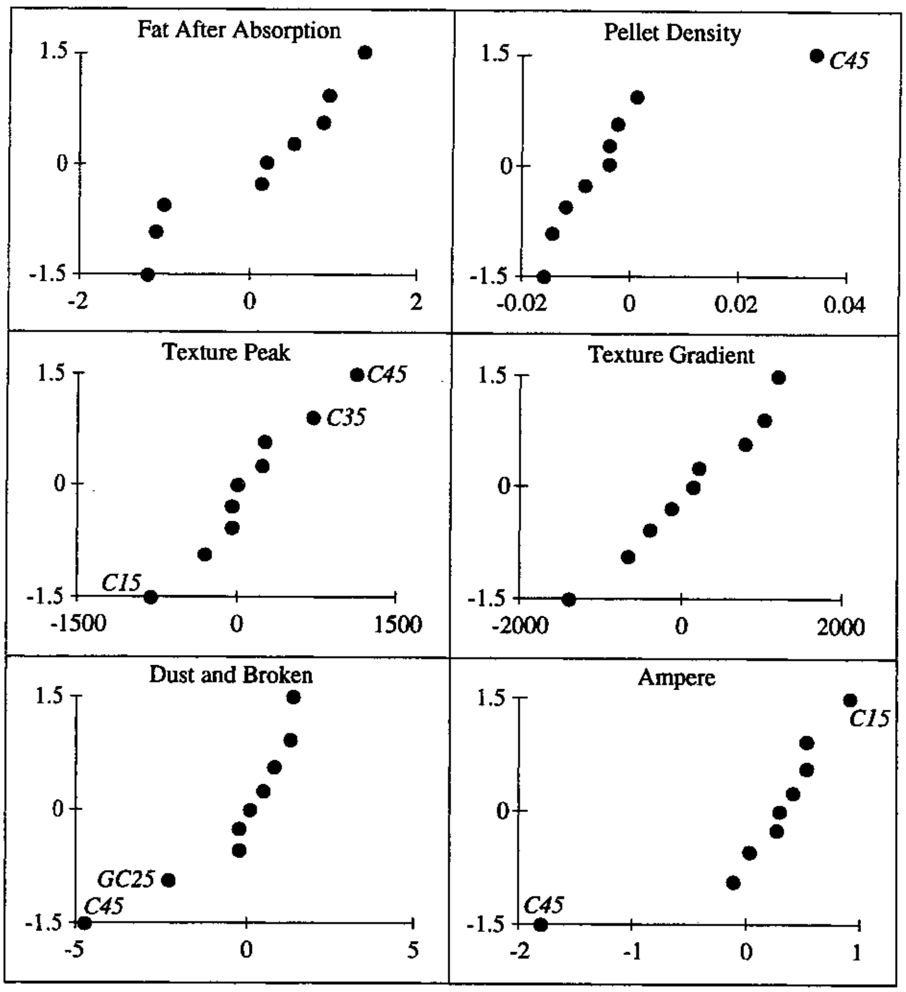

***Figure 4.** The normal plots of whole plot effects for the different responses found important in the PCA analysis.*

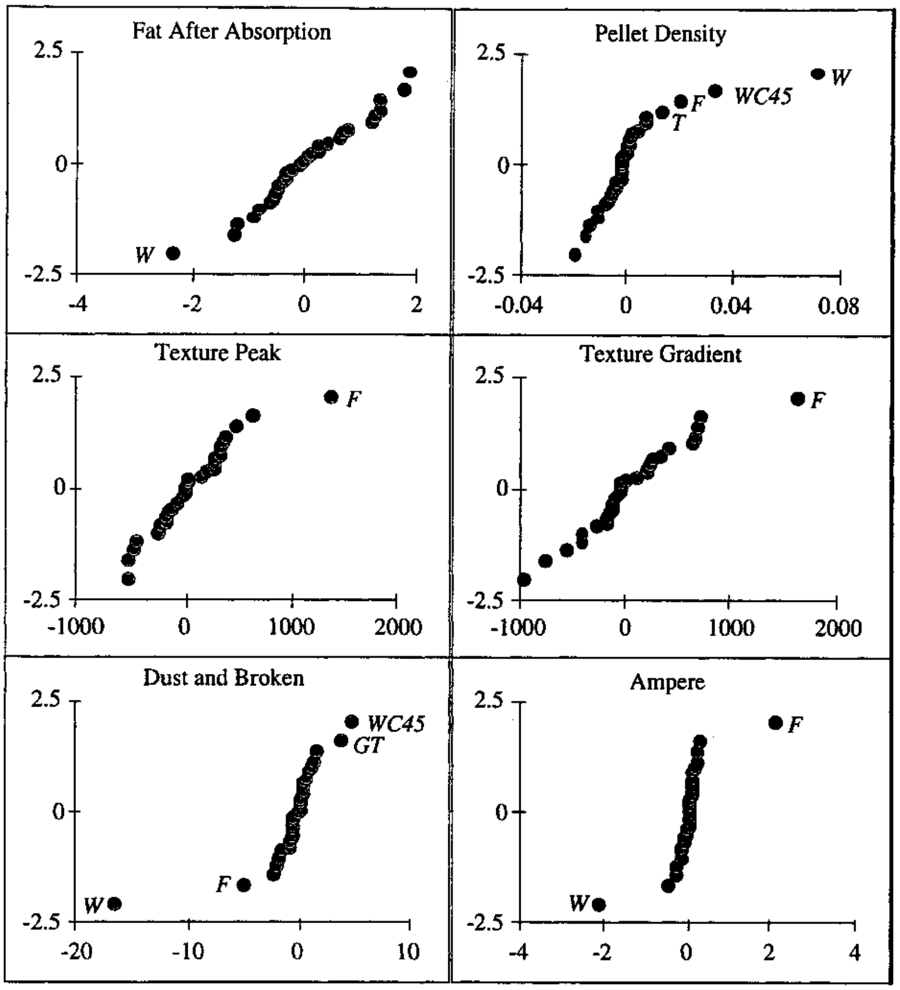

Figure 5. The normal plots of subplot effects for the different responses found important in the PCA analysis.

In Figure 4 only the probability plot of whole plot effects using fat after absorption as the response appears to be ill behaved. The pattern of effects in this plot may indicate a bad value (see Box, 1991). However, in this case we were unable to identify a bad value. From Figure 5 we see that all the normal probability plots of subplot effects appear relatively well behaved. Effects deemed significant by us are labeled in Figures 4 and 5.

VII. A COMPARISON OF THE ANOVA AND PROBABILITY PLOT ANALYSES

The effects found significant from the ANOVA and probability plot analyses are listed below in Table 3. Table 3 shows that in both analyses none of the whole plot factors appear to influence fat after absorption or texture gradient. However, pellet density, texture peak and ampere seem to be influenced by changing the carbohydrate types. Thus, except for the discrepancies associated with dust and broken, there is good agreement between these whole plot analyses. One possible explanation for the discrepancy is that there may be a significant *GC25* interaction which if real would inflate the estimate of the whole plot expected MSE in the ANOVA analysis. This in turn may cause the *C* effect to appear insignificant. Since assuming a two-factor interaction to be inert is a relatively strong assumption we would tend to follow the results of the normal plot in this case. Now from the normal plots of whole plot effects we can also get information about how the alternative types of carbohydrate affect the responses.

Pellets produced from carbohydrate 4 had lower density, higher amount of dust and broken, lower texture peak and resulted in higher energy usage as compared to pellets produced from carbohydrate 5. All these characteristics of carbohydrate 4 are

Table 3. *The significant effects from the ANOVA analysis and from the probability plot analysis given in columns denoted* A *and* P *respectively.*

	Fat after abs.		Pellet density		Tex. peak		Tex. gradient		Dust & broken		Ampere	
	A	*P*	*A*	*P*	*A*	*P*	*A*	*P*	*A*	*P*	*A*	*P*
Whole Plot			*C*	*C45*	*C*	*C15* *C35* *C45*				*C45* *GC25*	*C*	*C15* *C45*
Sub Plot	*W* *FT=GW*	*W*	*T* *F* *W* *WC* *TW=GF* *FW=GT* *FT=GW*	*T* *F* *W* *WC45*	*F*	*F*	*F* *W*	*F*	*F* *W* *WC* *FW=GT* *FT=GW*	*F* *W* *WC45* *GT*	*F* *W*	*F* *W*

unwanted properties according to the manufacturer. Carbohydrates 1 and 3 seem also to cause different texture peak in the pellet as compared to carbohydrate 5. Carbohydrate 1 resulted in pellets of higher peak, which is wanted, whereas carbohydrate 3 gave pellets of lower texture peak as compared to using carbohydrate 5.

Looking again at Table 3 we see that there is also good agreement between the subplot analyses. There are, however, a few interaction effects that appear marginally significant in the ANOVA analysis but do not appear significant when looking at the normal plots. One possible explanation for the discrepancy is that the F-test can be a liberal test of effect significance when the expected MSE is estimated by pooling the smallest effects (see Berk and Picard, 1991). Thus it is reasonable to confine our interpretation of the effects to only those that appear significant in both analyses.

These results indicate that it would be important to control water % (W). A high water % should be used since that decreased the amount of dust and broken pellets, gave pellets with high density (=100% sinkage), low ampere (=low production costs) and wanted texture properties. The use of carbohydrate 1, 2, 3 and 5 as raw materials resulted in pellets with desirable quality characteristics and low ampere. By using these four carbohydrates as raw materials it might be possible to produce a pellet of consistent quality. The use of high feeder rate (F) in combination with high water % (W) was preferable regarding minimization of dust and broken pellets, increased pellet density and texture peak, but also resulted in higher ampere (=higher production costs) than using low feeder rate in combination with high water %. Temperature (T) and degree of grinding (G) do not seem to have a large effect on either the quality of the pellet or the amount of energy used to produce it. Thus, it might be possible to select the level for these variables which is most economical or convenient.

VIII. CONCLUSION

In this paper we have shown that PCA can be an effective tool for reducing the number of response variables necessary to describe the main variation among the samples. In addition, PCA provided information about the variation among the samples which was verified by the ANOVA and Normal probability plot analysis. Furthermore, it was shown that different methods of analysis, in this case the ANOVA and normal plot, can be used as complementary sources of information. For the fish food example, the significance of most of the effects was shown to be robust to these two methods of analysis. Thus the assumption of inert effects for the ANOVA and the correlation of effects in the normal plot analysis did not appear to adversely impact the overall analysis.

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APPENDIX A

Table A1. *The coding scheme used to compute effects for use with the normal plots.*

	Whole Plot Effects								SubPlot Effects					
Run	*C15*	*C25*	*C35*	*C45*	*G*	*GC15*	...	*GC45*	*T*	*F*	*W*	*TC15*	...	*WG*
1	–	0	0	0	–	+		0	–	–	+	+		+
2	–	0	0	0	–	+		0	+	–	–	–		–
3	–	0	0	0	–	+		0	–	+	–	+		+
4	–	0	0	0	–	+		0	+	+	+	–		–
5	–	0	0	0	+	–		0	–	–	+	+		–
6	–	0	0	0	+	–		0	+	–	–	–		+
7	–	0	0	0	+	–		0	–	+	–	+		–
8	–	0	0	0	+	–		0	+	+	+	–		+
9	0	–	0	0	–	0		0	–	–	+	0		+
10	0	–	0	0	–	0		0	+	–	–	0		–
11	0	–	0	0	–	0		0	–	+	–	0		+
12	0	–	0	0	–	0		0	+	+	+	0		–
13	0	–	0	0	+	0		0	–	–	+	0		–
14	0	–	0	0	+	0		0	+	–	–	0		+
15	0	–	0	0	+	0		0	–	+	–	0		–
16	0	–	0	0	+	0		0	+	+	+	0		+
17	0	0	–	0	–	0		0	–	–	+	0		+
18	0	0	–	0	–	0		0	+	–	–	0		–
19	0	0	–	0	–	0		0	–	+	–	0		+
20	0	0	–	0	–	0	...	0	+	+	+	0	...	–
21	0	0	–	0	+	0		0	–	–	+	0		–
22	0	0	–	0	+	0		0	+	–	–	0		+
23	0	0	–	0	+	0		0	–	+	–	0		–
24	0	0	–	0	+	0		0	+	+	+	0		+
25	0	0	0	–	–	0		+	–	–	+	0		+
26	0	0	0	–	–	0		+	+	–	–	0		–
27	0	0	0	–	–	0		+	–	+	–	0		+
28	0	0	0	–	–	0		+	+	+	+	0		–
29	0	0	0	–	+	0		–	–	–	+	0		–
30	0	0	0	–	+	0		–	+	–	–	0		+
31	0	0	0	–	+	0		–	–	+	–	0		–
32	0	0	0	–	+	0		–	+	+	+	0		+
33	+	+	+	+	–	–		–	–	–	+	–		–
34	+	+	+	+	–	–		–	+	–	–	–		+
35	+	+	+	+	–	–		–	–	+	–	–		–
36	+	+	+	+	–	–		–	+	+	+	–		+
37	+	+	+	+	+	+		+	–	–	+	+		–
38	+	+	+	+	+	+		+	+	–	–	+		+
39	+	+	+	+	+	+		+	–	+	–	+		–
40	+	+	+	+	+	+	...	+	+	+	+	+	...	+