DISCLAIMER

This research was funded by the Midwest Regional University Transportation Center. The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the Department of Transportation, University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof. The contents do not necessarily reflect the official views of the Midwest Regional University Transportation Center, the University of Wisconsin, the Wisconsin Department of Transportation, or the Federal Highway Administration at the time of publication.

The United States Government assumes no liability for its contents or use thereof. This report does not constitute a standard, specification, or regulation.

The United States Government does not endorse products or manufacturers. Trade and manufacturers names appear in this report only because they are considered essential to the object of the document.
2. Government Accession No.
3. Recipient’s Catalog No.
   CFDA 20.701
4. Title and Subtitle
   Infrastructure Management Decision-Making with Condition Data Generated by Remote Sensors:
   A Time-Series Framework
5. Report Date
   August 31, 2004
6. Performing Organization Code
7. Author/s
   Pablo Durango-Cohen and Naveen Tadepalli
   MRUTC 04-03
9. Performing Organization Name and Address
   Midwest Regional University Transportation Center
   University of Wisconsin-Madison
   1415 Engineering Drive, Madison, WI 53706
10. Work Unit No. (TRAIS)
11. Contract or Grant No.
   DTRS 99-G-0005
12. Sponsoring Organization Name and Address
   U.S. Department of Transportation
   Research and Special Programs Administration
   400 7th Street, SW
   Washington, DC 20590-0001
13. Type of Report and Period Covered
   Research Report [Dates]
15. Supplementary Notes
   Project completed for the Midwest Regional University Transportation Center with support from the Wisconsin Department of Transportation.
16. Abstract
   Recent developments in remote sensing and communications technologies allow agencies to install sensors within infrastructure facilities, such as pavement segments and bridges in order to collect condition-related data in real-time. In theory, such data can be processed, analyzed and displayed on-line as a key component for maintenance, and repair decision-making. The reality facing public works agencies that have adopted these technologies is that vast amounts of data related to the structural and functional condition of infrastructure are accumulated, but not used to address management needs. The research presented herein, therefore, is to develop methodological tools to support the management of transportation infrastructure systems given recent developments in facility-condition data collection technologies. In particular, the objectives of this research study are to develop tools that will allow agencies to process and exploit the data to support IM&R decision-making, and to provide a framework to evaluate different strategies for deploying sensing technologies.
17. Key Words
18. Distribution Statement
   No restrictions. This report is available through the Transportation Research Information Services of the National Transportation Library.
19. Security Classification (of this report)
   Unclassified
20. Security Classification (of this page)
   Unclassified
21. No. Of Pages
   -0-
22. Price
   -0-

Form DOT F 1700.7 (8-72)  Reproduction of form and completed page is authorized.
## Contents

1 Introduction ................................. 5
   1.1 Motivation and Objectives 5
   1.2 Project Description and Outline 5

2 Background and Literature Review .......... 8
   2.1 Background: Transportation Infrastructure/Asset Management 8
   2.2 Motivation: Data Collection Using Remote Sensors and Other Advanced Technologies 9
       2.2.1 Using advanced technologies for condition assessment 10
   2.3 Infrastructure Management Decision-Making 13
       2.3.1 Computational limitations of the Latent-MDP approach: An Example 16

3 Model Formulation and Solution .......... 17
   3.1 Problem Description 17
   3.2 Model Formulation 18
       3.2.1 Assumptions 19
       3.2.2 Optimization Problem 20
   3.3 Solution Procedure 20
       3.3.1 Optimization Problem 20
       3.3.2 State Estimation Problem 21

4 Application Case Studies .................. 23
   4.1 Numerical Example: State-Estimation Problem 23
   4.2 State-Estimation using Sensor Data 25
   4.3 Empirical Study of the Effect of Uncertainty on Life-Cycle Costs 26
   4.4 Combining Multiple Technologies for Condition Assessment 27
       4.4.1 Numerical Results 28

5 Summary and Conclusions ................. 30

A Cost Parameter Generation ............... 31

B Latent Performance and Measurement-Error Models 32
   B.1 Estimation of Precisions Associated with Technologies: A Note on Linear Regression 33
List of Figures

1. Asset Management Process (Taken from FHWA (1999)) ........................................... 9
2. Latent Performance Modeling Approach ................................................................. 15
3. Economic Trade-offs Associated with M&R Investments ........................................ 18
4. Updated state-distribution: First moments ............................................................ 24
5. Updated state-distribution: Second moments ......................................................... 25
6. Updated state-distribution: First moments ............................................................ 26
7. Life-Cycle Costs vs. Deterioration Process Variance ............................................. 27

List of Tables

1. Cost Parameters ..................................................................................................... 26
2. Expected Costs for all Technology Combinations .................................................. 29
3. Discretization and Transformation of PCI Scale ..................................................... 31
4. Agency and User Costs from Madanat and Ben-Akiva (1994) ($/m^2) .................. 32
5. Descriptive statistics ............................................................................................... 33
6. Parameter Estimation Results .................................................................................. 33
7. Precision Estimates ................................................................................................. 34
1 Introduction

This document is the final report for project 04-03 sponsored by the Midwest Regional University Transportation Center (MRUTC). The project consists of developing an optimization framework to provide support for investments in preservation and improvement of transportation infrastructure facilities that are inspected periodically with sensors or other advanced technologies. In the remainder of this section, we first state the motivation for our work and the objectives of our study. We then present an overview of the tasks that we carried out as part of the project and provide an outline for the report.

1.1 Motivation and Objectives

This research project is motivated by recent developments in remote sensing and communications technologies that allow public works agencies to install sensors within infrastructure facilities, such as pavement sections and bridge decks, in order to collect condition-related data. In addition, a plethora of non-destructive inspection technologies, e.g., video, radar, and laser, have become commonplace in evaluating and measuring distresses on transportation infrastructure. In theory, such data should be processed and used as a key component to support maintenance and repair (M&R) decision-making. The reality facing agencies that have adopted these technologies is that vast amounts of data are accumulated, but not used to address management needs. The goal of the research described herein, therefore, is to develop methodological tools to exploit the extensive capabilities of advanced monitoring technologies to support M&R investment decisions. In particular, the objectives of the study are to develop a framework that allows agencies to:

1. Process condition data efficiently and use them to support M&R decision-making; and

2. Quantify the value of combining different monitoring technologies, which means that framework can be used as a tool to support the development of strategies to deploy advanced inspection technologies.

1.2 Project Description and Outline

The project consists of five tasks which we describe below. We also provide an outline for the remainder of the report which roughly is consistent with the tasks.

Task 1: Literature Review  This task consists of conducting an extensive literature review to identify possible approaches to address the research problem. The challenges involved in developing optimization models to support M&R investment decisions with condition data generated by sensors or other (advanced) technologies are related to the potentially vast amounts of data that can be obtained. These data have to be processed with tools that are both statistically rigorous and computationally efficient. In addition, the algorithm to solve the underlying optimization problem
needs to be tractable. These requirements, for the most part, constitute important limitations of the available tools to address the research problem that we study. These issues are discussed extensively in Section 2 of this report. In particular, we present an example that illustrates the computational shortcomings of the current state-of-the-art approach to address the problem. We begin Section 2 by putting our research in the context of the “Transportation Asset Management Framework” presented in The Asset Management Primer (FHWA, 1999). We also review examples of agencies and initiatives that are collecting condition data using sensors.

Task 2: Model Formulation and Solution We present an optimization framework to support M&R investment decisions for transportation infrastructure facilities. The framework involves formulating the underlying decision problem as a discrete-time, stochastic optimal control problem and consists of two components: a state-estimation problem that involves processing vast arrays of condition data and using them to develop condition forecasts; and an optimization problem whose solution yields M&R investment policies. Our approach differs from the literature in that both elements are fully integrated. This, in turn, leads to a framework that is both statistically rigorous and computationally efficient, i.e., capable of providing effective decision-support. The model is presented in detail in Section 3 of this report.

Task 3: Application Case Studies Section 4 of this report presents four application case studies. The objectives of the empirical studies are as follows:

1. Provide numerical examples to illustrate the methodology presented above to address the state-estimation problem in the above framework. In particular, we show how the Kalman Filter processes distress measurements to update the state distribution. For this part of the study we use both a set of simulated data as well as sensor data. The sensor data was provided by the Minnesota Road Research Project (MnROAD).

2. Show how the the framework can be used to study the effect of uncertainties in the deterioration process and in the process of collecting distress measurements on the optimal life-cycle cost of managing infrastructure facilities.

3. Illustrate how the framework can be used to quantify the value of combining different technologies for condition assessment.

Initially, our objective was to compare the methodology we developed to state-of-the-art models. However, early on we determined that the existing framework is inadequate to process data generated simultaneously by multiple technologies. We use the example presented in Section 2.3.1 to illustrate the limitations associated with the existing approach.

Task 4: Preparation of Reports and Deliverables In addition to this report and four quarterly progress reports delivered earlier, we have prepared (and are preparing) the following
materials to disseminate the results of this research effort.

1. A paper submitted for presentation at the Transportation Research Board 84th Annual Meeting to be held January 9–15, 2005 in Washington, D.C. The paper was also submitted for publication in Transportation Research Record (the journal of the Transportation Research Board).

2. A paper in preparation to be submitted to a leading journal in the area of transportation systems analysis.

3. A report for educational purposes that summarizes the results of this effort. The report is in the form of a powerpoint presentation that has been delivered at institutions, conferences, and meetings. These include invited presentations at the Third International Symposium on Infrastructure Management and Financing (Kyoto, Japan – September 2003), and at the Industrial and Systems Engineering Department at Lehigh University (Bethlehem, PA – May 2004); contributed presentations at the Annual Meeting of the Institute for Operations Research and Management Sciences (Informs) (Atlanta, GA – October 2003). In addition we prepared a poster for the MRUTC’s reception held at the 83rd Annual Meeting of the Transportation Research Board (Washington, D.C. – January 2004). The powerpoint presentation is available directly from the P.I. and can be made available through the MRUTC.

In addition, the P.I. would be happy to participate/teach in a MRUTC-organized workshop to instruct users and agencies about the work described herein.

**Task 5: Exploration of Technology Transfer**  We had hoped to use the MRUTC as a liaison to establish partnerships with public works agencies. Through this effort we established contacts at the Michigan DOT. Unfortunately, Michigan’s efforts to use advanced technologies to monitor transportation infrastructure are in their infancy. Through our own efforts we obtained pavement management data from the states of Arizona and Washington. These data, however, were not generated using advanced technologies. Eventually, we established a partnership with the Minnesota DOT through MnROAD. They have provided extensive data and technical assistance and have expressed interest in partnering with the P.I. to both continue with the research effort and to disseminate the results. We use data provided by MnROAD in the example presented in Section 4.2.
2 Background and Literature Review

This section addresses Task 1 of the project (Literature Review). To put our research in the context of “Transportation Asset Management”, we begin this section by presenting a brief overview of the broadly accepted and highly regarded framework presented in The Asset Management Primer (FHWA, 1999). We proceed to motivate the relevance of the research herein, by briefly describing examples of agencies and initiatives that are currently using remote sensors and other advanced technologies to inspect infrastructure facilities. To conclude the section, we review the literature on infrastructure management decision-making. In particular, we focus on the shortcomings associated with existing methodologies that motivate the need for the research presented in this report.

2.1 Background: Transportation Infrastructure/Asset Management

The nation’s transportation infrastructure serves as the backbone of a complex network for supplying goods and services in an increasingly competitive and distributed economy. The quality and efficiency of this infrastructure, through its ability to provide mobility, and consequently, access to people, goods, services and resources, impacts quality of life and the continuity of economic and business growth. Indeed, economists (c.f. Small et al. (1989) and Hulten (1996)) have argued that investments in the management and efficient use of infrastructure have a greater impact than investments in additional infrastructure. Consequently, with an aging transportation infrastructure whose replacement value is estimated at $1 trillion (Kane, 2000), the development of effective and efficient policies to allocate resources for the construction, operation, preservation and improvement of such facilities takes on unprecedented social and economic value.

Transportation Asset Management as defined in The Asset Management Guide (FHWA, 1999) is a strategic approach to manage transportation infrastructure which encompasses a broad array of business functions, activities and decisions. It provides a systematic framework to support the resource allocation decisions that are motivated by the trends mentioned in the previous paragraph. The primer describes the asset management process as follows:

“First, performance expectations, consistent with goals, available resources, and organizational policies, are established and used to guide the analytical process, as well as the decision-making framework. Second, inventory and performance/condition data are collected and analyzed. This information provides input on future system requirements. Third, the use of analytical tools and reproducible procedures produces viable cost-effective strategies for allocating budgets to satisfy agency needs and user requirements, using performance expectations (condition forecasts) as critical inputs. Alternative choices are then evaluated, consistent with long-range plans, policies, and goals. The entire process is reevaluated annually through performance monitoring and systematic processes.”
The research described in this report relates to the third step in the process. That is, we have developed a methodological tool that can exploit the extensive capabilities of advanced inspection technologies to support investments in M&R of transportation infrastructure. Our work recognizes that the steps in the asset management process are interconnected, i.e., that fundamental changes in the condition assessment and performance prediction step (Step 2) warrant improvements in the optimization models that are used to support M&R investments (Step 3) in order to fully take advantage of the enhanced capabilities.

2.2 Motivation: Data Collection Using Remote Sensors and Other Advanced Technologies

The main barrier for the implementation and use of the model presented herein would seem to be to convince public works agencies to collect condition data using sensors and other advanced technologies. Here we provide several examples of agencies and initiatives that have deployed advanced technologies for condition assessment of transportation infrastructure. The purpose is to illustrate the extent of the potential users for the model we developed. Overall, agencies have adopted these technologies in the last 10 years. Even though their use has primarily been directed toward experimental infrastructure facilities, it does seem reasonable to assume that technological developments in areas such as fiber optics, micro-electrical-mechanical systems (MEMS), radar, laser, satellite imaging, image processing, etc. will increase the availability and cost-effectiveness of using them for condition assessment of facilities that are in use. In the remainder of this section we proceed to describe initiatives and agencies that use advanced technologies to monitor/inspect transportation
Prior to discussing the use of advanced technologies for condition assessment, we mention that such technologies have also been widely used in a slightly different context, to inventory transportation infrastructure. Satellite imaging, for example, has been used to support planning decisions such as prioritizing corridors for development, and evaluating overall condition after natural disasters (floods, earthquakes, etc.). Examples of agencies and initiatives that have been involved in these efforts include the National Consortium for Remote Sensing in Transportation (NCRST)\(^1\) and a Commercial Remote Sensing Products and Spatial Information Technologies Program\(^2\) which is a partnership between the USDOT and NASA.

### 2.2.1 Using Advanced technologies for Condition Assessment

Here we describe initiatives and agencies that use advanced technologies to monitor/inspect transportation infrastructure. Inspection/condition assessment in this context refers to the process of measuring distresses on transportation infrastructure periodically. Distress measurements can be collected manually or automatically and are comprised of multiple measurements and/or (subjective) ratings that can be either discrete or continuous. Examples of distresses in pavement management include roughness, type and extent of cracking, rut depth and profile, extent of surface patching, and raveling.

**The Infrastructure Technology Institute at Northwestern University (ITI)** is recognized as a leader in transferring remote sensing and communications technologies to the inspection structural elements in infrastructure facilities. For example, the institute has pioneered the development and deployment of acoustic, strain, and optical sensors to monitor the growth of cracks in structural members of steel bridges and other infrastructure facilities. Currently, about 30 facilities around the country (20 in the Midwest), mostly bridges, are being monitored. The major emphasis of these projects is to provide continuous remote monitoring. Most of these structures are critical for safety and need continuous monitoring of the structural fitness and condition of the sub-surface environment.

The ITI has been successful in developing and applying Acoustic Emission and strain gage monitoring to steel bridges and Time Domain Reflectometry and Impulse Echo to geotechnical applications. Relevant projects include: successful deployment strain gages and clinometers to monitor crack development, in the fracture critical components, of a 70-year-old Michigan Street Lift Bridge in Sturgeon Bay, Wisconsin (Prine and Fish, 2003). The ITI has also deployed strain and temperature sensors on the Hoan Bridge, Milwaukee to test if thermally driven stresses would

---

\(^1\)http://www.ncgia.ucsb.edu/nrst
\(^2\)http://scitech.dot.gov/research/remote
induce fatigue cracking in the structure. Apart from these, the ITI has deployed Time Domain Reflectometers to monitor the crack growth and thus, the structural stability of the rock beneath I-70 in Souteastern Ohio. Additional information and publications can be obtained from the institutes website.3

The Minnesota Road Research Project  MnROAD is at the forefront of using advanced monitoring technologies for condition assessment of pavements. They have an extensive network of over 4,572 sensors spread over two pavement segments that run parallel to Interstate 94 near Ostego, Minnesota. The “mainline” section is 3.5 miles in length and the “low volume” road way consists of a 2.5 mile closed loop where controlled weight and traffic volume simulate rural road conditions. Static and dynamic sensors record pavement response to traffic loading such as deflections, strains, stresses, etc. Environmental sensors are used to measure characteristics such as temperature, precipitation, wind velocity and atmospheric pressure. MnROAD also uses uses probes that are equipped with lasers, radar and other technologies to measure pavement roughness, cracking, raveling, and rutting (depth and profile). The main objectives of the project are to evaluate the effects of heavy vehicles on pavements, evaluating the effects of seasonal changes in paving materials, and to improve the design and performance of low-volume roadways.

MnROAD has successfully completed 10 years in operation. The vast amount of data collected has been used to validate empirical models for pavement design. Using these models MnROAD has developed software programs (e.g.: Pavecool, MnPave), which help in designing new pavements for various climatic, traffic and structural conditions. In addition, MnROAD has tested various aggregates and crack sealants such as recycled concrete aggregates and carbonate aggregates. For further details the reader is referred to the MnROAD’s website.4

The Smart Road Project  in the state of Virginia is another project which has deployed sensors to monitor the condition of the pavement. This project is 9.6-km connector highway between Blacksburg and I-81 in southwest Virginia. This project tried to address limitations of test facilities such as climate control, control of traffic speed and loading and acceleration for loading. Accordingly, the first 3.2-km stretch is designated as a controlled test facility. This facility allows for testing of various hypotheses on pavement material performance and characteristics. Using the “All Weather Testing Facility”, pavement materials can be tested in different environmental conditions. The weather conditions are simulated using 76 snow towers which can simulate snowfalls upto 100 mm/hr and rainfall upto 50 mm/hr. The objectives of the project are to enhance the methodologies for design and construction of pavements and to evaluate the concepts, technologies and products of Intelligent Transportation Systems.

3http://www.iti.northwestern.edu/publications
4http://www.mrr.dot.state.mn.us/research/mnresearch.asp
Hot-Mix Asphalt Strain Gages, Aggregate Dynamic Strain Gages, Vibrating Wire Static Strain Gages are used to measure strains in the various layers of the pavement. Pressure cells are deployed to collect pressures in the various layers of the pavement and Thermocouples are used to measure the heat flow inside the pavement system. These constitute the dynamic measurement sensors. The static measurements constitute of environmental data such as temperature, moisture and frost depth. The construction of this project was recently completed and hence most of the work is in its infancy.

**National Consortium for Remote Sensing in Transportation (NCRST)** is at the forefront of developing Remote Sensing applications for transportation. The objective of this agency is to focus on testing and implementation of commercial remote sensing technologies and methods to meet future transportation requirements.

Many projects are being undertaken to compare and evaluate various techniques and their effectiveness in pavement management. As a part of these projects, studies were carried out on Laser Scanning for applications in Construction and Bridge Maintenance. A pilot study conducted by Iowa Department of Transportation has shown that Ground Laser, which is a very accurate way of imaging, is useful in developing as-built- 3-D infrastructure data. The study showed that we can obtain 2-6 mm precision images. But, it was found that this technique was costly by 30 percent when compared to its competing technique namely, aerial photogrammetry. The investigators believe that this technique could be made competitive by elevating this scanner on a boom truck and scanning both sides of the divided roadway (Jaselskis et al., 2003).

Pavement Health Surveys have also been carried out using equipments like hyperspectral sensor, hand held spectrometers etc. A study is being conducted by University of California Santa Barbara (UCSB) in joint collaboration with Iowa State University to find a correlation between remotely sensed parameters (like spectral reflectance) and physical characteristics like rutting and cracking. The listings of other projects by this agency can be found at the project’s website.

**FHWA’s Non Destructive Evaluation Validation Center (NDEVC)** is an another agency which is actively involved in developing and implementing automated pavement and bridge evaluation techniques. The objective of this center is to develop NDE tools and techniques that are both accurate and efficient. Apart from this, the center also tests the reliability of the NDE technologies in its laboratories. These laboratories help simulate field conditions. Apart from these, NDEVC has five decommissioned bridges to evaluate the NDE tools and techniques under realistic environ-

---

5 [http://www.cee.vt.edu/program_areas/tise/smart/overview.html](http://www.cee.vt.edu/program_areas/tise/smart/overview.html)
6 [http://www.ncgia.ucsb.edu/ncrst/research.html](http://www.ncgia.ucsb.edu/ncrst/research.html)
mental conditions.

High Speed Electromagnetic Roadway Measurement and Evaluation System (HERMES), Ground Penetrating Radar, Laser Bridge Deflection Measurements, Ultrasonic Stress Measurements, X-ray computed tomography are some of the tools developed by this center (Washer, 2000).

2.3 Infrastructure Management Decision-Making

In this section, we present an overview of optimization models used to support investment decisions for M&R of transportation infrastructure. We also discuss the limitations that motivate the need to develop a framework that can exploit the capabilities of advanced monitoring technologies.

Optimization models to support M&R of transportation infrastructure systems constitute applications, perhaps the most successful, of the “Equipment Replacement Problem” introduced by Terborgh (1949) and formulated as a dynamic control problem by Bellman (1955) and Dreyfus (1960). Friesz and Fernandez (1979) and Golabi et al. (1982) extended the models to support M&R of transportation infrastructure. State-of-the-art optimization models are formulated as Markov Decision Processes (c.f. Murakami and Turnquist (1985), Carnahan et al. (1987), and Carnahan (1988)). Golabi et al. (1982), for example, present a mixed-criteria, constrained, Markov Decision Process (MDP) for pavement management in the state of Arizona (a network of 12,000 kilometers of highways). Savings of $14 million were reported in the first year of implementation, and $101 million was forecast for the following four years. The same optimization model drives Pontis (Golabi and Shepard, 1997), a bridge management system used in over 40 states. The success and impact of these models is related to the magnitude of investments in M&R of transportation infrastructure which in the United States is on the order of tens of billions of dollars per year. Recent reviews of optimization models for transportation infrastructure management are presented in Gendreau and Soriano (1998) and Durango (2002).

Optimization models to support M&R investments must evaluate both the short and long-term consequences associated with M&R actions. For this reason, they must incorporate information about the effect of actions on current and future infrastructure condition. Information about current condition is obtained through distress measurements. Distress measurements can be collected manually or automatically and are comprised of multiple measurements and/or (subjective) ratings that can be either discrete or continuous. Examples of distresses in pavement management include roughness, type and extent of cracking, rut depth and profile, extent of surface patching, and raveling. Information about future condition, i.e., condition forecasts, are generated with statistical deterioration models. A deterioration model relates condition to a set of explanatory variables such as design characteristics, traffic loading, environmental factors, and history of M&R investments. Models to support M&R investments based on the MDP framework rely on indices/ratings that
combine condition data into a single quantity. Examples include the Concrete Bridge Deck Condition Ratings, the Present Serviceability Index (PSI) and the Pavement Condition Index (PCI) developed by FHWA (1979), HRB (1962) and Shahin and Kohn (1981), respectively. Unfortunately, these indices lack rigorous justification, have poor explanatory/predictive power, and rely on predetermined sets of distress measurements which precludes incorporating new ones. In spite of the computational efficiency of this approach, it is clear that relying on it to process condition data and to support M&R investments may negate the benefits of using advanced inspection technologies for condition assessment, and of using standard statistical methods to process condition data.

Ben-Akiva et al. (1991) introduced the latent performance modeling approach to address the problems of assessing and forecasting condition when multiple technologies are used to collect condition data. The approach relates distress measurements to the system’s current condition through a measurement-error model. The system’s condition is represented by latent/unobservable variables which capture the ambiguity that exists in defining (and consequently in measuring) a system’s condition. The measurement-error model accounts for uncertainties inherent in the data-collection process as well as for how different technologies and distress measurements relate to each other. As discussed by Humplick (1992), the uncertainties in the measurement process can be attributed to the precision and accuracy of measurement technologies because other biases can be corrected for. Latent performance models also include a deterioration model that describes the relationship between a set of explanatory variables and the system’s condition, and captures the randomness inherent in the system’s deterioration process.

The latent performance modeling approach is illustrated in Figure 2. The solid arrow represents the deterioration model and the dashed arrow represents the measurement-error model.

Empirical studies (Ben-Akiva and Ramaswamy, 1993; Ben-Akiva and Gopinath, 1995) have shown that latent performance models are appropriate to generate condition forecasts of transportation infrastructure, i.e., the goodness-of-fit measures are better than those reported using other other statistical methods. This lead Madanat and Ben-Akiva (1994) to include latent performance models into a framework to support M&R investments by formulating the underlying optimization problem as a latent MDP. The measurement-error model in latent MDP formulations is represented by a (discrete) probability mass function that relates the condition variable to the distress measurements. Mathematically,

$$\text{Prob}(Z_t = k | X_t = i), \ i, k \in S, t = 1, \cdots, T + 1$$

(1)

where $i$ and $k$ are elements in a finite set of possible conditions $S$, and Expression (1) represents
the conditional probability of collecting a measurement $Z_t = k$ at the start of period $t$ given that the true condition is $X_t = i$.

Unfortunately, discrete measurement-error models are virtually useless when multiple technologies are used simultaneously to measure different distresses (i.e.: when an array of measurements $\vec{Z}_t$ is collected) because it is necessary to specify a probability for every possible combination of measurements (a number that grows exponentially with the number of technologies and the number of distresses being measured). The computational complexity to find optimal M&R policies also increases exponentially with the size of $\vec{Z}_t$. An example and further discussion of limitation is addressed further in the following section. To a large extent, this difficulty explains why previous studies in the literature have only considered the case of inspections that yield a single distress measurement (c.f. Madanat and Ben-Akiva (1994), Smilowitz and Madanat (2000), Guillaumot, Durango-Cohen, and Madanat (2003)). In any case, it is clear that the emergence of advanced monitoring technologies poses serious methodological and computational challenges because of the potentially large quantities of data being generated. This limitation serves as motivation for the framework proposed in this paper which constitutes an alternative to the latent MDP that is both statistically rigorous and computationally efficient.
2.3.1 Computational Limitations of the Latent-MDP approach: An Example

Here we present an example to illustrate the computational problems that are associated with discrete measurement-error models. Consider a situation where five technologies are used to collect five distress measurements at the start of each period. Let's assume that each technology collects continuous measurements in a range $[0, R]$ and that each of the ranges is discretized into 11 points, $\{0, 1, 2, \cdots, 10\}$. This means that measurements are rounded up or down when they are collected. A measurement-error model for this situation must specify a probability for every possible combination of measurements. That is, $11^5 = 161,051$ probabilities need to be specified. This, in turn, poses two fundamental problems:

1. Statistically, we note that the schemes to specify these probabilities are based on approximation schemes that induce errors; and

2. Computationally, we see that the number of probabilities that need to be specified increases exponentially with the number of technologies/distress measurements that are collected. Unfortunately, the computational effort required to obtain optimal M&R policies using the latent-MDP approach also increases exponentially with the number of technologies/distress measurements. This is because every possible outcome of the measurement process in every period needs to be considered to obtain optimal M&R policies. In dynamic programming, these problems are referred to as the “curse of dimensionality” and are described in detail in references such as Dreyfus (1977) and Bertsekas (1995).

The above leads to the observation that state-of-the-art optimization models to support M&R investments are not suited to address the challenges posed by developments that allow agencies to simultaneously collect multiple distress measurements using multiple technologies.
3 Model Formulation and Solution

This section addresses Task 2 of the project (Model Formulation and Solution). First, we describe in very specific terms the problem that we address. We then present a mathematical formulation for the problem and the approach we propose to solve it.

3.1 Problem Description

We consider an agency that manages a facility under a periodic review policy over \( T \) periods. At the start of every period, \( t = 1, \ldots, T \), the agency collects sets of distress measurements. The data are related to a facility’s state represented with the random variable \( X_t \). The measurements taken at the start of \( t \) are represented by the vector \( \tilde{Z}_t \). We use \( I_t \) to represent the set of information that an agency has at its disposal at the start of period \( t \). Using the above notation,

\[
I_t \equiv \{ \tilde{Z}_1, A_1, \tilde{Z}_2, A_2, \ldots, \tilde{Z}_{t-1}, A_{t-1}, \tilde{Z}_t \} = \{ I_{t-1}, A_{t-1}, \tilde{Z}_t \}, \quad t = 1, \ldots, T + 1, \text{ and, } I_0 \equiv \phi \tag{2}
\]

Based on the available information, an agency decides to apply an action to the system, \( A_t \), and incurs a cost, \( g(X_t, A_t) \in \mathbb{R} \), that depends both on the action and on the current state of the system. This cost structure can be used to capture both agency and operating costs. In the management of transportation infrastructure, agency costs correspond to the costs of applying M&R actions and operating costs to (a fraction of) the users’ vehicle operating costs. Vehicle operating costs depend on condition and are associated with travel time, fuel consumption, vehicle maintenance, etc. At the end of the planning horizon facilities have a salvage/residual value of \( s(X_{T+1}) \in \mathbb{R} \) that depends on the terminal state of the system.

The costs to operate a facility increase as it deteriorates. By making M&R investments, agencies can mitigate and even reverse the effects of deterioration and, consequently decrease current and future operating costs. As a result, an agency’s choice of M&R investments trades off investments with operating costs. These trade-offs are illustrated in Figure 3.

The figure depicts a situation where at the start of the third period an agency is choosing between either a small investment, \( S \), or a large investment, \( L \). The figure on the left is for facility condition vs. time. The one on the right is for cumulative discounted costs over time. As is illustrated, the large investment results in greater improvement in condition and in additional costs incurred at the start of the third period. However, as a result of the improvement in condition, the rate at which costs are accrued after the investment is greater for the small investment. Ultimately, an agency’s choice of actions is intended to minimize the sum of expected discounted (social) costs

\[7 \text{In the management of transportation infrastructure, planning horizons tend to be long and uncertain. Therefore, it is often acceptable to consider the case when } T \to \infty.\]
3.2 Model Formulation

We begin by presenting a general formulation that captures an agency’s decision problem as described in the previous section. The decision problem can be represented as:

**Objective Function:**

\[
\min \quad E[X_1, \ldots, X_{T+1}] = \sum_{t=1}^{T} \left[ \frac{1}{\delta^{t-1}} g(X_t, A_t) - \frac{1}{\delta^T} s(1+\delta)^{-T} \right]
\]

where the time-value of money is captured with a discount factor \( \delta \). Equation (3) corresponds to the sum of expected discounted costs incurred over the planning horizon. Equation (4) represents the dynamics of the system, i.e., its physical deterioration over the planning horizon. Equation (5) (1 + \( \frac{r}{1+r} \)) represents the discount/interest rate.

Equation (6) represents the initial state of the system.

Subject to:

**Constraints:**

\[
X_{t+1} = D_t(X_t, A_t, \beta_t), \quad t = 1, 2, \ldots, T
\]

\[
\bar{Z}_t = M_t(X_t, \Gamma_t), \quad t = 1, 2, \ldots, T + 1
\]

\[
\bar{Z}_1 = \bar{z}_1
\]

The decision problem described in the previous section can be represented as:

**Model Formulation**

We then present the assumptions that we make to solve the problem and the set of models that comprise the framework we propose to address the problem of developing optimal M&R policies.

Figure 3: Economic Trade-offs Associated with M&R Investments
process. The structure of the deterioration model, \( D(\cdot) \), is determined by factors such as material and construction quality, environmental conditions, etc. The arguments of the system equation may include deterministic or stochastic exogenous inputs that are captured in the vector \( \vec{\beta}_t \). These inputs may include environmental factors, traffic loadings, etc. Equation set (5) is a measurement error model. It establishes the relationship between the underlying, true state of the system and the distress measurements. The measurement error model \( M(\cdot) \) includes a set of exogenous (deterministic or stochastic) inputs captured in the matrices \( \Gamma_t \) (one vector associated with each distress measurement). Equation (6) specifies the initial condition of the system.

### 3.2.1 Assumptions

We proceed to state and discuss the assumptions that we use to solve the mathematical model used to compute M&R policies. The assumptions are:

1. We assume that \( X_t, A_t \in \mathbb{R}, t = 1, \ldots, T + 1 \) and that \( \vec{Z}_t \in \mathbb{R}^n, t = 1, \ldots, T + 1 \).

2. The period cost function can be represented (or approximated) by a second order polynomial, i.e.,
   \[
   g(X_t, A_t) = aX_t^2 + bX_tA_t + cA_t^2 + dX_t + eA_t + f.
   \]
   We define \( (a_t, b_t, c_t, d_t, e_t, f_t) = \delta^{t-1}(a, b, c, d, e, f) \). The cost structure may also be used to account for non-stationary costs.

3. The salvage value function can be represented (or approximated) by a second order polynomial, i.e.,
   \[
   s(X_{T+1}) = -p_{T+1}X_{T+1}^2 - q_{T+1}X_{T+1} - r_{T+1}.
   \]

4. The system equation can be represented (or approximated) by a linear polynomial, i.e.,
   \[
   D_t(X_t, A_t) = g_tX_t + h_tA_t + \epsilon_t.
   \]
   The polynomial can be obtained by estimating an AutoRegressive Moving Average with eXogenous input (ARMAX) model.

5. We assume that \( \epsilon_t \) follow a Normal Distribution with mean \( \bar{\epsilon}_t \) and finite variance \( \sigma_{\epsilon_t}^2 \).

6. We assume that the measurement error model can be represented (or approximated) by a linear polynomial, i.e.,
   \[
   M_t(X_t) = H_tX_t + \vec{\xi}_t.
   \]
   We discuss the nature of \( \vec{\xi}_t \) in Section 3.3.2.

Prior to discussing a solution procedure for the problem, we note that the above assumptions are not overly restrictive. Specifically,

- The linear structure assumed for \( D(\cdot) \) and \( M(\cdot) \) is actually quite general as a number of transformations can be employed to capture complex patterns/structures in the data. ARMAX models represent a broad class of time series models.

- The assumptions about costs are not restrictive because, for example, it is possible to obtain optimal M&R investment policies for general cost structures by solving a finite sequence of problems. In each problem the cost structure is approximated by a second-order Taylor Series expanded about a different point.
3.2.2 Optimization Problem

With the assumptions discussed in the preceding section, the optimization problem presented above can be formulated as a dynamic program with state-space \( I_t, t = 1, \ldots , T+1 \). The optimal objective function, \( v_t(I_t) \) is defined as the minimum expected discounted cost from the start of \( t \) until the end of the horizon given the information available at the start of \( t, I_t \). The recurrence relation is as follows:

\[
v_t(I_t) = \min_{A_t} \left\{ E_{X_t|I_t} \left[ a_t X_t^2 + b_t X_t A_t + c_t A_t^2 + d_t X_t + e_t A_t + f_t + E_{\epsilon_t} \left[ v_{t+1} (g_t X_t + h_t A_t + \epsilon_t) \right] \right] \right\}
\]

The boundary condition for the problem is as follows:

\[
v_{T+1}(I_{T+1}) = E_{X_{T+1}|I_{T+1}} \left[ p_{T+1} X_{T+1}^2 + q_{T+1} X_{T+1} + r_{T+1} \right]
\]

3.3 Solution Procedure

The solution procedure we propose involves solving two subproblems: an optimization problem and a state-estimation problem. Both are described below.

3.3.1 Optimization Problem

The dynamic programming formulation allows for a solution that can be obtained inductively. For the above problem, the solution can be expressed in closed-form with parameters computed recursively as follows:

\[
A_t = - \frac{[b_t + 2p_{t+1} g_t h_t]}{2c_t + 2p_{t+1} h_t^2} E[X_t|I_t] + \frac{2p_{t+1} h_t \epsilon_t + q_{t+1} h_t + e_t}{2c_t + 2p_{t+1} h_t^2}, \quad t = T, \ldots , 1
\]

\[
p_t = a_t + p_{t+1} g_t^2 - \left( \frac{[b_t + 2p_{t+1} g_t h_t]^2}{4[c_t + p_{t+1} h_t^2]} \right), \quad t = T, \ldots , 2
\]

\[
q_t = d_t + 2p_{t+1} \epsilon_t g_t + q_{t+1} g_t - \frac{[b_t + 2p_{t+1} g_t h_t][e_t + 2p_{t+1} h_t \epsilon_t + q_{t+1} h_t]}{2[c_t + p_{t+1} h_t^2]}, \quad t = T, \ldots , 2
\]

\[
r_t = f_t + p_{t+1} (\epsilon_t^2 + \sigma^2) + q_{t+1} \epsilon_t + r_{t+1} - \frac{[e_t + 2p_{t+1} h_t \epsilon_t + q_{t+1} h_t]^2}{4[c_t + p_{t+1} h_t^2]}, \quad t = T, \ldots , 2
\]

These equations are derived from the first-order/necessary conditions for the problem. The second-order/sufficiency conditions are satisfied because the objective function is convex. The equations are evaluated recursively noting that \( p_{T+1}, q_{T+1}, r_{T+1} \) are the parameters that define the salvage value function. Using the solution to the above system of equations allows us to write the
optimal objective value function as:

\[ v_t(I_t) = p_t \mathbb{E}[X_t|I_t]^2 + q_t \mathbb{E}[X_t|I_t] + r_t, \quad t = 1, \ldots, T \] (13)

We note that in order to compute the optimal policy, \( A^*_t, t = 1, \ldots, T \) and to evaluate the optimal objective value function it is necessary to compute the expected state given the set of information in each period, i.e., \( \mathbb{E}[X_t|I_t], t = 1, \ldots, T \). This step is referred to as the state estimation problem and it is discussed further in the following section. We note that the key to processing distress measurements generated simultaneously by multiple technologies is to compute these expectations efficiently.

### 3.3.2 State Estimation Problem

The state estimation problem consists of finding the expected state for a given information set. This is in general a hard problem, however, under the assumption that the error terms \( \epsilon_t \) and \( \bar{\xi}_t \) follow a Gaussian Distribution with zero mean and finite second moments, then the expectation can be computed with a recursive algorithm known as the Kalman Filter. These assumptions are mild because they are consistent with obtaining adequate estimations of the models (unbiased parameters). The algorithm is presented below:

```
Kalman Filter Algorithm

Repeat at the start of each period:
Given \( \mathbb{E}[X_{t-1}|I_{t-1}], \text{Var}(X_{t-1}|I_{t-1}), A_{t-1}, \) and \( Z_t = \tilde{z}_t \)
Define: \( \hat{X}_{t-1} \leftarrow \mathbb{E}[X_{t-1}|I_{t-1}], \)
\( P_{t-1} \leftarrow \text{Var}(X_{t-1}|I_{t-1}), \) and
\( I_t = \{I_{t-1}, A_{t-1}, \tilde{z}_t\} \)

**Time Update:**
\[ \hat{X}_t^- = g_{t-1} \hat{X}_{t-1} + h_{t-1} A_{t-1} \]
\[ P_t^- = g_{t-1}^2 P_{t-1} + \sigma_{t-1}^2 \]

**Measurement Update:**
\[ K_t = P_t^- H' (P_t^- H H' + R)^{-1} \]
\[ \mathbb{E}[X_t|I_t] \leftarrow \hat{X}_t^- + K_t (\tilde{z}_t - H \hat{X}_t^-) \]
\[ \text{Var}(X_t|I_t) \leftarrow (1 - K_t H) P_t^- \]
```

The time update step uses the system equation to project the estimates of the conditional expectation and variance. The measurement update step updates (with Bayes’ Law) the expectation and variance taking into account the new set of measurements obtained at the start of period \( t, \tilde{z}_t \). The computational complexity of the Kalman Filter increases polynomially with the size of the vectors \( \bar{Z}_t \) which means that the framework does not suffer from the shortcomings of the latent
MDP approach. This is because with it is only necessary to update the first two moments of the state-distribution as opposed to the probability mass function.
4 Application Case Studies

This section addresses Task 3. We present a computational study where we:

1. Provide numerical examples to illustrate the methodology presented above to address the state-estimation problem in the above framework. In particular, we show how the Kalman Filter processes distress measurements to update the state distribution. In Section 4.1 we use a set of simulated data and in Section 4.2 we use the framework to address the state-estimation problem using data collected with a strain sensor.

2. Use the framework to study the effect of uncertainties in the deterioration process and in the process of collecting distress measurements on the optimal life-cycle cost of managing infrastructure facilities.

3. Use the framework to quantify the value of combining different technologies for condition assessment.

4.1 Numerical Example: State-Estimation Problem

To illustrate how the Kalman Filter addresses the state-estimation problem in the above framework, we consider the management of a pavement over a 40-year planning horizon. The initial condition of the pavement is 10 given in a PCI-like scale with range [0, 100]. The deterioration and measurement-error models in the example are given by:

\[
X_{t+1} = X_t + 8 - A_t \quad \text{(14)}
\]

\[
Z_t = X_t + \xi_t; \text{ where } \xi_t \text{ is Normally distributed with } \mu_{\xi_t} = 0 \text{ and } \sigma^2_{\xi_t}. \quad \text{(15)}
\]

That is, we assume that the pavement deteriorates deterministically at a rate of 8 PCI units per year, and that the distress measurements correspond to the actual condition plus a random error term/white noise. As stated earlier, the parameter \(\sigma^2_{\xi_t}\) represents the precision of the technology used to collect the distress measurements.

We also assume that the pavement is restored to its initial condition every ten years, i.e.,

\[
A_t = \begin{cases} 
80; & t = 11, 21, 31, 41 \\
0; & \text{otherwise} 
\end{cases} \quad \text{(16)}
\]

Finally, we assume that the initial, estimated state-distribution is Normal with \(\mathbb{E}[X_1|I_1] = 25\) and \(\text{Var}(X_1|I_1) = 20\).

To illustrate how the Kalman Filter uses the sequence of distress measurements to update the state-distribution we simulated an instance of the above process. The solid line in Figure 4 rep-
represents the true condition of the pavement over time. The triangles represent a set of randomly generated distress measurements that are consistent with the measurement-error model in Equation (15). In this part of the study we use $\sigma^2_{\xi_t} = 10$. The dashed line corresponds to the first moment of the estimated state-distribution. The figure shows how the condition estimate converges to the true condition of the pavement (over time, the dashed line traces the solid line).

Figure 4: Updated state-distribution: First moments

Figure 5 shows how the Kalman Filter updates the second moment of the estimated state-distribution. In this part of the study we considered the effect of technologies of different precisions to collect distress measurements. Specifically, we considered cases where $\sigma^2_{\xi_t} = 0, 2, 10$, i.e., “perfect”, “fine”, and “coarse” technologies used to collect measurements. We also considered the case where two “coarse” technologies with $\sigma^2_{\xi_t} = 10$ where used to collect distress measurements simultaneously (the technologies were assumed to be independent of each other and therefore this case does not correspond to using the same technology to collect two sets of measurements). We observe that the variance in the estimated state-distribution becomes very small very quickly. The asymptote and the convergence rate are properties of the technologies. The key observation is that the variance in the estimated state-distribution is well within the precision of each technology, i.e., the procedure filters out the random error/noise in the measurements. For example, the variance in the state distribution when measurements are collected with the “coarse” technology ($\sigma^2_{\xi_t} = 10$) converges to approximately 1 after 10 years.

24
4.2 State-Estimation using Sensor Data

Here we repeat the experiment conducted above, except that we use data collected with a strain sensor.

The state estimates obtained are presented in Figure 6.

\[
\begin{align*}
\dot{x}_t + \epsilon_t &= Z_t \\
\dot{\epsilon}_t + \eta_t &= 1 + \eta_t
\end{align*}
\]

The state estimates obtained using MATLAB are given below. We used the data to obtain deterioration and measurement-error models using MATLAB.

Figure 5: Updated state-distribution: Second moments

The data are for the strain induced by the front axle. The strain measurements used in our analysis are for the strain induced by a five-axle truck. The strain measurements used in our analysis are for the strain induced by a five-axle truck that is part of the the MnROAD Superpave project. The term Superpave (SUperior PERforming Asphalt Pavements) refers to strict criteria for properly designing and building hot-mix asphalt. The data used to perform better under the extremes of temperature and heavy truck loadings. The data consist of a resistance strain gauge embedded within a strip of glass-fiber-reinforced epoxy, with transverse steel anchors at each end of the strip. The sensor collects horizontal strain measurements induced by a five-axle truck. The sensor is coded C33LE001 on cell 33 which is part of the MnROAD Superpave project. The term Superpave (SUperior PERforming Asphalt Pavements) refers to strict criteria for properly designing and building hot-mix asphalt.
Table 1: Cost Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.024</td>
</tr>
<tr>
<td>$b$</td>
<td>-0.006</td>
</tr>
<tr>
<td>$c$</td>
<td>0.003</td>
</tr>
<tr>
<td>$d$</td>
<td>-1.262</td>
</tr>
<tr>
<td>$e$</td>
<td>0.795</td>
</tr>
<tr>
<td>$f$</td>
<td>14.650</td>
</tr>
</tbody>
</table>

In Appendix A, the parameters needed to specify the cost function are presented in Table 1. We set the residual value of a facility to zero (i.e., $s(x_{T+1}) = 0$) and used a discount rate of 5% ($\delta = 0.9524$). In this part of the study, we consider instances of managing the pavement described in the previous section. In order to use the framework that we proposed, it is necessary to specify a second-order polynomial that represents the per-period cost function $g(X_t, A_t)$. To this end, we adapted the cost functions presented in Madanat and Ben-Akiva (1994) (the details are presented in Appendix A). The type of study allows agencies to quantify the value of using different condition-assessment methodologies for condition-assessment techniques and therefore provides critical decision-support in the process of adopting advanced inspection procedures. This type of study allows agencies to quantify the value of using different condition-assessment methodologies for condition-assessment procedures. We present an empirical study to investigate the effect of uncertainties on Life-Cycle Costs.
In this part of the study we considered the same technology choices that we used in the previous section. However, instead of considering a deterministic deterioration process we let $\sigma_{\epsilon_t}^2$ be $0.1, 1, 2, 4,$ and $8$. For each combination of technology and deterioration process variance we calculated the average optimal expected cost of managing 100 pavement sections. The optimal policies were obtained by solving the optimization model presented earlier. The results appear in Figure 7.

![Figure 7: Life-Cycle Costs vs. Deterioration Process Variance](image)

From the figure we observe that, as expected, the costs to manage the pavements increase as the uncertainty in the deterioration process grows. Also as expected, “coarser” data collection technologies result in higher costs incurred. As mentioned earlier, this is the first study in the area of transportation infrastructure management to quantify the costs incurred when inspection technologies are combined for condition assessment. We notice that the cost of using the combination of “coarse” technologies falls roughly in between the costs of using the “coarse” or the “fine” technologies independently. This type of analysis can be used together with the costs of adopting technologies to obtain an effective data collection strategy. For example, it is conceivable that the cost of adopting the “fine” technology is not justified by the benefits that will accrue from using it in the management process. In the next section, we further explore the issue of quantifying the value of combining different technologies for condition assessment.

### 4.4 Combining Multiple Technologies for Condition Assessment

As discussed earlier, agencies often use multiple technologies to collect distress measurements simultaneously. A question that arises is: what is the use of collecting these data? From the previous
section, technology choice would seem to be dictated by precision. If this were true, then what would be the value of collecting additional data (with “coarse” technologies? Here, we show that combining various technologies can result in benefits even when compared to a single technology that with high precision. The analysis also provides insights that can be used when adopting inspection technologies.

To obtain a measurement-error model for the analysis, we build on the statistical work in Ben-Akiva and Gopinath (1995). They consider the case of collecting five distress measurements: roughness (RQI), cracking (CRX), rutting (RDMN), surface patching (SPAT), and raveling (RAV). The model is presented in Appendix B.

4.4.1 Numerical Results

In our study we considered the same setup as in Section 4.3 (deterioration model, cost functions, planning horizon, and discount rate). We assumed that the deterioration model described the progression of roughness. In order to highlight the effect of technology precision we set the variance in the deterioration process to be $\sigma^2_{\xi_t}$ to be 0.1.

Table 2 presents the average (over 100 instances) of the minimum expected costs for all possible combinations of the different technologies. The third column in the table corresponds to the percentage of the difference between the costs of using a particular combination of technologies and a case of perfect inspections yielding the true condition of the pavement (average minimum expected costs $337.4802$). This difference is taken relative to the difference in costs that results when the system is managed while collecting only raveling measurements.

The main observations from the simulation are as follows:

- We can see from Table 2 that the least minimum expected cost occurs when all the five technologies are combined together. This shows that we do obtain a better performance by combining different technologies.

- We also notice that the costs are not only dependent on $\sigma^2_{\xi_t}$ but, also on the $\lambda$’s in the measurement error model. This shows that we not only need precise measurements but also relevant measurements of the latent condition. In this case, measurements that are highly related to roughness. For example, the technology used to collect measurements of surface patching (SPAT) is highly precise when compared to other technologies. However, the value of $\lambda_4$ is 0.167. Cracking (CRX) is the least accurate of all the technologies but $\lambda_2 = 1.503$ which means that the measurements are closely related to the latent variable that we are trying to measure. We notice that collecting measurements of cracking is more cost-effective than collecting measurements of raveling.
Table 2: Expected Costs for all Technology Combinations

<table>
<thead>
<tr>
<th>Technology</th>
<th>Minimum Expected Cost</th>
<th>Percentage of Largest Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQI CRX RDMN SPAT RAV</td>
<td>338.5218</td>
<td>4.89</td>
</tr>
<tr>
<td>RQI CRX RDMN RAV</td>
<td>338.5797</td>
<td>5.16</td>
</tr>
<tr>
<td>RQI CRX RDMN SPAT</td>
<td>338.5975</td>
<td>5.25</td>
</tr>
<tr>
<td>RQI CRX RDMN</td>
<td>338.663</td>
<td>5.56</td>
</tr>
<tr>
<td>RQI CRX SPAT RAV</td>
<td>338.9166</td>
<td>6.75</td>
</tr>
<tr>
<td>RQI CRX RAV</td>
<td>339.0244</td>
<td>7.26</td>
</tr>
<tr>
<td>RQI CRX SPAT</td>
<td>339.045</td>
<td>7.36</td>
</tr>
<tr>
<td>RQI RDMN SPAT RAV</td>
<td>339.1679</td>
<td>7.93</td>
</tr>
<tr>
<td>RQI RDMN RAV</td>
<td>339.1701</td>
<td>7.94</td>
</tr>
<tr>
<td>RQI RDMN SPAT</td>
<td>339.3278</td>
<td>8.69</td>
</tr>
<tr>
<td>CRX RDMN SPAT RAV</td>
<td>339.3692</td>
<td>8.88</td>
</tr>
<tr>
<td>RQI RDMN</td>
<td>339.4972</td>
<td>9.48</td>
</tr>
<tr>
<td>CRX RDMN RAV</td>
<td>339.5526</td>
<td>9.74</td>
</tr>
<tr>
<td>CRX RDMN SPAT</td>
<td>339.5679</td>
<td>9.81</td>
</tr>
<tr>
<td>CRX RDMN</td>
<td>339.7869</td>
<td>10.84</td>
</tr>
<tr>
<td>RQI SPAT RAV</td>
<td>340.1278</td>
<td>12.45</td>
</tr>
<tr>
<td>RQI RAV</td>
<td>340.463</td>
<td>14.02</td>
</tr>
<tr>
<td>RQI SPAT</td>
<td>340.4689</td>
<td>14.05</td>
</tr>
<tr>
<td>CRX SPAT RAV</td>
<td>340.6569</td>
<td>14.93</td>
</tr>
<tr>
<td>RQI</td>
<td>340.8862</td>
<td>16.01</td>
</tr>
<tr>
<td>CRX SPAT</td>
<td>341.1373</td>
<td>17.19</td>
</tr>
<tr>
<td>CRX RAV</td>
<td>341.1445</td>
<td>17.23</td>
</tr>
<tr>
<td>RDMN SPAT RAV</td>
<td>341.6213</td>
<td>19.47</td>
</tr>
<tr>
<td>CRX</td>
<td>341.7704</td>
<td>20.17</td>
</tr>
<tr>
<td>RDMN SPAT</td>
<td>342.365</td>
<td>22.96</td>
</tr>
<tr>
<td>RDMN RAV</td>
<td>342.4338</td>
<td>23.29</td>
</tr>
<tr>
<td>RDMN</td>
<td>343.4865</td>
<td>28.23</td>
</tr>
<tr>
<td>SPAT RAV</td>
<td>349.6943</td>
<td>57.42</td>
</tr>
<tr>
<td>SPAT</td>
<td>356.5109</td>
<td>89.46</td>
</tr>
<tr>
<td>RAV</td>
<td>358.7532</td>
<td>100.00</td>
</tr>
</tbody>
</table>
5 Summary and Conclusions

We have developed an optimization framework to provide support for investments in preservation and improvement of transportation infrastructure facilities that are inspected periodically with sensors or other advanced technologies. This work is motivated by recent developments in remote sensing and communications technologies that have increased the availability and cost-effectiveness of using advanced technologies; and by statistical and computational limitations associated with existing optimization models to support investment decisions. These limitations are related to their inability to process condition data collected by simultaneously by multiple technologies.

The framework we present involves formulating the underlying decision problem as a discrete-time, stochastic optimal control problem and consists of two components: a state-estimation problem that involves processing vast arrays of condition data and using them to develop condition forecasts; and an optimization problem whose solution yields M&R investment policies. Our approach differs from the literature in that both elements are fully integrated. This, in turn, leads to a framework that is both statistically rigourous and computationally efficient, i.e., capable of providing effective decision-support.

Through four application case studies, we provide numerical examples to illustrate the methodology presented above to address the state-estimation problem in the above framework. In particular, we show how the Kalman Filter processes distress measurements to update the state distribution. For this part of the study we use both a set of simulated data as well as sensor data. The sensor data was provided by the MnROAD project. We then show how the the proposed framework can be used to study the effect of uncertainties in the deterioration process and in the process of collecting distress measurements on the optimal life-cycle cost of managing infrastructure facilities. We also illustrate how the framework can be used to quantify the value of combining different technologies for condition assessment. This is the first study in the area of transportation infrastructure management to quantify the costs incurred when inspection technologies are combined for condition assessment.

Finally, we gratefully acknowledge the support for this project. This support enabled Naveen Tadepalli, a graduate student in the Department of Civil and Environmental Engineering at Northwestern University, to complete the requirements for a M.S. degree.
A Cost Parameter Generation

The objective in the numerical study presented in Section 4.3 is to illustrate how the methodology can be used to obtain optimal M&R policies and to quantify the benefits of using different combinations of inspection technologies. To interpret the results of the study as being representative, we chose to use data “inspired by” studies in the pavement management literature. To estimate the parameters needed to specify the period cost function $g(X_t, A_t)$, we used data from the empirical study presented in Madanat and Ben-Akiva (1994) with minor modifications. In that study, the states used to represent pavement condition are obtained by discretizing the PCI scale into 8 states. From this discretization we constructed a modified roughness scale to be consistent with the assumption that as the variable used to represent condition, $X_t$, increases, the condition worsens. The two scales are shown in Table 3.

<table>
<thead>
<tr>
<th>State</th>
<th>PCI Range</th>
<th>Modified Roughness Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0–20</td>
<td>80–100</td>
</tr>
<tr>
<td>1</td>
<td>20–40</td>
<td>60–80</td>
</tr>
<tr>
<td>2</td>
<td>40–50</td>
<td>50–60</td>
</tr>
<tr>
<td>3</td>
<td>50–60</td>
<td>40–50</td>
</tr>
<tr>
<td>4</td>
<td>60–70</td>
<td>30–40</td>
</tr>
<tr>
<td>5</td>
<td>70–80</td>
<td>20–30</td>
</tr>
<tr>
<td>6</td>
<td>80–90</td>
<td>10–20</td>
</tr>
<tr>
<td>7</td>
<td>90–100</td>
<td>0–10</td>
</tr>
</tbody>
</table>

Table 3: Discretization and Transformation of PCI Scale

The agency and user costs used in the aforementioned study are presented in Table 4 and are a function of the discrete states shown above and four M&R actions. Each entry in the table is labeled with a corresponding state-action pair in the domain of the period cost function. The “do nothing” action was assumed to have no effect on facility condition, i.e., $A_t = 0$. “Routine maintenance” was assumed to prevent the facility from deteriorating, i.e., $A_t = 8$. The effects of the other two actions on improvements (measured as reductions on the modified roughness scale) was obtained by calculating the expected improvement using the transition probabilities in Madanat and Ben-Akiva (1994).

To obtain the parameters presented in Table 1 we assumed that $g(X_t, A_t)$ could be represented as a second-order polynomial and estimated the parameters using linear regression. The data come from Table 4.
Table 4: Agency and User Costs from Madanat and Ben-Akiva (1994) ($/m^2)

<table>
<thead>
<tr>
<th>State</th>
<th>Do Nothing Maintenance</th>
<th>Routine 2” overlay</th>
<th>Reconstruction</th>
<th>User Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(90, 0) → 0</td>
<td>(90, 8) → 6.9</td>
<td>(90, 51.5) → 21.81</td>
<td>(90, 91.5) → 25.97</td>
</tr>
<tr>
<td>1</td>
<td>(70, 0) → 0</td>
<td>(70, 8) → 2</td>
<td>(70, 41.5) → 12.31</td>
<td>(70, 71.5) → 25.97</td>
</tr>
<tr>
<td>2</td>
<td>(55, 0) → 0</td>
<td>(55, 8) → 1.4</td>
<td>(55, 36.5) → 10.69</td>
<td>(55, 56.5) → 25.97</td>
</tr>
<tr>
<td>3</td>
<td>(45, 0) → 0</td>
<td>(45, 8) → 0.83</td>
<td>(45, 36.5) → 9.06</td>
<td>(45, 46.5) → 25.97</td>
</tr>
<tr>
<td>4</td>
<td>(35, 0) → 0</td>
<td>(35, 8) → 0.65</td>
<td>(35, 36.5) → 6.64</td>
<td>(35, 36.5) → 25.97</td>
</tr>
<tr>
<td>5</td>
<td>(25, 0) → 0</td>
<td>(25, 8) → 0.31</td>
<td>(25, 26.5) → 4.11</td>
<td>(25, 26.5) → 25.97</td>
</tr>
<tr>
<td>6</td>
<td>(15, 0) → 0</td>
<td>(15, 8) → 0.15</td>
<td>(15, 16.5) → 3.91</td>
<td>(15, 16.5) → 25.97</td>
</tr>
<tr>
<td>7</td>
<td>(5, 0) → 0</td>
<td>(5, 8) → 0.04</td>
<td>(5, 6.5) → 3.81</td>
<td>(5, 6.5) → 25.97</td>
</tr>
</tbody>
</table>

B Latent Performance and Measurement-Error Models

The performance and measurement-error models from Ben-Akiva and Gopinath (1995) are presented below.

\[
X = \alpha_1 \frac{AGER}{SNC} + \alpha_2 \frac{ESAX}{SNC} + \alpha_3 \frac{CP}{SNC} + \epsilon \quad (19)
\]

\[
\begin{bmatrix}
RQI \\
CRX \\
RDMN \\
SPAT \\
RAV
\end{bmatrix} = \begin{bmatrix}
\lambda_1 X + \xi_1 \\
\lambda_2 X + \xi_2 \\
\lambda_3 X + \xi_3 \\
\lambda_4 X + \xi_4 \\
\lambda_5 X + \xi_5
\end{bmatrix} \quad (20)
\]

where \(X\) is the latent variable representing condition. The condition is specified to be influenced by the following factors:

**AGER:** The lapsed time since the last major rehabilitation.

**SNC:** The pavement structural number.

**ESAX:** The cumulative equivalent standard axle loads since the last rehabilitation.

**CP:** The cumulative precipitation since the last rehabilitation.

In the measurement error model, the distress measurements correspond to:

**RQI:** Roughness (in quarter-car index with units in count/km)

**CRX:** Cracking (in percentage of surface area)

**RDMN:** Rut Depth (in mm)
SPAT: Patching (in percentage of surface area)

RAV: Raveling (percentage of surface area).

The models were estimated using pavement deterioration data gathered in Brazil between 1975 and 1982. Statistics that describe the data used to estimate the model are presented in Table 5.

<table>
<thead>
<tr>
<th>Condition Indicator</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQI</td>
<td>40.53</td>
<td>15.36</td>
</tr>
<tr>
<td>CRX</td>
<td>15.16</td>
<td>25.70</td>
</tr>
<tr>
<td>RDMN</td>
<td>3.71</td>
<td>2.19</td>
</tr>
<tr>
<td>SPAT</td>
<td>2.47</td>
<td>8.65</td>
</tr>
<tr>
<td>RAV</td>
<td>7.61</td>
<td>19.70</td>
</tr>
</tbody>
</table>

Table 5: Descriptive statistics

The estimated parameters are presented in Table 6.\(^8\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td>3.562</td>
<td></td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>3.897</td>
<td></td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>1.654</td>
<td></td>
</tr>
<tr>
<td>(\lambda_1)</td>
<td>1.000</td>
<td>0.37</td>
</tr>
<tr>
<td>(\lambda_2)</td>
<td>1.503</td>
<td>0.35</td>
</tr>
<tr>
<td>(\lambda_3)</td>
<td>0.167</td>
<td>0.45</td>
</tr>
<tr>
<td>(\lambda_4)</td>
<td>0.256</td>
<td>0.09</td>
</tr>
<tr>
<td>(\lambda_5)</td>
<td>0.531</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 6: Parameter Estimation Results

The \(\lambda\)'s describe how the different technologies complement each other. In order to compute the expected costs associated with different combinations of technologies it is also necessary to obtain the precisions associated with each of the technologies. The next subsection describes how we estimated the precisions associated with the technologies.

B.1 Estimation of Precisions Associated with Technologies: A Note on Linear Regression

A regression equation is of the form:

\[ Y = mX + \epsilon \]  \hspace{1cm} (21)

\(^8\)SMC for structural model = 0.62
where \( Y \) is the observed variable, \( X \) is a vector of explanatory variables, and \( \epsilon \) is a random error term (assumed to have zero mean and finite variance). \( m \) is the set of parameters that are estimated.

Linear regression models try to explain the variance of \( Y \) using the variance of \( X \) and the variance of the error term. The variance of \( Y \) is termed total variance (SST), and the variance of \( mX \) is called the explained variance (SSR) and the error variance is called the unexplained variance (SSE). The relationship between these is as follows:

\[
SST = SSR + SSE
\]  

(22)

The goodness-of-fit for a regression equation is measured using a statistic called \( R^2 \), defined as the proportion of the variance of \( Y \) explained by \( X \). Mathematically, this can be represented as:

\[
R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}
\]  

(23)

Ben-Akiva and Gopinath (1995) present the Squared Multiple Correlation (SMC) for each of the measurement error model. SMC is analogous to \( R^2 \). From the definition of \( R^2 \), we have that \( SSE = SST(1 - SMC) \). SSE is nothing but the variance of error term in the regression model. We use SST and SMC to estimate the precisions of each of the technologies. The results are presented in Table 7.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roughness</td>
<td>148.6356</td>
</tr>
<tr>
<td>Cracking</td>
<td>429.3185</td>
</tr>
<tr>
<td>Rut Depth</td>
<td>7.570225</td>
</tr>
<tr>
<td>Surface Patching</td>
<td>68.088</td>
</tr>
<tr>
<td>Raveling</td>
<td>372.5664</td>
</tr>
</tbody>
</table>

Table 7: Precision Estimates
References


Hulten, C. (1996). “Infrastructure capital and economic growth: How well you use it may be more important than how much you have.” Mimeo, University of Maryland.


