

**APPLICATION OF OBJECTIVE VISUAL COMPLEXITY MEASURES TO BINARY
DASYMETRIC MAPS**

by

Daniel C. Reynolds

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ABSTRACT:

Dasymetric maps have been shown to be an improvement over more common mapping techniques in terms of spatial and attribute accuracy. This research is a step towards evaluating dasymetric maps in terms of their effectiveness in communicating information to map readers. The effectiveness of a map depends, in part, on the visual complexity of the pattern it presents to the map reader. Previous research has evaluated the visual complexity of maps using objective measures, but such measures have not been applied to dasymetric maps.

In this study a set of classed binary dasymetric population density maps was constructed for testing. As complexity is widely recognized as multifaceted, the maps were evaluated using a variety of objective measures. Several results from previous research with choropleth maps were confirmed. In particular, this study found a) there is considerable redundancy among complexity measures, b) that map complexity is affected profoundly by the data classification method, and c) that generally speaking complexity increases as the number of class increases. A finding unique to binary dasymetric maps is that complexity increases as the percentage of ancillary features increases.

Another part of the study compared a set of choropleth test maps with dasymetric maps. Four out of the six complexity measures indicate that dasymetric maps are more complex than choropleth maps. In particular, dasymetric maps create a more fragmented pattern with a greater disparity in the size of enumeration units as well. It remains for future research to determine if the differences in complexity found here have significant implications for map use.

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INTRODUCTION

Often cartographers wish to display geographic patterns using areal symbols. For thematic maps using such symbology, there are three major options: choropleth, isopleth, and dasymetric.

Perhaps the least understood of these three is the dasymetric map. In particular, these maps have not been studied in terms of visual complexity. Visual complexity, also known as pattern complexity, is defined as “looking intricate or involved” (MacEachren 1982b). Since the visual complexity of a map can affect map readability, it is important to explore the visual complexity of dasymetric maps (MacEachren 1982b). This research aims to characterize the visual complexity of dasymetric maps and examine the relative complexity of dasymetric and choropleth maps.

The Nature of Dasymetric Maps

In order to understand the nature of dasymetric maps, it is helpful to have an understanding of the similar choropleth map. The construction of choropleth maps is relatively straightforward which contributes to their popularity (Maantay et. al. 2007). There are two general forms of choropleth maps: classed and un-classed (Robinson et al. 1995). A classed choropleth map is constructed by grouping the data values of the enumeration units into a relatively small number of categories using any of a large number of algorithms. Each category is assigned a color or pattern and enumeration units are rendered accordingly (see Figure 1). In an un-classed choropleth map, the data values are not categorized and so each enumeration unit receives a unique color or pattern. Throughout this thesis, “color” will refer to the symbology (visual treatment) of enumeration units, but the reader should understand the discussion, methods, and conclusions apply equally well to cross-hatching, dot fills, and other ways of rendering data values for the map’s areal units.

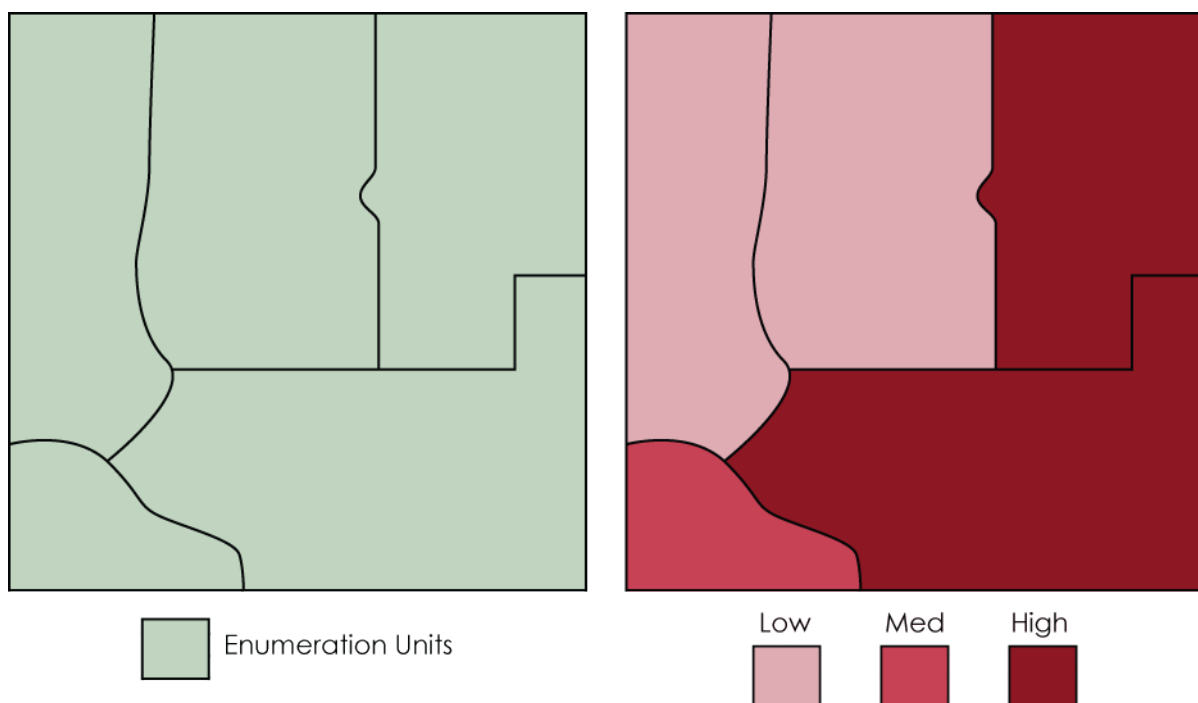


Figure 1: A base map (left) and a choropleth map constructed from that base map.

The differences between choropleth maps and dasymetric maps start with the fact that dasymetric maps use ancillary data to modify the boundaries of enumeration units. For example, a population density map could use the location of public lands and water bodies to subtract out areas in which there is zero population and recalculate densities accordingly. The dasymetric map is then colored in the same way as a choropleth in a classed or un-classed fashion (see Figure 2). This technique is known as the binary method (Eicher and Brewer 2001).

Throughout this thesis, the term “dasymetric” will refer to this method of construction unless otherwise noted. Although other methods of dasymetric map construction exist, this study focuses on the binary method because of the relative simplicity of construction and applicability of existing complexity measures.

Note that a binary dasymetric map can be displayed in one of three ways: with all of the new enumeration unit boundaries showing, with the boundaries between units with values of zeros,

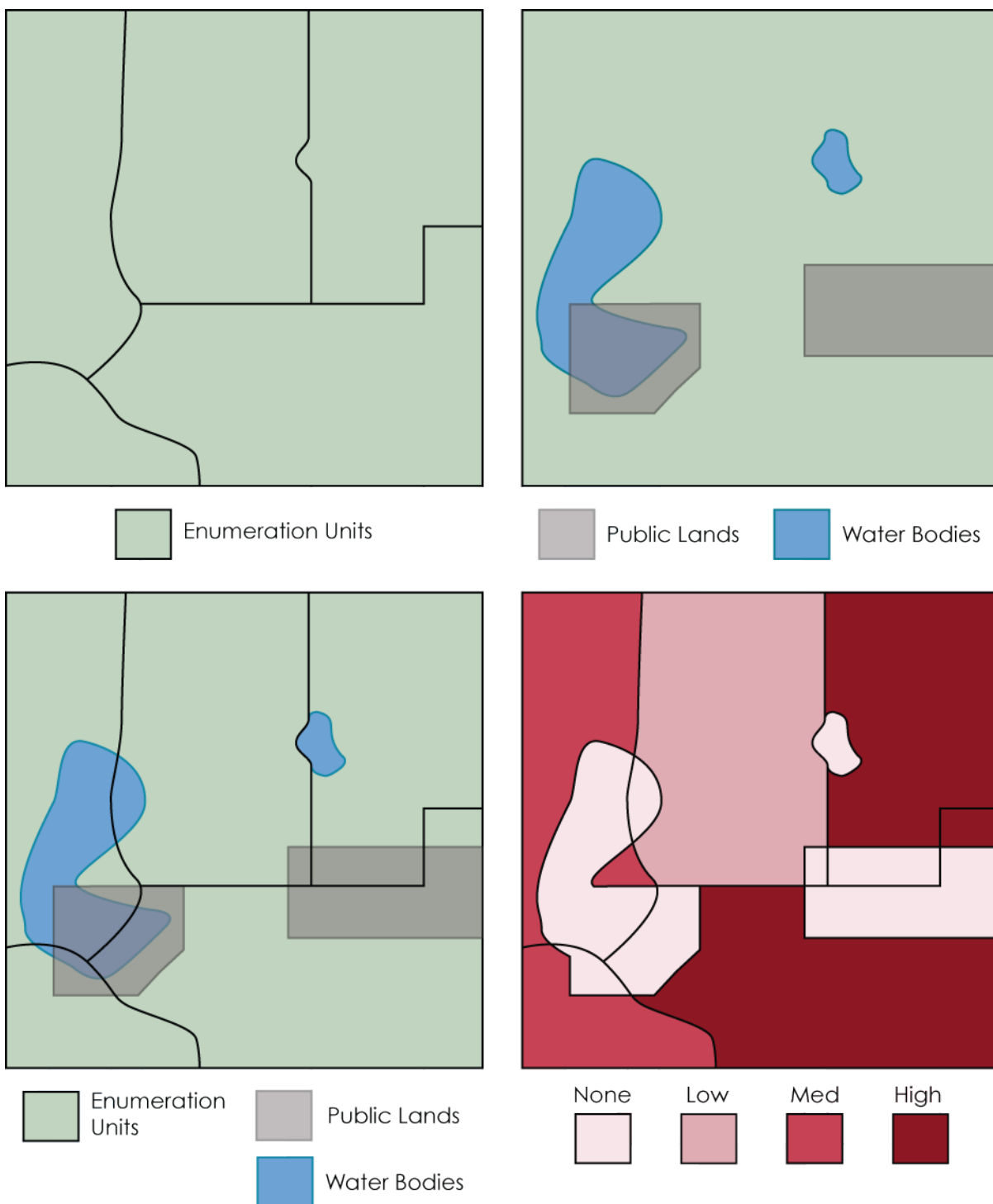


Figure 2: A base map (upper left) is combined with ancillary data (upper right) to form a new set of enumeration units (lower left). The resulting base map is classified and colored similar to a choropleth map (lower right).

or with boundaries dissolved between all units falling into the same category (see Figure 3, Figure 4, and Figure 5). Each of these methods has a purpose. Retaining all boundaries may be useful if the intended audience would find the boundaries of the original enumeration units helpful as reference. Dissolving boundaries between zero-valued neighbors recognizes that the dasymetric map changes the spatial distribution of the data without lumping enumeration units of different data values that fall into the same category together. This particular approach is best used with an un-classed approach. The last method, dissolving boundaries completely, fully embraces the change in spatial distribution that occurs due to the nature of dasymetric map construction.

While dasymetric maps are not limited to mapping population densities, it is the most common use (see Maantay et al. 2007, Holt et al. 2004) and helps provide context in the exploration of the characteristics of dasymetric maps. For example, in Figure 2, it is obvious that negligible populations will be found within a water body or area of public land, so the resulting dasymetric map better represents a given spatial distribution when compared to a choropleth map of the same area made using the same data. In terms of attributes, the dasymetric map helps overcome situations where populations reside in only a small portion of the original enumeration unit. That is, the assumption with a choropleth map is that the phenomenon in question is spread evenly throughout the enumeration unit. This, however, is rarely the case. A dasymetric map enables the cartographer to create more internally homogeneous enumeration units and, as a result, more closely represent the true spatial nature of the dataset in question.

There are, of course, some drawbacks to using dasymetric maps. Because they require the use of ancillary data, the cartographer must understand the GIS processes used to create the final

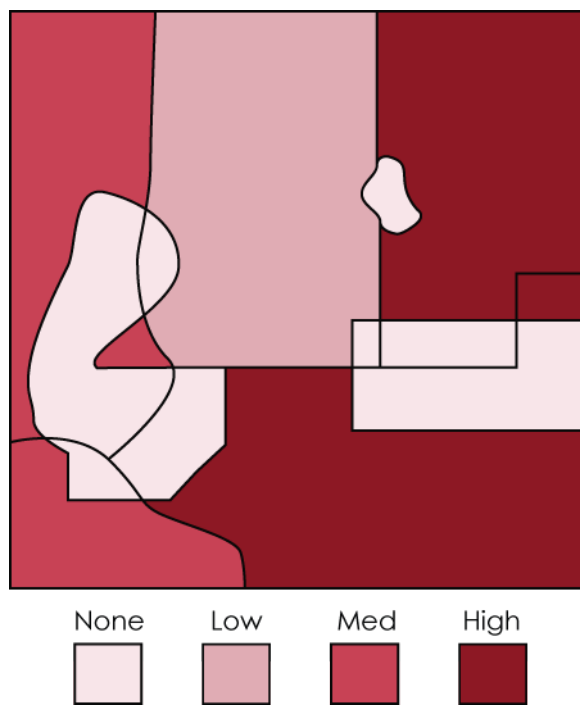


Figure 3: Dasymetric map displayed with all boundaries retained.

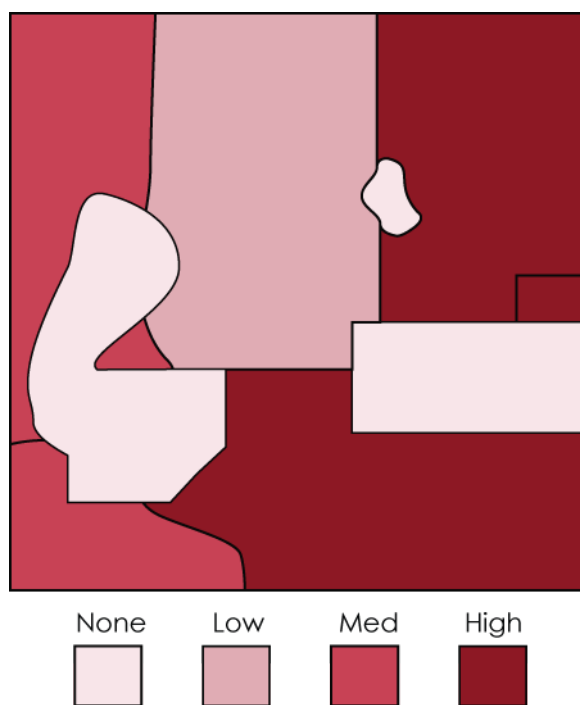


Figure 4: Dasymetric map displayed with boundaries between neighboring zero-valued polygons dissolved.



Figure 5: Dasymetric Map displayed with boundaries between adjacent polygons of the same category dissolved.

enumeration units and reassign the data values. Also, since the steps required to incorporate the ancillary data into the base choropleth map require the use of GIS overlay operations, sliver polygons are likely to be introduced. This occurs if there are topological inconsistencies between the ancillary layers and the base map (see Figure 6). These added steps in the map creation process increase the difficulty of construction and introduce a subjective element since there are no rules as to what qualifies as suitable ancillary data and sliver resolution methods are many (Eicher and Brewer 2001). The net result of these factors is that more work is required to create a dasymetric map than is required for a choropleth. Lastly, the visual appearance of a dasymetric map varies from that of a choropleth map due to the change in base enumeration units. It is unknown whether the change in visual appearance will or will not inhibit the ability of a map reader to effectively utilize these maps.



Figure 6: The dasymetric map (upper left) has numerous sliver polygons. In the insets (below), the slivers may be observed at boundaries between enumeration units where there are alignment issues between the county boundaries and the boundaries of the ancillary data.

In short, the theoretical arguments that show dasymetric maps to be more accurate than choropleth in terms of spatial and attribute characteristics do not guarantee that a map user would be able to interpret a dasymetric map as effectively as they would a choropleth map. Therefore, an analysis of visual complexity will be performed to explore this issue. More specifically, this research will seek to understand the following:

1: Can the visual complexity of a dasymetric map be characterized from the inputs of the dasymetric map?

2: Is there redundancy among complexity measures applied to dasymetric maps?

3: Are dasymetric maps more or less visually complex than choropleth maps constructed from the same base map? If differences exist, how large are they?

One of the potential uses of complexity measures is to predict map effectiveness. Given the increased difficulty of construction of a dasymetric map, it may be useful to be able to predict the complexity of a map based on the inputs so that multiple maps need not be constructed to find one of acceptable complexity. If the resulting complexity measures calculated from test maps are sensitive to sliver resolution methods then more suitable data need to be found. It has been shown that some visual complexity measures applied to choropleth maps are highly correlated and therefore redundant (Bregt and Wopereis 1990). Question two seeks to examine this possibility for dasymetric maps.

BACKGROUND AND REVIEW OF LITERATURE

History of Dasymetric Maps

There is some debate as to the origin of the dasymetric technique although J. K. Wright is often cited as the originator. In writing about this method, Wright said the term dasymetric (meaning density measuring) comes from “the Russians” implying that his work was not the first to address dasymetric maps (1936). Mennis and Hultgren cite a 1922 map by Semenov Tian-Shanskyteva as the earliest use of dasymetric mapping, lending weight to Wright’s nod to “the Russians.” (2006). Another possible origin of the technique is the work of Henry Drury Harness (1804-1883). Arthur Robinson states that while Harness did not name the method, he did in fact utilize the basic concept behind dasymetric mapping (i.e. using ancillary data to inform the map) in 1837 (1955). So, while the dasymetric technique has uncertain origins, this much may be said: despite a long existence in one form or another, dasymetric mapping has not enjoyed nearly as much popularity as has the more familiar choropleth technique (Eicher and Brewer 2001. See also Crampton 2004).

Relevance

Why then should dasymetric maps be discussed now? Recently, there has been something of a surge in the literature arguing for the use of dasymetric maps over choropleth maps. Crampton has argued that the dasymetric map is preferable to the more popular choropleth map due to the improvement in spatial and attribute accuracy achieved with a dasymetric map (2004). Robinson et al. note that GIS has “greatly simplified the dasymetric technique and is popularizing its use” (1995). Many have begun to explore the possibilities afforded by dasymetric maps (notably Langford and Unwin 1994, Mennis 2003, Eicher and Brewer 2004, and Holt, Lo, and Hodler 2004). It is unknown, however, whether or not dasymetric maps can be used effectively in place

of more common mapping techniques. It is therefore important to investigate the complexity of dasymetric maps to better understand their nature and appropriate use.

Previous Studies Involving Dasymetric Maps

Many studies of dasymetric maps focus on the use of the various construction techniques available. Some contend that dasymetric maps lack popularity because there is no standardization in construction methods (Eicher and Brewer 2001). For example, Langford and Unwin utilize a binary construction technique that uses land cover data for the ancillary source as an intermediate step in modeling population densities as a surface (1994). Maantay et al. applied a technique they termed the Cadastral-based Expert Dasymetric System (CEDS) to improve models of the relationship between asthma and air pollution (2007). The CEDS technique differs from other methods by using cadastral data as the ancillary data to better model urban areas (Maantay et al. 2007). Mennis has shown that the technique of using land cover data to inform a dasymetric map can be made more accurate by using the area of polygons to weight the redistribution of the mapped variable (2003). Langford and Higgs used dasymetric maps to model population densities and assess access to healthcare services and found them to be more effective than other techniques (2006).

Another focus of studies involving dasymetric maps is determining the functional relationship between the ancillary data and the base map used to construct such a map. Essentially, the goal of such research is to find an objective method to determine the influence that ancillary layers should have on the dasymetric map. Prominent in this focus of study is the work of Mennis and Hultgren 2006 and Eicher and Brewer 2001.

Definition of Complexity

There exist a number of definitions for map complexity in the literature. Generally speaking, complexity is comprised of two facets: cognitive complexity and visual complexity. This research will focus on the visual aspect of complexity and the term “complexity” as used in this research will refer to visual complexity. Perhaps the most widely used definition of visual complexity is that given by MacEachren: “...the degree to which the combination of map elements results in a pattern that appears to be intricate or involved” (1982a). Traditionally, the visual complexity of a map is assessed using objective measures (such as spatial autocorrelation or complexity indices), subjective user testing (which examines users’ reaction to how busy or complicated a map appears), or a combination of both. This research will utilize only objective measures of complexity.

Among the first approaches to quantifying visual map complexity are Monmonier’s work on measurements for choropleth maps and Olson’s use of spatial autocorrelation (1974 and 1975 respectively). Castner and Eastman measured eye-movement parameters in order to analyze complexity (1985). More recently, Fairbairn examined the possibility of using file compression as a measure of complexity (2006). Complexity, for the purposes of this research, will be examined by measures that are functions of the distribution being mapped (i.e. the base map and the underlying data distribution) and the symbolization used to represent that distribution (i.e. the classification algorithm used and the number of classes) (MacEachren 1982a and 1982b).

Studies Examining Visual Complexity and Map Effectiveness

The utility of map complexity measures lies in their hypothesized ability to predict map effectiveness. It has been shown that complexity may have a negative effect on map readability and therefore the effectiveness of a map in communicating information (Monmonier 1974,

MacEachren 1982a, Bregt and Wopereis 1990). Muller examined how complexity affects map comparison (1976). Other studies examining map effectiveness as related to map complexity include Mersey 1990, and Bregt and Wopereis 1990. Mersey confirmed the results of MacEachren 1982b using color maps. Bregt and Wopereis compared multiple measures of complexity and found them to be redundant in predicting subjective complexity scores.

The given definition of map complexity as a function of map inputs does not encompass all factors influencing map effectiveness. For example, Mersey explored the impact of color selection on map effectiveness (1990). Other factors include the prior experience of the map user, the task for which the map is being used, and the media in which the map is presented (Bunch and Lloyd 2006). In short, while visual complexity should not be seen as the only factor in map effectiveness, it may be helpful as a map design tool in combination with other techniques.

Of particular interest to this research is MacEachren's work which was the first to examine complexity in terms of the resulting changes in effectiveness (1982a and 1982b). MacEachren created isopleth and choropleth maps using the same data. He then calculated objective complexity scores and correlated them with subjective scores. The study determined that choropleth maps are viewed as more visually complex than a corresponding isopleth map and that the distribution and number of classes used are significant factors of map complexity (MacEachren 1982a). Following up on this research, the maps were tested for effectiveness through the use of a questionnaire. The key result of this follow-up study was that visual complexity does not hinder effectiveness in all map reading tasks (MacEachren 1982b).

Generally speaking, map users are usually able to interpret general patterns in a complex map but have difficulty extracting specific information. Though he only focused on choropleth and

isopleth maps, MacEachren states, “The attempt to improve the communication effectiveness of various forms of map symbolization is a worthwhile endeavor, but it seems that a determination of the relative effectiveness of the various forms of symbolization currently used should also be given attention” (1982b). Thus, in an attempt to meet MacEachren’s recommendation, this research will compare the relative complexity of choropleth and dasymetric maps.

Measures of Complexity

There are a number of measures that have been used to describe visual map complexity. For this study, a variety of measures were selected based on ease of understanding, ease of calculation, and applicability to dasymetric maps. Monmonier argues that there are three dimensions of map complexity for choropleth maps: “pattern fragmentation and autocorrelation, size inequality of map regions, and the general spatial trend and its autocorrelation or strength” (1974). With this understanding, measures were also selected based on the aspect of complexity that each measure captures. It is important to note that these measures are not appropriate for all forms of dasymetric maps. In fact, dependant upon the construction method and display method selected, the results may be constant regardless of the input rendering the results useless. For this reason, the display method retaining all boundaries was selected for this study. It must be noted that the utility of such measures has not been fully realized. In particular, while measures have been correlated with map user’s subjective judgment of complexity, the impact of varying degrees of complexity has not been fully studied. That is to say that it is unknown how large a difference in complexity must be before it changes the effectiveness of a map.

Graph Theory

A number of complexity measures use a field of mathematics known as Graph Theory. A graph, in Graph Theory, is a collection of vertices, edges, and faces (MacEachren 1982a). A map can be transformed into a graph by treating polygons as faces, boundaries between adjacent polygons as edges, and intersections of boundary lines as vertices (see Figure 7). For this study, the definition of adjacency used is that of Rook Contiguity, meaning two polygons are adjacent only if they share a line segment. Note that more complex relations are possible to describe with graphs than those seen in Figure 7. For example, edges can be encoded with the nature of the relationship between two faces.

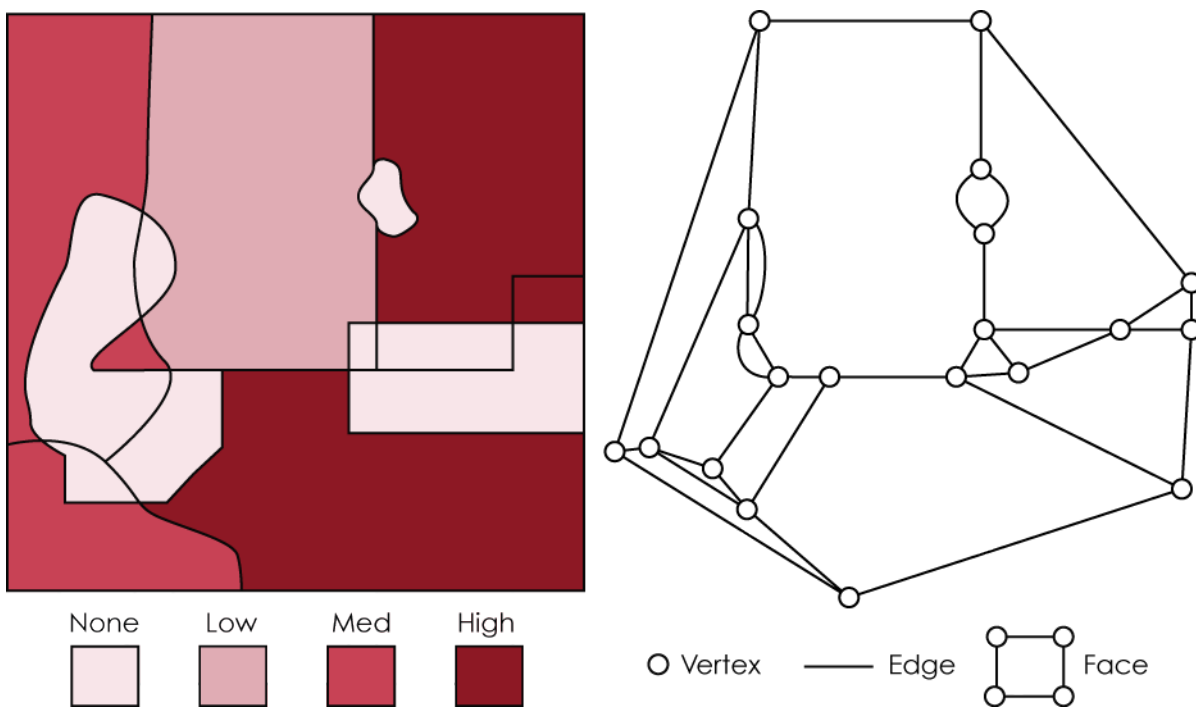


Figure 7: A map and the graph transform of that map.

There are some drawbacks to the graph theoretic approach. For the scenario in which two polygons are contiguous in multiple locations (multi-adjacency), the graph transform of the map reflects this. This characteristic, however, is not captured in the calculation of adjacency for

maps when software is used (see Figure 8). In other words, unless complexity measures are calculated by hand, the impact of multi-adjacent polygons does not factor into the calculation. Another drawback to measures using graph theory is that the shape of the boundaries between polygons is not considered. This is a flaw as it may be that the sinuosity of boundaries has an impact on complexity.

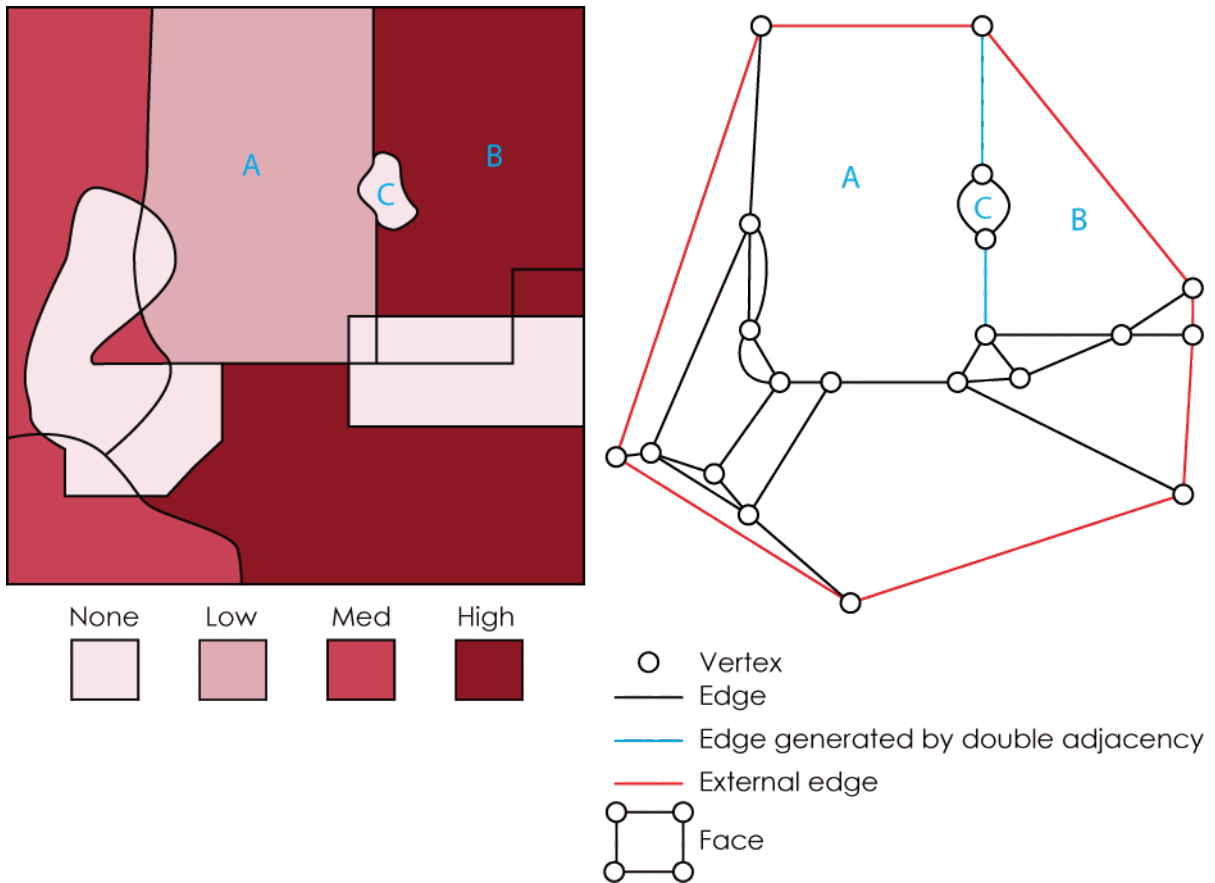


Figure 8: Polygons A and B are adjacent. Polygon C splits this connection so there are two edges between face A and B. Calculating the adjacency matrix for the map would not reveal this case of two instances of adjacency. The red lines are edges that do not separate two faces. These edges correspond to the external boundaries of the map.

Aggregation Index

The Aggregation Index (AI), originally defined by Muller, is, in a way, a measurement of spatial autocorrelation (1976). It is designed to measure how aggregated a map appears. It is calculated as follows:

$$AI = \frac{\sum_{i=1}^n \sum_{j=1}^n b_{ij}}{\sum_{i=1}^n \sum_{j=1}^n a_{ij}}$$

Where:

n = the number of enumeration units

$b_{ij} = \begin{cases} 1 : \text{if enumeration units } i \text{ and } j \text{ are adjacent and have the same category value} \\ 0 : \text{if enumeration units } i \text{ and } j \text{ are not adjacent or if } i = j \end{cases}$

$a_{ij} = \begin{cases} 1 : i \text{ and } j \text{ are adjacent} \\ 0 : i \text{ and } j \text{ are not adjacent} \end{cases}$

In terms of graph theory, the denominator is equivalent to the number of edges in the graph transform of the map, excluding exterior edges. The numerator is the number of edges in the graph transform of the map that separate faces with the same category value. Note that multi-adjacency (as seen in Figure 8) is only counted once due to the manner in which adjacency is calculated. The AI is undefined for the trivial case of a single enumeration unit or if there are no adjacent polygons in the map. This measure ranges from 0 to 1 with lower values indicating greater complexity.

Assuming ignorance of multi-adjacency, the AI for the map in Figure 7 would be calculated as:

$$\begin{aligned}\sum b_{ij} &= 9 \\ \sum a_{ij} &= 26 \\ AI &= \frac{9}{26} \\ &= 0.346\end{aligned}$$

Examining the graph transform of the map (see Figure 9) would change the denominator to 27 as it does not ignore multi-adjacency. This would change the calculation for the example map to the following:

$$\begin{aligned}\sum b_{ij} &= 9 \\ \sum a_{ij} &= 27 \\ AI &= \frac{9}{27} \\ &= 0.333\end{aligned}$$

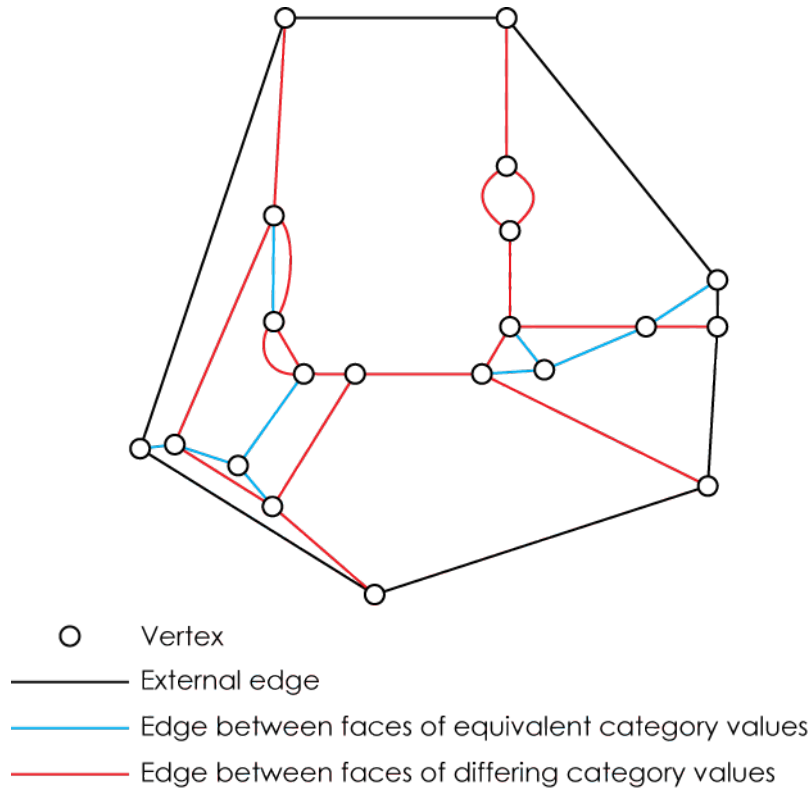


Figure 9: Reference graph for example AI calculation.

Boundary Contrast Index

Muller defined a trivalent difference as three mutually adjacent enumeration units that fall into three different classes. He then defined complexity as the total number of trivalent differences in a map (1975). The Boundary Contrast Index, defined by Bregt and Wopereis, (BCI) builds on Muller's complexity definition by dividing Muller's definition of complexity by the total number of mutually adjacent polygons in a map (1990). The divisor is the same as the maximum possible value for Muller's definition of complexity. Thus, the BCI is calculated as:

$$BCI = \frac{T}{\max(T)}$$

Where:

$$T = \text{Muller's definition of complexity}$$

$$\text{max}(T) = \text{the maximum possible value of } T$$

For the map in Figure 7, the BCI would be calculated as:

$$T = 2$$

$$\text{max}(T) = 4$$

$$BCI = \frac{2}{4}$$

$$= 0.500$$

While this calculation ignores multi-adjacency, it is consistent with the algorithm used in this study. A more correct calculation would incorporate multi-adjacency and give (see Figure 10):

$$T = 3$$

$$\text{max}(T) = 5$$

$$BCI = \frac{3}{5}$$

$$= 0.600$$

Importantly, because this study uses Rook Contiguity as the definition of adjacency, the BCI will ignore intersections of four or more polygons as they will not be considered mutually adjacent. The BCI ranges in value from 0 to 1 with higher values indicating greater complexity. This measure is undefined if no three polygons are mutually adjacent in the map.

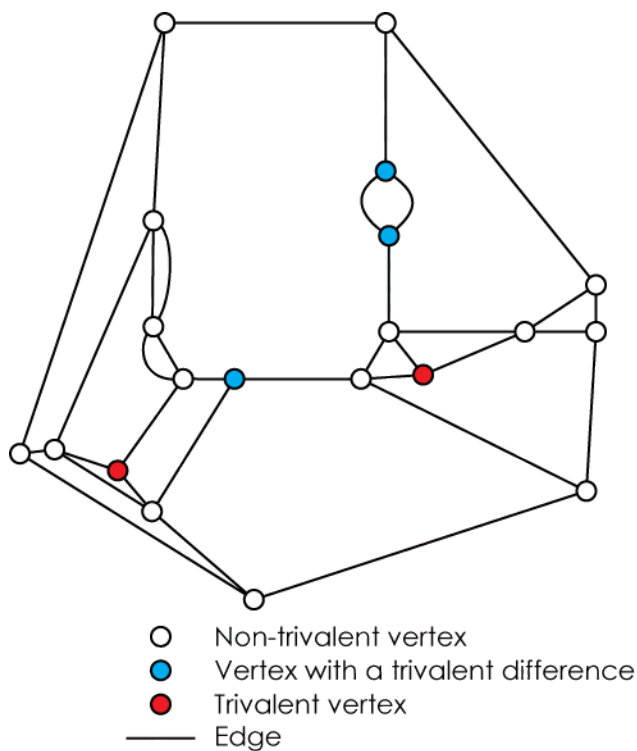


Figure 10: Reference graph used for example BCI calculation.

Fragmentation Index

The Fragmentation Index (FI) was originally utilized by Monmonier in 1974. It is designed to capture how fragmented a map appears. The FI is calculated as:

$$FI = \frac{M - 1}{N - 1}$$

Where:

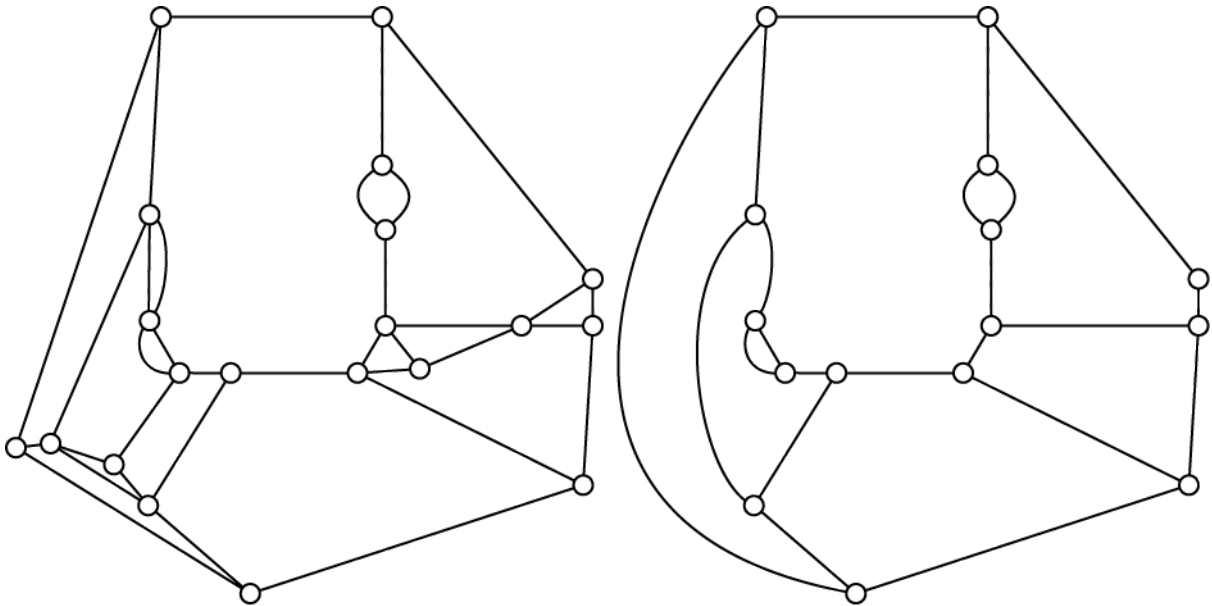
M = the number of polygons after dissolving boundaries between enumeration units of the same category

N = the number of enumeration units

The FI is undefined for the trivial case of the map consisting of a single enumeration unit. It ranges from 0 to 1. A higher value is indicative of greater complexity. For the map in Figure 7, the FI is calculated as:

$$\begin{aligned}
 M &= 8 \\
 N &= 15 \\
 FI &= \frac{8 - 1}{15 - 1} \\
 &= \frac{7}{14} \\
 &= 0.500
 \end{aligned}$$

This measure can also be calculated from the graph transforms of the base map and the map after dissolving boundaries between polygons with the same category value (see Figure 11).



Graph of base map: Number of Faces: 15 Graph of dissolved map: Number of Faces: 8

Figure 11: Reference graphs for example calculation of FI. The graph on the left is from the base map. The graph on the right is from the map after boundaries between polygons with the same category value are dissolved.

Pattern Variability Index

Muller describes a map as redundant if a large proportion of enumeration units belong to the same category (1976). To examine this, he designed the Redundancy measure. A more appropriate name is the Pattern Variability Index (PVI). The PVI is the probability that any two randomly selected enumeration units are represented with the same symbol. It is calculated as:

$$PVI = P(C_i = C_j) = \frac{\sum_{i=0}^n \sum_{j=0}^n b_{ij}}{n^2 - n}$$

Where:

C_i = the category value of enumeration unit i

C_j = the category value of enumeration unit j

n = the number of enumeration units

$$b_{ij} = \begin{cases} 1 & : \text{if } C_i \text{ and } C_j \text{ are the same and } i \neq j \\ 0 & : \text{if } C_i \text{ and } C_j \text{ are different or } i = j \end{cases}$$

The numerator of the PVI may also be expressed as:

$$\sum b_{ij} = \sum_{i=1}^{N_c} P_i^2 - P_i$$

Where:

N_c = the number of classes

P_i = the number of polygons in class i

Either method gives the number of polygon pairs with the same category value divided by the total number of polygon pairs while ignoring the case of pairing polygons with themselves.

The PVI is undefined in the trivial case of the map consisting of 0 or 1 enumeration units. The maximum value of the PVI is 1 and the minimum is 0 with lower values indicating higher complexity. For Figure 12, the PVI is:

$$\begin{aligned}
 \sum b_{ij} &= 8^2 + 1^2 + 3^2 + 3^2 \\
 &= 64 + 1 + 9 + 9 \\
 &= 83 \\
 n &= 15 \\
 PVI &= \frac{83}{15^2 - 15} \\
 &= \frac{83}{210} \\
 &= 0.395
 \end{aligned}$$

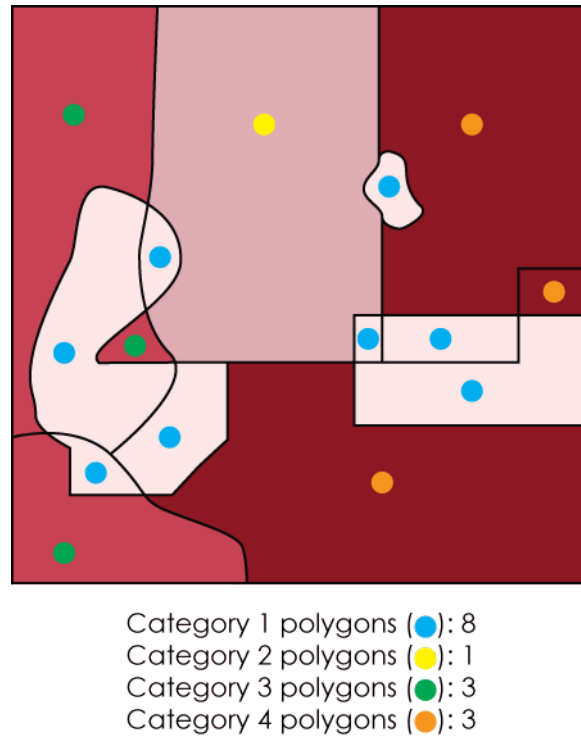


Figure 12: Reference Map for example of PVI calculation.

Size Disparity Index

The Size Disparity Index (SDI) described by Monmonier is used to examine the size inequality of map regions (1974). This measure is based on the Lorenz curve. To calculate the SDI, the areas of all polygons are sorted and normalized by the total area. Next, a vector of cumulative normalized areas called the Cumulative Proportional Area Vector (CPAV) is calculated. The SDI is defined as the area between these values and the equation $y = x$. The values given by this measure range from 0 to 0.5. Bregt and Wopereis state that lower values generally indicate higher complexity (1990). For this research, the SDI was calculated on maps after dissolving boundaries between enumeration units belonging to the same category. This was done to attempt to incorporate the natural grouping that occurs with proximal objects of similar appearance.

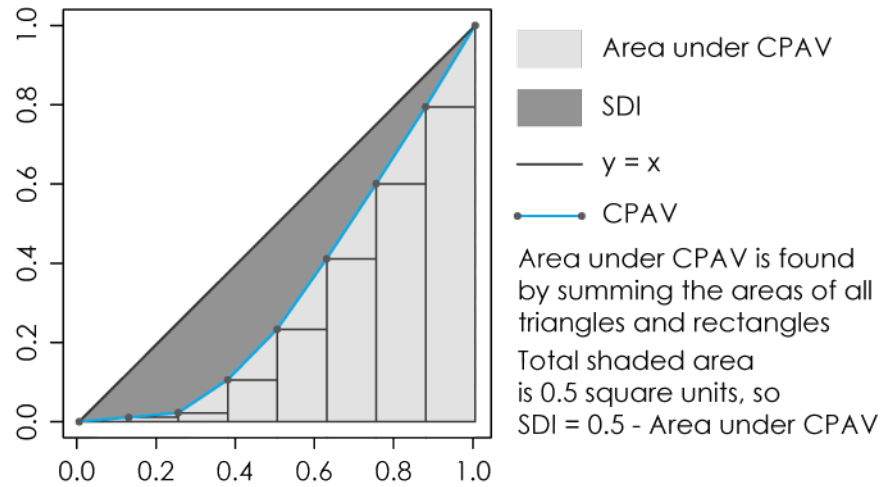


Figure 13: CPAV and $y = x$. The SDI is the area between the two lines.

Note that since the area under $y = x$ from $x = 0$ to $x = 1$ is 0.5, the SDI is equivalent to 0.5 – the area under the curve defined by the CPAV. The area under the CPAV is more readily calculated than the actual SDI by breaking the area down into triangles and rectangles as seen in Figure 13 and summing the areas of these triangles and rectangles. The map in Figure 14 generates the cumulative proportional area vector seen in Table 1. For those areas, the SDI is:

$$\begin{aligned} \sum \text{Triangular Areas} &= 0.272 \\ \sum \text{Rectangular Areas} &= 0.063 \\ \text{SDI} &= 0.5 - (0.272 + 0.063) \\ &= 0.5 - 0.335 \\ &= 0.165 \end{aligned}$$

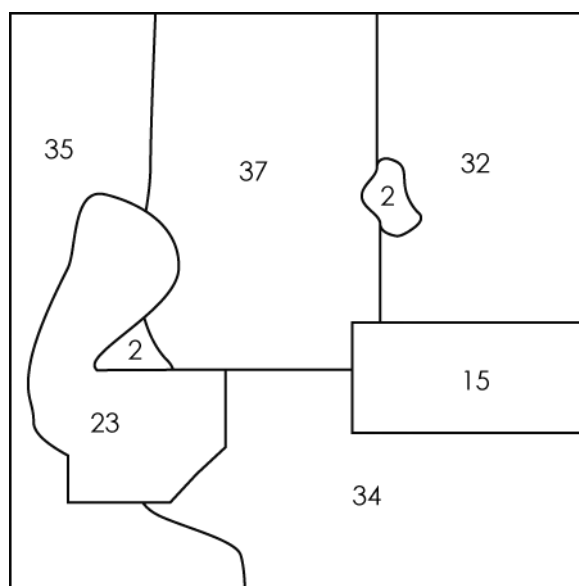


Figure 14: Reference map for example of SDI calculation. Areas are estimated.

Areas	2	2	15	23	32	34	35	37
Proportional Areas	0.011	0.011	0.083	0.128	0.178	0.189	0.194	0.206
CPAV	0.011	0.022	0.106	0.233	0.411	0.600	0.794	1.000
Rectangle Height	0.000	0.011	0.022	0.106	0.233	0.411	0.600	0.794
Triangle Height	0.011	0.011	0.083	0.128	0.178	0.189	0.194	0.206
Rectangle Area	0.000	0.001	0.003	0.013	0.029	0.051	0.075	0.099
Triangle Area	0.001	0.001	0.005	0.008	0.011	0.012	0.012	0.013

Table 1: Calculations for SDI example.

Weighted Edge Ratio

In addition to the above measures, Mersey modified the edge ratio measure used by Olsen to take into account adjacent enumeration units of the same category by using a weighted edge ratio (WER) model (1990). This measure is meant to enable easier comparison between different geographical distributions (Mersey 1990). The WER assumes that the category values of the map polygons are ordinal in nature. It is calculated as follows:

$$WER = \frac{\sum_{j=1}^{N_c} \sum_{i=1}^{N_c} N_{ij} W_{ij}}{E}$$

Where:

N_c = the number of classes

N_{ij} = the number of boundaries between polygons of class i and j

W_{ij} = the weight assigned to a boundary between polygons of class i and j

$$= \frac{1}{|i - j| + 1}^k$$

k = an arbitrary exponent which determines the rate at which the influence of neighbors diminishes

E = the number of boundaries between polygons in the map

The calculation of the WER may be simplified as:

$$WER = \frac{\sum_{|i-j|=0}^{N_c-1} N_{|i-j|} W_{|i-j|}}{E}$$

Where:

$|i - j|$ = the absolute difference between category values i and j

N_c = the number of classes

$N_{|i-j|}$ = the number of boundaries between polygons with a difference in categories of $|i - j|$

$W_{|i-j|}$ = the weight assigned to boundaries between polygons with a difference in categories of $|i - j|$

$$= \frac{1}{|i - j| + 1}^k$$

E = the number of boundaries between polygons in the map

Using either form of the WER, it is obvious that it will be undefined for maps with no adjacency (e.g. maps consisting of a group of islands). The WER ranges from 0 to 1 with lower values indicating greater complexity. For the map in Figure 7, Table 2 contains the steps in calculating values for the WER assuming ignorance of multi-adjacency. The remaining calculations are:

$$\begin{aligned}
 k &= 1 \\
 E &= 26 \\
 \sum N_{|i-j|} W_{|i-j|} &= 15.33 \\
 WER &= \frac{15.33}{26} \\
 &= 0.590
 \end{aligned}$$

As with the AI and BCI, the WER could be calculated factoring in cases of multi-adjacency, but algorithmically, this is not the case. To calculate the WER acknowledging multi-adjacency, the calculations shown in Table 3 would be used to give:

$$\begin{aligned}
 k &= 1 \\
 E &= 27 \\
 \sum N_{|i-j|} W_{|i-j|} &= 15.67 \\
 WER &= \frac{15.67}{27} \\
 &= 0.580
 \end{aligned}$$

$ i-j $	$N_{ i-j }$	$W_{ i-j }$	$N_{ i-j } W_{ i-j }$
0	9	1.00	9.00
1	7	0.50	3.50
2	4	0.33	1.33
3	6	0.25	1.50

Table 2: Calculations for the WER.

$ i-j $	$N_{ i-j }$	$W_{ i-j }$	$N_{ i-j }$	$W_{ i-j }$
0	9	1.00	9.00	
1	7	0.50	3.50	
2	5	0.33	1.67	
3	6	0.25	1.50	

Table 3: Calculations for the WER acknowledging multi- adjacency.

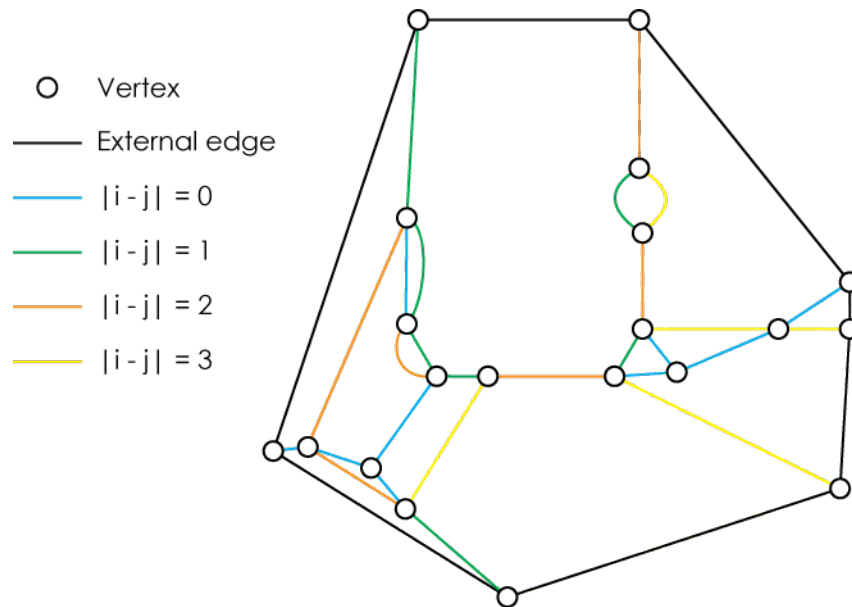


Figure 15: Reference graph for example of calculation of the WER.

Discussion of Visual Complexity Measures

These measures were selected for the diversity of ways in which they describe visual complexity. For example, it may be that dasymetric maps are more likely to have a large disparity in the size of polygons, but are more aggregated than corresponding choropleth maps. By using multiple measurements, the complexity of dasymetric maps will be better characterized. It should be noted that not all of the measures have been corroborated by subjective user testing. None of the measures discussed have been applied to dasymetric maps to the author's knowledge.

The calculation of each measurement is dependent on a number of inputs. Table 4 summarizes the inputs that affect each measure as well as the range of possible values for each measure and what values indicate a visually complex map. For those complexity measures which have the category value of the enumeration units as an input, it should be understood that these categories are dependant upon the data being mapped, the classification algorithm being used and the number of classes being displayed (see Figure 16). It is also important to note that, for dasymetric maps, the value of each measure is sensitive to the display method used. For example, the FI for a map with all boundaries between polygons of the same class dissolved will always give a value of 1 and decreased values (indicating a lesser degree of complexity) for the other two methods (retaining all boundaries and dissolving boundaries between zero-valued neighbors).

	AI	BCI	FI	PVI	SDI	WER
Enumeration Unit Areas					●	
Classification of Polygons	●	●	●	●		●
Base Map Structure	●	●	●			●
Range	[0,1]	[0,1]	[0,1]	(0,1)	(0,0.5]	(0,1]
Value indicitave of Complexity	low	high	high	low	low	low

Table 4: Summary of complexity measure characteristics.

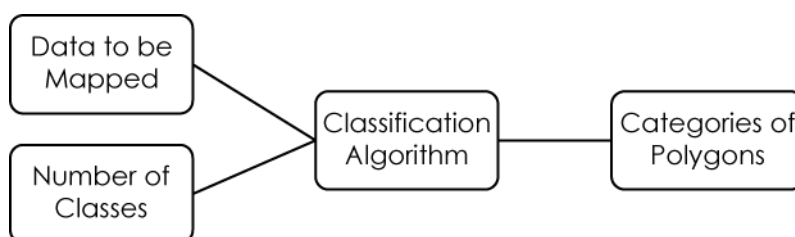


Figure 16: Factors affecting the category value of enumeration units.

Unfortunately, some complexity measures do not adequately take into account the effect that islands may have on pattern complexity. MacEachren points out that features in the same

general location are often grouped in a map reader's mind (2004). This mental grouping is made even stronger if those features share an attribute such as color. Recognition of this fact underlies the reasoning behind many measures of complexity. For example, the AI assumes that adjacent polygons with the same category value are likely to be mentally grouped by a map reader and therefore a map which has many instances of this is less complex. In general, however, measures which attempt to account for map structure are affected by islands in surprising ways. We can examine these effects by looking at the measures generated by a map retaining islands and that same map with islands removed post-classification (See Figure 17 and accompanying Table 5). The AI, and WER are unaffected by the removal of islands unless some of those islands are split into more than one enumeration unit. In other words, an infinite number of islands could be added to a map without affecting the AI or WER. The BCI is unaffected by the removal of islands unless there is an island with three mutually adjacent polygons. For the FI, removing islands means that M and N are reduced by an equal amount. This means that the FI for a map with islands will always be greater than a map with islands removed. Assuming that map readers find maps with islands more fragmented in appearance, the FI is successful in gauging map complexity. The PVI and SDI may also adequately account for islands as adjacency of polygons is not a factor in their calculation. For this study, because the impact of islands on visual complexity is not more fully understood, islands have been removed from all base maps prior to classification. While this has a definite impact on the resulting calculations, consistency is desirable over uncertainty.

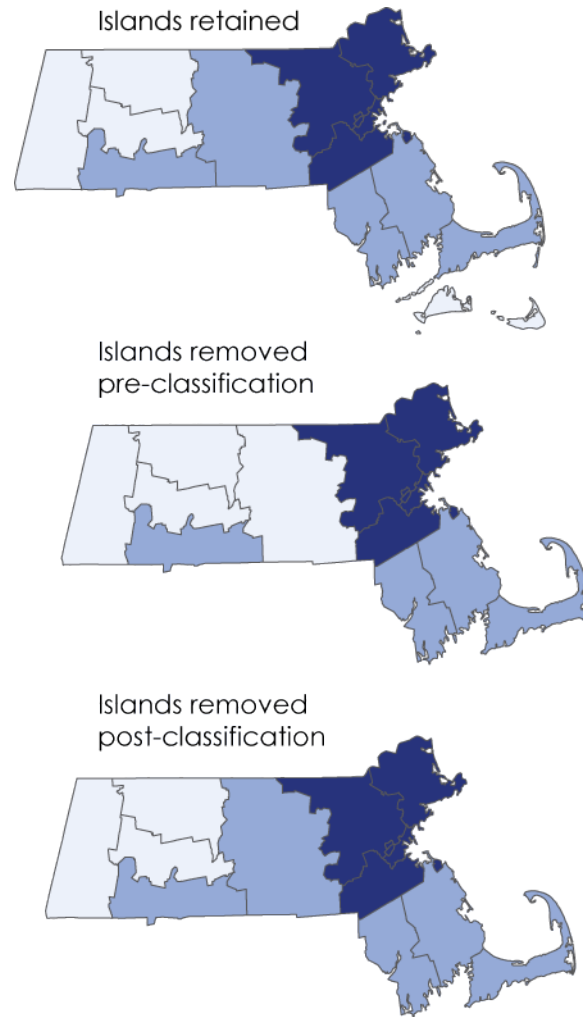


Figure 17: Example maps for calculation of complexity measures with and without islands.

	AI	BCI	FI	PVI	SDI	WER
Islands retained	0.640	0.000	0.710	0.325	0.289	0.820
Islands removed pre-classification	0.680	0.000	0.286	0.297	0.220	0.827
Islands removed post-classification	0.640	0.000	0.286	0.308	0.220	0.820

Table 5: Resulting complexity measures for maps with and without islands.

METHODS

Region of Study

The region of study consists of county boundaries of the United States with some omissions that will be described later. Ancillary data for this same region were also obtained, including federal lands boundaries, wilderness area boundaries (a subset of federal lands), lakes, and water bodies. These data were obtained as shapefiles from the national atlas data repository (<http://www.nationalatlas.gov>). The nominal scale for the data is 1:2,000,000 so generalization decisions were made based on this as the viewing scale. All shapefiles were re-projected from the National Atlas projection to Alber's Equal Area since equivalent projections are preferred for choropleth mapping (Slocum et al. 2005).

Choropleth Map Preparation

Census population data for the year 2000 were also obtained through the national atlas. Utilizing ArcGIS, the population data were joined to the county enumeration units based on FIPS codes. The county layer was clipped using state boundaries resulting in an individual base map for each state. Islands were deleted as the complexity measures do not account for their impact. Densities were calculated as persons per square mile. The counties were then examined for multi-part polygons that were then separated into single-part polygons prior to complexity measure calculation. This step was necessary for the calculation of complexity measures. Adjacency information was calculated using a script written in Python that utilizes the `arcgisscripting` module (see Appendix A). Complexity measures were calculated for each map using a varying number of classes and different classification methods. That is each map was classified using three, four, five, six, and seven categories using the Quantile, Equal Interval, and Fisher-Jenks methods resulting in fifteen different maps. In addition, the Standard Deviation

classification was also used though this always resulted in empty classes so it was omitted. The algorithms used for classification come from the Python Spatial Analysis library (PySAL). If the classification failed or there were fewer than $2 \cdot n$ polygons to classify (where n is the number of classes), that particular combination of classification method and number of classes was omitted. This decision was made to limit the number of instances of empty classes. Omitting empty classes is desirable to guarantee that comparisons between n -class maps were consistent. With one base map, there are $3 \cdot 5$ possible choropleth maps. Since there are 50 original basemaps, this gives a total of 750 possible choropleth maps (see Figure 18). The actual number is fewer since classification sometimes failed due to a small number of features or processing error. This processing error is a result of the PySAL implementation of the Fisher-Jenks algorithm and the cause of the error is unknown.

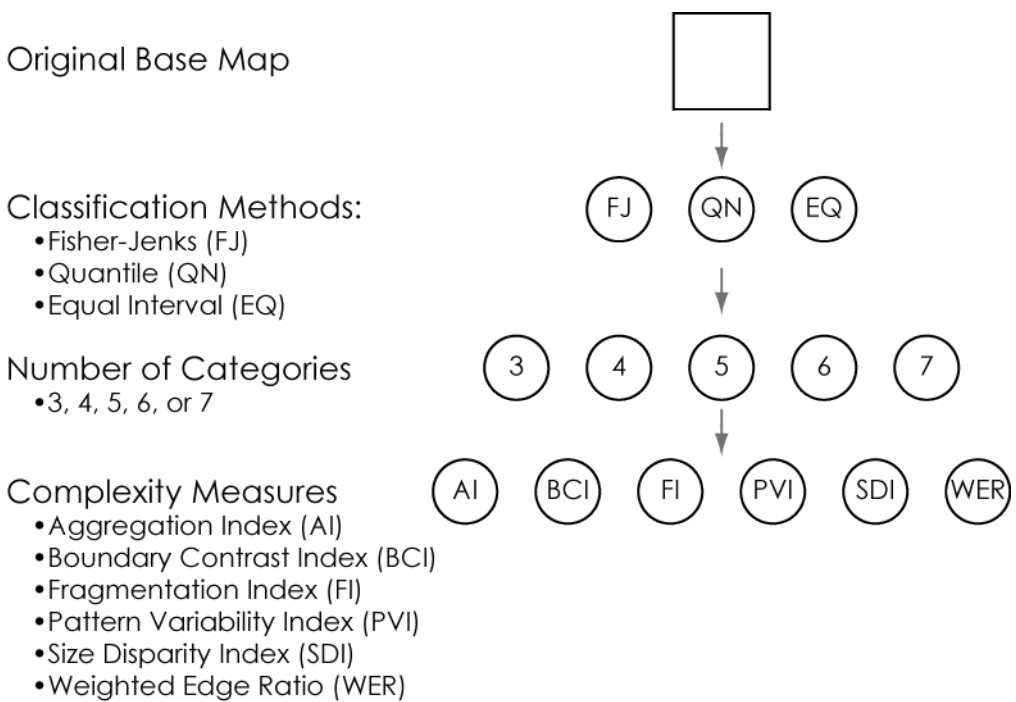


Figure 18: The possible combination of inputs for choropleth maps and resulting complexity measures.

All complexity measures were calculated utilizing scripts written in the Python programming language. These scripts were built on modules available in the PySAL library (see Appendix A).

Dasymetric Map Preparation

Preparation of ancillary data required some editing of the original files. In order to speed up processing and make an objective attempt to generalize the maps, all polygons with an area less than ten square miles were removed. Any extraneous features were also removed (i.e. features which did not overlap with the county layer). The three base ancillary layers (lakes, federal lands, and wilderness areas) were combined, forming two new ancillary layers, lakes and federal lands, and lakes and wilderness areas. A combination of all three was deemed redundant to the combination of federal lands and lakes since wilderness areas are a subset of federal lands.

Similarly, the combination of federal lands and wilderness areas is redundant to federal lands by itself. These ancillary layers were overlaid with the county layer to form the dasymetric maps (see Figure 19). Each ancillary layer was used to erase from the county layer, creating a layer containing only populated areas. Next, a clip was conducted with the ancillary layers clipping the county layer. This resulted in a layer of unpopulated areas. The populated and unpopulated layers were then combined using a union function to form the dasymetric maps.

Further processing was still necessary to prepare the maps for complexity measurement generation. First, all islands were deleted from the layers. Next, the layers were clipped using state boundaries. Sliver polygons resulting from the overlay operations (erase, clip, and union) were handled using three methods: 1) Merging slivers with their largest neighbor. 2) Merging slivers with their smallest neighbor. 3) Merging slivers with a randomly selected neighbor. A sliver was defined as any polygon with an area of less than 5 square miles. In all three methods,

polygons were not merged unless a neighbor could be found that was not a sliver polygon as well. This resulted in three sets of dasymetric maps that included five maps for each state, one

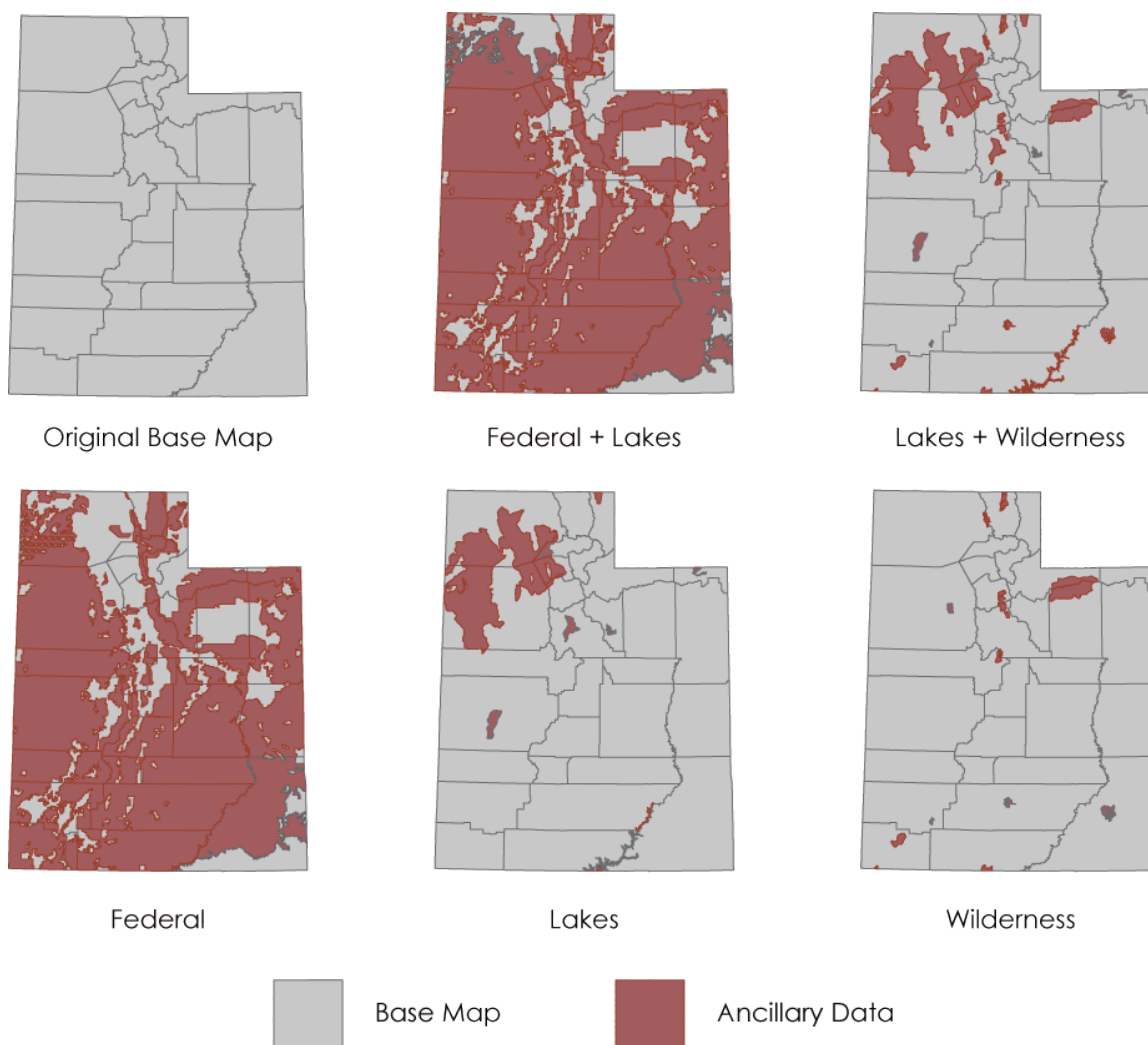


Figure 19: Examples of the combinations of ancillary data and base map for dasymetric map creation.

for each ancillary layer. With one base map, this gives $5 \times 3 \times 4 \times 5$ possible dasymetric maps. Since there are 50 original basemaps, this gives a total of 15000 dasymetric maps (see Figure 20). As with choropleth maps, the actual number is fewer since classification sometimes failed due to a small number of features or processing error. Densities were calculated as persons per square mile in populated areas only. Unpopulated areas were assigned a density of zero. Complexity

measures were generated for each resulting map based on the density calculations as with the choropleth maps. The only difference is that zero values (i.e. those polygons that show unpopulated areas) were automatically assigned to their own class and the remainder of the data were classified using one of four classification algorithms. This was done under the assumption that the areas masked by ancillary data are of interest.

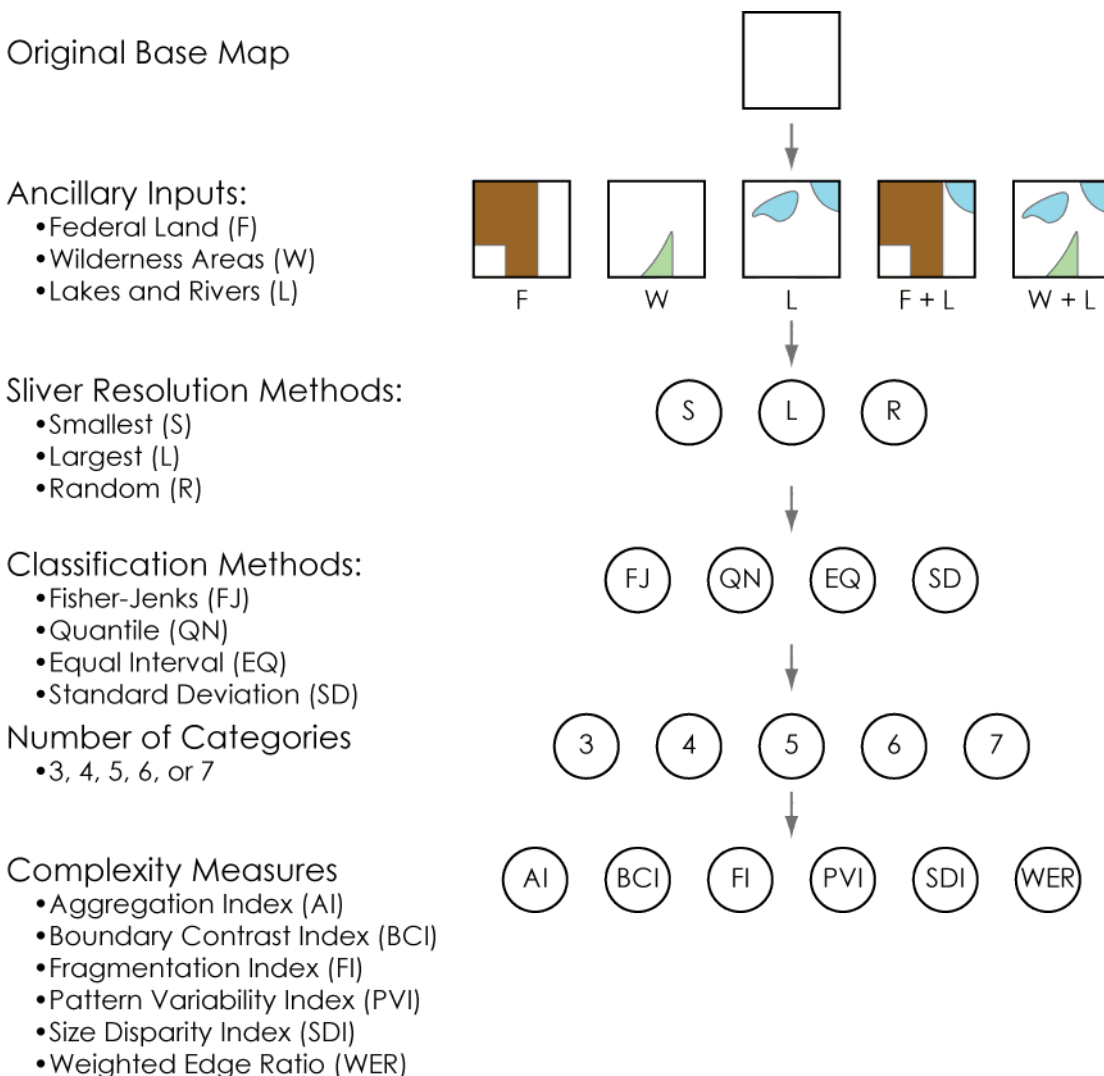


Figure 20: The possible combination of inputs for dasymetric maps resulting complexity measures.

Testing Strategy

Prior to analysis of the resulting complexity measures, an examination of the impact of resolving slivers will be made to ensure that results are not sensitive to sliver resolution methods. The results for each sliver resolution method can be compared across methods by pairing maps based on inputs. That is, a map generated using a particular ancillary input, classification method, number of classes, and sliver resolution method may be paired with another map generated using the same ancillary input, classification method and number of classes, but a different sliver resolution method. A paired t-test will be conducted for each measurement between the three combinations of sliver resolution methods (i.e. small vs. large, small vs. random, and large vs. random).

The primary research questions will be examined as follows:

1: Can the visual complexity of a dasymetric map be characterized from the inputs of the dasymetric map?

Inputs to be examined include the classification method used, the number of classes used, the number of ancillary features relative to the total features, and the area of all ancillary features relative to the area of all features. Scatter plots and box plots will be examined to determine what effect these various aspects of map construction may have on visual complexity.

2: Is there redundancy among complexity measures applied to dasymetric maps?

This question will be examined using scatter plots between measures and appropriate measures of association between complexity measures.

3: Are dasymetric maps more or less visually complex than choropleth maps constructed from the same base map? If differences exist, how large are they?

To answer this question, differences between complexity measures for choropleth maps and dasymetric maps constructed using the same base map, classification method, and number of categories will be examined using a paired t-test to examine the mean difference between choropleth and dasymetric complexity. Comparing a three-class choropleth map to a three-class dasymetric map means that one of the classes in the dasymetric map contains only zero-valued data points with the remainder being split into two classes using the same algorithm that was used for the choropleth map. If map type (choropleth versus dasymetric) does not affect resulting complexity measures, then the mean difference between measures will be zero. Additionally, scatter plots examining the relationship between choropleth maps and corresponding dasymetric maps will be constructed. Measures of association between the complexity measures calculated for choropleth maps and dasymetric maps will also be calculated. Another approach that will be used is to examining the difference between choropleth and dasymetric map complexity with regard to the trend in complexity seen as the number of classes changes.

RESULTS

Sliver Resolution Method

If there were no difference between sliver methods, the average difference between complexity measures would be zero. Examining the average difference between the three sliver resolution methods shows that the largest average difference is 0.003 (seen in Table 6 for the FI). It is reasonable to assume that such a difference is unlikely to be detectable by a map user. The results of the paired t-tests examining the average difference between the sliver resolution methods are seen in Table's 6, 7, and 8. The hypothesis for these tests was:

$$H_O : d = 0$$

$$H_A : d \neq 0$$

In this test, d is the average difference between measures. In all cases, the PVI is the only measure for which the null hypothesis is not rejected at an alpha of five percent. While this would seem to indicate that visual complexity measures are sensitive to sliver resolution method, it can be argued that the average difference is much smaller than a perceptible difference. In fact, the statistical significance of the results can be attributed to the large number of data values. In Figure 21, maps showing the largest observed differences are seen (see also Table 9). These all occur for comparisons between the smallest and largest merge methods. Given that the differences are barely noticeable between map pairs, it can be assumed that the impact of sliver resolution method is negligible for this study. Under this assumption, the remaining tests will be examined utilizing only the results from maps created using the large sliver resolution method.

	d	t	p-val	95% Confidence Interval	
				lower	upper
AI	0.0025	24.047	0.0000	0.0023	0.0027
BCI	-0.0026	-15.477	0.0000	-0.0029	-0.0023
FI	-0.0029	-19.980	0.0000	-0.0031	-0.0026
PVI	0.0001	1.385	0.1662	0.0000	0.0001
SDI	-0.0013	-19.293	0.0000	-0.0015	-0.0012
WER	0.0016	24.066	0.0000	0.0014	0.0017

Table 6: Summary of paired t-test: Smallest VS Largest

	d	t	p-val	95% Confidence Interval	
				lower	upper
AI	0.0007	9.494	0.0000	0.0006	0.0009
BCI	-0.0011	-8.912	0.0000	-0.0013	-0.0008
FI	-0.0010	-12.665	0.0000	-0.0012	-0.0009
PVI	0.0001	1.525	0.1275	0.0000	0.0001
SDI	-0.0005	-14.105	0.0000	-0.0005	-0.0004
WER	0.0004	8.961	0.0000	0.0003	0.0005

Table 7: Summary of paired t-test: Smallest VS Random

	d	t	p-val	95% Confidence Interval	
				lower	upper
AI	-0.0018	-24.218	0.0000	-0.0019	-0.0016
BCI	0.0015	11.267	0.0000	0.0013	0.0018
FI	0.0018	16.034	0.0000	0.0016	0.0021
PVI	0.0000	-0.147	0.8833	0.0000	0.0000
SDI	0.0009	15.196	0.0000	0.0008	0.0010
WER	-0.0011	-23.847	0.0000	-0.0012	-0.0010

Table 8: Summary of paired t-test: Largest VS Random

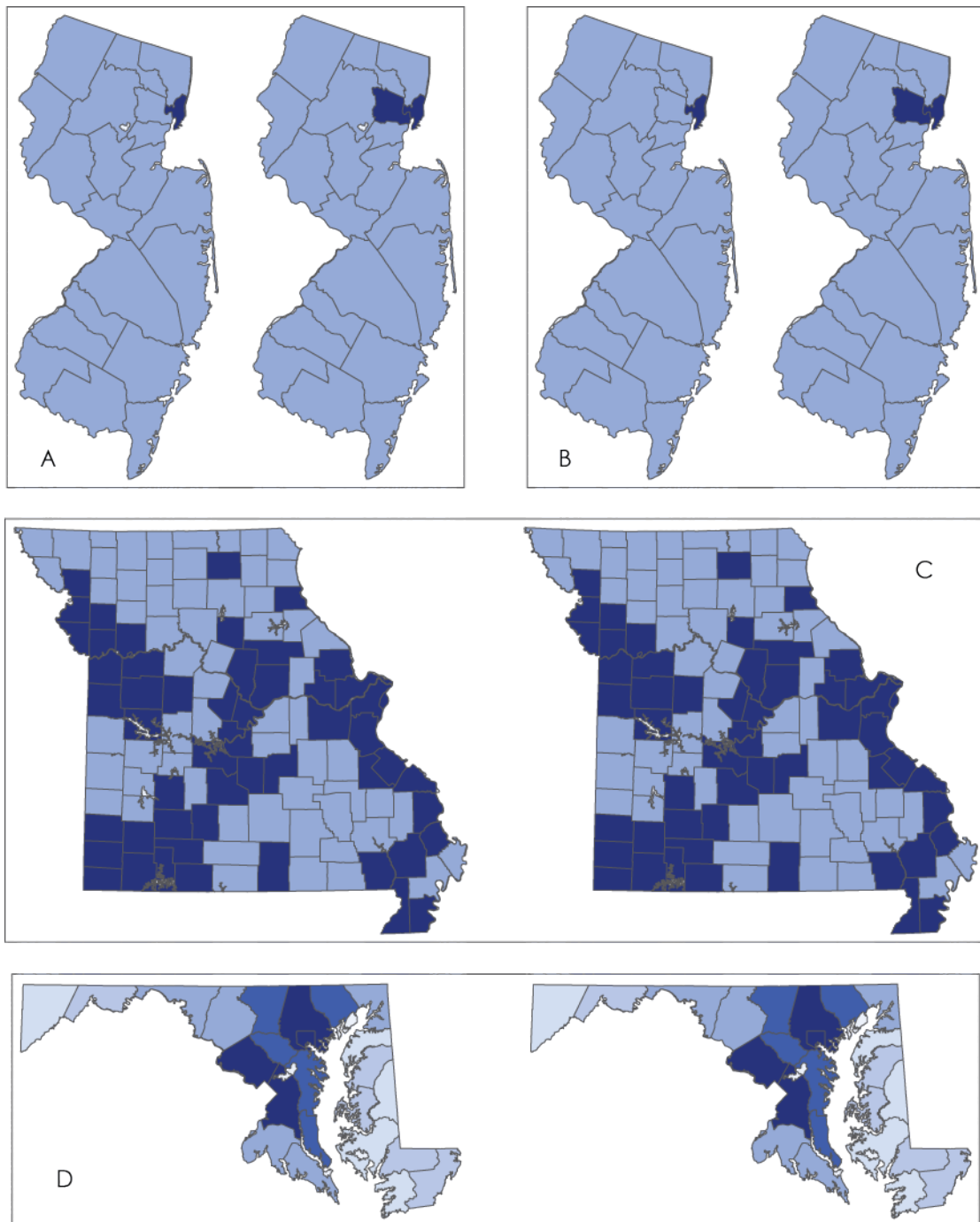


Figure 21: Map pairs depicting maximum differences in complexity measures for the three sliver resolution methods. See Table 9 for the associated measures.

	Map Pair	Smallest	Largest	Absolute Difference
AI	A	0.8333	0.7447	0.0887
BCI	D	0.1333	0.2500	0.1167
FI	D	0.5938	0.6563	0.0625
PVI	B	0.6992	0.6341	0.0652
SDI	C	0.3984	0.4355	0.0371
WER	D	0.9132	0.8688	0.0444

Table 9: Complexity measures with the maximum difference between sliver resolution methods.

Characterization of Dasymetric Map Complexity

For none of the complexity measures is the full range of possible values observed using the test maps (see Figure 22). This is due to the fact that most of the measures reach maximum complexity only for un-classed maps and minimum complexity for maps with only a single class. Neither of these conditions applies here as this study only examined classed maps with the number of classes ranging from three to seven.

Examining box plots of the inputs for dasymetric maps versus the resulting measures, number of classes as well as the selection of a classification method have an obvious impact on resulting complexity measures (see Figures 23 and 24). Specifically, as the number of classes increases, the complexity of the map increases. This coincides with Mersey's assertion that number of classes and complexity are interchangeable (1990). The Quantile classification method generally results in a more complex map, Equal Interval results in a less complex map and Fisher-Jenks and Standard Deviation fall in the middle. This is somewhat intuitive as using the Quantile method will always result in a map with approximately equal numbers of enumeration units in each class regardless of the nature of the data. Equal Interval results in a map with relatively more enumeration units falling into the lower class since the population data are generally

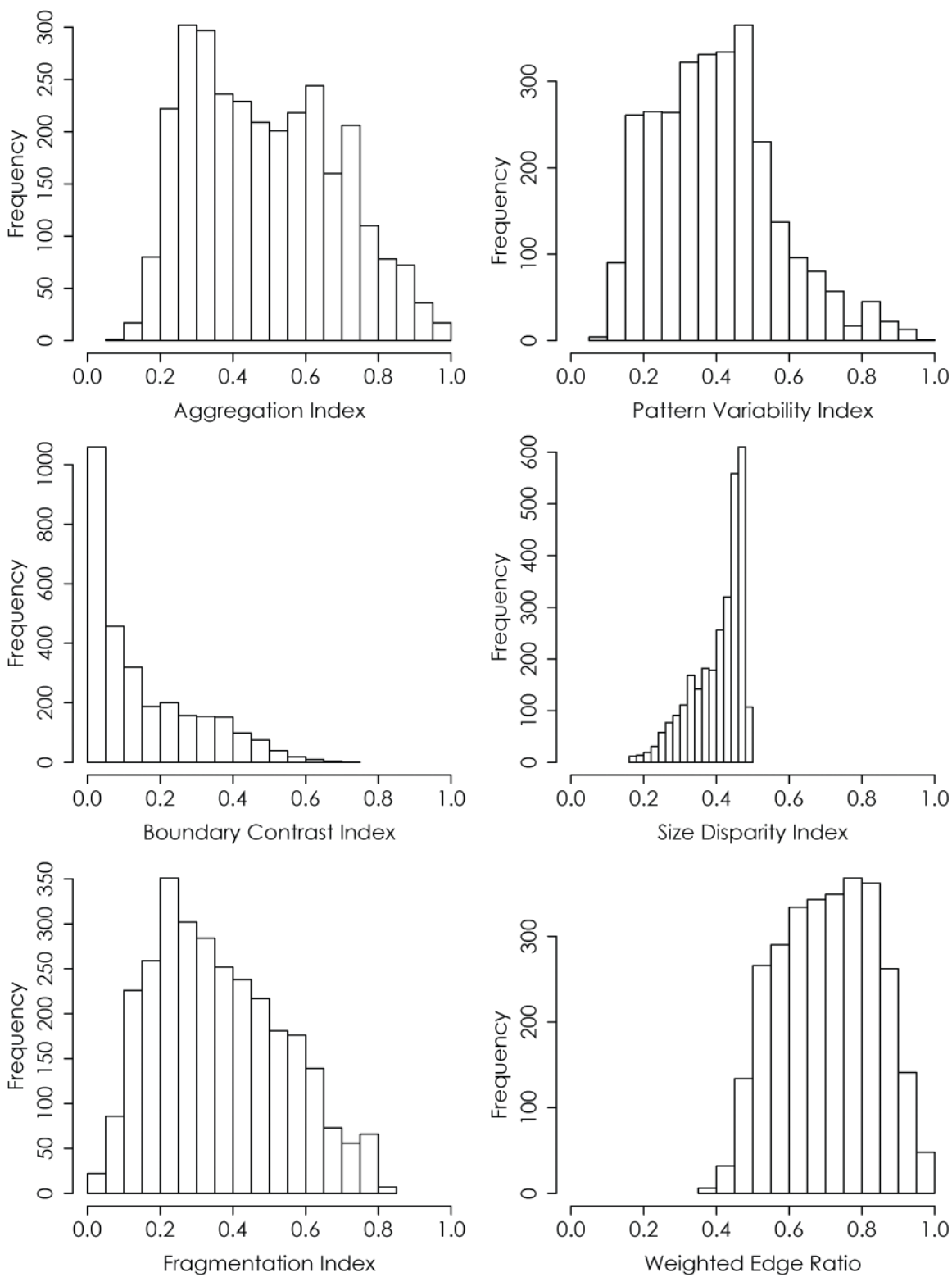


Figure 22: Frequency histograms for complexity measures calculated on dasymmetric maps.

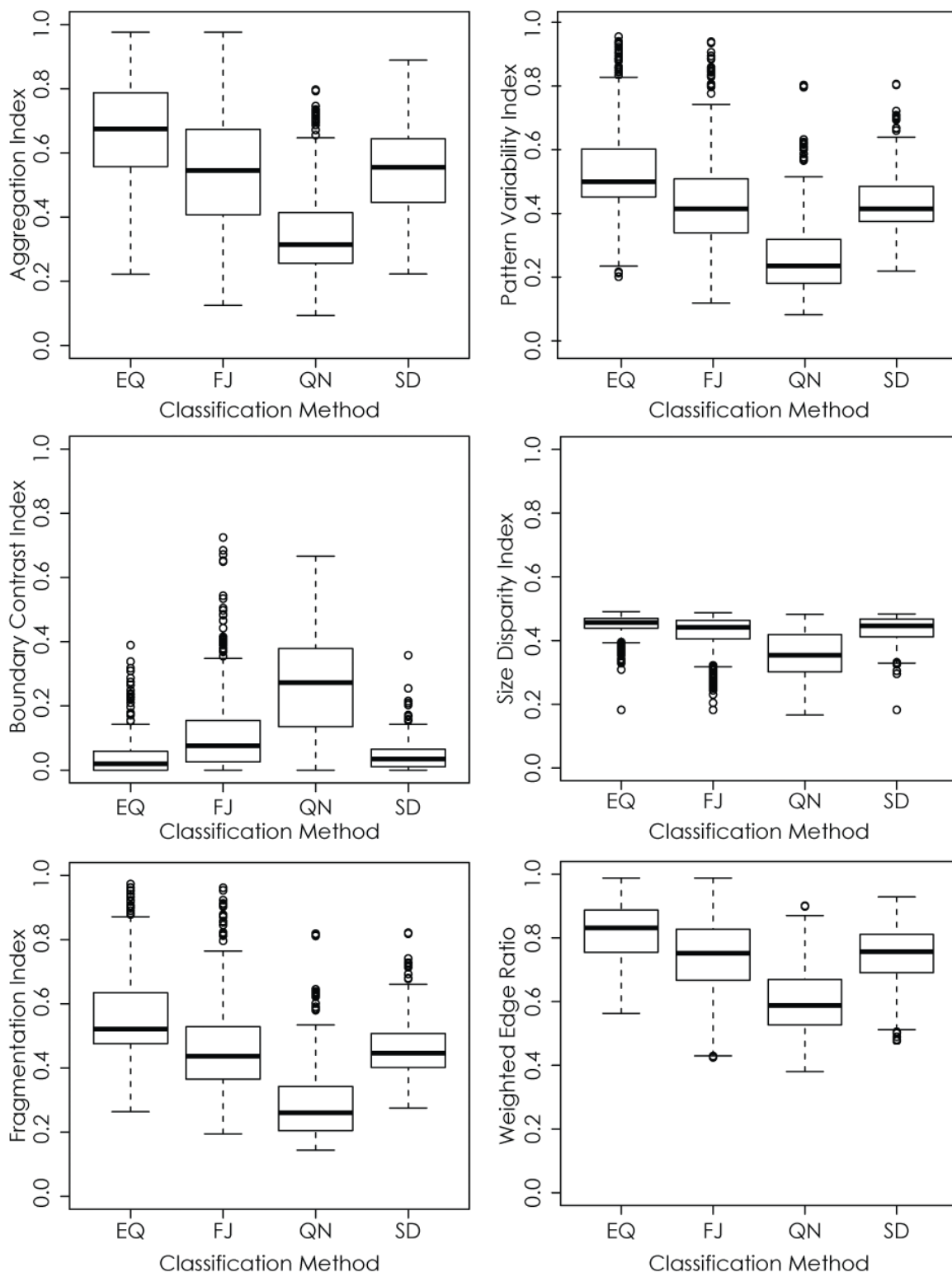


Figure 23: Box plots of complexity measures calculated on dasymetric maps versus classification method.

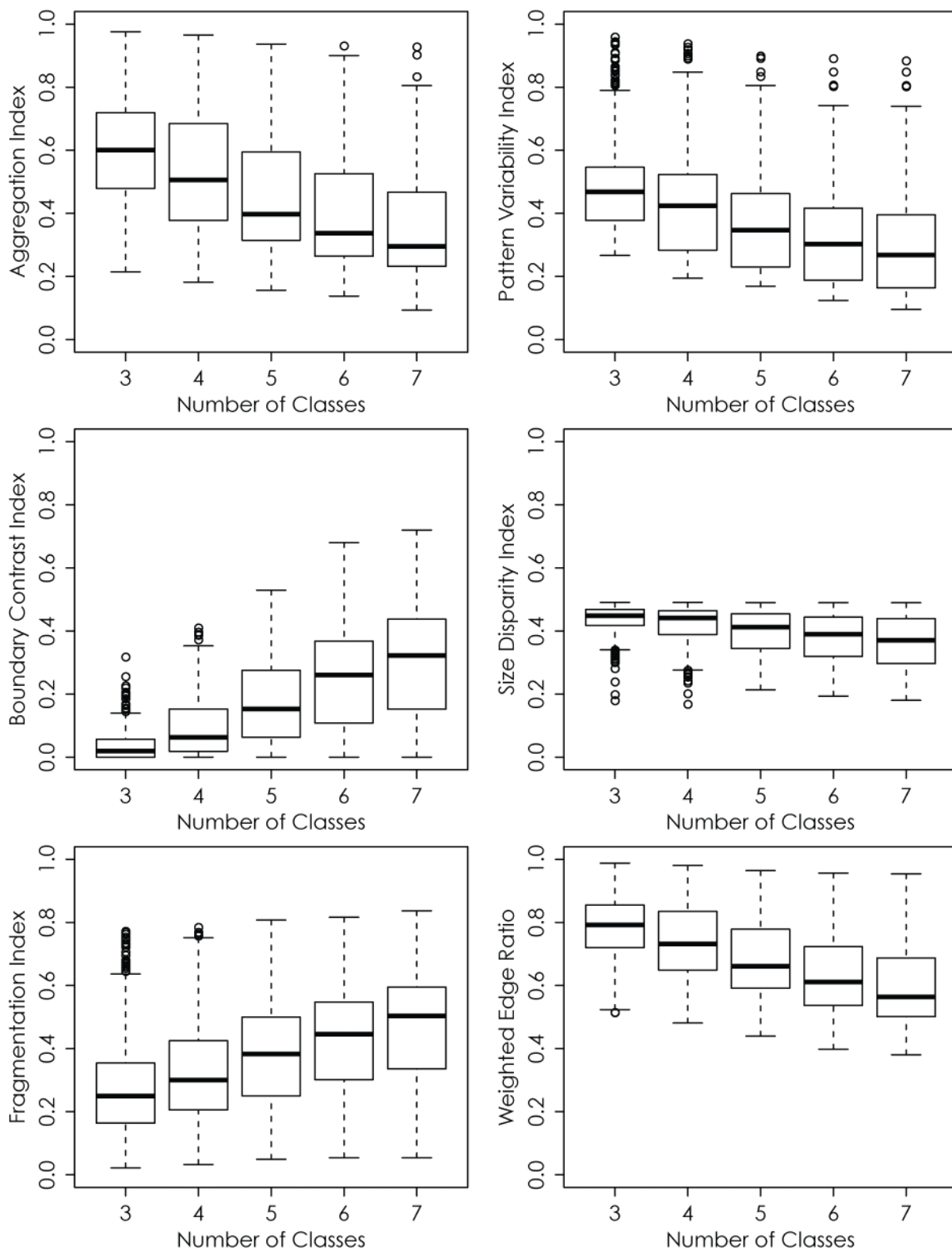


Figure 24: Box plots of complexity measures calculated on dasymmetric maps versus number of classes used.

positively skewed. The type of ancillary data used to create the map does not seem to have an obvious impact on the visual complexity of the resulting map (see Figure B-1).

The relationship between area and map complexity is not very clear. Examining total area, ancillary area, or the ratio of ancillary area to total area does not show any clear relationship (see Figures B-2, B-3, and B-4). Examining how the number of features affects complexity shows more promise (see Figures 25, 26, and 27). Briefly, as the number of ancillary features increases relative to the total number of features, complexity tends to increase.

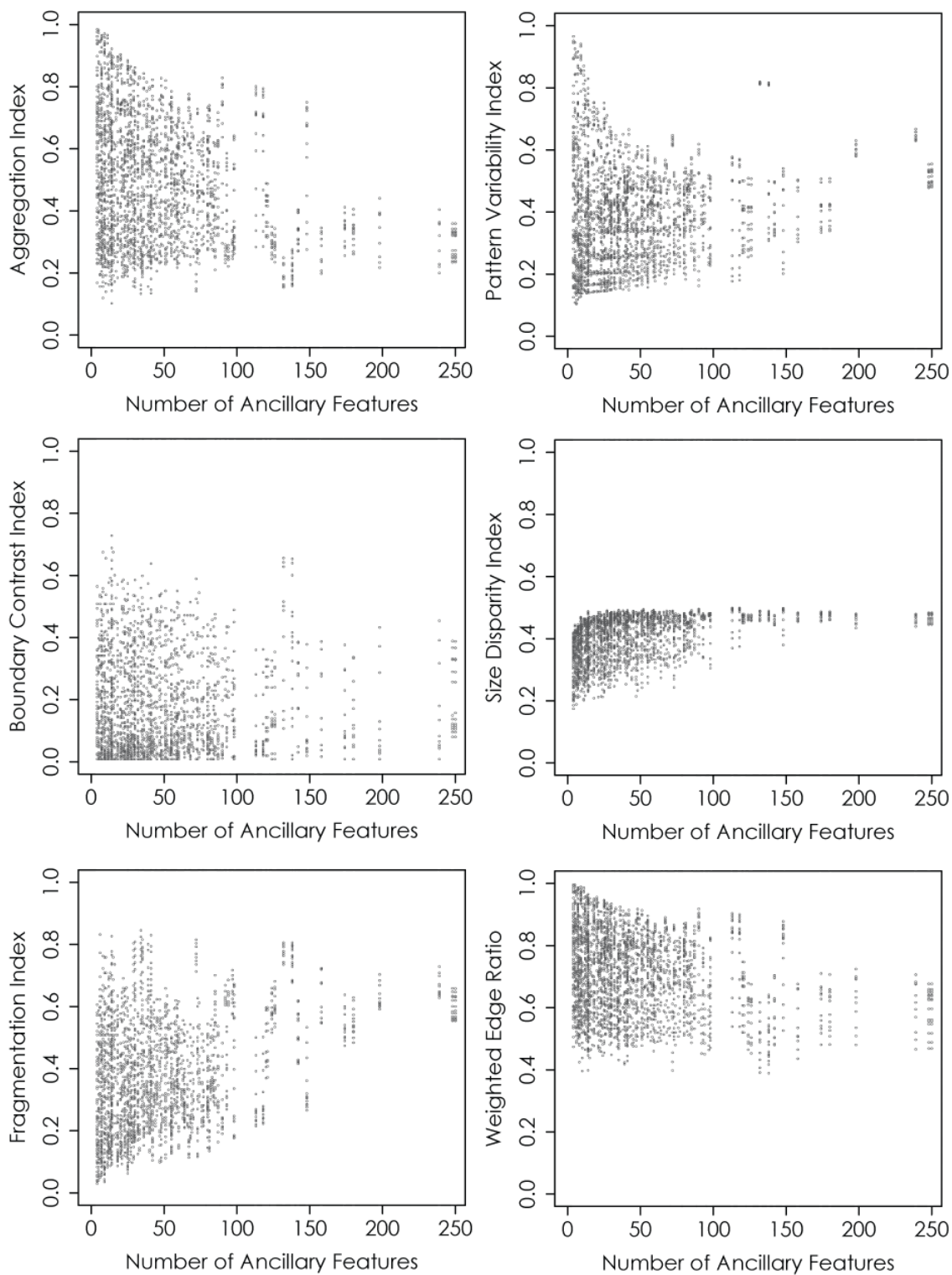


Figure 25: Scatter plots of complexity measures calculated from dasymmetric maps versus number of ancillary features.

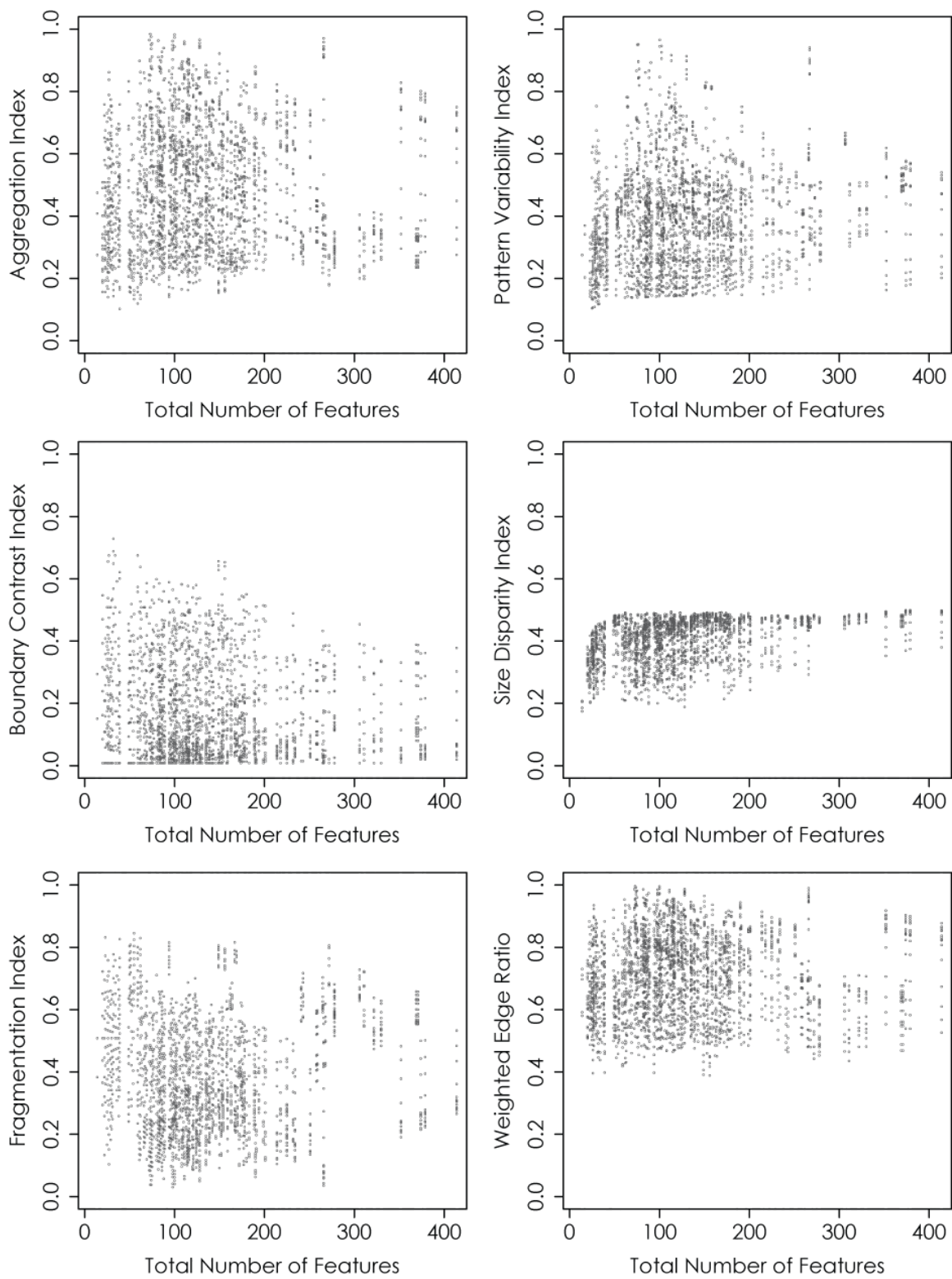


Figure 26: Scatter plots of complexity measures calculated from dasymmetric maps versus number of features.

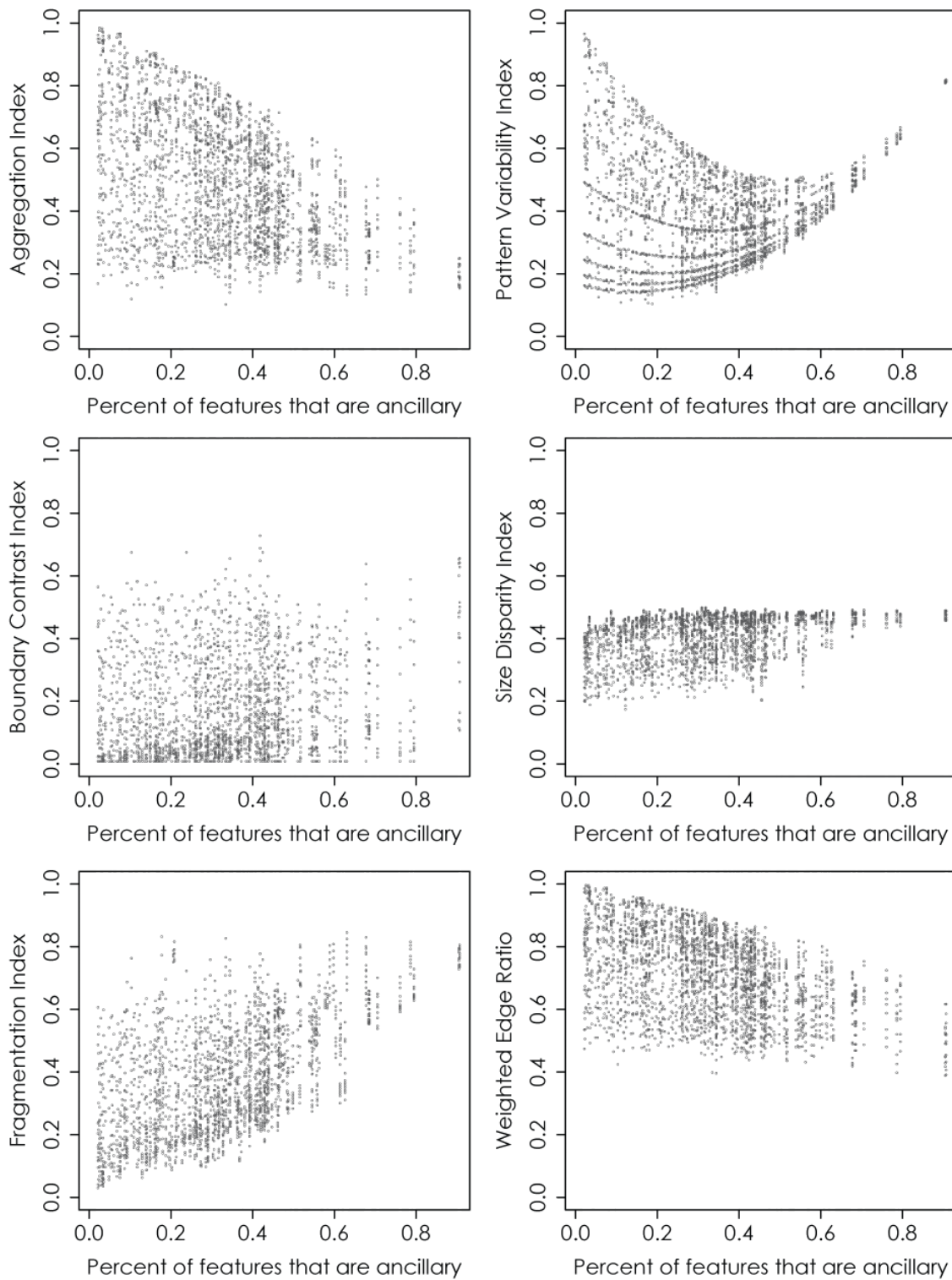


Figure 27: Scatter plots of complexity measures calculated from dasymmetric maps versus the ratio of ancillary features to total number of features.

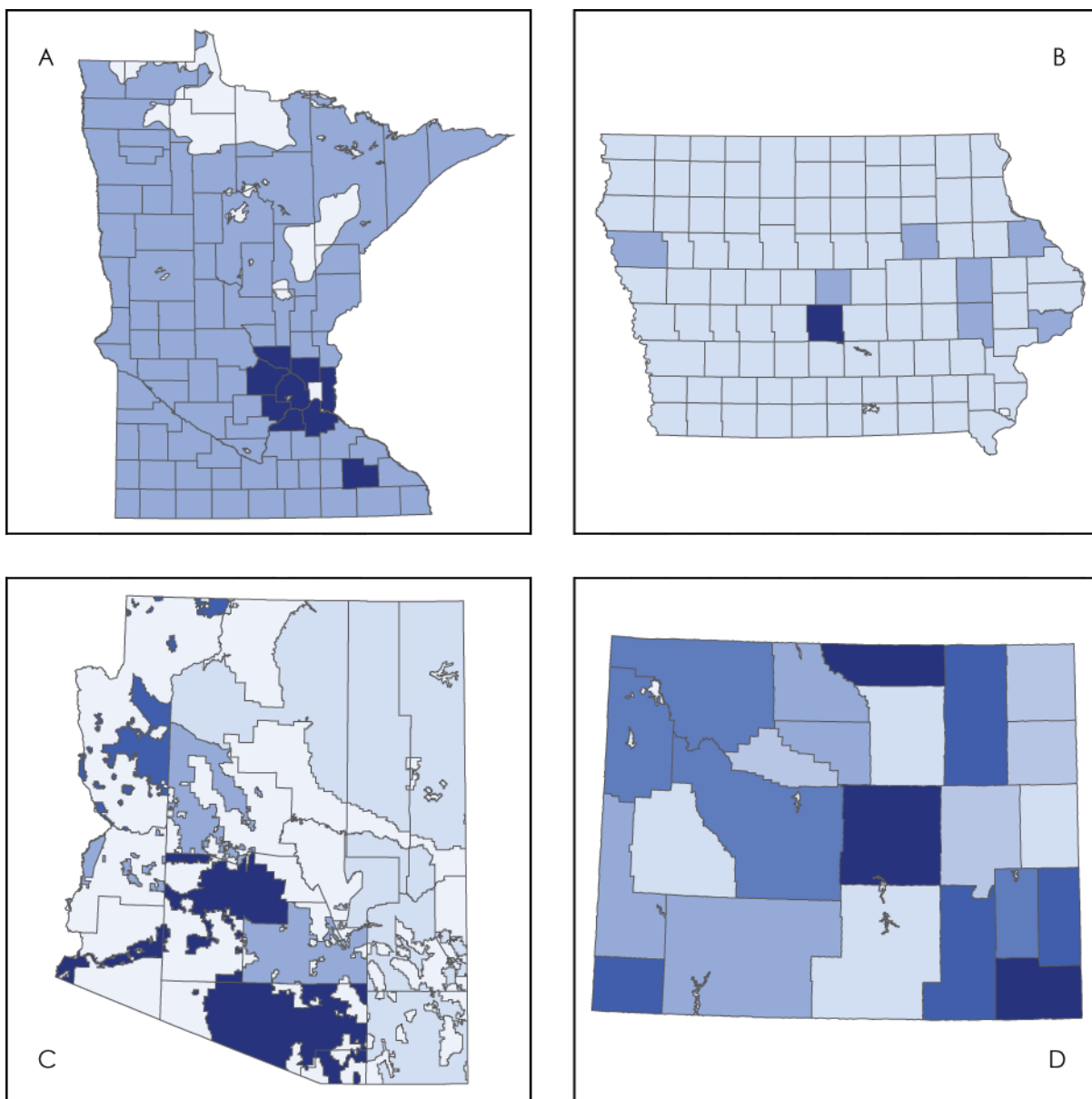


Figure 28: Example dasymetric maps. The resulting complexity measures are found in Table 10.

Map	AI	BCI	FI	PVI	SDI	WER
A	0.714	0.000	0.226	0.466	0.471	0.830
B	0.792	0.032	0.138	0.738	0.453	0.891
C	0.242	0.327	0.633	0.370	0.457	0.505
D	0.094	0.613	0.818	0.157	0.295	0.390

Least complex map according to the given measure
 Most complex map according to the given measure

Table 10: Complexity measures calculated from the maps in Figure 28.

Redundancy of Complexity Measures

As in previous research, there is some redundancy between measures. If all measures capture the same aspect of complexity, the sign of association between each measure would be as seen in Table 11. While the expected signs are seen in the calculation of Spearman's Rho (see Table 12), the scatter plots show that the relationships between measures are not always linear in nature (see Figure 29). In addition, while some measures are strongly associated, they do not create a consistent rank-order of complexity for a given set of maps (see Figure 28 and Table 10).

BCI	-				
FI	-	+			
PVI	+	-	-		
SDI	+	-	-	+	
WER	+	-	-	+	+
	AI	BCI	FI	PVI	SDI

Table 11: Expected signs for the association between complexity measures.

Examining both Table 12 and Figure 29, it is obvious that the two most redundant measures are the AI and WER. As the only difference between the two is the weighting of boundaries in the WER, this is unsurprising. In addition, it appears that measures that are dependant upon the topological structure of the base map (AI, BCI, FI, and WER) are all highly redundant. The SDI is the least redundant measure, with six of the seven lowest values for Spearman's Rho. This is unsurprising as it is unique among the selected measures in that the area of enumeration units is a factor in it's calculation.

Choropleth Versus Dasymetric

To test the relative complexity of choropleth and dasymetric maps, a paired t-test was conducted examining complexity measures from choropleth maps and complexity measures from

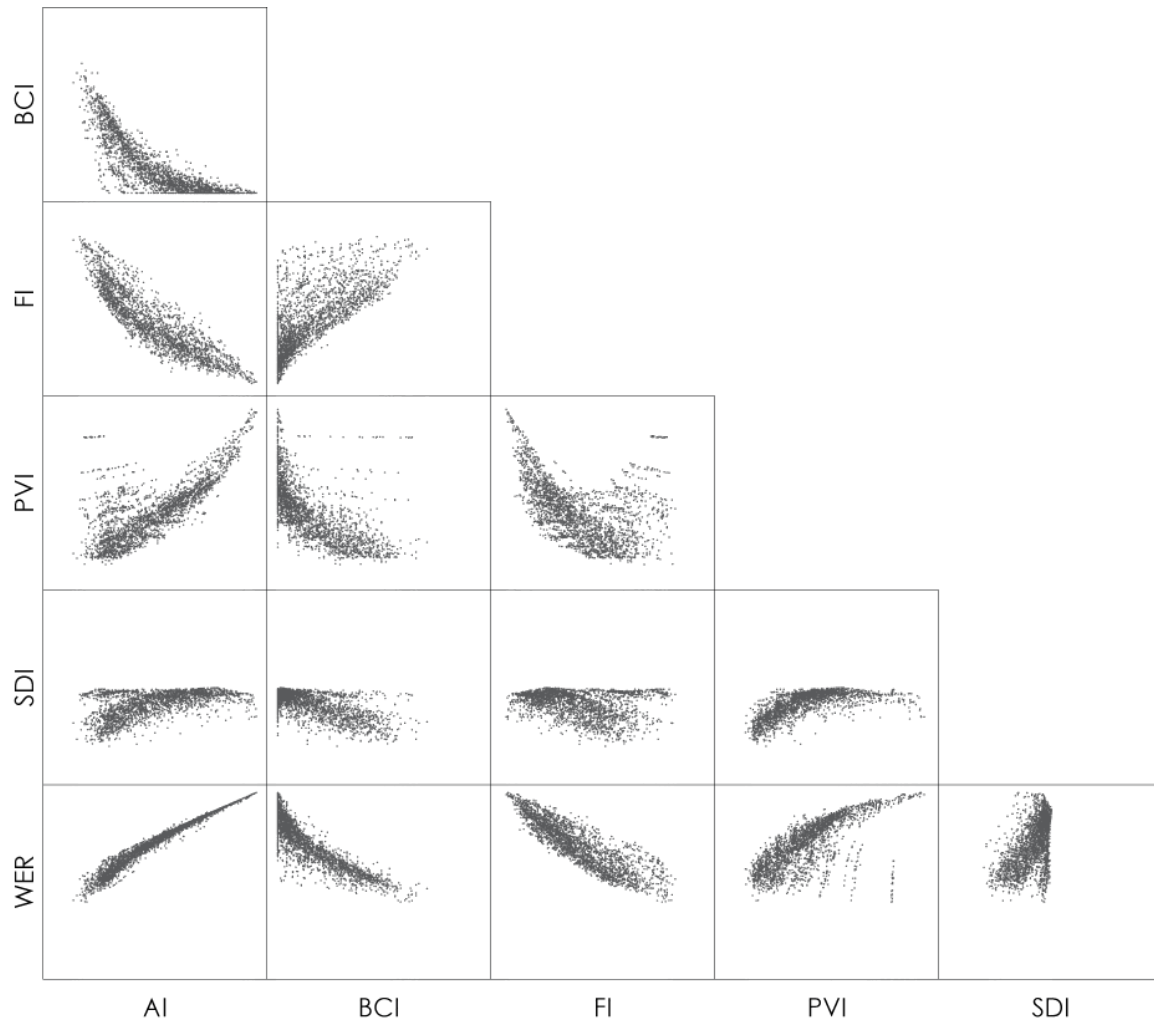


Figure 29: Scatter plot matrix of complexity measures for dasymetric maps.

BCI	-0.837				
FI	-0.912	0.706			
PVI	0.723	-0.753	-0.571		
SDI	0.436	-0.570	-0.198	0.701	
WER	0.986	-0.865	-0.894	0.749	0.466
	AI	BCI	FI	PVI	SDI

Table 12: Spearman's Rho for dasymetric complexity measures.

corresponding dasymetric maps. The hypothesis, for the BCI and FI was as follows:

$$H_O : d \geq 0$$

$$H_A : d < 0$$

For the AI, PVI, SDI, and WER, the hypothesis was:

$$H_O : d \leq 0$$

$$H_A : d > 0$$

Where d is the average difference between dasymetric measure and corresponding choropleth measure. In other words, if dasymetric maps are more complex than choropleth maps as hypothesized, we would expect d to be positive or negative depending on the nature of the complexity measure in question (see Table 13).

	AI	BCI	FI	PVI	SDI	WER
Expected sign	-	+	+	-	-	-
Actual sign	-	-	+	-	+	-

Table 13: Expected and actual signs for the average difference between complexity measures calculated on dasymetric and choropleth maps.

For all complexity measures, the average difference is very unlikely to be zero. The magnitude of the difference ranges from 0.0146 for the WER to 0.0998 for the SDI (see Table 14). While this is certainly a greater difference than that seen for sliver resolution methods, unfortunately it is unknown how large a difference of complexity must be to be detectable by a map reader.

These results show that dasymetric maps are less aggregated and more fragmented than choropleth maps. The results for the WER and PVI, as with the AI and BCI indicate that dasymetric maps are more complex than choropleth maps. The test conducted for the SDI indicates that dasymetric maps are less complex than choropleth maps, but have greater disparity in size of enumeration units. The BCI also indicates that dasymetric maps are less complex than choropleth maps.

	d	t	p-val	95% Confidence Interval	
				lower	upper
AI	-0.0317	-11.376	0.0000	-0.0371	-0.0262
BCI	-0.0464	-28.505	0.0000	-0.0496	-0.0432
FI	0.0622	22.582	0.0000	0.0568	0.0676
PVI	-0.0437	-12.286	0.0000	-0.0507	-0.0367
SDI	0.0998	80.293	0.0000	0.0974	0.1022
WER	-0.0146	-10.007	0.0000	-0.0174	-0.0117

Table 14: Results of t-test examining the difference between choropleth and dasymetric maps.

The scatter plots seen in Figure 30 also show that the relationship between the complexity of choropleth maps and the complexity of corresponding dasymetric maps are not necessarily linear in nature. In addition, not all dasymetric maps are more complex than the corresponding choropleth map. Though it was not specifically examined, it may be that the degree to which the ancillary data modifies the base choropleth map has an impact on whether or not the resulting dasymetric map is more or less complex. The measures of association in Table 15 do indicate that all measures calculated for choropleth maps are positively associated with the measures calculated for dasymetric maps.

	Pearson Spearman	
	Pearson	Spearman
AI	0.834	0.848
BCI	0.906	0.888
FI	0.740	0.751
PVI	0.766	0.818
SDI	0.710	0.680
WER	0.888	0.892

Table 15: Measures of association between choropleth and dasymetric maps for complexity measures.

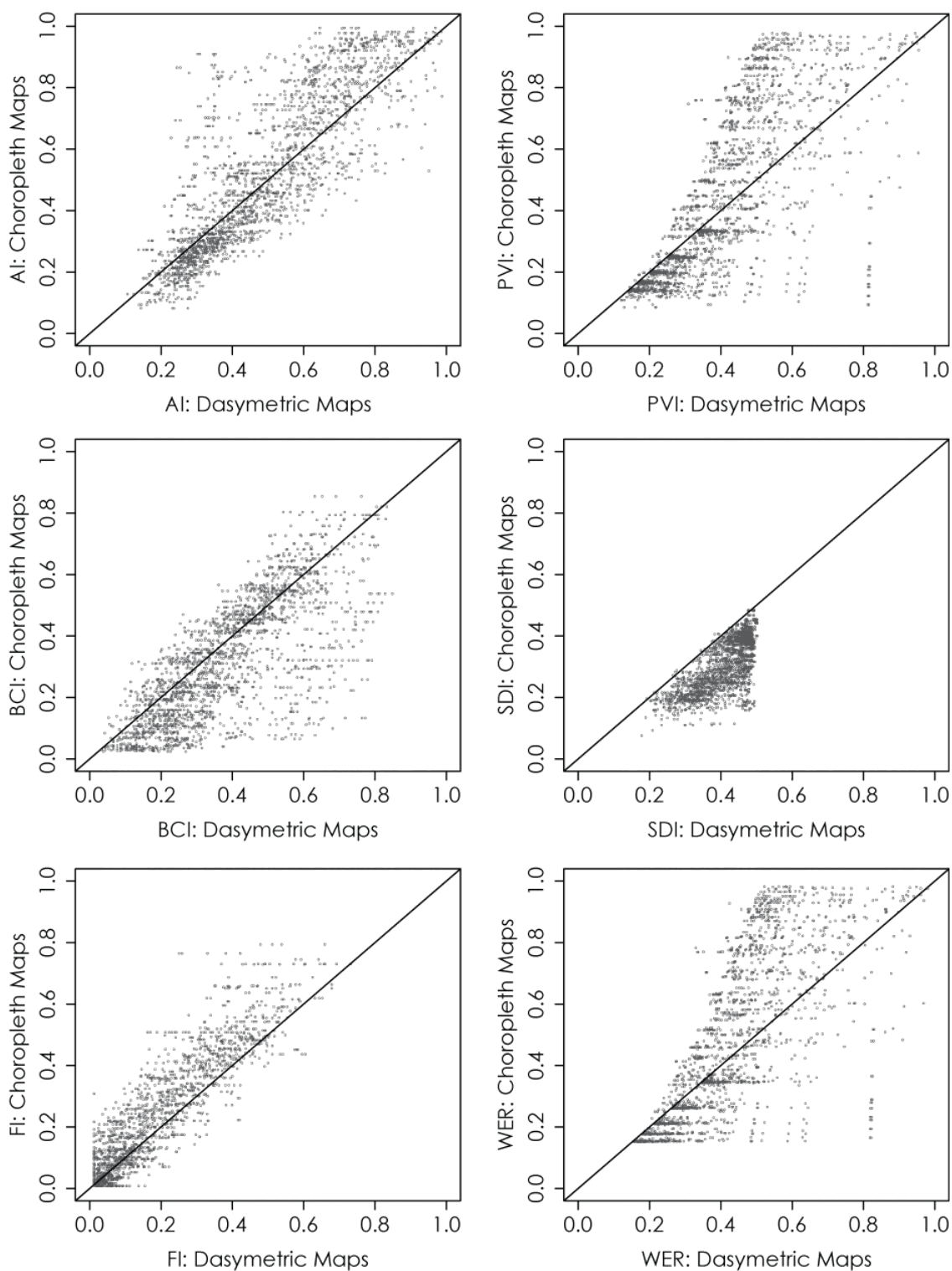


Figure 30: Scatter plots of complexity measures calculated from choropleth maps versus complexity measures calculated from corresponding dasymetric maps.

In examining the trend of complexity measures as the number of categories increases, it becomes clear that the trends are different for choropleth and dasymetric maps (see Figure 31). For the AI, FI, PVI and WER, dasymetric maps with three classes are more complex than choropleth maps with three classes. For dasymetric maps with seven classes, complexity is less than that of choropleth maps. That is to say, the range is compressed for dasymetric maps. Regarding the BCI and the SDI, the rate of change in complexity as the number of categories changes appears to be similar for dasymetric and choropleth maps though again, dasymetric maps are clearly less complex according to this measure.

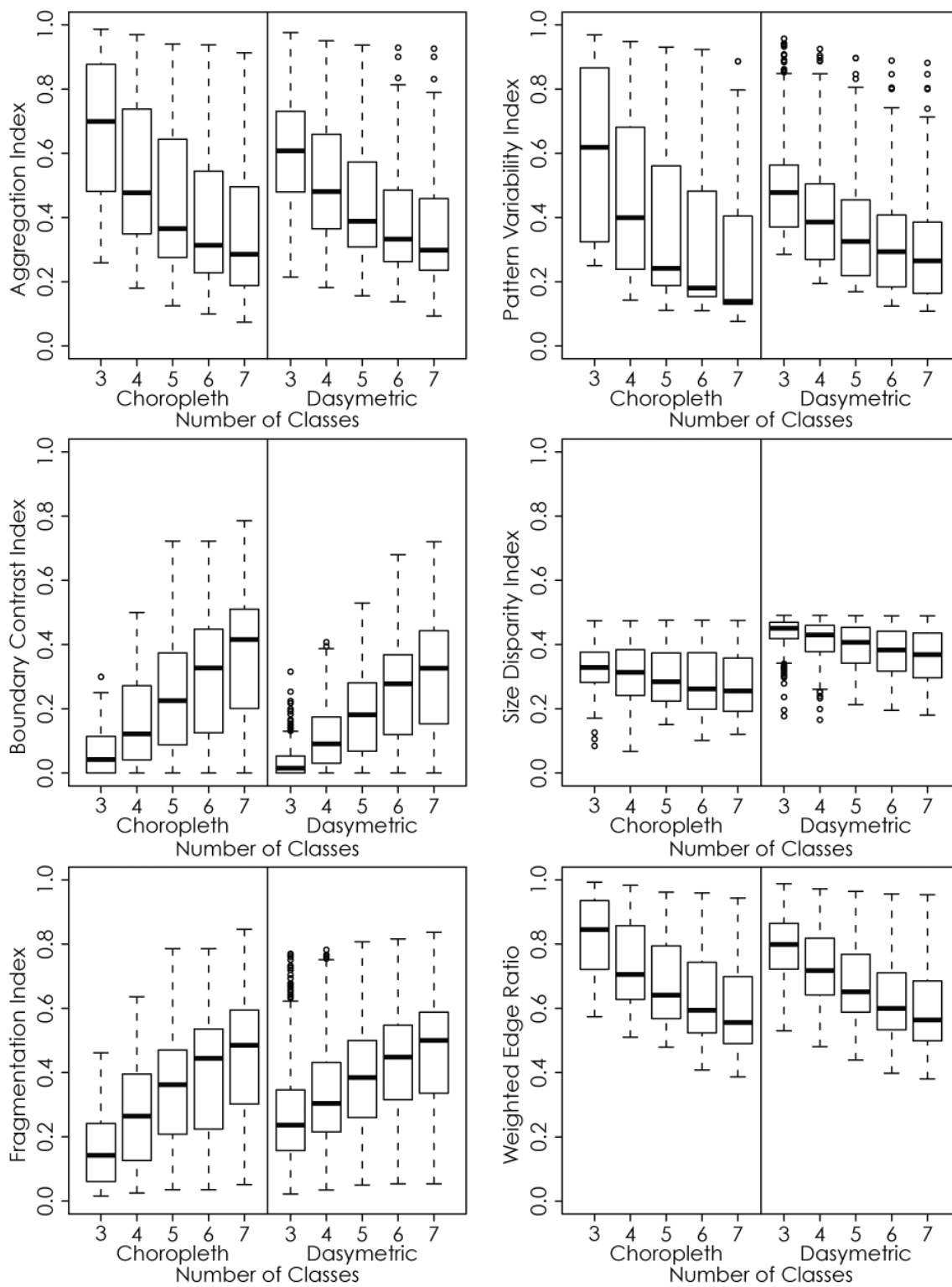


Figure 31: Box plots examining the trend in complexity measures versus number of classes for choropleth and dasymetric maps.

Caveats and Assumptions

The results of this research are not applicable to all dasymetric maps. Obviously, the choice of dataset is relevant in this regard. In addition, certain decisions in the map construction process clearly affect the results. For example, the decision was made to isolate any zero values in their own category. This was done under the assumption that a desirable goal in constructing a binary dasymetric map would be to highlight areas masked by the ancillary data.

In calculating adjacency for complexity measure calculation, it was decided to use the rook contiguity instead of queen contiguity. It has not been determined if map users perceive bishop contiguity as adjacent. Previous research has also used rook contiguity.

The data used for this research spanned the forty-eight contiguous United States. The results should not be assumed to apply to dasymetric maps of this region as islands were deleted and generalization was handled programmatically to enable generation of a large number of sample maps. As generalization is highly subjective, it must be pointed out that results may vary from those seen here if different generalization decisions are used. In particular, the decisions to remove small features prior to construction of the dasymetric maps and the handling of sliver polygons were highly subjective decisions. There are many other methods for sliver resolution and the creation of a map for publication is likely to include other generalization procedures such as line simplification.

One other decision that has had a significant impact on resulting complexity measures is the decision to calculate the measures using the polygons of the base map before instead of after dissolving boundaries between polygons falling into the same category with the exception of the SDI. This decision was made on the basis that it may be desirable to show county boundaries to allow the user some point of familiar reference. It could also be argued that, since a dasymetric

map modifies the underlying base map; such boundaries should be dissolved to emphasize the more accurate spatial distribution. The difference between these two options is quite drastic. As was noted previously, the calculation of a complexity measure for a dasymetric map is dependent upon the display method used. For example, the AI will always be 0 and the FI and BCI will be 1 (indicating maximum complexity) if calculated on a dasymetric map with all boundaries dissolved or if calculated on an un-classed map. The numerator for the AI will always be zero in this case as there will not be any neighboring polygons in the same category. A dasymetric map constructed in this manner will always attain the maximum value for the FI as the number of apparent faces will be the same as the number of actual faces. The numerator for the BCI will likewise be equal to the maximum possible number of trivalent differences. For the SDI, if calculated for a map with all boundaries retained, it will be a constant regardless of the classification method and number of categories used for the final display.

Lastly, the nature of the underlying data associated with the enumeration units may have influenced the results. The data used were highly skewed (see Figure 32). Data that follow some other distribution may give different results.

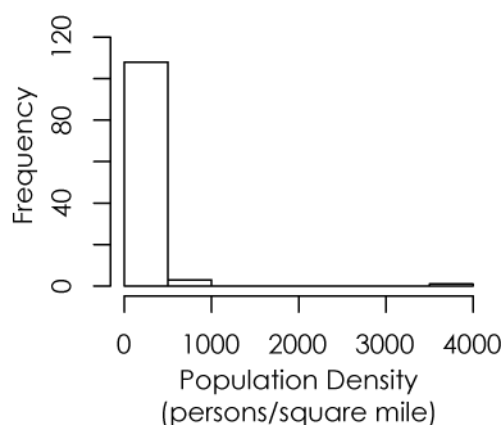


Figure 32: Typical distribution of the population density data (From a dasymetric map of Wisconsin with federal land as ancillary data).

Discussion

It is clear that there are many factors involved in the complexity of dasymetric maps. As with choropleth maps, one of the most influential factors is the number of classes used in the construction of the map. The classification algorithm used also has an impact. As these factors also have a significant impact on the complexity of choropleth maps, this is unsurprising. The relationship between map area and map complexity is unclear as is the relationship between ancillary area and map complexity. More useful to understanding dasymetric map complexity is examining the impact of the number of ancillary features and overall number of features. As the number of ancillary features increases, the range of complexity values seen appears to shift to be more complex. Examining the ratio of ancillary features to the overall number of features shows that this increase in complexity is more pronounced as the number of ancillary features increases relative to the overall number of features.

For the PVI, an increase in complexity is indicative of a classification becoming more similar to the Quantile method. That is to say that dividing all polygons equally among the number of classes will result in maximum complexity according to the PVI (see Figure 34). Examining the change in PVI compared to the number of features for maps classed using the Quantile method, we would expect to see something like Figure 33. Instead, for dasymetric maps, we see the pattern shown in Figure 26. This pattern is due to the fact that, for this study, zero-valued polygons were always isolated in their own class. As the number of zero-valued polygons approaches the number of polygons in other classes, the PVI will approach the minimum for the given number of features and classes. For this reason, we see the parabolic pattern in Figure 27.

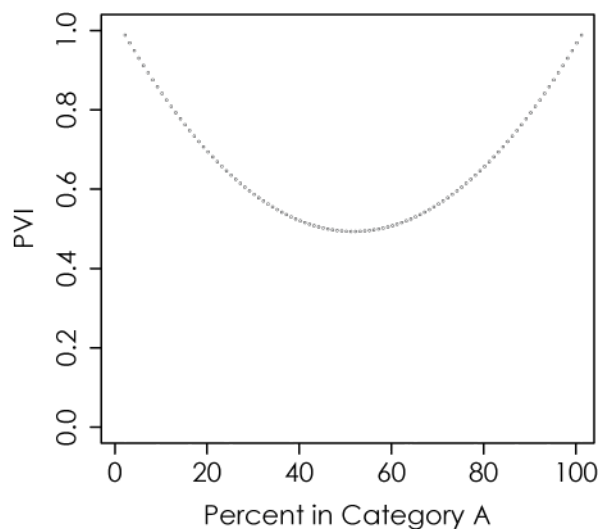


Figure 33: The behavior of the PVI as the proportion of polygons divided between two classes varies. Maximum complexity occurs when the polygons are split evenly between the two classes.

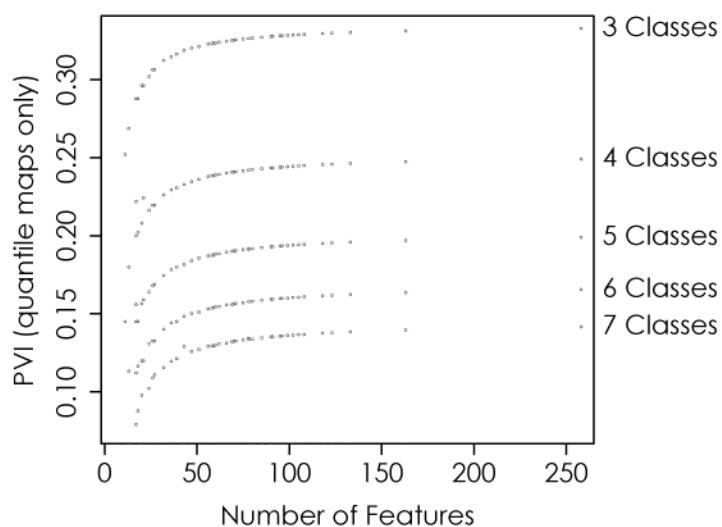


Figure 34: The behavior of the PVI for choropleth maps classed using the Quantile algorithm

For the remaining measures, the decrease in complexity is likely due to the selected display method. Recall that zero-valued polygons, created by ancillary data, were isolated in their own class. As the percentage of features belonging to a single category increases, it becomes more likely that neighboring polygons belong to the same class and less likely that mutually adjacent

trios of polygons belong to distinct categories. It follows that this equates to a decrease in complexity.

The results of this research show that, as with choropleth maps, some existing complexity measures are redundant. The AI, FI, and WER are the most redundant of the calculated measures. This is unsurprising as the AI and WER differ only in that the WER weights the magnitude of the difference between category values. The FI attempts to measure the opposite of the AI; that is fragmentation instead of aggregation. In other words, it is not surprising that the two measures are inversely related.

Still, the strength of association between the SDI and other measures was relatively weak. Given Monmonier's assertion that complexity is comprised of "pattern fragmentation and autocorrelation, size inequality of map regions, and the general spatial trend and its autocorrelation or strength," it seems wise to use a combination of complexity measures to assess the complexity of a map (1974). In particular, a measure such as the FI or AI combined with the SDI will likely give a good overall feel for the complexity of a given map.

Not all complexity measures used indicated that dasymetric maps are less complex than choropleth maps, specifically, the BCI and SDI. Recall that a value closer to 0.5 for the SDI means that polygons are more similar in area and a value closer to zero indicates dissimilarity in area and an increased visual complexity. For this study, the SDI was calculated on maps after dissolving boundaries between adjacent polygons of the same category value. The decreased average complexity indicated by the SDI then, indicates that the regions defined by category values in dasymetric maps do not vary as much those in choropleth maps.

A decrease in complexity as measured by the BCI is due to one of three scenarios. First, T can decrease as $\max(T)$ remains the same. Second, if $\max(T)$ decreases, the BCI decreases if T

decreases sufficiently. Third, if $\max(T)$ increases, T must decrease or increase sufficiently little for the BCI to insure that the BCI decreases (see Figure 35). Since dasymetric maps are less complex than choropleth maps on average according to the BCI, one of these three situations must be true on average. Figure 36 clearly shows that $\max(T)$ has increased moving from choropleth to dasymetric. This means that the number of sets of three mutually adjacent polygons increases due to the use of the dasymetric method. So, since the BCI indicates a decrease in complexity for dasymetric maps, the number of trivalent differences has not increased sufficiently to indicate increased complexity.

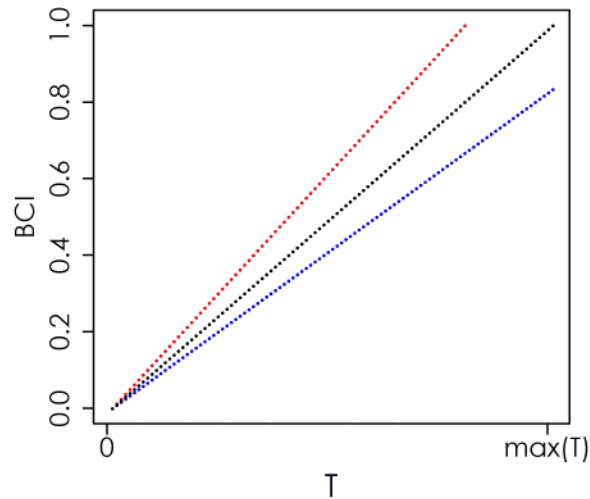


Figure 35: The behavior of the BCI as T varies from 0 to $\max(T)$ in black. In blue is an increase in $\max(T)$, in red, a decrease in $\max(T)$.

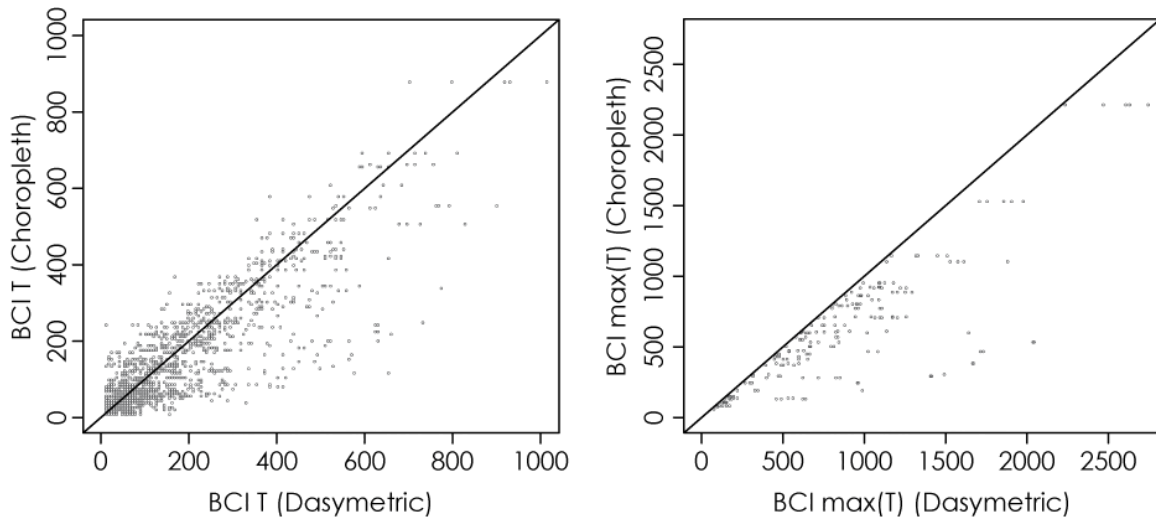


Figure 34: Scatter plots comparing the values of T and $\max(T)$ (the numerator and denominator of the BCI) in choropleth and dasymetric maps.

The AI and WER indicated that dasymetric maps are more complex than choropleth maps. Figures 36 and 37 show that there is an increase in the number of edges in a dasymetric map relative to a choropleth map. For the AI and WER to decrease (indicating an increase in complexity), the number of those edges that belong to the same category (in the case of the AI) or similar categories (for the WER) must decrease relative to the overall number of edges. This aspect of both measures is captured by the numerator for each. Figures 36 and 37 show that there is an increase in the numerator for both measures, so it must be assumed that this increase is not large enough relative to the total number of edges. In other words, there are fewer instances of neighboring enumeration units being placed into the same category value in dasymetric maps.

The FI increases in complexity if fewer adjacent polygons belong to the same category relative to the total number of polygons. Since a dasymetric map increases the number of features and the FI indicates that dasymetric maps are more complex than choropleth maps, the percentage of

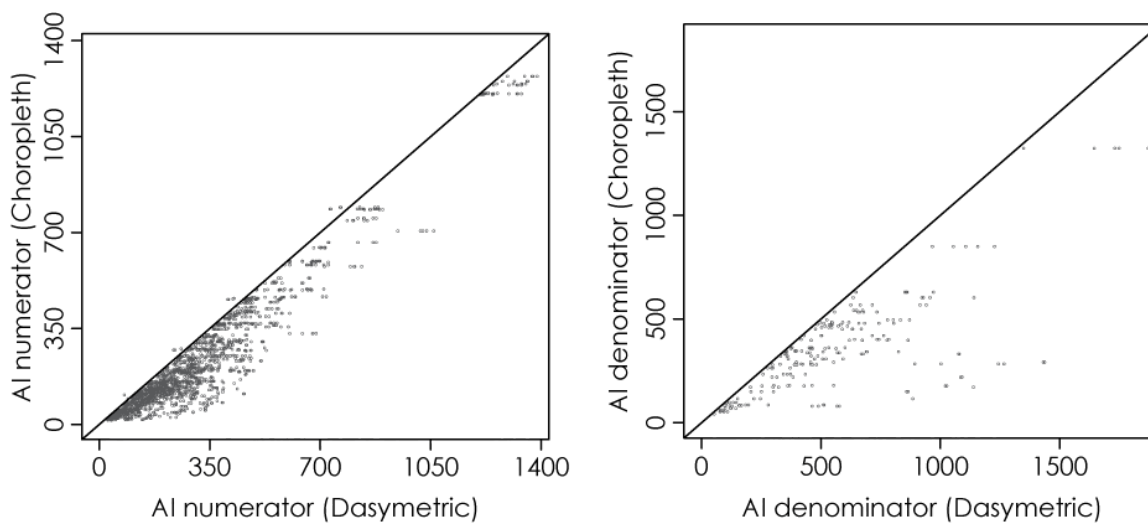


Figure 36: Scatter plots comparing the values of the numerator and denominator of the AI in choropleth and dasymetric maps.

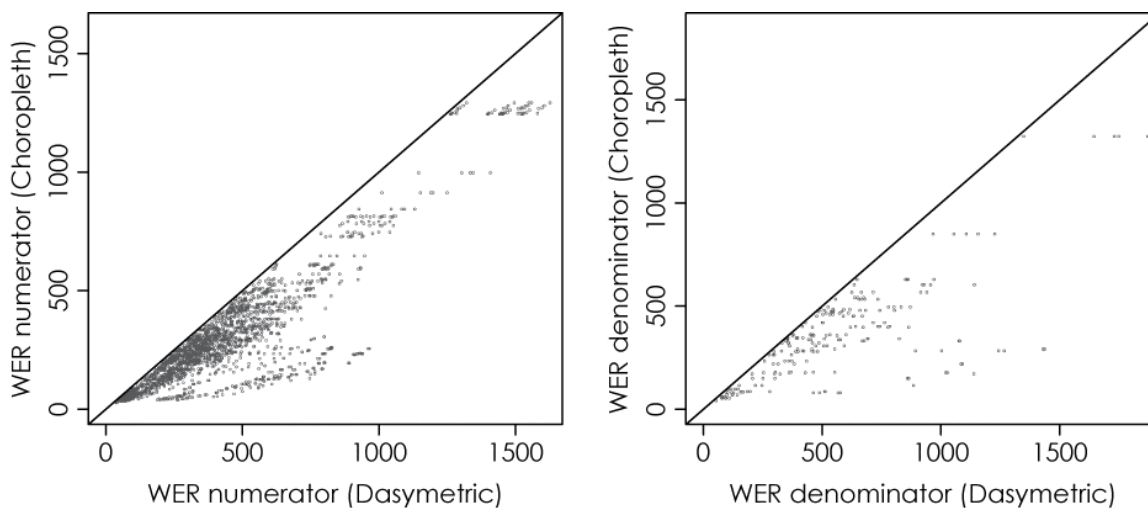


Figure 37: Scatter plots comparing the values of the numerator and denominator of the WER in choropleth and dasymetric maps.

adjacent enumeration units in a dasymetric map that fall into the same category must decrease (see Figure 38).

As mentioned before, as the PVI decreases (indicating greater complexity), the number of polygons in each category begins to mimic the results of the Quantile classification method.

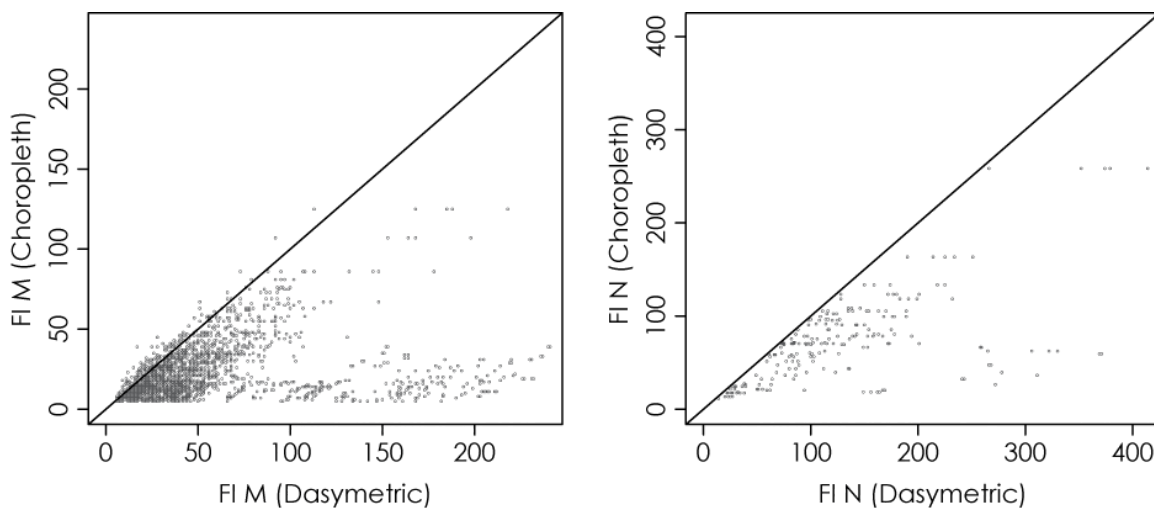


Figure 38: Scatter plots comparing the values of M and N (the numerator and denominator of the FI) in choropleth and dasymetric maps.

Another way to look at this is to examine how the numerator and denominator of the PVI change going from choropleth to dasymetric. Figure 39 shows that both the numerator and the denominator (approximated by the number of features) of the PVI increase for dasymetric maps relative to choropleth maps. This means that while dasymetric maps increase the number of

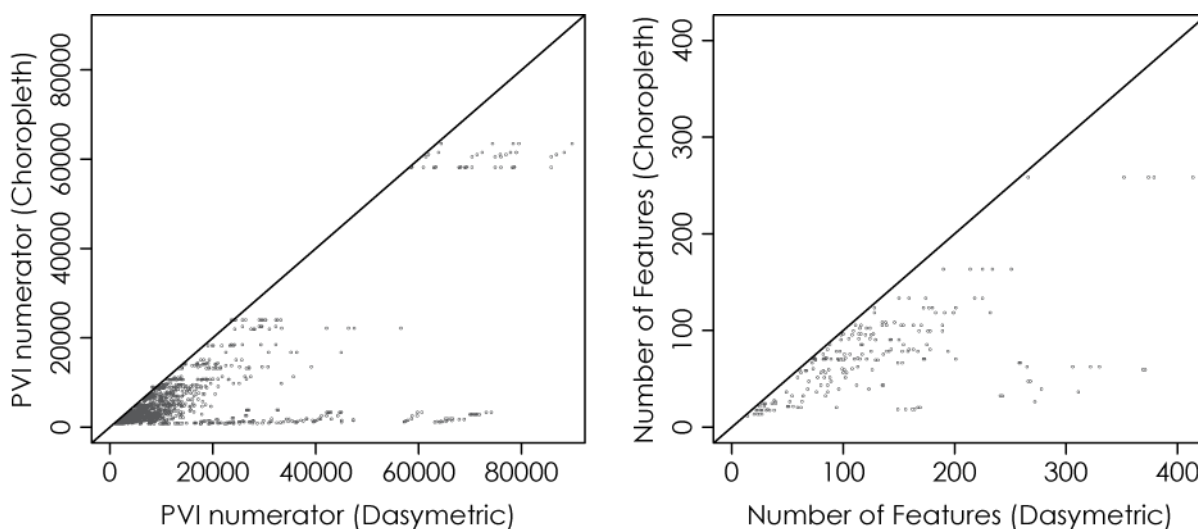


Figure 39: Scatter plots comparing the values of the numerator of the PVI and the number of features (approximating the denominator of the PVI) in choropleth and dasymetric maps.

features relative to choropleth maps, category values of the polygons in a dasymetric map are spread more evenly among classes.

Conclusion

The advantages of the dasymetric map over the more common choropleth map make it tempting to always select this technique. This research indicates, however, that dasymetric maps are more visually complex than choropleth maps. As the significance of the magnitude of this difference is unknown, the reader is advised to apply sound judgment in cartographic theory rather than avoid dasymetric maps altogether. In particular, the proper selection of a classification algorithm and number of classes will go a long way in communicating the desired message of the map.

Future Directions

Perhaps the most important step to take in the future would be to confirm the results of this research with subjective user testing. In particular, it would be useful to determine the minimum perceptible complexity difference in order to assess the functional significance of the differences between choropleth and dasymetric maps found here.

Given that there are many techniques for construction of dasymetric maps, future research should also examine the visual complexity of other techniques. Likewise, there are different display methods and these should be examined as well. In addition, it is important to examine complexity measures in the context of data with distributions other than that commonly found in population density.

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APPENDIX A: CODE

Generation of adjacency data

```

# Import dependencies
import arcgisscripting
import os
# Create the Geoprocessor object
gp = arcgisscripting.create()
# Load required toolboxes...
gp.AddToolbox("C:/Program Files (x86)/ArcGIS/ArcToolbox/Toolboxes/Data Management
Tools.tbx")
def genGAL(fileName):
    """Generates a gal object (adjacency information) and writes to file"""
    if gp.Exists(fileName):
        #Generate a feature layer object to store selections
        layer = 'layer'
        if not gp.Exists(layer):
            gp.MakeFeatureLayer_management(fileName,layer,',','FID FID
VISIBLE')
        #Cycle through all features and collect adjacency data
        shpCursor = gp.SearchCursor(fileName)
        currFeature = shpCursor.Next()
        adjList = []
        while currFeature:
            #Select a feature
            currFID = currFeature.FID
            gp.SelectLayerByAttribute_management(layer, "NEW_SELECTION", "\"FID\"
= " + str(currFID))
            #Select features adjacent to features in layer and store in layer
            gp.SelectLayerByLocation_management(layer,"SHARE_A_LINE_SEGMENT_WITH", layer, "",
"NEW_SELECTION")
            #Use a search cursor to find the FIDs of adjacent features
            sc = gp.SearchCursor(layer)
            row = sc.Next()
            temp = []
            while row:
                if row.FID != currFID:
                    temp.append(row.FID)
                    row = sc.Next()
            adjList.append(temp)
            currFeature = shpCursor.Next()
        #Write out the adjacency information in .gal format
        outPath = fileName.replace('.shp','.gal')
        outFile = open(outPath,'wt')
        #Write the header
        outFile.writelines(str(len(adjList)) + '\n')
        for i in xrange(0,len(adjList)):
            #Write the current FID and number of adjacent features
            outFile.writelines(str(i) + ' ' + str(len(adjList[i])) + '\n')
            #Generate and write the adjacency information
            fidList = ''
            for fid in adjList[i]:
                fidList = fidList + str(fid) + ' '

```

```
        fidList.rstrip(' ')
        fidList = fidList + '\n'
        outFile.writelines(fidList)
#Memory management
gp.Delete_management(layer)
outFile.close()
layer = None
shpFile = None
shpCursor = None
```

Miscellaneous Functions

```

def getUnique(data):
    """Returns a list of the unique values in a list object"""
    uniqueValues = []
    for eachValue in data:
        if eachValue not in uniqueValues:
            uniqueValues.append(eachValue)
    return uniqueValues

def getCatRegion(gal,categories,initialID):
    """Finds all adjacent polygons in the same category as
    the polygon with ID initialID and groups them into a region"""
    regionCat = categories[int(initialID)]
    region = []
    region.append(initialID)
    added = True
    while added:
        for ID in region:
            addedID = False
            connectedIDs = gal[str(ID)]
            for each in connectedIDs:
                if each not in region:
                    if categories[int(each)] == regionCat:
                        region.append(each)
                        addedID = True
            if not addedID:
                added = False
    return region

def getCatRegions(gal,cats):
    """Groups all polygons into regions based on adjacency and category"""
    unusedIDs = []
    for i in xrange(0,gal.n):
        unusedIDs.append(str(i))
    regions = []
    while len(unusedIDs) > 0:
        initialID = unusedIDs[0]
        tempRegion = getCatRegion(gal,cats,initialID)
        for ID in tempRegion:
            unusedIDs.remove(ID)
        regions.append(tempRegion)
    return regions

```

Aggregation Index

```

def getAI(gal,cats):
    """Returns a value AI which is the Aggregation Index
    AI = (number of adjacent polys in the same category)/(number of adjacent
    polys)"""
    n = gal.n
    numerator = 0
    denominator = 0
    for i in xrange(0,n):
        #Find polygons adjacent to the current polygon
        currID = str(i)
        currCat = cats[i]
        connectedIDs = gal[currID]
        #Determine the number of adjacent polys with the same category
        #as the current polygon
        for ID in connectedIDs:
            denominator += 1
            if cats[int(ID)] == currCat:
                numerator += 1
    #Calculate the AI
    if denominator != 0:
        AI = float(numerator)/denominator
    else:
        AI = -1
    return AI

```

Boundary Contrast Index

```
def getBCI(gal,cats):
    """returns a value BCI which is the Boundary Contrast Index
    BCI = (number of trivalent differences)/(number of mutually adjacent
    trios)"""
    T = 0
    Tmax = 0
    n = gal.n
    for i in xrange(0,n):
        iConnected = gal[str(i)]
        for j in xrange(0,n):
            jConnected = gal[str(j)]
            for k in xrange(0,n):
                kConnected = gal[str(k)]
                #Check for mutual adjacency
                if str(k) in iConnected and\
                   str(k) in jConnected and\
                   str(j) in iConnected:
                    Tmax += 1
                #Check for three unique categories
                if cats[i] != cats[j] and\
                   cats[i] != cats[k] and\
                   cats[j] != cats[k]:
                    T += 1
    #Calculate the BCI
    if Tmax != 0:
        BCI = float(T)/Tmax
    else:
        BCI = -1
    return BCI
```

Fragmentation Index

```
def getFI(gal,cats,N):
    """Returns a value FI which is the Fragmentation Index
    FI = ((number of polygons after dissolving boundaries) - 1)/((number of
    enumeration units) - 1)"""
    #Determine the number of polygons after dissolving boundaries
    regions = pysalUtilities.getCatRegions(gal,cats)
    M = len(regions)
    #Calculate the FI
    if (N-1) != 0:
        FI = float(M-1)/(N-1)
    else:
        FI = -1
    return FI
```

Pattern Variability Index

```
def getPVI(cats):
    """Returns a value PVI which is the Pattern Variability Index
    PVI =  $P(C_i == C_j)$  where  $C_i$  is the category of poly  $i$  and  $C_j$  is the
    category of poly  $j$ """
    n = len(cats)

    #Count the number of i,j pairs with the same category
    count = 0
    i = 0
    while i < n:
        j = 0
        while j < n:
            if cats[i] == cats[j] and i != j:
                count += 1
            j += 1
        i += 1

    #Calculate the PVI
    if n*n-n != 0:
        PVI = float(count)/(n*n - n)
    else:
        PVI = -1
    return PVI
```

Size Disparity Index

```

def getSDI(gal,cats,areas):
    """Returns a value SDI which is the Size Disparity Index
    SDI is the area between y = x and the Cumulative Proportional
    Area Vector (CPAV)"""
    #Find the areas of regions after dissolving boundaries
    #between polygons with the same category
    regions = pysalUtilities.getCatRegions(gal,cats)
    regionAreas = []
    for region in regions:
        area = 0
        for ID in region:
            area += areas[int(ID)]
        regionAreas.append(area)

    #Calculate the proportional areas
    totalArea = sum(areas)
    pAreas = []
    for area in regionAreas:
        pAreas.append(float(area)/totalArea)
    pAreas.sort()

    #Calculate the CPAV
    curr = 0
    cpav = []
    for i in xrange(0,len(pAreas)):
        curr += pAreas[i]
        cpav.append(curr)
    integral = 0
    if len(pAreas) != 0:
        deltaX = 1./len(pAreas)
    else:
        return -1

    #Calculate the integral of the CPAV
    lastHeight = 0
    for height in cpav:
        deltaY = height - lastHeight
        #Triangle Area
        area1 = 0.5 * deltaX * deltaY
        #Rectangle Area
        area2 = deltaX * lastHeight
        integral += (area1 + area2)
        lastHeight = height

    #Calculate the SDI
    SDI = 0.5 - integral
    return SDI

```

Weighted Edge Ratio

```
def getWER(gal, cats):
    """Returns a value WER which is the Weighted Edge Ratio
    WER = SUM[(number of edges of a given weight)*(given weight)]/(number of
    edges)"""
    n = gal.n
    numerator = 0
    #Find all possible i,j pairs for categories
    dList = []
    nCats = len(getUnique(cats))
    for i in xrange(0, nCats):
        for j in xrange(0, nCats):
            dList.append(abs(i-j))
    dList = getUnique(dList)

    #Count the number of edges for each possible i,j pair
    denominator = 0
    for d in dList:
        Nij = 0
        for f in xrange(0, n):
            currCat = cats[f]
            connectedIDs = gal[str(f)]
            for ID in connectedIDs:
                otherCat = cats[int(ID)]
                if abs(otherCat - currCat) == d:
                    Nij += 1
        #Calculate the weight of the current i,j pair
        weight = 1./(d + 1)
        numerator += (float(Nij) * weight)
        denominator += Nij

    #Calculate the WER
    if denominator != 0:
        WER = float(numerator)/denominator
    else:
        WER = -1
    return WER
```

APPENDIX B: GRAPHICS

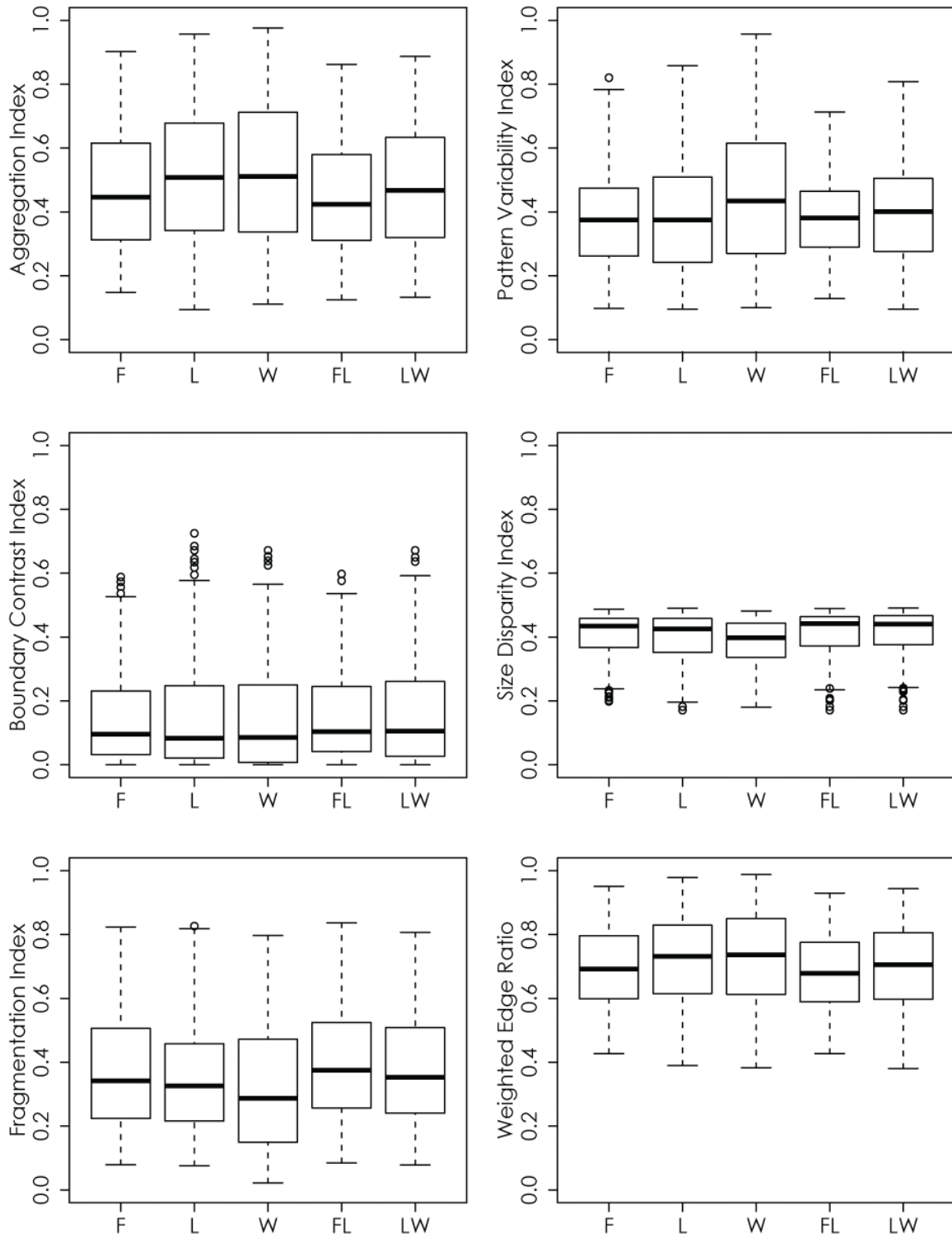


Figure B-1: Box plots of complexity measures calculated on dasymetric maps versus ancillary data used.

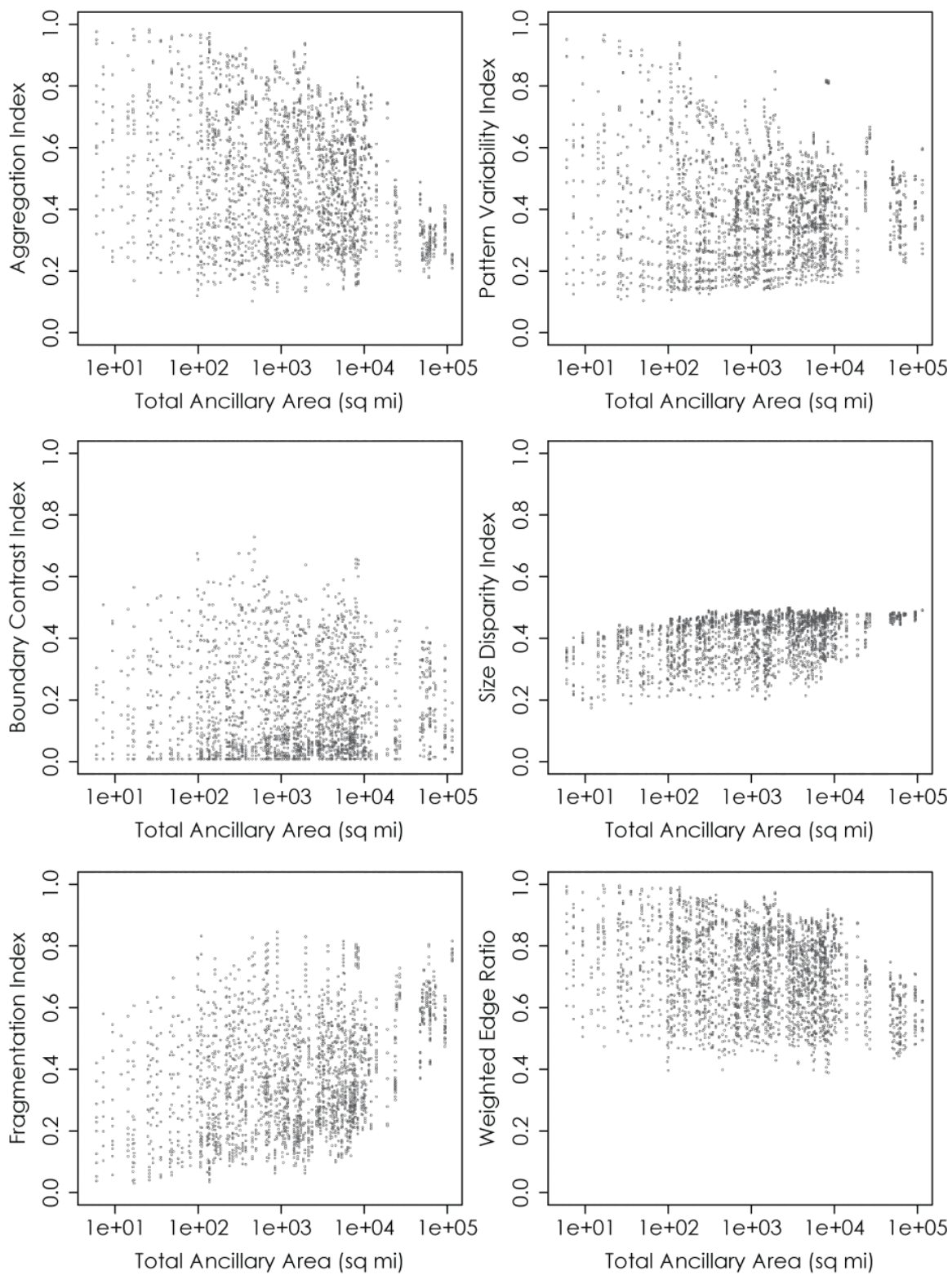


Figure B-2: Scatter plots of complexity measures calculated from dasymetric maps versus total ancillary area.

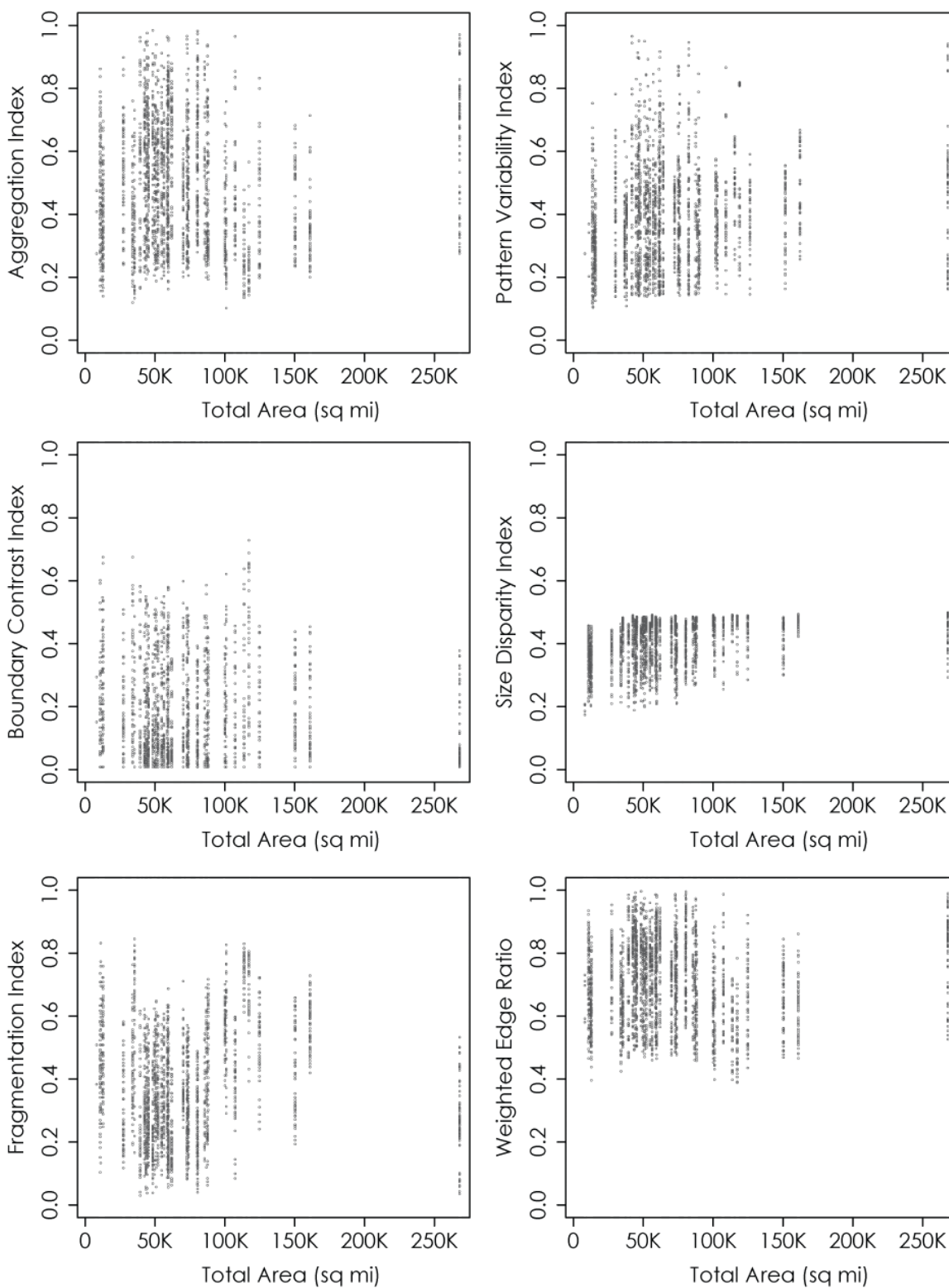


Figure B-3: Scatter plots of complexity measures calculated from dasymmetric maps versus total map area.

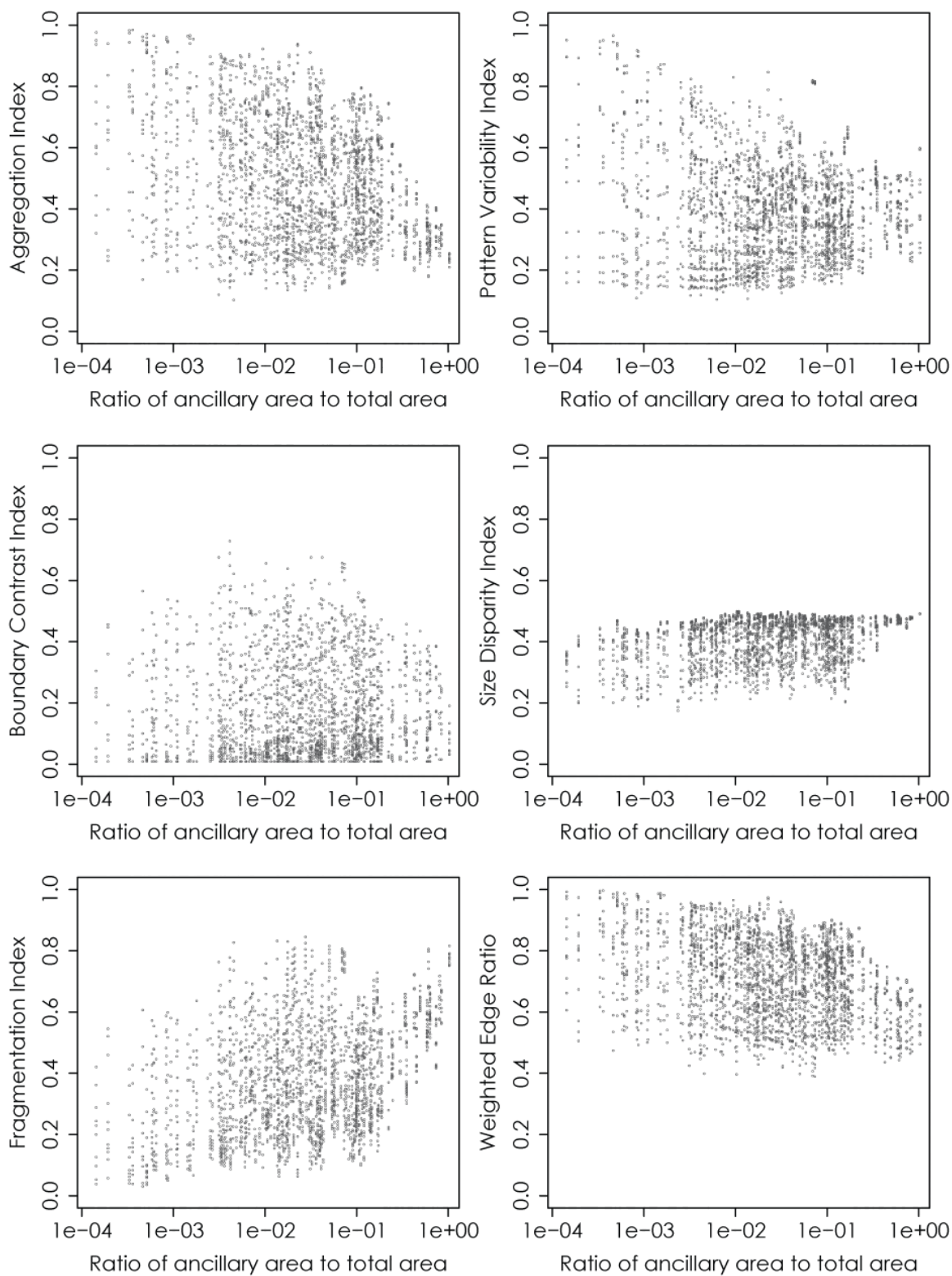


Figure B-4: Scatter plots of complexity measures calculated from dasymetric maps versus the ratio of ancillary area to total map area.