THERMAL RESISTIVITY DRY-OUT CURVES FOR THIRTEEN SANDY SOILS

By

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EXECUTIVE SUMMARY

The objective of this study was to identify which physical properties impact the thermal resistivity dry-out curve (TRDC) of natural sandy soils. The TRDC is a relationship between soil thermal resistivity and degree of wetness (e.g., volumetric water content, gravimetric water content, or degree of saturation). TRDCs of 13 sandy soils were investigated using modified hanging column tests. The tests were also used to investigate co-effects of the soil-water characteristic curve (SWCC), which represents the hydraulic properties of unsaturated soil. The physical properties evaluated in this research included: (1) degree of saturation, (2) soil particle size ($D_{10}$ and $D_{50}$), (3) fines content, (4) soil type, (5) soil density ($\gamma_{\text{dmax}}, e_{\text{max}}$, and $e_{\text{min}}$) and gradation ($C_u$), (6) quartz content, and (7) particle shape (sphericity and roundness). In the TRDCs, three analysis points—thermal resistivity ($\rho$) at the fully dried condition, critical degree of saturation, and fully saturated condition—were selected for analysis. Correlations between the three points of interest on the TRDC and the physical properties were supported with high-resolution images obtained by synchrotron X-ray computed tomography (CT) and statistical analysis, including, ANOVA and stepwise regression. Results included the significant effects of the measured soil physical parameters on the TRDC in addition to the well-recognized parameter of degree of saturation as reported in the literature.

Impacts of degree of saturation on the TRDC were related to thermal resistivity of three phases of soil systems ($\rho_{\text{solid}}, \rho_{\text{liquid}},$ and $\rho_{\text{air}}$). Due to high $\rho_{\text{air}}$, the highest $\rho_{\text{soil}}$ was observed at the fully dried condition. As degree of saturation increases, thin films and liquid bridges among soil particles are formed, resulting in a rapid decrease in thermal resistivity. Near the critical degree of saturation of a TRDC, which is located near the knee point of the SWCC where moisture exists as adsorbed films (McQueen and Miller, 1977), changes in $\rho_{\text{soil}}$ are more rapid. After liquid bridges form, $\rho_{\text{soil}}$ decreases gradually.
as degree of saturation increases. The lowest $\rho_{\text{soil}}$ was measured at the saturated condition.

Soil thermal resistivity also decreased with increase in particle size as evaluated by $D_{10}$ and $D_{50}$. This was primarily related to size and thermal resistivity of the solid phase. In high-resolution images, for example, larger solid particles provide larger heat transfer paths, while smaller solid particles (e.g., silt-sized) consist of smaller heat transfer paths with a more tortuous void structure. Consequently, $\rho_{\text{soil}}$ was affected by particle size as related to the thermal resistivity of the solid and void phases in addition to the tortuosity of the matrix.

In contrast, $\rho_{\text{soil}}$ increased with increasing fines content. Reasons for the effect of fines were similar to those of particle size. At a constant void ratio, soil that included higher fines content, such as SM soils, had relatively small solid particles with tortuous voids compared with SP or SW soils, which do not include significant fines content. Smaller solid particles and tortuous voids led to a decrease in $\rho_{\text{soil}}$.

In the modified hanging column test and ANOVA, thermal resistivity values of the 13 sandy soils were unaffected by the type of sandy soil regardless of the point of comparison. Parallel with laboratory tests, statistical analyses indicate that slight differences among soil types are not statistically significant regardless of the point of interest evaluated. At the dried, critical, and saturated condition, statistical significance by soil types per ANOVA were 0.061, 0.174, and 0.268. Therefore, a larger database of soil that represents the full spectrum of gravel, silt, and clay is required to fully investigate the effect of soil type on the TRDC.

The effect of soil density on TRDC was analyzed using four density parameters: (a) maximum dry unit weight ($\gamma_{\text{dmax}}$), (b) minimum void ratio ($e_{\text{min}}$), (c) maximum void ratio ($e_{\text{max}}$), and (d) coefficient of uniformity ($C_u$). Soil thermal resistivity decreased as $\gamma_{\text{dmax}}$ and
$C_u$ increased, while soil thermal resistivity increased with increasing $e_{\text{min}}$ and $e_{\text{max}}$. In other words, $\rho_{\text{soil}}$ decreased as soil density increased because of closer particle contacts and a reduction of the volume of air. On the other hand, $\rho_{\text{soil}}$ increased with increase in $e_{\text{min}}$ and $e_{\text{max}}$, which represent a decrease in soil density, due to less particle contacts and greater air volume.

Because quartz is amongst the best heat conducting minerals that is common in natural soils (Winterkorn, 1962), influence of quartz content on the TRDC was analyzed. Soil thermal resistivity at each of the points of comparison (dry, critical, and saturated) decreased as quartz content increased.

Effect of particle shape on TRDC was analyzed based on sphericity and roundness of particles. Higher thermal resistivity was measured for prismoidal particle shapes as compared to spherical particle shapes. Soil packing with prismoidal particle shape included more voids than soil packing with spherical particle shape. Roundness of the 13 specimens ranged between 1.08 (well-rounded shape) and 1.13 (rounded shape). Soil thermal resistivity increased slightly with increasing roundness because moisture adsorption is enhanced when particle shape changes from a well-rounded shape to a rounded shape (Likos and Jaafar, 2013).

In stepwise regression, $D_{10}$ was the only significant factor in terms of $\rho_{\text{sat}}$, and $D_{50}$ was only significant factor in terms of $\rho_{\text{dry}}$. The regression model for $\rho_{\text{sat}}$ and $\rho_{\text{dry}}$ resulted in $R^2$ of 0.245 (24.5%) and 0.519 (51.9%), respectively. To investigate further statistical significance among the physical properties, a greater database of measurements for the TRDCs of sandy soils would be required.

In comparing thermal resistivity with parameters from the van Genuchten (VG) model, thermal resistivity increased slightly with the $\alpha$ and $n$ parameters, both of which are indicative of the shape of SWCC. However, one of the correlations—dry thermal
resistivity to the $\alpha$ parameter—decreased with the two SWCC parameters. Because of similar physical properties, however, increases in thermal resistivity were small. Therefore, additional tests with an expanded database of soils—including gravel, sand, silt, and clay—is recommended to more fully investigate the correlation between thermal resistivity and the VG parameters.

Among the three points of analysis on the TRDC of the 13 sandy soils, $\rho_{\text{soil}}$ at the fully dried condition was most affected by soil physical properties; to be specific, the dry thermal resistivity values ranged from about 150 °C·cm/W to about 330 °C·cm/W. Heat transfer in unsaturated soil systems directly depends on the matrix of solid particles and air voids, with large differences in resulting thermal resistivity. In contrast, thermal resistivity of the 13 specimens in terms of the physical properties changed only slightly at fully and partially saturated conditions ($\rho_{\text{soil}}$ ranged from about 40 °C·cm/W to about 80 °C·cm/W). These findings indicate that degree of saturation, particularly dry of the critical saturation, is the most significant factor for thermal resistivity of sandy soils with similar physical properties. A larger range of soil types with varying gravel content and percentage of coarse- and fine-sized sand is required to fully investigate the effect of soil physical properties on the TRDC at partially and fully saturated conditions.
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1. INTRODUCTION

The thermal conductivity of a soil (or, the thermal resistivity, the inverse of the thermal conductivity) plays a fundamental role in heat transfer. Soil thermal conductivity \( \lambda_{\text{soil}} \) and soil thermal resistivity \( \rho_{\text{soil}} \) are selectively applied for varying applications, depending on historical analogs. Electrical engineers historically use resistance terms due to analogies to electrical resistance in circuits. Emerging applications, such as collectors at wind energy sites, thermally active geo-structures, and heat exchange elements, use \( \rho_{\text{soil}} \) as a parameter for the design. For example, soil that has higher \( \rho_{\text{soil}} \) can be used as a thermal insulation. Backfill soil around hot water pipes in a cold region requires higher \( \rho_{\text{soil}} \) to effectively retain heat. Soil with lower \( \rho_{\text{soil}} \), on the other hand, can be used as a material to dissipate heat efficiently. For example, soil used as backfill in trenches with buried high-voltage power cables at wind energy sites is specified with lower \( \rho_{\text{soil}} \) to prevent cable overheating and to allow for economical cable selection.

The importance of using soil with suitable thermal properties is broadly recognized. Historical and ongoing research on soil thermal properties has been driven by theoretical approaches and emerging experimental methods (e.g., Donazzi et al., 1979; Dayan and Merbaum, 1984; Fan et al., 2007; Woodward and Tinjum, 2012). Farouki (1981) provides a comprehensive review of soil thermal properties and summarizes the major factors that influence heat transfer processes in soil, including an overview of experimental and modeling methods available for quantifying the thermal behavior of soil and the thermal resistivity dry-out curve (TRDC), which is the relationship between thermal resistivity and degree of saturation. To obtain single-point \( \rho_{\text{soil}} \) or \( \lambda_{\text{soil}} \) measurements, procedures that use transient thermal probe methods are described in ASTM International Test Standard ASTM
D5334-08 and Institute of Electrical and Electronics Test Standard IEEE 442-1981. Several laboratory testing methods have been developed for application along drying and wetting paths, although there is no internationally recognized specific standardized approach yet for measuring the TRDC. Reasonable experimental approaches include the multiple-specimen method (Campbell, 2011), the staged-drying method (Woodward and Tinjum, 2012; Woodward et al., 2013), and the instrumented Tempe cell or hanging column methods (e.g., Smits et al., 2010; Likos et al., 2012). Woodward and Tinjum (2012) and Likos et al. (2012) examined influences of inherent properties to the TRDC measurement, including gravity-induced moisture migration, drying temperature, drying time, sensor location, sensor orientation, and sample heterogeneity. Because of the complexity and expense in experimental determination of the TRDC, a variety of approaches for modeling or estimating the TRDC have been proposed, most of which are based on empirical correlation to more easily measured soil properties (e.g., Farouki, 1981; Campbell, 1985; Côté and Konrad, 2005). Computed tomography (CT) scanning techniques including synchrotron X-ray computed tomography have been also been proposed to precisely investigate the influence of soil physical properties—the type of soil based on mineral content and particle size, content and migration of moisture, and tortuosity and radius of curvature of water among particles—at a micro-scale on $\rho_{\text{soil}}$. CT is a process that uses X-ray energy to produce three-dimensional representations of the soil matrix. Soil thermal properties have thus been developed and extended with theoretical approaches, experimental data, modeling, and new micro-technique, such as X-ray computed tomography.

Despite this historical development, there are few comprehensive experimental studies available that systematically evaluate the influences of soil physical properties, such as grain shape and distribution, on the general behavior of the TRDC for coarse-grained soil.
Moreover, many studies employ a limited sample set and/or use artificial/manufactured soil. Therefore, the purpose of this study was to identify the influences of soil physical properties using 13 well-characterized sandy soils, as well as fundamental material properties including grain surface roundness and sphericity, mineralogy, fines content, and compaction condition. In this study, TRDCs of the 13 sandy soils was investigated using a modified hanging column test, as well as tests for soil physical properties. Statistical analyses, including analysis of variance (ANOVA) and stepwise regression, were used to statistically assess how the physical properties of soil influence TRDCs. This thesis describes the findings of this study. Section 2 provides an overall background in terms of heat transfer and thermal resistivity of soil. Materials and methods are described in Sections 3 and 4, respectively. Results and analyses are provided in Section 5. Summary and conclusions are provided in Section 6.
2. BACKGROUND

2.1 HEAT TRANSFER

Heat transfer is classified by the mechanisms of:

(i) thermal conduction (heat transfer facilitated by the kinetic and potential energies of objects at the microscopic scale due to a temperature gradient; e.g., vibrations and collisions of molecules, propagation and collisions of phonons, and diffusion and collisions of free electrons);

(ii) thermal convection (heat transfer via the mass flow of an advection fluid, such as gas or liquid); and

(iii) thermal radiation (energy emitted by electromagnetic waves).

Thermal conduction is the most significant property for heat transfer in solids. In a solid, the network of relatively close, fixed spatial relationships among atoms helps to transfer energy via vibrations, while a fluid has relatively large spacing between atoms. Therefore, thermal conduction primarily affects heat transfer in a porous medium, such as soil, relative to thermal convection and radiation (Kaviany, 1991).

In thermal conduction, the temperature changes gradually from one location to another until temperature equilibrium is achieved. Until thermal equilibrium, transient conduction occurs, which is also called the non-steady state. Steady-state conduction indicates that the spatial distribution of temperatures in the conducting object is constant. In the steady state, the amount of entering and exiting heat is constant and equilibrium is maintained.

2.1.1 Fourier’s Law
Thermal conduction follows the well-known and established Fourier’s law (1822). Fourier’s law originated from observed phenomena rather than derivation from first principles (Incropera et al., 2011). Fourier developed the relationship of heat transfer under steady-state conditions from an experiment performed using a cylindrical rod that was maintained at different temperatures at each end. Fourier observed that the heat transfer rate: (i) was directly proportional to the cross-sectional area when the temperature difference and the rod length were constant; (ii) changed inversely with the rod length when the temperature difference and the cross-sectional area were constant; and (iii) was directly proportional to the temperature difference when the cross-sectional area and the rod length were constant. These three observations were coupled and expressed as:

\[ q_x \propto A \frac{\Delta T}{\Delta x} \]  

(2.1)

where \( q_x \) is the heat transfer rate in the x-direction, \( A \) is the cross-sectional area, \( \Delta T \) is the temperature difference, and \( \Delta x \) is the rod length. Fourier’s experimental observation was performed with a cylindrical rod; however, if a material with constant \( A \) and \( \Delta T \) changes (e.g., from metal to wood), the heat transfer rate changes due to the thermal properties of the material. Thus, the proportional equation (Eqn. 2.1) is converted to equality by the material’s thermal conductivity (Eqn. 2.2):

\[ q_x = \lambda A \frac{\Delta T}{\Delta x} \]  

(2.2)

where \( \lambda \) is the thermal conductivity of material. The variation of thermal conductivity with temperature is generally small over a significant range—such as 16 W/(m·k) at 25 °C, 17 W/(m·k) at 125 °C, and 19 W/(m·k) at 225 °C for stainless steel—of temperatures for common materials; however, \( \lambda \) occasionally varies in anisotropic materials.

The heat rate (Eqn. 2.3) and the heat flux (Eqn. 2.4) are obtained by evaluating Eqn. 2.2 in the limit as \( \Delta x \to 0 \).
\begin{equation}
q_x = -kA \frac{dT}{dx} \tag{2.3}
\end{equation}

\begin{equation}
q_x^* = \frac{q_x}{A} = -k \frac{dT}{dx} = -k \nabla T \tag{2.4}
\end{equation}

where \( \frac{dT}{dx} \) is the local temperature gradient, \( \nabla \) is the three-dimensional del operator, and the minus sign is used to offset the negative temperature gradient. The right-hand side of Eqn. 2.4 presents Fourier’s law as a vector quantity, while the left-hand side is described in one-dimensional form for simplicity. According to Eqn. 2.3 and Eqn. 2.4, Fourier’s law states that the heat transfer rate through a unit area per unit time is proportional to the negative gradient of the temperature.

Local thermal equilibrium is applied for evaluation of heat transfer in porous media, such as rock, soil, and wood (Kaviany, 1991; Quintard and Whitaker, 1995). Accordingly, the heat transfer processes in soil systems are frequently described by a single heat conduction equation. However, when the differences in thermal properties of solid and fluid are large, separate heat conduction equations for each material type may be required. In unsaturated soil, for example, the soil systems consist of soil solids, water, and air. Typical thermal resistivity of soil particles and water are about 4 °C·cm/W and 165 °C·cm/W respectively, while typical thermal resistivity of air is about 4000 °C·cm/W (Kersten, 1946; Winterkorn, 1962; Salomone et al., 1979). The large difference of thermal resistivities for each phase may lead to an incorrect application of the local thermal equilibrium to transient heat transfer processes in unsaturated soil systems. However, the continuity equation can be used to account for the large difference of thermal resistivity.

\section*{2.1.2 Continuity Equation}
The continuity equation describes the transport of conserved quantities, such as mass, energy, momentum, electric charge, and other natural quantities. The differential form for the general continuity equation is defined as follows:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot j = \sigma \]  

(2.5)

where \( \nabla \cdot \) is the divergence, which measures the magnitude of a vector field’s source or sink at a given point based on a signed scalar, \( t \) is time, \( j \) is flux, and \( \sigma \) is the generation of \( q \) per unit volume per unit time, terms that generate \((\sigma > 0)\) or remove \((\sigma < 0)\) \( q \) are referred to as a “sources” and “sinks”, respectively. Eqn. 2.5 can be used to derive any continuity equation.

The continuity equation as defined in terms of thermodynamic laws is

\[ \nabla \cdot q + \frac{\partial u}{\partial t} = 0 \]  

(2.6)

where \( u \) is the local energy density (energy per unit volume), and \( q \) is the energy flux (i.e., transfer of energy per unit cross-sectional area per unit time as a vector).

According to Campbell and Norman (1998), Fourier’s law can be used to calculate heat transfer from one layer to the next; however, the continuity equation has to be solved simultaneously to find temperature variation with distance and time.

\[ \rho_s c_s \frac{\partial T}{\partial t} = -\frac{\partial G}{\partial z} \]  

(2.7)

where \( \rho_s \) is soil density, \( c \) is soil specific heat, \( \rho_s c_s \) is the volumetric heat capacity, and \( G \) is heat flux density in the soil. The rate of heat storage in a layer of soil and the heat flux divergence (i.e., rate of change of heat flux density with distance) are represented on the left-hand side and the right-hand side of Eqn. 2.2, respectively. The continuity equation (Eqn. 2.7) combines with Fourier’s law (Eqn. 2.4) as

\[ \rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) \]  

(2.8)
If thermal conductivity is constant with distance, the conductivity of the material can be taken outside the derivative. Both sides are divided by the volumetric heat capacity to obtain a more familiar form of the heat equation:

\[
\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2}
\]

where \( \kappa = \frac{k}{\rho_s c_s} \) is the soil’s thermal diffusivity. According to Eqn. 2.9, the location of the largest temperature gradient is equivalent to the location of the fastest temperature change within a soil. In other words, the temperature gradient is directly proportional to the thermal resistivity that resists heat transfer in soil.

### 2.1.3 Thermal Conductivity and Resistivity

Both thermal conductivity and thermal resistivity are material properties that quantify thermal properties. Thermal conductivity (\( \lambda \)) is the intrinsic property of a material to conduct heat, and the thermal resistivity (\( \rho \)) is the reciprocal of thermal conductivity (or the intrinsic property of a material to resist heat flow). Thermal conductivity is measured in SI units as W/(m·K) or in IP units as Btu/(hr·ft·°F). The thermal ohm is a unit of resistivity that is defined as the number of degrees centigrade of temperature drop that occurs when heat flows through 1 cm\(^3\) at a rate of 1 W (Salomone et al., 1984). The unit of thermal resistivity is °C·cm/W (100 °C·cm/W = 1 m·K/W and 0.01731 °C·cm/W = 1 hr·ft·°F/Btu).

### 2.1.4 Heat Transfer in Soil

According to Kersten (1946), Winterkorn (1962), and Salomone et al. (1979), bulk soil thermal resistivity collectively results from contributions of each soil phase: \( (i) \) the phase of soil solids is about 4 °C·cm/W; \( (ii) \) the phase of water is about 165 °C·cm/W; and \( (iii) \) the
phase of air is about 4000 °C-cm/W. Average thermal resistivity values of soil constituents are presented in Table 2.1.

Heat flows primarily through soil solids and water, which have relatively lower thermal resistivity than air. Heat transfer mechanisms in soils are schematically shown in Fig. 2.1, with red arrows that indicate the amount of heat transfer as a thickness. Soil thermal resistivity related to heat transfer is affected by both fundamental differences of thermal resistivity related to each soil phase and physical factors of soil, such as moisture content, density, and soil type.

2.2 FACTORS INFLUENCING SOIL THERMAL RESISTIVITY

2.2.1 Moisture Content

Previous studies on the effect of moisture content on \( \rho_{\text{soil}} \) typically divide an unsaturated soil system into regimes, such as the pendular water-retention regime. Fig. 2.2 characterizes the relationship between thermal resistivity and moisture content for crushed quartz. As moisture content increases, moisture forms a thin film around soil particles, replaces the air in the voids with pore water, and leads to a decrease in \( \rho_{\text{soil}} \) regardless of soil type (Salomone et al., 1979; Farouki, 1981; Salomone and Kovacs, 1984; Brandon and Mitchell, 1989; Gangadhara Rao and Singh, 1999; Smits et al., 2010; Likos et al., 2012; Woodward and Tinjum, 2012). Fig. 2.3 describes heat transfer mechanisms in terms of moisture content. In Fig. 2.3(a), contacts between solid particles, which efficiently transfer heat, are restricted to small regions. As moisture content increases, heat transfer paths widen significantly, thereby leading to higher thermal conductivity (Roth, 2012).

2.2.2 Density
Soil thermal resistivity decreases with an increase in soil density. The relationship between $\rho_{\text{soil}}$ and soil density for thermal sand is shown in Fig. 2.4. As soil density increases, the overall volume of voids decreases, contacts among soil particles increase, and a reduction in $\rho_{\text{soil}}$ occurs (Salomone et al., 1979; Farouki, 1981; Salomone et al., 1982; Salomone and Kovacs, 1984; Brandon and Mitchell, 1989; Gangadhara Rao and Singh, 1999; Smits et al., 2010). In addition, the increase in soil density leads to a higher capillary force, which holds the water in the soil. The reduced moisture migration due to the higher capillary force also affects decreases in $\rho_{\text{soil}}$.

2.2.3 Soil Type

The type of soil (as defined based on mineral type, particle size, and particle-size distribution) influences water adsorption to the grain surface, absorbed water into the three-phase soil system, capillary force in the soil, and contact area among soil particles. These collective influences of water on soil lead to a change in $\rho_{\text{soil}}$ (Salomone et al., 1979; Farouki, 1981; Salomone and Kovacs, 1984; Gangadhara Rao and Singh, 1999; Smits et al., 2010; Likos et al., 2012). Fig. 2.5 shows $\rho_{\text{soil}}$ versus degree of saturation for a variety of soil types.

Montmorillonite, which is a type of clay mineral, has a large potential for adsorbing water, while kaolinite has a smaller potential. Salomone et al. (1979) and Farouki (1981) indicate that adsorbed water in montmorillonite forces soil particles apart through swelling action, thus increasing the thermal resistivity of the bulk soil.

Quartz is one of the most abundant minerals on Earth and a principal constituent of many sands. Quartz has the lowest thermal resistivity among minerals commonly found in sand at 11 °C·cm/W (Winterkorn, 1962). Knowledge of quartz content is particularly
important to evaluate and estimate the thermal properties of a soil (Winterkorn, 1962; Salomone et al., 1979; Farouki, 1981; Brandon and Mitchell, 1989).

Particle size has an effect on $\rho_{\text{soil}}$ as well as the mineralogical and physicochemical characteristics. Fine-grained soils, such as clay and silt, have higher capillary forces that lead to a large amount of retained water in the soil, thus fomenting less contact among soil particles in comparison to granular soils. As the representative particle size decreases, $\rho_{\text{soil}}$ increases at any given moisture content (Salomone et al., 1979; Farouki, 1981; Gangadhara Rao and Singh, 1999; Smits et al., 2010; Likos et al., 2012). Smits et al. (2010) investigated the statistical effect of sandy soils on $\lambda$ in terms of particle size. Particle size is relatively insignificant for the particle size range of typical sands, but particle size may become more significant when gravel-, pebble-, and boulder-size particle ranges are involved.

### 2.2.4 Temperature

As mentioned in the Introduction, soil temperature can change through interaction with underground structures such as buried high-voltage power cables, geothermal exchangers, and gas pipes. The change of soil temperature influences moisture migration in soil and thermal resistivity of the soil components including air and crystalline minerals. Fig. 2.6(a) indicates that thermal resistivity of quartz increases with increasing temperature, while thermal resistivity of water and saturated pore water decreases. Accordingly, soil thermal resistivity at the saturated condition decreases as temperature increases, while soil thermal resistivity at the dried condition increases with increasing temperature.

The kinetic energy of water molecules increases as temperature increases. Pore moisture evaporates or diffuses at higher temperatures to a cooler region where it condenses with heat transfer (Van Rooyen and Winterkorn 1957; Salomone et al., 1979;
Radhakrishna et al., 1980; Farouki, 1981). The extent of moisture migration in the form of vapor diffusion depends on the vapor pressure gradient and vapor permeability (Radhakrishna et al., 1980). The thermal conductivity of ice, in the range from 0 °C to -200 °C, increases as temperature decreases (Seigo, 1977).

The diffusion coefficient of water vapor in air increases with increasing temperature. Increased water vapor in the air leads to a decrease in thermal resistivity of air (Farouki, 1981; Brandon and Mitchell, 1989). Moreover, thermal resistivity of most crystalline minerals, excepting feldspars, increases as temperature increases (Farouki, 1981; Brandon and Mitchell, 1989). The thermal resistivity of moist sand based on the crystalline minerals decreases as temperature increases, while the thermal resistivity of dry sand increases with temperature (Flynn and Waston, 1969; Radhakrishna and Steinmanis, 1981; Brandon and Mitchell, 1989). The decrease in the thermal resistivity of moist sand with increasing temperature is interpreted to enhance heat transfer through the saturated pore air. The increase in the thermal resistivity of dry sand with temperature, on the other hand, is affected by evaporation or dispersion in water vapor.

### 2.3 THERMAL RESISTIVITY DRY-OUT CURVE

A typical thermal conductivity (the inverse of resistivity) dry-out curve (TCDC) and soil-water characteristic curve (SWCC) are shown in Fig. 2.7. The TRDC represents the nonlinear relationship between thermal resistivity and water content [gravimetric (ω) or volumetric (θ)] or degree of saturation (S) of soils. The thermal resistivity is generally some maximum at zero saturation where air is fully represented in voids among the three-phase soil system. The air phase, which has a much higher thermal resistivity, prevents heat flux in the solid-liquid matrix. Thin films around soil particles are developed as the degree of
saturation increases slightly, and these films cause a rapid decrease in thermal resistivity through the increase in the contact areas among soil particles. This decrease in thermal resistivity continues until liquid bridges among the thin films are formed. The liquid bridges aid heat conduction through solid-liquid-solid paths. After the liquid bridges form, the thermal resistivity decreases gradually up to complete saturation. Decreasing thermal resistivity with increases in S reflects the presence of the more conductive water phase in the multiphase unsaturated soil system.

The SWCC represents the correlation of water storage capacity at various matric suctions (i.e., negative soil pressure) through the relationship between degree of saturation and matric suction. The SWCC is characterized with an air-entry suction ($\psi_a$) and residual moisture content ($\theta_r$). The $\psi_a$ is the suction in which air first starts to enter the soil’s largest pores and desaturation commences. The $\theta_r$ indicates the point in which little pore water exists on the particle surface due to molecular bonding mechanisms, and very large suction increments are required to remove additional water from the system. The SWCC changes depending on soil physical factors including density, composition, and grain-size distribution and includes the phenomenon of hysteresis, which includes desorption and sorption because of the entrapment of occluded air bubbles (Tinjum et al., 1997; Lu and Likos, 2004, Lu and Likos, 2006). The SWCC may be used with the TRDC to describe unsaturated and partially saturated mechanisms in terms of heat transfer and moisture migration in soils (Salomone and Kovacs, 1984).

2.3.1 Critical Moisture Content

Critical moisture content is designated at the knee in the TRDC, where liquid bridges between soil particles break down, resulting in a disproportionate increase in the thermal
resistivity with small reduction in moisture content. The critical moisture content is affected by the grain-size distribution, particle shape, and degree of soil compaction (Radhakrishna et al., 1980). Fig. 2.8 shows that the critical moisture content also depends on density. As density decreases, the critical moisture content increases.

The soil condition below the critical moisture content is often relatable to thermal instability. Soil thermal stability is defined as the moisture condition above the critical moisture content in which thermal resistivity changes slightly with change in moisture content. According to Radhakrishna et al. (1980), sustained moisture migration along a thermal gradient occurs below the critical moisture content for which vapor permeability increases to a point that vapor outflow exceeds liquid inflow, thus causing progressive drying. Sustained moisture migration causes thermal instability. Thermal instability can be predicted with the physical parameters of soil suction, optimum moisture content, and plastic limit (Salomone and Kovacs, 1984). The upper flex point in the SWCC is located near the critical moisture content (e.g., Fig. 2.7). Thus, the upper flex point provides a good estimate of the critical moisture content (Jones and Kohnke, 1952; Abdel-Hadi and Mitchell, 1981; Mitchell et al., 1981; Salomone and Kovacs, 1984). The critical moisture content is estimated with optimum moisture content and maximum dry density (Salomone and Kovacs, 1984). Plastic limit can be used to determine the critical moisture content of fine-grained soil that has low dry density because the plastic limit of a low-density soil (e.g., < 1.6 Mg/m³) is only slightly above the optimum moisture content (Salomone and Kovacs, 1984).

2.3.2 Implications and Use of the TRDC in the Practice of Energy Geotechnics

The TRDC combined with the SWCC provides fundamental information required to describe the geo-mechanical behavior of unsaturated soil including heat flux and moisture
migration. Various emerging applications, such as artificial ground freezing, agricultural water management, frost penetration, buried utilities, thermally active geo-structures, and heat exchange elements, rely on unsaturated soil mechanics (most notably heat transfer). Accordingly, the TRDC and the SWCC are widely used for design and performance of these applications (Donazzi et al., 1979; Dayan and Merbaum, 1984; Fan et al., 2007).

For example, construction of high-voltage transmission lines via buried cables may be required as smart grids and renewable power generation sites develop and where the aesthetics of overhead transmission lines is questioned. Due to the high cost of metal used in electrical conductors (typically aluminum or copper), the cost of transmission line construction is significantly impacted by the cost of the conductor. Conductor sizing depends on the ability of the cable to maintain a stable operating temperature and conductivity and the required amperage (Neher and McGrath, 1957; Milne and Mochlinski, 1964; Martin and Black, 1981; IEC, 2006). The controlling factor is the ability of soil to transmit heat energy away from the cables (Adams & Baljet 1968; Mitchell 1991); particularly, the ampacity of high-voltage transmission lines is highly dependent on the thermal resistivity of the medium surrounding cables (Jorgensen, 2012). Emerging applications in energy geotechnics use the ground to supply constant-temperature fluids for direct heating/cooling or for heat pump applications (Brandl, 2006; Ortan et al., 2009; Wu, 2013). Geothermal exchange systems require soil that has a higher thermal conductivity (or a lower thermal resistivity) to effectively reject or extract heat, depending if operation is in the cooling or heating mode, respectively. To obtain higher thermal conductivity, there are various options, such as controlling compaction or maintaining moisture sources within native soils, or through the use of a high-conductivity thermal backfill (Jorgensen, 2012). However, improvement of backfill may add
cost relative to backfilling with native soils. Therefore, a cost sensitivity analysis may be performed to determine if the backfill improvement will reduce overall backfilling costs.

Even though the influences of water content, density, soil type, and temperature on the TRDC have been recognized by earlier studies, these influences have not been systematically explored for naturally occurring sandy soils. There are still uncertainties about the effects of soil physical properties such as roundness and sphericity of soil particles in addition to index properties including particle diameter corresponding to 10% finer \((D_{10})\), particle diameter corresponding to 50% finer \((D_{50})\), coefficient of uniformity \((C_u)\), and fines content on the TRDC. This thesis attempts to address this gap in the literature through the systematic characterization of 13 naturally occurring sandy soils.
3. MATERIALS AND METHODS

3.1 MATERIALS

Sandy soils are widely distributed and commonly used as backfill. To identify the influences of sandy soils on the TRDC, 13 sandy soils were used in this study: (i) nine sandy soils from the University of Wisconsin soil bank, which were collected from an in situ layer at various field sites, (ii) three artificially created (i.e., remixing of portions of sieved soil from a concrete sand and fines from SM1 of the soil bank) well-graded sands, and (iii) a sandy soil from Grand Marsh, Wisconsin. Locations and origins of the 13 sandy soils are shown in Fig. 3.1. These soils originated from weathered sandstone, glacial outwash, ice-contact stratified deposits, and other deposits of glacial origin (Bareither et al., 2008).

Specimens taken from these 13 sandy soils were classified using mechanical sieve analysis (ASTM D422 and ASTM D2487) as (i) Poorly Graded Sand, SP; (ii) Well-Graded Sand, SW; (iii) Well-Graded Sand with Silt, SW-SM; (iv) Poorly Graded Sand with Silt, SP-SM; and (v) Silty Sand, SM. Fig. 3.2 presents grain-size distribution curves for the samples. A summary of the index properties including particle diameter corresponding to 10% finer ($D_{10}$), particle diameter corresponding to 50% finer ($D_{50}$), coefficient of uniformity ($C_u$), coefficient of curvature ($C_c$), fines (%), and specific gravity ($G_s$) for the 13 specimens is summarized in Table 3.1. The specific gravity was investigated using ASTM D854.

Physical properties for the 13 specimens were investigated using laboratory tests, and those are also shown on Table 3.1: (i) maximum dry unit weights ($\gamma_{d,max}$) were obtained by standard proctor compaction test (ASTM D698); (ii) minimum void ratios ($e_{min}$) and maximum void ratios ($e_{max}$) were obtained by ASTM D4254; and (iii) representative roundness and sphericity of soil particles were investigated per procedures described in Janoo (1998) and Alsaleh (2004). Fig. 3.3 indicates range of void ratio for the 13 specimens.
As described in the Background, the thermal resistivity for soil particles partially depends on the mineralogy (Table 2.1). Mineralogy of the specimens was analyzed by X-ray diffraction. The X-ray diffraction data were acquired using a Rigaku D/Max Rapid II X-ray diffraction system with a curved, two-dimensional imaging plate (2D IP). Testing was performed at an acceleration voltage of 50 kV, a current of 50 mA (rated at 2.5 KW), and an exposure time of 10 min. Results of the mineralogical analyses is summarized in Table 3.2.

3.2 MODIFIED HANGING COLUMN EXPERIMENT

3.2.1 Apparatus

A modified hanging column, which goes by the trade name, Tempe cell (Smits et al., 2010; Likos et al., 2012), was used to measure SWCCs and TRDCs concurrently along an initial drying path (i.e., drainage from $S = 1$). The modified hanging column (Fig. 3.4) consists of three parts: (i) a top cap, (ii) an acrylic confining sleeve, and (iii) a perforated bottom plate. The top cap includes two holes and a groove for sensors. In the bottom plate, a brass screen and an o-ring are used to support a high-air-entry nylon membrane (diameter = 142 mm, pore size = 0.2 μm).

As suction is increased via the hanging water column, matric suction, moisture content, temperature, and thermal resistivity/conductivity are measured using four sensors that are directly installed in the specimen. The matric suction sensor (I) is a small-tip tensiometer inserted through a plastic fitting on the side wall of the cell and embedded into the mid of specimen. A differential pressure transducer (Model P55D, Validyne Engineering Corp., Northridge, CA) and data-logger system were connected with the sensor to measure matric suction. The thermal sensor (II) was a dual-needle transient thermal probe (SH-1), and soil thermal properties (conductivity/resistivity) were saved in a KD-2 Pro data-
acquisition system (Decagon Devices, Pullman, WA). The moisture sensor (III) is a dual-prong dielectric moisture sensor (ECH2O EC-5, Decagon Devices, Pullman, WA), and is connected to an Em50 data logger. Raw data was independently calibrated using the two-point α-mixing model of Sakaki et al. (2008) and back-calculation. The temperature sensor (IV) was embedded into the top portion of the soil connected to the Em50 data logger. Measurements acquired from the four sensors were interpreted to produce a continuous SWCC and TRDC along either a continuous drying or wetting path.

3.2.2 Modified Hanging Column Procedures

Fully dried sand, which was dried in an oven at 105 °C for 24 h, was prepared for the test. The target void ratio of the samples was 0.6 based on recorded $e_{\text{min}}$ and $e_{\text{max}}$ (see Table 3.1). For a void ratio of 0.6, the dry mass of sand was calculated and compacted directly into the confining sleeve in four equal layers to a height of about 1 cm below the top edge of the cell; however, three specimens (SP6, SW1, and SW-SM2) did not reach the height due to lower minimum void ratios as shown in Fig. 3.3. The void ratios for SP6, SW1, and SW-SM2 were 0.46, 0.47, and 0.47, respectively. The tensiometer (I), SH-1 (II), and ECH2O EC-5 (III) were embedded in the third layer. After compaction of the four layers, the top cap was covered, and the temperature sensor (IV) was installed into the top portion of the sample through a vent in the top cap.

After packing, the water column was filled with water to control the suction. As valve 1 (Fig. 3.4a) was opened, water in the water column started to saturate the specimen from bottom to top until approximately 1 cm of water ponded on top of the sand surface. The saturation was maintained for about 1 h to remove air. The water level of the cell and the water column was then kept equilibrium at the midpoint of the cell where the tensiometer,
moisture sensor, and thermal probe were located. The suction head was then increased slowly and continuously to the specimen as valve 2 was partially opened to produce a slow drip at a rate of 6 to 10 s/drop. The top cap of the cell was removed, and a mechanical fan and a dehumidifier were set up near the specimen to evaporate the moisture in the specimen after the water in the water column was completely drained at a suction head of \(~126.5\) cm H\(_2\)O. Matric suction, volumetric water content, and thermal resistivity were continuously monitored. At low saturation (0.1 to 0.2), continued drainage via gravity becomes difficult (Likos et al., 2012). Tightly adsorbed water usually is not removed by natural processes (McQueen and Miller, 1977). To achieve lower saturations in this study, a mechanical fan and dehumidifier were coupled with the gravity-induced drainage. In this study, the termination criterion was when volumetric water content of the sand reached a value of about 0.01. However, SM1 (terminal volumetric water content: 0.049) did not reach the termination criterion despite use of the mechanical fan and dehumidifier. This is because water is retained by larger-ranged capillary mechanisms that depend on geometry of the pores and short-ranged physical and chemical sorption mechanisms that occur near the solid surfaces (Likos and Jaafar, 2013).

3.3 STATISTICAL ANALYSIS

In the literature, there is uncertainty about the relationship between index or physical properties of soil and the TRDC. In this study, evaluated physical properties included fines and quartz content, roundness and sphericity of particles, particle diameter at 10% finer (\(D_{10}\)), particle diameter at 50% finer (\(D_{50}\)), coefficient of uniformity (\(C_u\)), maximum dry unit weight (\(\gamma_{d\text{max}}\)), and minimum void ratio (\(e_{\text{min}}\)) and maximum void ratio (\(e_{\text{max}}\)). The effect of particle size on the TRDC has been reported for specimens with larger particles, ranging
from 2 mm to 4 mm (Midttomme and Roaldset, 1998; Smits et al., 2010). In comparison, the effect of smaller grain size is relatively insignificant compared to correlations with larger particles. Findings from the literature—Midttomme and Roaldset (1998) and Smits et al. (2010)—imply the necessity for precise analysis of the effect of smaller particle size. Accordingly, in relation to the particle size, the specimens of this study only relatively small particle sizes, ranging from 0.11 mm to 0.77 mm; thus, statistical significance is examined to define the effect of smaller particle size on TRDC. In this research, statistical approaches—analysis of variance (ANOVA) and stepwise regression—are used to investigate the influences of index and physical properties on the TRDC, as well as the effect of smaller particle size on the TRDC.

3.3.1 Variables

Although three regimes in terms of degree of saturation are considered in certain studies (e.g., Smits et al., 2010; Gouda et al., 2011), the TRDC is typically separated into two zones divided by the critical degree of saturation (the critical moisture content): thermal instability on the dry side and thermal stability on the wet side (Salomone et al., 1979; Radhakrishna et al., 1980; Farouki, 1981; Salomone and Kovacs, 1984; Brandon and Mitchell, 1989; Gangadhara Rao and Singh, 1999). Three points of comparison were chosen at the fully saturated condition (first measurement in the TRDC), the dry condition (terminal measurement in the TRDC, at volumetric moisture content of 0.01), and the critical degree of saturation, as representative of the continuous data of TRDC (Fig. 3.5). The knee of the SWCC and TRDC is commonly defined as the point of intersection of the two lines that best fit the linear wet and dry portions of the curves (Radhakrishna et al., 1980; Abdel-Hadi and Mitchell, 1981; Salomone and Kovacs, 1984). Consistent with these methods from the
literature, the critical analysis point for this study was determined as the intersection of the extended lines extrapolated from the two linear portions of TRDC. The three points of comparison (saturated, critical, and dry) and oven-dry thermal resistivity are considered as dependent variables. Independent variables taken from index and physical properties of the 13 specimens and the independent variables are shown in the Table 3.3.

3.3.2 Analysis of Variance (ANOVA) and Stepwise Regression

Analysis of variance (ANOVA) is a statistical analysis technique used to determine whether or not differences exist between the means of several observation groups and their categorical factors. In this study, ANOVA was performed to determine the differences among TRDCs of the 13 specimens based on soil type using IBM SPSS Statistics, which is a widely used software package for data mining, text analytics, and collaboration and deployment. The significance was tested by the $F$-statistic (Draper and Smith, 1981) and the significance level set at 0.05. The $F$-statistic is the ratio

$$ F = \frac{M_{S_r}}{s^2} \quad (3.1) $$

where $M_{S_r}$ is mean square due to regression and $s^2$ is residual variance. The mean square due to regression is the component of the total variance that can be explained by the linear trend. Thus, trends with greater significance have a higher $F$ (Benson et al., 1994).

While ANOVA analyzes the differences among TRDCs, multiple regression is a statistical method for determining which dependent variables are most influential on independent variables. For this study, the multiple regression was also conducted using IBM SPSS Statistics to estimate correlations between TRDCs and physical properties of soil and to investigate which physical property is most impactful on TRDC. There are varying methods for an optimized regression equation: (a) enter method, (b) forward selection
method, (c) backward elimination method, and (d) stepwise selection method (Field, 2013).

Because stepwise regression allows the removal and addition of independent variables, stepwise regression is one of the most popular methods used to derive the regression equations. Thus, stepwise regression was selected in this study to clarify the statistical significance of physical properties on the TRDC. The significance was tested by the $F$-statistic and the significance level set at .05. The regression model takes the form:

$$\ln K_g = a_0 + a_1X_1 + a_2X_2 + \cdots + a_iX_i + \cdots + a_nX_n + \varepsilon$$  \hspace{1cm} (3.2)

where $a_i$ are the coefficients, $X_i$ are the independent variables, and $\varepsilon$ is a mean-zero Gaussian random-error term.
4. RESULTS AND ANALYSES

TRDCs and SWCCs obtained from the modified hanging column tests are presented in Fig. 4.1 and Fig. 4.2, respectively. The 13 sandy soils have similar TRDCs. The main difference is the slope after $\rho_{\text{crit}}$ is reached. Fig. 4.1 (b) indicates oven-dry thermal resistivity values and TRDCs near the critical and dried condition. Although the results are continuously measured as curves, oven-dry thermal resistivity and the three points of comparison described in Section 3.3.1 are used for analyses, including the statistical analysis.

Some ranges on the TRDCs include minor errors that were typically associated with: (i) loss of contact with sensors, (ii) rapid moisture migration near sensors, and/or (iii) external impacts, such as temperature, humidity, and battery failure of KD-2 Pro. For example, inaccurate measurements occurred for thermal resistivity values of SM1 near the fully saturated condition when rapid moisture migration occurred from initial drainage and bad contact existed between the sensors and soils. Moreover, the data for SM1 from $S = 0.45$ to $S = 0.24$ was lost due to a dead battery in the KD-2 Pro. Therefore, on Fig. 4.1, errors and lost data were extrapolated and represented as dashed lines.

4.1 INFLUENCE OF SOIL PHYSICAL PROPERTIES

4.1.1 Degree of Saturation

Effects of degree of saturation on the TRDC in this study present similarly to previous test results (Salomone et al., 1979; Farouki, 1981; Salomone and Kovacs, 1984; Brandon and Mitchell, 1989; Gangadhara Rao and Singh, 1999; Smits et al., 2010; Likos et al., 2012; Woodward and Tinjum, 2012). The lowest thermal resistivity is measured at the fully saturated condition and increases continually with the decrement of degree of
saturation until the critical degree of saturation is reached, at which point liquid bridges among soil particles break down (Radhakrishna et al., 1980) and $\rho_{soil}$ increases rapidly. The highest thermal resistivity is at the fully dried condition. This result is interpreted to reflect average thermal resistivity values described in Table 2.1 (e.g., $\rho_{air}$ is the highest at about 4000 $^\circ$C·cm/W, while thermal resistivity values for the soil particles and the water phase are 4 $^\circ$C·cm/W and 165 $^\circ$C·cm/W, respectively).

The critical degree of saturation, as reported by Radhakrishna et al. (1980), is an important threshold for changes to the TRDC ($S_{\rho,\text{crit}}$). Degree of saturation at the knee of SWCC ($S_{\psi,\text{crit}}$) indicates a capillary limit (McQueen and Miller, 1974). In this study, the critical degree of saturation is analyzed with SWCCs. Critical degrees of saturation on the 13 TRDCs (i.e., $S_{\rho,\text{crit}}$) are located below the degree of saturation at knee points of the 13 SWCCs (i.e., $S_{\rho,\text{crit}} < S_{\psi,\text{crit}}$); degrees of saturation at knee points of the TRDCs and the SWCCs are summarized in Table 4.1. According to a conceptual model suggested by McQueen and Miller (1974), regimes in the SWCC can be classified using straight-line segments [Fig. 4.3 (a)]: (i) tightly adsorbed regime where moisture is tightly adsorbed to the particle surface and usually is not removed by natural processes, (ii) adsorbed film regime where moisture is retained as films on the particle surface, and (iii) capillary regime where water is lightly held. Below the capillary regime of the SWCC, water still exists. Therefore, the critical degree of saturation of the 13 TRDCs where liquid bridges among particles break down is located below the knee of the 13 SWCCs. In addition, the critical degrees of saturation of the TRDCs are on the tightly adsorbed regime or the adsorbed film regime [Fig. 4.3 (b)]. Due to the slight water adsorption, therefore, thermal resistivity of soils greatly increases below critical degrees of saturation (Radhakrishna et al., 1980; Salomone and Kovacs, 1984; Lu and Likos, 2004; Smits et al., 2010).
A SWCC can change shape through the phenomenon of hysteresis. Due to the "ink bottle effect", wetting of soil is controlled by pore diameter, while drying of soil is influenced by smaller pore throats (Likos and Lu, 2004). Because suction increases as pore radius decreases, drying a soil requires more suction than wetting a soil at the given moisture content. In contrast, the TRDC reverts to the same thermal resistivity regardless of being on a drying or wetting path. SP3 and SW-SM3 (see Fig. 4.4) are representative of this lack of hysteresis in the TRDC. In Fig. 4.4, degree of saturation increases abruptly near about 0.28 and about 0.18 for SP3 and SW-SM3, respectively. During the drying process for specimen SP3 and SW-SM3, infiltration and evaporation occurred at the same time. Moisture migration by infiltration and evaporation caused an unexpected increase in degree of saturation. Despite this increase in degree of saturation, there was no difference in soil thermal resistivity when the degree of saturation returned to the initiation point of the hysteretic loop. The main reason relates to the media of heat transfer. While moisture migration occurs through the voids, heat transfer fundamentally transmits through soil particles as well as liquid within voids. Smits et al. (2010) also investigated the hysteresis of TCDCs. Fig. 2.7 (a) describes primary drainage, secondary wetting, and secondary drainage, and the lack of the hysteresis phenomenon in the TCDC.

Heat transfer processes based on solid-liquid-solid paths were analyzed with high-resolution images obtained by synchrotron X-ray computed tomography (CT) conducted at the Advanced Photon Source of Argonne National Laboratory. Two specimens—SP5 and SM2—at various degrees of saturation were used to acquire high-resolution images. Because specimens for the images were prepared in small aluminum tubes (diameter = 2 mm and length = 3 cm), the target void ratio of 0.6 corresponding to the void ratio from the modified hanging column test was not uniformly achieved. Accordingly, void ratios and
degrees of saturation for each image were recalculated using Image J, which is an image analysis software package. The acquired images and the recalculated properties are shown in Fig. 4.5. In the acquired images, light grey, dark grey, and black regimes indicate soil particles, water, and air, respectively.

Through the high-resolution images, heat transfer in soils may be visualized. In the left-most image for each specimen, heat migrates smoothly through solid-liquid-solid paths. The lowest $\rho_{\text{soil}}$ (ranging from $\approx 35 \, ^\circ\text{C} \cdot \text{cm}/\text{W}$ to $\approx 55 \, ^\circ\text{C} \cdot \text{cm}/\text{W}$ based on the TRDC) is observed in this regime. As degree of saturation decreases, subsequently, the heat migration is partially interrupted by air phases that have thermal resistivity of $4000 \, ^\circ\text{C} \cdot \text{cm}/\text{W}$ (the middle two images). Finally, heat transfer in the right-most image primarily occurs through solid-air systems. The highest $\rho_{\text{soil}}$ (ranging from $\approx 150 \, ^\circ\text{C} \cdot \text{cm}/\text{W}$ to $\approx 300 \, ^\circ\text{C} \cdot \text{cm}/\text{W}$) is observed in this regime.

From Table 4.1, $S$ at the knee points of the TRDCs and the SWCCs for SP5 are 0.061 and 0.144, respectively; degrees of saturation at the knee points of the TRDCs and the SWCCs for SM2 are 0.039, and 0.248, respectively. These degrees of saturation are located between the third images and the right-most images in which the liquid phase exists as adsorbed water on the surface of the grain or absorbed water in the three-phase system of soil. The lack of water including the liquid bridges thus leads to significant increments of thermal resistivity and matric suction (Salomone et al., 1979; Radhakrishna et al., 1980; Salomone and Kovacs, 1984).

### 4.1.2. Thermal Resistivity of Oven-dried Soil and Dried Soil

Oven-dry (volumetric water content equals zero) thermal resistivities of the 13 specimens were measured with separate tests for comparisons to $\rho_{\text{soil}}$ obtained from the
modified hanging column device at volumetric water content of about 0.01. The target void ratio (typically 0.6) for the oven-dry, static measurement of $\rho_{\text{soil}}$ was the same for the two methods.

Comparison and plotting of oven-dry thermal resistivity values measured in additional tests and dry thermal resistivity values determined from the TRDCs are described in Fig. 4.6. Oven-dry thermal resistivity values ($\rho_{\text{oven-dry}}$) excluding SM1 are higher than the dry thermal resistivity values ($\rho_{\text{dry}}$). In the modified hanging column tests, average degree of saturation at the terminal measurement, was 0.012. Soil at the terminal condition ($S_{\text{avg}} = 0.012$) include adsorbed water as thin films on particle surfaces [Fig. 2.3 (b)], while water in the oven-dried soils is fully dried from the energy provided with the higher temperature oven temperature [Fig. 2.3 (a)]. In addition, fines may migrate and cement on soil particles during drainage and drying. This residual water and migration and cementation of fines could lead to lower $\rho_{\text{soil}}$ due to better and closer connections of particles. High-resolution images for the dried/drained soils and oven-dried soils would be required to visualize and support this interpretation.

### 4.1.3 Particle Size ($D_{10}$ and $D_{50}$)

$D_{10}$ and $D_{50}$ are indicative of effective sizes of particles in a soil and are generally used as indicators of soil behavior. Correlations between the particle effective size ($D_{10}$ and $D_{50}$) and the thermal resistivity at the three analysis points are analyzed with trend lines shown in Fig. 4.7. In Fig. 4.7, $\rho_{\text{soil}}$ decreases as the particle diameter increases, which observationally corresponds with previous studies by Midttomme and Roaldset (1998) and Smits et al. (2010). This finding is interpreted to reflect the smallest thermal resistivity of soil.
particles (4 °C·cm/W) among the three phases of soil. In Fig 4.5, for example, $D_{10}$ and $D_{50}$ of SP5 are 0.215 mm and 0.145 mm, respectively; $D_{10}$ and $D_{50}$ of SM2 are 0.13 mm and 0.05 mm, respectively. Particles from specimens of SP provide larger average particle sizes and are considered as larger thermal conductors compared to the smaller particles in a specimen of SM. In other words, heat transfer in the SM occurs through the liquid or air phases more frequently than in the SP. That is, the portion of liquid and air phases per unit volume increases with the decrement of particle size per unit volume when other properties, such as void ratio, are kept constant. Consequently, the difference in media size influences soil thermal resistivity based on the lower thermal resistivity of the soil particles.

The most remarkable change of thermal resistivity observed through scatter plots is at the fully dried condition and is caused by large differences of thermal resistivity between solid particles (4 °C·cm/W) and air (4000 °C·cm/W). Due to absence of water, which has thermal resistivity of 165 °C·cm/W, heat transfer involves the air phase (thermal resistivity of 4000 °C·cm/W) and depends on the soil-air matrix. Thus, the most remarkable changes and scatterings are observed amongst the three degree of saturation points of interest is at the fully dry condition. On the other hand, thermal resistivity at the partially saturated or fully saturated condition changes slightly because of liquid phases, such as adsorbed water on the surface of the grain, absorbed water onto the three-phase system of soil, and liquid bridges. This finding is significant for the TRDCs of sandy soil—as sandy soils have similar physical properties, degree of saturation is more important than soil particle size. However, before a definitive conclusion may be reached, additional laboratory tests are required with soils that have the same physical properties, with only mean particle size (e.g., $d_{10}$, $d_{30}$, $d_{60}$) changing.
4.1.4 Fines Content

Fines are defined as soil particles that are less than 0.075 mm in size (ASTM D422 and ASTM D2487). Fines content is a common parameter, particularly in soil classification. In this study, \( \rho_{\text{soil}} \) is affected by fines content. Correlations between fines content and \( \rho_{\text{soil}} \) of the 13 specimens are portrayed in Fig. 4.8. Soil thermal resistivity increased with increase in fines content, most notably at the fully dried condition. As shown in Fig. 4.5, near the target void ratio of 0.6, SM2, which includes 19.87% fines, has a more tortuous voids structure than SP5, which consists of 1.14% fines. The tortuous void structure of SM2 obstructs heat transfer in soil systems and eventually leads a higher thermal resistivity of soil.

4.1.5 Soil Type

Fig. 4.9(a) shows thermal resistivity based on soil types at the three points of saturation (i.e., saturated, critical saturation, and completely dry). At the critical and saturated condition, there is no significant difference among soil thermal resistivity values for the 13 specimens. For example, \( \rho_{\text{soil}} \) at \( S_{\text{crit}} \) range from 58.5 \(^{\circ}\text{C} \cdot \text{cm/W} \) to 82.5 \(^{\circ}\text{C} \cdot \text{cm/W} \) regardless of soil type. Soil thermal resistivity at the \( S_{\text{sat}} \) range from 35.8 \(^{\circ}\text{C} \cdot \text{cm/W} \) to 55.3 \(^{\circ}\text{C} \cdot \text{cm/W} \), and the low and high values were both for an SP soil. This finding indicates the significant effect of the liquid phase on the thermal resistivity of sandy soil at the same void ratio.

At the fully dried condition, \( \rho_{\text{soil}} \) is indicative of soil type. For example, average thermal resistivity of SM (283.1 \(^{\circ}\text{C} \cdot \text{cm/W} \)) is higher than SP and SW (196.2\(^{\circ}\text{C} \cdot \text{cm/W} \) and 194.1 \(^{\circ}\text{C} \cdot \text{cm/W} \), respectively) because of the smaller particle size and tortuous voids. However, thermal resistivity of certain SP soils, such as 149.2 \(^{\circ}\text{C} \cdot \text{cm/W} \) of SP6 and 257.1 \(^{\circ}\text{C} \cdot \text{cm/W} \) of SP5, indicate similarly with 157.1 \(^{\circ}\text{C} \cdot \text{cm/W} \) of SW1 or 259.7 \(^{\circ}\text{C} \cdot \text{cm/W} \) of SM2.
Therefore, the effect of soil type at the fully dried condition requires a statistical analysis for a full evaluation.

Results of ANOVA for the 13 specimens indicate that TRDCs based on soil type show no statistical significance regardless of degree of saturation. Table 4.2 indicates significances ($\rho_{\text{ooven-dry}} = 0.066$, $\rho_{\text{dry}} = 0.110$, $\rho_{\text{crit}} = 0.137$, and $\rho_{\text{wet}} = 0.225$). Although the 13 sandy soils are classified differently as SP, SP-SM, SW, SW-SM, and SM, the 13 specimens consist of similar physical properties (e.g., roundness of the 13 specimens ranges from 1.08 to 1.13). The less distinctive physical properties eventually do not affect TRDCs. However, the results were located in typical ranges of $\rho_{\text{soil}}$ in terms of soil type [Fig. 4.9(b)]. Further investigation with different kinds of soils, such as gravel, silt, and clay, are required to fully investigate and report on the effect of soil type on the TRDC.

### 4.1.6 $\gamma_{\text{dmax}}$, $e_{\text{max}}$, $e_{\text{min}}$, $C_u$, and Relative Density

Maximum dry unit weight ($\gamma_{\text{dmax}}$), minimum void ratio ($e_{\text{min}}$) and maximum void ratio ($e_{\text{max}}$), and coefficient of uniformity ($C_u$) are related to density of soil. Specifically, soil density increases with $\gamma_{\text{dmax}}$ and $C_u$, while soil density decreases as $e_{\text{min}}$ and $e_{\text{max}}$ increases. Soil thermal resistivity is investigated with the density parameters—$\gamma_{\text{dmax}}$, $e_{\text{min}}$, $e_{\text{max}}$, and $C_u$—in this study. Fig. 4.10 shows that: (i) $\rho_{\text{soil}}$ decreases as $\gamma_{\text{dmax}}$ increases; (ii) $\rho_{\text{soil}}$ increases with $e_{\text{min}}$ and $e_{\text{max}}$; and (iii) $\rho_{\text{soil}}$ decreases as $C_u$ increases. The three cases indicate that $\rho_{\text{soil}}$ decreases with increase in soil density. These results concur with previous studies (Salomone et al., 1979; Farouki, 1981; Salomone et al., 1982; Salomone and Kovacs, 1984; Brandon and Mitchell, 1989; Gangadhara Rao and Singh, 1999; Smits et al., 2010). This is because contacts among soil particles are improved by incremental increases in soil density. For example, the void ratio for the left-most image of Fig. 4.5(a) is 0.52, while the void ratio
of the right-most image of Fig. 4.5(a) is 0.7. The left-most (void ratio of 0.42) and middle-right (void ratio of 0.55) images of Fig. 4.5(b) are similar. Visible differences of particle contacts are confirmed from the two examples. Better contacts consequently lead to better heat transfer in the soil system.

To calculate relative densities of the 13 specimens, three different approaches based on \( \gamma_{d_{\text{max}}} \), \( e_{\text{min}} \), and \( e_{\text{min}} \) and \( e_{\text{max}} \) are used (Table 4.3). Fig. 4.11 indicates correlations between the relative densities and \( \rho_{\text{soil}} \). \( \rho_{\text{dry}} \) increases with relative density regardless of the approach. This finding is opposed to the correlations between \( \rho_{\text{soil}} \) and the three density parameters \( (\gamma_{d_{\text{max}}}, e_{\text{max}}, e_{\text{min}}, \text{and } C_u) \) and indicates an insignificance of relative density related to \( \rho_{\text{soil}} \). For a more nuanced investigation of the effect of relative density on TRDC, additional tests at a constant condition (mineralogy, grain size distribution, etc.) are required.

In the three saturation cases, \( \rho_{\text{soil}} \) changes most significantly at the fully dried condition. The more significant changes in soil thermal resistivity at the dry condition is interpreted to reflect importance of liquid, which has an intermediate thermal resistivity of 165 °C·cm/W. In contrast, changes of thermal resistivity at partially and fully saturated conditions indicate a slight decrement by an increase or equivalency in density. Similarly for changes in particle size, slight changes occur due to: (i) the four density parameters are less significant than moisture content and (ii) the four density parameters of the 13 specimens are not enough to affect \( \rho_{\text{soil}} \) at the partially and fully saturated conditions.

As described in Modified Hanging Column Procedures section, void ratios for SP6, SW1, and SW-SM2 were 0.46, 0.47, and 0.47, respectively. The relatively low void ratios compared to target void ratio of 0.6 lead to low \( \rho_{\text{dry}} \) even though \( \rho_{\text{crit}} \) and \( \rho_{\text{sat}} \) are similar with other specimens (\( \rho_{\text{dry}} \) of SP6 = 149.2 °C·cm/W, \( \rho_{\text{dry}} \) of SW1 = 157.1 °C·cm/W, and \( \rho_{\text{dry}} \) of
SW-SM2 = 176.8 °C·cm/W). That is, the \( \rho_{\text{dry}} \) is achieved by low void ratio consisted of closer particle contacts with decreases in air.

### 4.1.7 Quartz Content

Sandy soil consists of various mineralogies, including quartz, anorthite, and dolomite. Soil mineralogy has specific thermal resistivity values, and these resistivity values influence the entire \( \rho_{\text{soil}} \). Among soil mineralogies, quartz is the best heat conductor (Winterkorn, 1962); thus, quartz content is used as a parameter in this study. \( \rho_{\text{soil}} \) decreased slightly as quartz content increased regardless of \( S \) (Fig. 4.12). Quartz influences the TRDCs as a heat conductor. In addition, correlations between quartz content and fully dried thermal resistivity are relatively scattered because heat transfer only occurs through soil particles and air.

### 4.1.8 Sphericity and Roundness of Particles

Sphericity and roundness related to particle shape are defined in Fig. 4.13. Sphericity is classified as ranging from discoidal shaping to prismoidal shaping. Roundness ranges from the well-rounded shape to a very-angular shape as roundness increases (Powers, 1982; Alsaleh, 2004). Correlations between particle shape and \( \rho_{\text{soil}} \) are shown Fig. 4.14 between (a) thermal resistivity and sphericity and (b) thermal resistivity and roundness. In Fig. 4.14 (a), the lowest thermal resistivity is observed at sphericity of around 0.4, and thermal resistivity increases with sphericity. According to Fig. 4.13, spherical shape ranges from 0.2 to 0.5, and the spherical shape changes as prismoidal shape with increment of the sphericity value; that is, the lowest thermal resistivity is measured at the spherical particle shape, while the highest thermal resistivity is observed at the sub-prismoidal shape. Soil
packing with the sub-prismoidal particle shape includes more voids than soil packing with the spherical particle shape. The void structure directly affects $\rho_{\text{soil}}$.

Effects of roundness on thermal resistivity are shown in Fig. 4.14(b). Ranges of roundness for the 13 specimens were from 1.08 to 1.13 (from well-rounded shape to rounded shape). At the fully dried condition, $\rho_{\text{soil}}$ increased slightly with roundness due to better water adsorption on particle surfaces (Likos and Jaafar, 2013). However, $\rho_{\text{soil}}$ was constant at the fully and partially saturated condition; also, the changes at the fully dried condition are scattered. A full statistical analysis with a larger database of TRDCs is required to determine whether roundness of soil particles influences $\rho_{\text{soil}}$.

4.2 STATISTICAL ANALYSIS

Although there is no statistical significance of effect of soil physical properties on TRDC in ANOVA, stepwise regression includes two statistical significances. The following two models were obtained:

\[ \rho_{\text{dry}} = 289.541 - 193.039 \times D_{50} \]  \hspace{1cm} (4.1)

\[ \rho_{\text{sat}} = 49.834 - 44.296 \times D_{10} \]  \hspace{1cm} (4.2)

The two results are as follows: (a) $D_{50}$ is only significant factor in terms of $\rho_{\text{dry}}$ ($R^2 = 0.519$, meaning that 51.9% of the variance in thermal resistivity is explained by the regression model) and (b) $D_{10}$ is the only significant factor in terms of $\rho_{\text{sat}}$ ($R^2 = 0.245$, meaning that 24.5% of the variance in thermal resistivity is explained by the regression model). The negative coefficients on $D_{50}$ and $D_{10}$ infer that $\rho_{\text{dry}}$ and $\rho_{\text{sat}}$ increase as $D_{50}$ and $D_{10}$ decrease. These findings indicate the TRDCs for sandy soils are affected by particle size ($D_{50}$ and $D_{10}$) corresponding with the analysis for effects of particle size on TRDC obtained from the laboratory tests, with $\rho_{\text{soil}}$ decreasing with increase in representative particle size
As indicated in Smits et al. (2010), multiple measurements for the TRDCs at similar conditions are required for a confirmation of the statistical significance.

4.3 van Genuchten’s PARAMETERS AND THERMAL RESISTIVITY

A result of experimental methods for SWCC often consists of a series of discrete data points. The experimental measurements are also a relatively demanding and often expensive endeavor (Tinjum et al., 1997; Lu and Likos, 2004). Therefore, several models, such as Brooks and Corey (1964), van Genuchten (1980), and Fredlund and Xing (1994), were developed to represent or predict the SWCC. Because three parameters in van Genuchten’s (VG) model allow for a reasonable fit (Tinjum et al., 1997; Lu and Likos, 2004), the 13 SWCCs were fitted with the VG model. The VG model equation is as follows:

\[
\theta = \frac{\theta_s - \theta_r}{\theta_s - \theta_r} = \left(\frac{1}{1 + (\alpha \psi)^n}\right)^m
\]

(4.3)

where \(\alpha\), \(n\), and \(m\) are fitting parameters. The \(m\) parameter is related to the overall symmetry of the SWCC and provides stability of fitting and permits a closed-form solution of the hydraulic conductivity function despite a reduction in the flexibility of the VG model (van Genuchten et al., 1991). \(\alpha\) and \(n\) are related to the shape of SWCC, most notably the pore-size distribution.

Using Eqn. 4.3, \(\alpha\) and \(n\) parameters of the 13 SWCCs are tabulated in Table 4.4. The parameters were similar in range because of the similar physical properties of the 13 specimens. Correlations between the two VG parameters and thermal resistivity at the three points of comparison are described in Fig. 4.16. In terms of the \(\alpha\) parameter, thermal resistivity values at conditions of critical and saturation increased with an increase in the
parameter, while thermal resistivity at the dried condition decreased as the parameter increased. In addition, $\rho_{\text{soil}}$ increased slightly with the $n$ parameter regardless of the three conditions. As the $\alpha$ and $n$ parameters increase, the SWCC has a flatter shape (i.e., poorly graded soil). On the other hand, a smooth shape in the SWCC and relatively high air-entry pressure (i.e., well-graded soil) are recognized to produce smaller $\alpha$ values (Lu and Likos, 2004). The correlations between $\rho_{\text{soil}}$ and the two parameters excluding dry thermal resistivity in terms of the $\alpha$ parameter correspond with laboratory test results. However, the correlation between dry thermal resistivity and the $\alpha$ parameter is discordant with the laboratory tests. Moreover, increments of thermal resistivity with the two parameters are small because of similar physical properties of the data set. Therefore, additional tests, including specimens with greater percentages of gravel, silt, and clay, are required to investigate the correlation between thermal resistivity and VG parameters.
5. SUMMARY AND CONCLUSIONS

This study evaluated the measured thermal resistivity dry-out curves (TRDCs) of 13 sandy soils. The objective was to identify effects of soil physical properties on TRDC using modified hanging column tests, high-resolution images by synchrotron X-ray computed tomography (CT), and statistical analyses, such as ANOVA and stepwise regression. Three \( \rho_{\text{soil}} \) points at the fully dried, critical, and fully saturated conditions were selected on TRDCs obtained from modified hanging column tests to analyze effects of soil physical properties. Soil physical properties used in the analyses were as follows: (1) degree of saturation, (2) soil particle size \( (D_{10} \text{ and } D_{50}) \), (3) fines content, (4) soil type, (5) soil density \( (C_u, \gamma_{\text{dmax}}, e_{\text{max}} \text{ and } e_{\text{min}}) \), (6) quartz content, and (7) particle shape (sphericity and roundness). The following conclusions result from observed and analyzed results:

1. The first soil physical property, degree of saturation, affected TRDCs of the 13 specimens. The lowest \( \rho_{\text{soil}} \) was observed at fully saturated condition due to thermal resistivity values of the soil particle phase of approximately 4 \(^{\circ}\text{C}\cdot \text{cm/W} \) and liquid phase of 165 \(^{\circ}\text{C}\cdot \text{cm/W} \). As degree of saturation decreased, \( \rho_{\text{soil}} \) increased gradually due to the thermal resistivity of the air phase of 4000 \(^{\circ}\text{C}\cdot \text{cm/W} \). Because heat transfer in soil was primarily occurs through the three-phase system, the TRDC did not include a hysteresis phenomenon. Soil thermal resistivity increased more rapidly after the critical degree of saturation was reached. In this range, liquid bridges among soil particles break down (Radhakrishna et al., 1980). The highest \( \rho_{\text{soil}} \) was eventually measured at the fully dry condition. High-resolution images at the fully dried condition consisted of soil particles and air phases, which led to the highest \( \rho_{\text{soil}} \). Moreover, a knee point of SWCC where moisture exists as adsorbed films (McQueen and Miller, 1977) was located near the critical degree of saturation of...
TRDC. Therefore, existence of liquid phases including liquid bridges was a significant factor for the resulting \( \rho_{\text{soil}} \) in terms of degree of saturation.

2. As soil particle size \((D_{10} \text{ and } D_{50})\) increased, \( \rho_{\text{soil}} \) decreased most notably at the fully dried condition. This was because larger heat transfer elements were provided as particle size increased at the target void ratio of 0.6 (e.g., SP and SM of Fig. 4.5). On the other hand, smaller particle size of sandy soil, such as SM, consisted of more tortuous void structures, as well as smaller heat transfer elements.

3. Soil thermal resistivity increased with fines content. At the same targeting void ratio, soils that had high fines content, such as SM2 of 19.87\%, consisted of small heat transfer elements and a tortuous void structure compared to soils that had low fines content, such as SP5 of 1.14\%. Size of heat transfer elements and voids affected \( \rho_{\text{soil}} \). For example, thermal resistivity of SM2 at dried, critical, and saturated condition were 259.7 \( ^\circ \text{C} \cdot \text{cm}/\text{W} \), 78 \( ^\circ \text{C} \cdot \text{cm}/\text{W} \), and 47.3 \( ^\circ \text{C} \cdot \text{cm}/\text{W} \), respectively. Thermal resistivity of SP5 at dried, critical, and saturated condition were 257.1 \( ^\circ \text{C} \cdot \text{cm}/\text{W} \), 67 \( ^\circ \text{C} \cdot \text{cm}/\text{W} \), and 42.7 \( ^\circ \text{C} \cdot \text{cm}/\text{W} \), respectively.

4. In terms of soil types, there was no statistical significance among TRDCs obtained from the laboratory tests regardless of the three analysis conditions.

5. Increase of soil density based on \( C_u, \gamma_{d_{\text{max}}}, e_{\text{max}}, \text{ and } e_{\text{min}}, \) which indicated closer contacts and reductions of air phases among soil particles, led to a decrease in \( \rho_{\text{soil}} \).

6. Soil thermal resistivity decreased with increase of quartz content because quartz had the lowest thermal resistivity among common mineralogies of sand.

7. Particle sphericity and roundness affected the TRDCs. Soil thermal resistivity increased as sphericity of particles changed from the spherical particle shape to the prismoidal particle shape. Soil consisting of prismoidal particle shapes are packed
via a more-complicated void arrangement. Results of roundness analyses indicated that $\rho_{\text{soil}}$ increased slightly when particle shape changed from a well-rounded shape to a rounded shape, which was likely caused by adsorbed water on the particle surface.

8. Statistical analyses in terms of correlations among the physical properties were as follows: (a) there was no significance among TRDCs based on soil type, (b) $D_{10}$ was the only significant factor in terms of $\rho_{\text{sat}}$, and (c) $D_{50}$ was only significant factor in terms of $\rho_{\text{dry}}$. The model for $\rho_{\text{sat}}$ and $\rho_{\text{dry}}$ had $R^2$ of 0.364 (36.4%) and 0.516 (51.6%), respectively.

9. Soil thermal resistivity increased slightly as the $\alpha$ and $n$ parameters in the van Genuchten model increased. The correlation between dry thermal resistivity and the $\alpha$ parameter was discordant with other correlations, as well as the laboratory tests. Moreover, the increments of thermal resistivity were small because of similar physical properties of the soil set.

In laboratory testing, thermal resistivity values of the 13 sandy soils at the fully dried condition were significantly affected by soil physical properties. This implies that the physical properties of sandy soil are significant in arid places or shallow subsurfaces where moisture migration frequently occurs by evaporation and infiltration. In contrast, $\rho_{\text{soil}}$ values of the 13 specimens at critical and saturated condition were slightly affected by the physical properties; in other words, degree of saturation was the most significant property on TRDCs of the 13 specimens. Although additional studies with a variety of soils (such as gravel, silt, and clay) are required to investigate the full effect of soil physical properties on the TRDC at partially and fully saturated conditions, this study provides comprehensive analyses of TRDCs of
sandy soils based on laboratory tests, high-resolution images, and statistical analyses including ANOVA and stepwise regression.
REFERENCES


Roth, K. (2012). Soil physics lecture notes V2.2. Institute of Environmental Physics, Heidelberg University.


TABLES
Table 2.1. Average resistivity of soil constituents (Winterkorn 1962)

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Resistivity ($^\circ$C·cm/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz (-)</td>
<td>7.9</td>
</tr>
<tr>
<td>Quartz (+)</td>
<td>14.9</td>
</tr>
<tr>
<td>Quartz</td>
<td>11.0</td>
</tr>
<tr>
<td>Quartz glass</td>
<td>79.0</td>
</tr>
<tr>
<td>Granite</td>
<td>26-58</td>
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<tr>
<td>CaCO$_3$ (+)</td>
<td>26.3</td>
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<tr>
<td>Marble</td>
<td>34-48</td>
</tr>
<tr>
<td>Limestone</td>
<td>45</td>
</tr>
<tr>
<td>Ice</td>
<td>45</td>
</tr>
<tr>
<td>Sandstone</td>
<td>50</td>
</tr>
<tr>
<td>Dolomite</td>
<td>58</td>
</tr>
<tr>
<td>Slate</td>
<td>67</td>
</tr>
<tr>
<td>Water</td>
<td>165</td>
</tr>
<tr>
<td>Mica (+)</td>
<td>170</td>
</tr>
<tr>
<td>Air</td>
<td>4,000</td>
</tr>
</tbody>
</table>

Note: ‘+’ is perpendicular to crystallographic axis and ‘-’ is parallel to crystallographic axis
Table 3.1. Properties of the 13 soil specimens

<table>
<thead>
<tr>
<th>Sample</th>
<th>USCS(^a)</th>
<th>USCS</th>
<th>(D_{50}) (^b) (mm)</th>
<th>(D_{10}) (^b) (mm)</th>
<th>(C_u) (^a)</th>
<th>(C_c) (^a)</th>
<th>Fines (%)</th>
<th>(G_s) (^c)</th>
<th>(\gamma_{d,max}) (^d) (kN/m(^3))</th>
<th>(e_{min}) (^e)</th>
<th>(e_{max}) (^e)</th>
<th>Roundness</th>
<th>Sphericity</th>
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<td>0.18</td>
<td>2.39</td>
<td>1.01</td>
<td>0.57</td>
<td>2.65</td>
<td>17.89</td>
<td>0.49</td>
<td>0.82</td>
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<tr>
<td>SP2</td>
<td>SP</td>
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<td>0.175</td>
<td>1.83</td>
<td>0.94</td>
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<td>16.28</td>
<td>0.55</td>
<td>0.73</td>
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<td>0.64</td>
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<tr>
<td>SP3</td>
<td>SP</td>
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<td>0.115</td>
<td>3.39</td>
<td>1.63</td>
<td>4.75</td>
<td>2.68</td>
<td>18.61</td>
<td>0.46</td>
<td>0.77</td>
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<td>2.66</td>
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<td>0.64</td>
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<td>1.10</td>
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<td>0.97</td>
<td>1.77</td>
<td>2.65</td>
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<td>0.43</td>
<td>0.66</td>
<td>1.08</td>
<td>0.43</td>
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<td>1.32</td>
<td>7.36</td>
<td>2.70</td>
<td>16.60</td>
<td>0.47</td>
<td>0.72</td>
<td>1.13</td>
<td>0.53</td>
<td></td>
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<tr>
<td>SW1</td>
<td>SW</td>
<td>0.62</td>
<td>0.13</td>
<td>7.54</td>
<td>1.02</td>
<td>3.13</td>
<td>2.66</td>
<td>19.53</td>
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<td>1.12</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>SW-SM2</td>
<td>SW-SM</td>
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<td>0.078</td>
<td>8.72</td>
<td>1.43</td>
<td>9.12</td>
<td>2.68</td>
<td>19.39</td>
<td>0.34</td>
<td>0.64</td>
<td>1.13</td>
<td>0.61</td>
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<tr>
<td>SW-SM3</td>
<td>SW-SM</td>
<td>0.51</td>
<td>0.10</td>
<td>7.00</td>
<td>1.04</td>
<td>7.82</td>
<td>2.72</td>
<td>18.46</td>
<td>0.35</td>
<td>0.59</td>
<td>1.12</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>SM1</td>
<td>SM</td>
<td>0.11</td>
<td>0.049</td>
<td>2.55</td>
<td>1.12</td>
<td>20.79</td>
<td>2.68</td>
<td>16.27</td>
<td>0.51</td>
<td>0.86</td>
<td>1.10</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>SM2</td>
<td>SM</td>
<td>0.13</td>
<td>0.05</td>
<td>3.20</td>
<td>1.01</td>
<td>19.87</td>
<td>2.75</td>
<td>16.38</td>
<td>0.55</td>
<td>0.81</td>
<td>1.13</td>
<td>0.48</td>
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</tbody>
</table>

\(^a\)USCS=Unified Soil Classification System (ASTM D 2487); \(C_u\)=coefficient of uniformity; \(C_c\)=coefficient of curvature.

\(^b\)Particle size determined by ASTM D 422: \(D_{50}\)=particle diameter at 50% finer; \(D_{10}\)=particle diameter at 10% finer.

\(^c\)Specific gravity \((G_s)\) determined by ASTM C 127 and ASTM D 854.

\(^d\)\(\gamma_{d,max}\)=maximum dry unit weight.

\(^e\)\(e_{max}\)=maximum void ratio; \(e_{min}\)=minimum void ratio.

Note: \(e_{min}\) determined by ASTM D 4253, \(e_{max}\) determined by ASTM D 4254, and \(\gamma_{d,max}\) determined by ASTM D 698 (standard Proctor).
Table 3.2. Crystalline mineralogy of the 13 soil specimens

<table>
<thead>
<tr>
<th>Sample</th>
<th>Quartz (%)</th>
<th>Albite (%)</th>
<th>Anorthite (%)</th>
<th>Hematite (%)</th>
<th>Calcite (%)</th>
<th>Dolomite (%)</th>
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<tbody>
<tr>
<td>SP1</td>
<td>97.2</td>
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<td>2.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>SP2</td>
<td>82.7</td>
<td>0.0</td>
<td>3.8</td>
<td>0.0</td>
<td>0.0</td>
<td>13.5</td>
</tr>
<tr>
<td>SP3</td>
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<td>0.0</td>
<td>11.9</td>
<td>0.0</td>
<td>0.0</td>
<td>8.2</td>
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<td>SP4</td>
<td>77.4</td>
<td>0.0</td>
<td>5.3</td>
<td>0.0</td>
<td>1.9</td>
<td>15.4</td>
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<td>14.7</td>
<td>1.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>SP6</td>
<td>69.1</td>
<td>3.5</td>
<td>26.4</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>SP7</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>SP-SM8</td>
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<td>35.0</td>
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<td>10.7</td>
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<td>34.3</td>
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<tr>
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<td>0.0</td>
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<td>SM2</td>
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<td>6.4</td>
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<td>0.0</td>
<td>40.9</td>
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* Percentages shown for the crystalline component and does not represent the amorphous content of the total specimen.
Table 3.3. Parameters for statistical analyses

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<th>Sample</th>
<th>#</th>
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<th>Dependent</th>
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<td></td>
<td>ρ_{dry}</td>
<td>ρ_{crit}</td>
<td>ρ_{sat}</td>
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<td>SP1</td>
<td>1</td>
<td>214.7</td>
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<tr>
<td>SP2</td>
<td>2</td>
<td>178.0</td>
<td>63.0</td>
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<tr>
<td>SP3</td>
<td>3</td>
<td>172.0</td>
<td>64.0</td>
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<td>SP4</td>
<td>4</td>
<td>231.6</td>
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<td>SP5</td>
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<td>67.0</td>
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<td>149.2</td>
<td>61.0</td>
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<td>SP7</td>
<td>7</td>
<td>170.8</td>
<td>60.0</td>
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<tr>
<td>SP-SM8</td>
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<td>64.0</td>
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<td>157.1</td>
<td>58.5</td>
</tr>
<tr>
<td>SW-SM2</td>
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<tr>
<td>SW-SM3</td>
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<td>248.4</td>
<td>77.5</td>
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<td>12</td>
<td>306.5</td>
<td>79.0</td>
</tr>
<tr>
<td>SM2</td>
<td>13</td>
<td>259.7</td>
<td>78.0</td>
</tr>
<tr>
<td>Sample</td>
<td>Knee point on SWCCs</td>
<td>Knee point on TRDCs</td>
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</tr>
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<td>---------------------</td>
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<td>0.034</td>
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<td>0.020</td>
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<td>SP-SM8</td>
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<td>0.094</td>
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</tr>
<tr>
<td>SW-SM2</td>
<td>0.201</td>
<td>0.171</td>
<td></td>
</tr>
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<td>SW-SM3</td>
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<td>0.062</td>
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<tr>
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<td>0.251</td>
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<td>SM2</td>
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<td>0.039</td>
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Table 4.2. Results of ANOVA

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<tr>
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<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
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<tr>
<td>Between Groups</td>
<td>10946.757</td>
<td>5473.379</td>
<td>3.599</td>
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<tr>
<td>Within Groups</td>
<td>15207.584</td>
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<tr>
<td>Total</td>
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<tr>
<td>Dry</td>
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<td>Between Groups</td>
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<td>Within Groups</td>
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<td>Total</td>
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<tr>
<td>Critical</td>
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<td>Between Groups</td>
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<td>Within Groups</td>
<td>623.135</td>
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<td>Total</td>
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<td>Saturation</td>
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<td>Within Groups</td>
<td>249.909</td>
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<td>Total</td>
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### Table 4.3. Relative Density (%) by Three Different Approaches

<table>
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<tr>
<th>Sample</th>
<th>Based on $\gamma_{d_{\text{max}}}$</th>
<th>Based on $e_{\text{min}}$</th>
<th>Based on $e_{\text{min}}$ and $e_{\text{max}}$</th>
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</thead>
<tbody>
<tr>
<td>SP1</td>
<td>90.8</td>
<td>93.1</td>
<td>66.7</td>
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<tr>
<td>SP2</td>
<td>100.9</td>
<td>96.9</td>
<td>72.2</td>
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<td>SP3</td>
<td>88.3</td>
<td>91.3</td>
<td>54.8</td>
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<td>SP4</td>
<td>89.9</td>
<td>87.5</td>
<td>25.9</td>
</tr>
<tr>
<td>SP5</td>
<td>99.2</td>
<td>94.4</td>
<td>62.5</td>
</tr>
<tr>
<td>SP6</td>
<td>96.2</td>
<td>91.1</td>
<td>43.5</td>
</tr>
<tr>
<td>SP7</td>
<td>91.6</td>
<td>89.4</td>
<td>26.1</td>
</tr>
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<td>SP-SM8</td>
<td>99.7</td>
<td>91.9</td>
<td>48.0</td>
</tr>
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<td>SW1</td>
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<td>89.1</td>
<td>51.5</td>
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<td>SW-SM2</td>
<td>92.2</td>
<td>91.2</td>
<td>56.7</td>
</tr>
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<td>SW-SM3</td>
<td>90.3</td>
<td>85.4</td>
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<td>SM1</td>
<td>101.0</td>
<td>94.4</td>
<td>74.3</td>
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<td>102.9</td>
<td>96.9</td>
<td>80.8</td>
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### Table 4.4. van Genuchten parameters for the 13 SWCCs

<table>
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<tr>
<th>Specimens</th>
<th>α (kPa)</th>
<th>n</th>
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<td>SP1</td>
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</tr>
<tr>
<td>SP3</td>
<td>0.4110</td>
<td>22.1263</td>
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<tr>
<td>SP4</td>
<td>1.1521</td>
<td>22.1153</td>
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<td>SP5</td>
<td>0.2799</td>
<td>22.1211</td>
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<td>SP6</td>
<td>0.7120</td>
<td>22.1199</td>
</tr>
<tr>
<td>SP7</td>
<td>0.5518</td>
<td>22.1190</td>
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<td>SP-SM8</td>
<td>0.2358</td>
<td>22.1182</td>
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<tr>
<td>SW1</td>
<td>0.5780</td>
<td>22.1210</td>
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<td>SW-SM2</td>
<td>0.4529</td>
<td>22.1208</td>
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<td>SW-SM3</td>
<td>0.5262</td>
<td>22.1201</td>
</tr>
<tr>
<td>SM1</td>
<td>0.1485</td>
<td>22.1186</td>
</tr>
<tr>
<td>SM2</td>
<td>0.1736</td>
<td>22.1135</td>
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</table>
Fig. 2.1. Heat transfer mechanisms in soils, with size of heat transfer arrow indicating quantity of heat transfer: (a) heat transfer in soil systems, (b) loose soil, and (c) dense soil
Fig. 2.2. Resistivity versus moisture content for several size ranges of crushed quartz sand at various dry densities (Winterkron, 1962)
Fig. 2.3. Heat transfer in a coarse-textured porous medium: (a) fully dried condition, (b) thin films on the particle surface, (c) liquid bridges between particles, and (d) fully saturated condition (after Roth, 2012)
Fraction of voids filled with water - M
Resistivity vs. moisture: thermal sand

(a)
Fig. 2.4. Correlation between soil thermal resistivity and soil density: (a) soil density (Winterkron, 1962), (b) porosity (Winterkron, 1962), and (c) void ratio (Campbell, 2006)
GAZZANA – PRIAROGGIA
A Simple Test to Determine the Possibility of Moisture Migration on Cable Backfill Materials, ICC Minutes, Apr 1971, App IIIA.


Westinghouse Laboratories

Insulated Conductors Committee Minutes, November 1974

Insulated Conductors Committee Minutes, November 1975

Gazman, Gazzana and Priaroggia.
Fig. 2.5. Correlation between soil thermal resistivity and soil type: (a) (Salomone et al., 1979), (b) (IEEE Std 442, 1981, reaffirmed 1996), and (c) (Campbell, 2006)
Fig. 2.6. Soil thermal resistivity affected by temperature: (a) three constituents of sand (Brandon and Mitchell, 1989), (b) thermal resistivity of surge sand (Brandon and Mitchell, 1989), and (c) thermal resistivity of loam soil at three different temperatures (Campbell, 2006)
Fig. 2.7. (a) Typical thermal conductivity dry-out curve and soil-water characteristic curve (Smits et al., 2010) and (b) soil-water characteristic curve and thermal resistivity dry-out curve (Salomone and Kovacs, 1984)
Fig. 2.8. Influence of dry density on critical moisture content for AMRL silty clay
(Salomone and Kovacs, 1984)
Fig. 3.1. Locations and origins of the 13 soil specimens
Fig. 3.2. Grain-size distribution curves for the 13 soil specimens
Fig. 3.3. Range of void ratio
Fig. 3.4. Modified hanging column apparatus: (a) schematic and (b) photo
Fig. 3.5. Three points of comparison adapted from Salomone and Kovacs (1984)
Fig. 4.1. (a) Thermal resistivity dry-out curves and (b) TRDCs near critical and dried condition
Fig. 4.2. Soil-water characteristic curves
Note: pF is forces retaining the moisture.
Fig. 4.3. (a) Definition diagram for method of approximating soil moisture characteristics from limited data (McQueen and Miller, 1974) and (b) regimes of TRDC and SWCC
Point of origination of hysteretic loop
Fig. 4.4. Hysteresis of TRDC: (a) SP3 and (b) SW-SM3
Fig. 4.5. High-resolution images: (a) SP5 and (b) SM2

Note: Red arrows indicate heat transfer based on thermal resistivity of the three phases.
(a)
Fig. 4.6. (a) Comparisons of $\rho_{\text{oven-dry}}$ and $\rho_{\text{dry}}$ and (b) plotting of oven-dry thermal resistivity and dry thermal resistivity.
Fig. 4.7. Correlation between particle size and soil thermal resistivity: (a) $D_{10}$ and (b) $D_{50}$
Fig. 4.8. Correlations between fines content and soil thermal resistivity
(a)
Fig. 4.9. Effect of soil types on TRDC

(b) (IEEE Std 442, 1981, reaffirmed 1996)
Fig. 4.10. Correlations between the parameters related to density and soil thermal resistivity: (a) $\gamma_{d_{\text{max}}}$, (b) $e_{\text{min}}$, and $e_{\text{max}}$, and (c) $C_u$. 
Fig. 4.11. Correlations between soil thermal resistivity and relative density: (a) based on $\gamma_{d_{\text{max}}}$, (b) based on $e_{\text{min}}$, and (c) based on $e_{\text{min}}$ and $e_{\text{max}}$
Fig. 4.12. Correlations between quartz content and soil thermal resistivity
Fig. 4.13. Modified visual comparison chart for estimating roundness and sphericity
(Powers, 1982; Alsaleh, 2004)
Fig. 4.14. Correlations between particle shape and soil thermal resistivity: (a) sphericity and (b) roundness.
Fig. 4.15. Correlations between van Genuchten’s parameters and soil thermal resistivity:
(a) $\alpha$ and (b) $n$