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Curriculum Project Statement

The purpose of this project is to provide mathematics teachers with a ready-to-use-activity-based probability and counting principles component of the statistics curriculum, which will help students better understand the mathematical concepts and ideas of probability and counting principles. Unlike traditional problems, most of which are designed to assess and review content mastery and tend to be passive activities, these activities actively engage students in learning. This curriculum invites students to explore counting principles through their own self-discovery, with real-life activities and without having to know or understand the mathematical rules and formulas first. Students will feel connected to their mathematics and be able to formulate the mathematics for themselves with a more nontraditional environment, in particular, the counting formulas that are a part of the statistics curriculum.

Statistics courses involve many formulas that require students to follow seemingly abstract rules. The students memorize the formulas and have no idea what they are doing or why. This can easily frustrate students and turn them off, especially when dealing with counting principles that arise in statistics courses. Students struggle to learn them and teachers have a difficult time teaching them. Textbooks offer an introduction to a formula, then give the formula and provide examples in which to use the formula. This does not allow for students to understand or formulate the mathematics. Therefore, the student only understands how to use the formula but does not comprehend the mathematics behind the formula.

Permutations and combinations can be difficult topics for students to understand and to solve without a clear conceptual understanding. Most of the time students are given the formal mathematics representation without being able to think about these problems sufficiently critically. For example, most students can analyze a problem and understand whether it involves
permutations or combinations. They are able to determine whether objects should be arranged in order for a permutation or whether objects are indistinguishable for a combination. However, they cannot solve simple problems involving permutations or combinations because they cannot identify which situation they’re dealing with. In addition, students complain that math is boring or that they have already seen this material before, and they struggle with the basic facts and concepts.

This project’s main goal is to help students learn and understand counting principles through activities. I believe that activities are a way to teach students concepts in a fun and more casual environment. Furthermore, I believe that through the use of activities students will learn the concepts, and gain confidence and motivation. Also, they will heighten their math skills by seeing the connections between math and the real world. Trying to find activities that are skill level appropriate and interesting, is however, extremely time-consuming. This project is an attempt to provide a number of classroom-ready activities for the teacher. A teacher may use the activities at his/her disposal, for an introductory activity, or for reinforcement of concepts already presented. The activities may be assigned as homework, or as a classroom activity. The students may work individually or in groups. Each activity is written as a worksheet which guides the student through the activity. There may need to be teacher interaction, help, or extensions, but that is described within the curriculum.

Osceola High School did not offer statistics until the spring of 2008. The course Osceola High School offered for students who had completed Algebra II and wanted to take another math course, but who did not want, nor need, pre-calculus, was called Algebra III. The purpose of Algebra III was to be similar to pre-calculus, just at a slower pace, continuing with functions from Algebra II and involving trigonometry. However, the student expectation of this course
was that of an ‘easy’ course. In other words, student expectations were quite different from that of the teacher.

There was a perception by our students of Algebra III, which is a credited college course, that it was an easy way to earn one’s fourth credit of math as an alternative to the more difficult pre-calculus. The students in my Algebra III courses would become frustrated when the material became difficult, despite my repeated warnings of, “this not an easy math course.” More than one student told me they were in Algebra III just because they wanted to avoid pre-calculus.

The administration and the mathematics department discussed our options and decided to discontinue our Algebra III and replace it with a semester of Trigonometry and a semester of Statistics and Probability. Our school supports curriculum development that is congruent with our needs, including an approach and a rationale for enhancing and developing students as holistic learners. For these reasons, we have chosen to have statistics and probability in the second semester in order to give our students more options. Any student in pre-calculus or calculus may switch into statistics and probability after the first semester if they no longer want to remain in their respective courses. Also, students may take statistics in addition to their respective calculus course.

Once we decided to discontinue algebra III, statistics became a clear choice for our new direction. We have become aware that colleges have added statistics as a graduation requirement. To reflect the growing demand in many disciplines, especially the sciences and business, universities around the world require their students to be literate in statistics (Peiris, 22). High schools are predominately assigned to prepare students for the next level of life, whether that is the workforce or college. Indeed, in order to prepare students for college, our
natural new direction leads to statistics. Scheaffer comments that, “making sense of data and dealing with uncertainty are skills essential to being a wise consumer, an enlightened citizen, and an effective worker or leader in our data-driven society” (Scheaffer, 56). Statistics is also a natural choice for those students preparing for the workforce directly after high school. Most often, statistics in high school mathematics is either embedded as part of algebra or set up as separate units in an integrated mathematics program. Peiris makes it clear that there is a need to strengthen research into statistics education since statistics is becoming more and more relevant in much of the workforce that is involved in the decision making processes, while minimizing the uncertainty in the options that are available to them (Peiris, 22). The ever growing presence of statistics across disciplines is playing out at top universities too: “A survey of offerings at the University of Minnesota, for example, turned up over 160 statistics courses—of which 40 were introductory—taught in 13 departments” (Garfield, 44). The process of collecting, organizing, and displaying data has applications to any occupation.

The curriculum at Osceola High School continues to pose challenges despite recent innovations. It is quite common for Osceola students to take trigonometry and statistics just to avoid pre-calculus, and that student expectations will not match teacher expectations in that course. However, now, when students at Osceola take the trigonometry and statistics sequence, they expect it to be rigorous. Moreover, the Osceola faculty feels that students will be better prepared with a better understanding of what the course entails, and the students at Osceola are better prepared for their next stage of life.
Literature Review

Even though the concept of counting seems extremely elementary, students have a difficult time understanding these concepts and teachers have a difficult time teaching them. Teaching itself is increasingly being recognized as a complex and multifaceted product of many known and unknown variables. Because of these unknown variables, teaching and learning statistics can be a very sour experience for many teachers and learners. The teaching of statistics has been the least emphasized component of mathematics. This may be due to lack of teacher preparation on the content and pedagogy, lack of good instructional materials, or possibly a lack of emphasis on standardized tests. In a high school curriculum, most often statistics is integrated into an algebra course, or into separate units in multiple courses where teachers may or may not implement the statistics units. Statistics is the key in the information age in making decisions and students tend to be interested in data on practical problems that relate to their lives. Scheaffer adds, “Youngsters and adults alike are confronted daily with situations involving statistical information. Making sense of data and dealing with uncertainty are skills essential to being a wise consumer, an enlightened citizen, and an effective worker or leader in our data-driven society” (Scheaffer, 56).

A set curriculum and a textbook series are very helpful in building a new course; however, they should not be the main focus of the teacher. Teachers must adopt and develop their own teaching style gained from personal experience. Once that is the goal, a teacher may begin to motivate students and educate them while taking into account the varying ability of the students, which helps everyone.
In a subject like statistics, what Peiris describes “as the first step in the teaching of any discipline” is not an easy one: “An optimal transmission of knowledge between the teacher and his/her student” (Peiris, 22). However, the importance of achieving this transmission of knowledge in statistics is very important, as there is a growing movement to introduce elements of statistics and probability into the secondary school curriculum “as part of the basic literacy in mathematics that all citizens in today’s world should have” (Garfield, 44). Moreover, data handling and chance are now integral components of the mathematics curriculum. However, they require some different approaches from both teachers and learners from other components of the curriculum. Indeed, “in reports concerning the curriculum, the Conference Board of the Mathematical Sciences described elementary data analysis and statistics as ‘more important’ than current advanced mathematics topics and recommended that these topics be included as early as the middle school curriculum” (Garfield, 44-45). Schools may need to begin to integrate statistics into their curriculum as early as the sixth grade. If so, the integration must be more than one unit per course or grade. The students will need the repetition of the material for the development of their mathematics knowledge. Kun elaborates: “Curriculum was a major key of educational management since it was determiner of every guideline relating to student development as an important instrument for specifying the educational future as well as directing device in national growth” (Kun, 450).

To achieve this optimal transmission of knowledge, the teacher must not only be knowledgeable in the subject matter, but also have the ability to adapt to each student’s different needs with differentiated instruction. Garfield analyzes the obstacles behind this as follows:

At any level, students appear to have difficulties developing correct intuition about fundamental ideas of probability for at least three reasons. First, many students have
an underlying difficulty with rational number concepts (i.e. fractions, decimals, and percents) and proportional reasoning, which are used in calculating, reporting, and interpreting probabilities. Second, probability ideas often appear to conflict with students’ experiences and how they view the world. Third, many students have already developed a distaste for probability through having been exposed to its study in a highly abstract and formal way” (Garfield, 47).

It is critical to understand that there are issues in teaching probability and statistics that are unique to these disciplines, namely, that the concepts of statistics are developed based on randomness or uncertainty, “which create difficulties in general quantitative thinking and hypothetical reasoning” (Garfield, 58). Students may not comprehend these difficulties, but they do comprehend that mathematics is difficult. Some students are therefore afraid of mathematics. To overcome a student’s ‘math-phobia’ the teacher must encourage a student’s interest and curiosity of the subject and convince them their learning is important and useful in their possible careers. A few ways to overcome the phobia is to i) introduce topics through activities and simulations, as opposed to theoretical generalizations, ii) promote the feeling that mathematics relates to the real world not just symbols and rules, iii) teach descriptive statistics first without probability, and iv) point out misuses of statistics, as in the media and advertisements.

Garfield explains, “The experience of most college faculty members in education and the social sciences is that a large proportion of university students in introductory statistics courses do not understand many of the concepts they are studying” (Garfield, 46). In context with these challenges of how students learn probability and counting, there are three key questions:

a) What conceptions of probability do children of various ages have?

b) How might these conceptions be changed?
c) Are there optimum teaching and learning techniques? (Garfield, 54)

As stated earlier, the concept of counting seems elementary, however students struggle with counting principles, especially with counting multiple objects at a time as in permutations and combinations. This curriculum is attempting to help students, and possibly other teachers, with this struggle. One attempt to bridge this gap is to incorporate real-life problems.

In particular, curriculum should be designed to support students in constructing their own mathematical ideas and connections. Students should solve problems, communicate ideas both orally and in writing, engage in mathematical reasoning, and search for mathematical connections. Today’s students demand that their lessons be real, interesting, relevant, and manageable. When students are taught only the permutation and combination formulas, they use the formulas in a mechanical way with little understanding. In addition, students have memorized the formulas only to forget them without being able to solve problems with counting principles. Busadee found that students who were taught to think about probability through sports—that is, actual settings before formal probability calculation—have gained a better understanding of the material and have retained it longer (Busadee, 373). To help guide a productive mathematical discussion of activities, the teacher should ask such questions as what did given group do to solve this problem? Can anyone find a counterexample for this conjecture? How do you know that this formula always works? Why is this outcome the answer? Is this result consistent with what we found earlier? Does anyone have another way of explaining or showing this result? Students can and will make good mathematical arguments if they are expected to do so.
Researchers at Mahidol University in Thailand, Busadee, Panijpan, Laosinchai, and Ruenwongsa created an experiment for a statistics course with four different inquiry-based instructional units on permutations and combinations. The four units were a traditional unit, a nontraditional word problem unit, a sport problem unit, and a probabilistic game unit. Four concepts were incorporated into each unit: linear permutations, permutations of similar things, circular permutations, and combinations. Linear permutations and circular permutations count the number of ways objects can be arranged in a line or a circle, respectively. Permutations of similar things, or sometimes call distinguishable permutations counts the number of ways one can arrange objects in a line where some objects are repeated, i.e. how many ways can the letters in the word “probability” be arranged? Combinations are permutations without regard to order.

The traditional unit was created by collecting exercises, examples, and practice problems from national standard textbooks, exercise books and educational websites, while the other units were all conceived by the researchers. In the nontraditional word problem unit the emphasis was on real-life problems designed to engage the students’ interest. For example, one such word problem was to ask students to find the meaningful permutations of the sentence “I do want to go home”; another was based on using DNA-related examples for introducing circular permutations. In the sport problem unit the emphasis was on word problems based on popular sports and sport situations. For example, a problem about lining up 4 different books on a shelf most likely will not spark the students’ interest as could a 4 x 100 relay team with counting or listing the possibilities of selecting the first to fourth baton and its prospects of winning or losing a race. In the probabilistic game unit the emphasis was on games of chance.

The project of the Thai group was an eight week intervention implemented in the four classes. One was randomly assigned to be the control group (traditional); the other three served
as the treatment groups (nontraditional, sport, and game). They were all taught by the same researcher using the inquiry approach. All the students were to learn the same concepts in permutations and combinations and to take the same tests - pre, post, comprehensive, and retention. During the learning activities, the students worked collaboratively in groups of five to share ideas and then to discuss and to derive the mathematical formulas from the given problems. After each concept, the teacher debriefed the students, and provided them with an opportunity to reflect on what they learned.

Their overall results were quite interesting. The nontraditional word problem unit and the traditional unit were the tops in all the test categories. The nontraditional word problems unit scored slightly higher on the pretest, posttest and comprehensive test than all the other units. The traditional unit scored slightly higher on the retention test then all the other units, which meant they had the smallest percentage reduction. The probabilistic game unit was higher than the sport problem unit on the comprehensive test, but lower than the pretest and posttests.

Of the four concepts covered on the posttest, the traditional unit scored the highest for linear permutations and permutations of similar things, while the sport problem unit scored the highest for circular permutations, and the nontraditional word problem unit scored the highest for combinations. Of the four concepts on the comprehensive test, the nontraditional word problem unit scored the highest for linear permutations, permutations of similar things, and combinations, while the traditional unit scored the highest for circular permutations.

Each student was administered a questionnaire at the end of his or her unit. They were asked to rate their unit on a 7-point scale [-3, 3] questionnaire and write comments/suggestions. All groups were highly satisfied with their instructional units; the average for each unit was
positive and all higher than 1.68. The highest, 2.34, was received by the traditional unit, then the nontraditional word problem unit, probabilistic game unit and sport problem unit respectively.

The researchers used an inquiry approach to their teaching. They thought this would provide a more meaningful and effective way for students to learn. They thought that students can be provided the opportunity to explore and experience a challenge to their own way of thinking. As stated in their research, “Asking and posing questions are the heart of the inquiry approach to learning” (Busadee, 414). With that, the learning in general is enhanced when the process of asking questions is encouraged.

Since the inquiry approach is based on asking and answering questions, the teacher must develop strong questions and/or problems that are applicable to each lesson. This will allow students to access the full traits of the inquiry process. Busadee, describes the traits as follows: “connecting former knowledge and experiences with the problems, designing plans to find an answer to the problem, investigating phenomena through conjecture, and constructing meaning through the use of logic, evidence, and reflection” (Busadee, 415).
Curriculum Project Summary

This curriculum project was created for a statistics and probability course taught at a rural high school. This project covers the probability and counting unit that is a two – three week unit in the 18 week semester course. The main focus of the project was on the counting principles that accompany this unit. These lessons are given in the curriculum and Appendix A. They were designed to be used as the standard classroom instruction and the worksheets were to be assigned as in-class activities and finished for homework and were assigned in the order in which they are presented. Students were able to offer their opinion on the overall lessons through a survey given at the end of the curriculum.

This project really begins with the Fundamental Counting Principle. This project has students count the number of license plates in a given state and zip codes. This leads them into the factorial process, beginning with the game of Sudoku, i.e. how many ways can you arrange nine numbers without repeats? Then students are given a problem about taking a photograph with their friends and how many different photos are there if they stand in different orders. Changing how many friends are there will lead students into finding the number of different arrangements of \( n \) people (\( n! \)). Continuing with the photograph theme, now the problem becomes a photograph of four people, however, there will only be three people in the photo at a time. This leads students into the permutation notation of \( _nP_r \), but does not introduce the permutation formula just yet, i.e. \( _4P_3 = 4*3*2 \) and \( _{10}P_4 = 10*9*8*7 \). Changing the concept just slightly, is to count without regards to order, i.e. how many committees of three can be formed from five people? This will introduce students to combinations and will be the first comparison between a combination and permutation.
The project has three more activities in which students discover the connection/difference between a permutation and combination. The activities compare the different arrangements of wallpaper possibilities, the arrangements of people in a track relay, and the arrangements of possible outcomes of a soccer match that ended in a score of 3-2. In addition, students discover the permutation and combination formulas and are given problems with repetition and problem with nontraditional word problems.

I am most proud of lesson eight on soccer scoring. It involves many of the concepts from the previous activities as well as a great comparison of permutations and combinations. Students receive repetition and an eye opener how a combination is the same as a permutation just divided by an additional \( r! \).

There were two sections of the statistics and probability course of 25 students each. The classes were a part of a modified block schedule. Every Monday, Tuesday and Friday are eight period days, and Wednesday and Thursday were on a block schedule with a four period day on alternating five day weeks. If school were not in session for a full five day week, Wednesday and Thursday would be an eight period day.

As a result of the counting principles curriculum, the 2013 cohort had the highest scores on the chapter 3 test out of any other year. Each cohort was given the exact same chapter 3 test and 2013 had the best average test score followed by the 2011, 2012, 2009, and 2010 cohorts respectively. Table 1 displays the average scores and standard deviations from each year, and all of the specific data can be found in the appendix A6. Compared to the similar project mentioned in my literature review, my project shows student improvement and comprehension with the increased use of nontraditional word problems. Given that my data reflects a small size with
various outside factors, I conclude that while it shows success, there are improvements that can still be made.

Table 1: Chapter 3 Mean Test Scores and Standard Deviations.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean</th>
<th>SD</th>
<th>Max/Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>23.7</td>
<td>5.55</td>
<td>29.3/ 18.2</td>
</tr>
<tr>
<td>2012</td>
<td>18.9</td>
<td>5.83</td>
<td>24.7/ 13.1</td>
</tr>
<tr>
<td>2011</td>
<td>22.8</td>
<td>4.86</td>
<td>27.7/ 17.9</td>
</tr>
<tr>
<td>2010</td>
<td>17.2</td>
<td>6.55</td>
<td>23.8/ 10.7</td>
</tr>
<tr>
<td>2009</td>
<td>17.9</td>
<td>4.82</td>
<td>22.7/ 13.1</td>
</tr>
</tbody>
</table>

Overall, students seemed to respond favorably to the activities. As expected, some students found the activities confusing or difficult and therefore not very helpful, but the majority of students enjoyed the activities. I found the activities very helpful from a teaching standpoint. When conducting a classroom activity, it becomes readily apparent when students do not comprehend something. I could gain a sense of how well or poorly an activity was going and could quickly and easily make adjustments. Also, the worksheets gave students opportunities to speak up and ask for help when they had any problems or questions.

While most students enjoyed the activities, some students did struggle. They either did not know how to get started, or became frustrated when they did not know what to do next, especially given that the worksheets were designed to make the students think critically and discover the formulas on their own. Their time was spent trying to overcome their frustration rather than engaging in the mathematics at hand. One problem I think for these students was a lack of familiarity with the content. As stated earlier, statistics and probability many times is pushed to the end of the curriculum or not taught at all. I encouraged students to work in groups
to help with this issue. Students like working with each other to take some pressure off of them knowing that there is another student to turn to for help. Also, being able to bounce ideas off another student and seeing another’s perspective can help solve the with problem solving.

Another issue was absences. There were quite a few students who missed a day or multiple days during the project. Some students were able to catch up right away, and others struggled once they were behind. However, the students who caught up were either able to handle the material on their own or they spent time receiving extra help before or after school, or during homeroom. The students who struggled did not come in for extra help outside of their regular class period. Also, I was forced to miss quite a few days due to my daughter being sick. This caused some confusion and struggles when I handed out worksheets and was not there to teach or give a lecture on the material required for that particular worksheet.

Another issue was to determine the appropriate amount to time each lesson required. There were times when I knew my students felt rushed. The hard part was deciding whether they really needed more time, or if they were just procrastinating. As this was the first time I had ever presented this material, I fought this battle every day.

The results of the students’ satisfaction with the counting principles curriculum are shown in Table 2. The results from the 7-point rating scale [-3, 3] illustrated that the majority of students were highly satisfied with the counting principles curriculum. The highest satisfaction was seen in their understanding of combinations and permutations and knowledge of how to use their formulas. They agreed that the curriculum helped them develop the conceptual understanding of permutations and combinations. I was especially pleased to see that the students who have already taken pre-calculus, and therefore have worked with permutations and
combinations prior, gave a high rating for the counting principles curriculum deepening their understanding of permutations and combinations.

Table 2: Scores of Students’ Satisfaction and Understanding.

<table>
<thead>
<tr>
<th>Questions for all students</th>
<th>Mean Score [-3,3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.) My self-assessment of my knowledge of permutations and combinations.</td>
<td>0.8</td>
</tr>
<tr>
<td>2.) I felt the counting principles worksheets were beneficial to my knowledge.</td>
<td>1.2</td>
</tr>
<tr>
<td>3.) The worksheets gave me a deep understanding of permutations and combinations.</td>
<td>0.5</td>
</tr>
<tr>
<td>4.) I found the counting principles worksheets were helpful.</td>
<td>1.1</td>
</tr>
<tr>
<td>5.) I understand what a permutation (nPr) and a combination (nCr) mean and know how to calculate them.</td>
<td>1.6</td>
</tr>
<tr>
<td>6.) I have a conceptual understanding of the nPr and nCr formulas.</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Questions for students who have already taken Pre-Calculus

<table>
<thead>
<tr>
<th>Questions</th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.) I remember permutations and combinations from pre-calculus.</td>
<td>1.2</td>
</tr>
<tr>
<td>B.) I felt the counting principles worksheets deepened my understanding of permutations and combinations.</td>
<td>1.1</td>
</tr>
</tbody>
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The following is a sampling of anecdotal data collected from the survey of students’ response to the statement “Please let me know what you enjoyed or disliked about the counting principles worksheets”:

+ “I liked the repetition of the problems so that the process could stay in my memory.”
+ “I like that they progressively got more complex, explaining it as they went.”
+ “I like how the worksheets built off one another, and when I got stuck I could use the previous ones as a reference.”
+ “I enjoyed the different scenarios for problems.”
+ “I really like the extra practice I got from all the worksheets. I felt like I learned a lot more by taking my time than I would have if I had rushed through a book assignment.”
+ “I like the real-life situations and was able to visualize and connect to the problems.”
+ “I liked them better than textbook assignments.”
  - “Needed more note-taking time/help from you.”
  - “It would have been helpful to be taught the lesson first before doing the worksheet wrong and remembering the wrong process.”
  - “I didn’t like how you made us teach it to ourselves before you showed us how.”
  - “More time on poker combinations.”
  - “There was confusion on the differences between permutations and combinations.”

The results of my project are directly in line with Vygotsky’s learning model of the “Zone of Proximal Development” (ZPD). Vygotsky argued that the best learning occurs when students are pushed just beyond where their current academic comfort level is, giving them enough support to be successful and not feel as though they are doomed to fail. The process of this curriculum is designed to help students find their ZPD instead of always relying on an adult to determine it for them. Instead of giving the students the concepts and ideas myself, my students were trying to discover them on their own first and then have assistance from an adult. Building on Vygotsky’s premise, I was able to create an environment for my students of discovery which allowed an option for failure and then the ability to recover from it. The focus was entirely on growth and to moving along the spectrum to develop the formulas themselves. Students were given support along the way to ensure they stayed on the current level of understanding as the class. The students were pushed beyond their comfort level with the teacher as their safety net if they were pushed too far.
My MSE coursework was very helpful in creating my curriculum project. The Probability and Statistics courses had a direct connection to the curriculum and instruction in my project. I relearned the different counting principles in those two courses. I was able to work through the complicated problems and see my own difficulties. Thinking about my own methods of counting and working with others to see the many different thought processes was the beginning of my ideas for this project.

The other courses were also extremely helpful in my project. Just being a student again; it had been three years since my last college course before my first course in the MSE program. Sitting in a desk listening to teachers lecture, inspire, and connect the mathematics was eye-opening. It was much different being a student again, because I had spent the three intermittent years as a teacher. I really paid attention to the different teaching styles of the professors. Also, all of these courses deepened my understanding of how mathematics is interconnected, allowing me to push my students’ level of understanding farther than I was able to before as I can see what their future learning endeavors require to be successful.

My professional development has greatly benefitted from this curriculum project. I have become increasingly involved in helping my other coworkers with understanding the counting principles and the ways to teach them. My classroom environment has changed. I now teach with more of an inquiry approach, where my students are investigating and asking questions about the mathematics. Students who are in my classroom are encouraged to “break out of their shell” in mathematics, reduce their anxiety and ask questions. I am confident that as I continue to incorporate this curriculum, that I will see an increase in achievement levels and confidence levels in my students.
Curriculum

Lesson One: Basic Concepts of Probability

Objective:
- The student will be able to identify the sample space of a probability experiment
- The student will be able to identify simple events
- The student will be able to create and use a tree diagram

Lesson Description:
This lesson begins with defining the basics of probability (probability experiment, outcomes, sample space, event, and a simple event) with examples of each. Students are given several guided practice examples in order for their comprehension. Next in the lesson is to demonstrate a tree diagram and its uses.

Level: Statistics and Probability

Pre-Learning: There is a prerequisite of Algebra II to take Statistics
Basic Concepts in Probability

1. Define the following concepts.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Probability Experiment:</td>
<td>Example:</td>
</tr>
<tr>
<td>b.</td>
<td>Outcome:</td>
<td>Roll a die</td>
</tr>
<tr>
<td>c.</td>
<td>Sample Space:</td>
<td>Roll a two</td>
</tr>
<tr>
<td>d.</td>
<td>Event:</td>
<td>{1, 2, 3, 4, 5, 6}</td>
</tr>
<tr>
<td>e.</td>
<td>Simple Event:</td>
<td>Roll an even</td>
</tr>
</tbody>
</table>

New Example:
Toss a head and roll a 3
[only one outcome - H3]
Toss a head and roll an even
is not simple
[H2, H4, H6]

2. Make a tree diagram of the following probability experiments.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Tossing a coin and rolling a die.</td>
</tr>
<tr>
<td>b.</td>
<td>Rolling a pair of dice consecutively.</td>
</tr>
</tbody>
</table>

A.)

```
          H |
       /   |
      /     |
     H      |
   /   /   |
  /     /   |
 1 2 3 4 5 6
```

B.)

```
          1 |
       /   |
      /     |
     2      |
   /   /   |
  /     /   |
 1 2 3 4 5 6
```
Lesson Two: Basic Concepts in Counting

Objective:
The student will be able to use the Fundamental Counting Principle to find the number of ways two or more events can occur.

Lesson Description:
This lesson begins with the discussion of counting. From the very basic (counting students) to the more difficult. Learning how to count the number of ways multiple objects can be counted in sequence, i.e. the Fundamental Counting Principle. Begin the lesson with the English Soccer League counting problem. Then, finish the lesson with many guided practice examples, including counting the number of license plates for a given state, and the number of zip codes and telephone numbers.

Level: Statistics and Probability

Pre-Learning: Basic Concepts of Probability
Basic Concepts in Counting

Soccer Scoring
In the English Premier Football League, 3 points are awarded to the winner of each match, 0 points to the loser, and 1 point to each team for a tie. Teams are ranked according to the total number of points accumulated. If a team has played 35 matches and has obtained 81 points, what are the possible results that the team could have had at this stage of the season (win, draw, losses)?

1. What is the Fundamental Counting Principle? When should you use it?

2. A license plate in Wisconsin consists of three letters followed by three numbers. How many possible license plates are there in Wisconsin?

3. A license plate in California consists of one number, followed by three letters and then three numbers. How many possible license plates are there in California?

4. A zip code is a series of five numbers in sequence. How many possible zip codes are there?

5. If a zip code had a five as the first number and there could be no repeats, then how many are possible?
Lesson Three: Photographs

Objective:
The student will be able to find the number of ways to arrange objects in a row.
The student will be introduced to and understand how to use factorial.

Lesson Description:
Start this lesson with one line of Sudoku. Find the number of ways you may arrange nine numbers without repeats, as this will introduce the students to factorial. Work into the photograph activity where students are figuring the ways a group of people can stand in a straight line for a photograph where different orders are counted as different photographs. Have students then generalize their findings with $n$ people.
(You may need to show students that $0! = 1$, but it can be taught in any of the next three lessons)

Level: Statistics and Probability

Pre-Learning: Basic Concepts in Counting
Photographs

1. How many different ways can you arrange a group of four people in a straight line for a photograph? Everyone wants to be in each photograph. Display your results in an organized manner.

2. How many different photographs can be taken if another person joins the group? How many for 6 people? How many for 8 people? Display your results in an organized manner.

3. How many different photographs can be taken with \( n \) people?

Basic Counting

4. Find the number of ways you can have a three-digit code so no number is repeated.

5. How many ways can you rearrange the letters AAAABBC?

6. How many distinguishable ways can you rearrange the same letters?

7. How many distinguishable ways can you rearrange the word PROBABILITY?
Lesson Four: Photographs Extension

Objective:
The student will be able to find the number of ways a group of objects can be arranged in order.

Lesson Description:
Refresh the photograph activity from the factorial lesson. Now have students figure how many ways $r$ people can stand in a line for a photograph out of a group of $n$ people. Then introduce the students to the permutation notation ($nP_r$).

Level: Statistics and Probability

Pre-Learning: Photographs
Photographs Extension

1. How many different photos can a group of 3 people stand in line for a photograph taken of 2 people at a time? List your results in an organized manner.

2. How many different photos can a group of 4 people stand in line for a photograph taken of 3 people at a time? List your results in an organized manner.

3. How many different photos can a group of 6 people stand in line for a photograph taken of 2 people at a time? List your results in an organized manner.

4. How many different photos can a group of 5 people stand in line for a photograph taken of 2 people at a time? List your results in an organized manner.

5. Find a mathematical way to get your answers from problems 1-4 using the Fundamental Counting Principle. Display your results in an organized manner.
6. Use the process you discovered in problem 5 to answer the following (show your work):
   a. How many different photos can a group of 5 stand in line for a photograph of 3 people at a time?

   b. How many different photos can a group of 6 stand in line for a photograph of 4 people at a time?

   c. How many different photos can a group of 10 stand in line for a photograph of 3 people at a time?

   d. How many different photos can a group of 10 stand in line for a photograph of 5 people at a time?

7. First find how many different photos can a group of 20 stand in line for a photograph of 8 people at a time. Then rewrite your formula where \( n \) will replace 20, and \( r \) will replace 8.

8. With your results from problem 7, display how one can find the number of different photos can a group of \( n \) stand in line for a photograph of \( r \) people at a time?
9. Find the following permutations. Show your work.

a. \( 6P_5 \)  

b. \( 12P_4 \)

c. \( 20P_9 \)  

d. \( 20P_{10} \)

e. \( 30P_3 \)  

f. \( 30P_{13} \)
Lesson Five: Committees

Objective:
The student will be able to find the number of ways a group of objects can be arranged without regard order.

Lesson Description:
Begin with the committee activity where students figure out the ways a group of people can form a committee where two people make a committee regardless of how they were chosen. Have students then generalize their findings with n people arranged r at a time. Introduce the combination notation \( \binom{n}{r} \).

Level: Statistics and Probability

Pre-Learning: Photographs
Committees

1. How many different committees of various sizes can be formed from a group of four people? List your results in an organized manner.
   a. Committees of size 4?
   b. Committees of size 3?
   c. Committees of size 1?
   d. Committees of size 2?
   e. Committees of size 0?

2. If another person joins the group, how many committees of sizes 5, 4, 3, 2, 1, and 0 can be formed? List your results in an organized manner.

3. Find the following permutations (show your work):
   a. \(4P_4\)
   b. \(4P_3\)
   c. \(4P_2\)
   d. \(4P_1\)
   e. \(5P_5\)
   f. \(5P_4\)
   g. \(5P_3\)
   h. \(5P_2\)
   i. \(5P_1\)

4. Describe any patterns that you see, and make at least three statements about committees.
Find the following using the pattern. Show your work, you need not list your results.

5. How many different committees can be formed from a group of 6 people into committees of size 4?

6. How many different committees can be formed from a group of 10 people into committees of size 3?

7. How many different committees can be formed from a group of 10 people into committees of size 5?

8. Find the following combinations:
   a. \( \binom{6}{3} \)
   b. \( \binom{7}{4} \)
   c. \( \binom{13}{5} \)
   d. \( \binom{8}{7} \)
   e. \( \binom{20}{6} \)
   f. \( \binom{16}{4} \)
**Lesson Six: Wallpaper / Track Relay**

**Objective:**
The student will be able to find the differences between counting with order (permutation) and counting without regard to order (combination).

**Lesson Description:**
Arrange students into groups. Describe the problem of putting wallpaper into three rooms with four different wallpaper patterns. They are to find the number of ways to wallpaper all three rooms. They should try all three different situations of the rooms, for example, whether they can use the same pattern in more than one room and whether order is important or not.

**Level:** Statistics and Probability

**Pre-Learning:** Committees and Photographs
Wallpaper

You are redecorating the upstairs in your house. There are three rooms upstairs to wallpaper. You have four patterns of wallpaper from which to choose. Once a particular pattern has been selected for a room, that same pattern will be used for the entire room. In how many ways can you wallpaper these three rooms?
(There are different situations below regarding whether you can use the same pattern in more than one room. Answer all three situations.)

1. How many ways can you wallpaper the three rooms if you cannot use the same pattern in more than one room and you feel that the order is important (i.e. ABC is a different way than CAB)? List all the ways below in an organized manner.

2. How many ways can you wallpaper the three rooms if you cannot use the same pattern in more than one room and you feel that the order is unimportant (i.e. ABC is the same as CAB)? List all the ways below in an organized manner.

3. How many ways can you wallpaper three rooms if you can use the same pattern in more than one room? Please do not list all the ways.
Track Relay

The high school girls track team can only select 4 runners for a 4 × 100 relay team but has 5 runners (A, B, C, D, E) to choose from.

4. How many different teams are possible when (a) order does not matter and (b) order does matter? List all the ways below for each answer in an organized manner.
Lesson Seven: Permutations

Objective:
The student will be able to find the number of ways a group of objects can be arranged in order.
The student will be able to use permutations to find probabilities.

Lesson Description:
Refresh the photograph extension activity from the beginning permutations lesson and the permutation notation \( \binom{n}{r} \). Then introduce the students to the permutation formula and the definition of a permutation in parts. End with using permutations to find probabilities. Emphasize throughout the lesson that a permutation is used when order is important and when counting multiple objects at a time. If not already, students must be shown that \( 0! = 1 \) for this lesson.

Level: Statistics and Probability

Pre-Learning: Beginning Permutations
Permutations

1. Find the following permutations using the permutation formula. Show the formula.
   a. \( 7P_3 \)  
   b. \( 7P_4 \)
   c. \( 8P_6 \)  
   d. \( 24P_6 \)
   e. \( 30P_{12} \)  
   f. \( 45P_{30} \)

2. What is a Permutation?

3. A jukebox has 50 songs on it, and you decide there are five songs that you like. Find the number of ways you can play your songs.

4. The Kentucky Derby has 20 horses in the race. How many ways can they finish first, second, and third?

5. The JV baseball team has nine players for the game today. How many different batting orders are possible using a permutation?

6. Find another way of achieving the same answer from problem 7 without using a permutation.
7. Find the number of ways of forming four-digit codes in which no digit is repeated.

8. What is the probability of guessing the correct code?

9. A student advisory board consists of 20 members. Four members serve as the board’s chair, vice chair, secretary and webmaster. Each member is equally likely to serve either of the positions. What is the probability of selecting at random the members that hold each position?
Lesson Eight: Combinations Part I

Objective:
The student will be able to find the number of ways a group of objects can be arranged without regard order.

Lesson Description:
Refresh the committee activity from the committee lesson and the combination notation \( \binom{n}{r} \). Then introduce the soccer scoring activity. After the students have finished the soccer scoring activity bring them back together as a class. Create a visual demonstration of a specific committee of the group of five people. Bring five students to the front and have them hold a sign of A, B, C, D, and E. Compare a committee of size three and the different photographs that could be taken in the 3! ways. In turn, with each committee of size 3, the students will see that a committee of 2 is formed by those people not on the committee of size 3.

Level: Statistics and Probability

Pre-Learning: Beginning Permutations and A1
Soccer Scoring

1. The final score of a soccer match was 3-2. Find all the possible scoring sequences that could have occurred during the match if either team can be the winner. (Hint: use different color cubes to represent the teams; 1 red cube means 1 goal scored by the home team and 1 yellow cube means 1 goal scored by the visiting team and then write down each combination.)

2. Select one scoring sequence from problem 1 where the home team wins and label the cubes Red 1, Yellow 1, Red 2, etc. Find the number of ways the cubes could be rearranged within a line. (Hint: would this be like a photograph or a committee?)

3. Now label only the red cubes and find the number of ways the cubes could be rearranged within a line. Why is the number of sequences reduced? How are the answers to question 2 and 3 related?

4. Now label only the yellow cubes and find the number of ways the cubes could be rearranged within a line. Why is the number of sequences reduced? How are the answers to question 2 and 3 related?

5. From question 1, how many possible ways can the home team win? How is this answer related to the answers from problems 2, 3, and 4?
Lesson Eight: Combinations Part II

Objective:
The student will be able to find the number of ways a group of objects can be arranged without regard order.
The student will be able to use counting principles to find probabilities

Lesson Description:
Refresh combinations part I. Then introduce the students to the combination formula and the definition of a combination in parts. End with using combinations to find probabilities. Emphasize throughout the lesson that a combination is used when order is not important and when counting multiple objects at a time. Introduce how to use complements to find probabilities. Either hand out, or have the students copy in notes, the summary of the counting principles.

Level: Statistics and Probability

Pre-Learning: Beginning Permutations and A1
Combinations

1. Find the following combinations using the combination formula. Show the formula.
   a. \( 24C_6 \)
   b. \( 15C_{14} \)
   c. \( 25C_{10} \)
   d. \( 33C_{12} \)
   e. \( 50C_{40} \)
   f. \( 100C_{66} \)

2. What is a combination?

3. A lottery has 52 numbers. In how many different ways can 6 of the numbers be selected?

4. From a group of 40 people, a jury of 12 people is selected. In how many ways can a jury of 12 people be selected?

5. In a certain state, each license plate consists of two letters followed by a four-digit number. How many distinct license plates can be formed if (a) there are no restrictions and (b) the letters O and I are not used? (c) What is the probability of selecting a license plate that ends in an even number?

6. You look over the songs on a jukebox and determine that you like 15 of the 56 songs.
   a. What is the probability that you like the next three songs that are played? Assume a song cannot be repeated.

   b. What is the probability that you do not like the next three songs that are played? Assume a song cannot be repeated.
7. A food manufacturer is analyzing a sample of 400 corn kernels for the presence of a toxin. In this sample, three kernels have dangerously high levels of the toxin. If four kernels are randomly selected from the sample, what is the probability that exactly one kernel contains a dangerously high level of the toxin?

8. Suppose 4 people are chosen at random from a group of 1200. What is the probability that all four would rate their financial shape as excellent? (Make the assumption that the 1200 people are represented by the pie chart.

![Rate Your Financial Shape](chart.png)

9. A warehouse employs 25 workers on first shift and 15 workers on second shift. Seven workers are chosen at random to be interviewed about the work environment. Find the probability of choosing
   a. All first shift workers.
   b. All second shift workers.
   c. Six first shift workers.
   d. Four second shift workers.

10. A shipment of 10 microwave ovens contains two defective units. In how many ways can a restaurant buy three of these units and receive (a) no defective units, (b) one defective unit, and (c) at least two defective units? (d) What is the probability of the restaurant buying at least two defective units?
<table>
<thead>
<tr>
<th>Principle</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Counting Principle</td>
<td>If one event can occur in ( m ) ways and a second event can occur in ( n ) ways, the number of ways the two events can occur in sequence is ( m \times n ).</td>
<td>( m \times n )</td>
</tr>
<tr>
<td>Factorial</td>
<td>The number of different \textit{ordered} arrangements of ( n ) distinct objects</td>
<td>( n! )</td>
</tr>
<tr>
<td>Permutation</td>
<td>The number of permutations of ( n ) distinct objects ordered ( r ) at a time, where ( r \leq n ).</td>
<td>( nP_r = \frac{n!}{(n-r)!} )</td>
</tr>
<tr>
<td>Distinguishable Permutation</td>
<td>The number of distinguishable permutations of ( n ) objects where ( n_1 ) are of one type, ( n_2 ) are of another type, …</td>
<td>( \frac{n!}{n_1! \cdot n_2! \ldots n_k!} )</td>
</tr>
<tr>
<td>Combination</td>
<td>The number of combinations of ( n ) distinct objects taken ( r ) at a time without regard to order.</td>
<td>( nC_r = \frac{n!}{(n-r)!r!} )</td>
</tr>
</tbody>
</table>
Lesson Nine: NBA Draft Lottery

Objective:
The student will be able to use combinations to answer questions about real-life applications.

Lesson Description:
Introduce students to the problem of the NBA draft. The association wants continuity of teams, so the worst teams have a greater shot at the new players. However, they must be wary of teams playing for the draft, i.e. losing on purpose, so they came up with the draft lottery. Then pass out the assignment.

Level: Statistics and Probability

Pre-Learning: Combinations, A1 and A2
NBA Draft Lottery

The National Basketball Association (NBA) uses a lottery to determine which team receives the first pick in its annual draft, in order to prevent teams from losing on purpose to receive the first pick. There are 30 teams in the NBA. The teams eligible for the lottery are the 14 non-playoff teams.

Fourteen Ping-Pong balls numbered 1 through 14 are placed in a drum. Each of the 14 teams is assigned several four number combinations that correspond to the numbers on the Ping-Pong balls. Four balls are drawn out of the drum to determine the first pick in the draft. The order in which the balls are drawn is not important. All of the four-number combinations are assigned to the 14 teams by a computer except for one four-number combination. If this four-number combination is drawn, the balls are put back in the drum and another drawing takes place. For example, if Team A has been assigned the four-number combination 3, 8, 10, 12 and the balls 8, 12, 3, 10 are drawn then Team A wins the first pick.

After the first pick of the draft is determined, the process continues to choose the teams that will select second and third. The remaining order of the draft is determined by the number of losses of each team.

**Answer the following questions; show your work in an organized manner.**

1. In how many ways can 4 of the 1 to 14 be selected if order is not important? How many sets of 4 numbers are assigned to the 14 teams?

2. In how many ways can 4 of the numbers be selected if order is important?
In the Pareto chart, the number of combinations assigned to each of the 14 teams is shown. The team with the most losses (the worst team, ranked 1st) gets the most chances to win the lottery, so they receive the greatest frequency of four-number combinations, 250. The team with the best record, ranked 14th, has the fewest chances, with 5 four-number combinations.

3. For each team, find the probability that the team will win the first pick.

4. What is the probability that the team with the worst record will win the first pick, given that the team with the best record, ranked 14th, wins the first pick?

5. What is the probability that the team with the second worst record will win the third pick, given that the team with the best record, ranked 14th, wins the first pick and the team ranked 2nd wins the second pick?

6. What is the probability that neither the first- nor the second-worst team will get the first pick?
**Lesson Ten:** Poker Combinations

**Objective:**
The student will be able to use counting principles to find the probabilities of realistic situations, mainly combinations for the hands of poker.

**Lesson Description:**
Refresh the classical probability and the combination formulas. Then describe to the students how to count the number of outcomes in an event, how to count the number of outcomes in the sample space, and how to divide the two. Show the students how to use complements to find probabilities. Use examples.

**Level:** Statistics and Probability

**Pre-Learning:** Combinations and A1
Poker Combinations

1. Mr. Early has just received his shipment of 100 calculators. After testing them all out, there were 6 defective calculators. How many ways can he give out the following:
   a. 3 defective calculators?
   b. 3 non defective calculators?
   c. 2 defective and 2 non defective calculators?

2. Using the same information from problem 1, find the PROBABILITY that Mr. Early will hand out the following:
   a. 3 defective calculators?
   b. 2 defective and 2 non defective calculators?
   c. At least one defective calculator (given he is handing our 4 calculators).

3. Find the probability of the following poker hands:
   a. 2-of-a-kind.
   b. Full house.
   c. 3-of-a-kind (the other two cards are different from each other).
   d. Diamond Flush.
   e. At least one king.
4. Best Buy has just received a new shipment of 200 televisions. If there were 5 defective televisions, find the probability of the following:
   a. 3 defective and 2 non defective televisions.

   b. 2 defective and 3 non defective televisions.

   c. At least one non defective television out of 5.

5. Find the probability of the following poker hands:
   a. 4-of-a-kind.

   b. Full house consisting of 3 kings and 2 queens.

   c. Two Clubs and one of each other three suits.
Lesson Ten: Quiz 2

Lesson Description:
Announce two days prior to this day there is a quiz (Possibly hold a review and/or study session of the topics covered). Have the students complete the quiz.

Level: Statistics and Probability

Pre-Learning: Permutations and Combinations (also A4 and A5)
1. The table shows the number (in thousands) of earned degrees in the United States in the year 2008 by level and gender

<table>
<thead>
<tr>
<th>Level Of Degree</th>
<th>Gender</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Total</td>
</tr>
<tr>
<td>Associate</td>
<td>260</td>
<td>405</td>
<td>665</td>
</tr>
<tr>
<td>Bachelor’s</td>
<td>595</td>
<td>804</td>
<td>1339</td>
</tr>
<tr>
<td>Master’s</td>
<td>230</td>
<td>329</td>
<td>559</td>
</tr>
<tr>
<td>Doctorate</td>
<td>25</td>
<td>23</td>
<td>448</td>
</tr>
<tr>
<td>Total</td>
<td>1110</td>
<td>1561</td>
<td>2671</td>
</tr>
</tbody>
</table>

A person who earned a degree is randomly selected. Find the probability of selecting someone who:

a. Earned a bachelor’s degree.

b. Earned a bachelor’s degree given that they are female.

c. Earned a bachelor’s degree given that the person is not a female.

d. Earned an associate degree or a bachelor’s degree.

e. Earned a doctorate given that the person is male.

f. Earned a master’s degree or is female.

g. Earned an associate degree and is male.

h. Is a female given that the person earned a bachelor’s degree.

2. Decide if the events are mutually exclusive. Then decide if the events are independent or dependent. Explain your reasoning.

   Event $A$: Selecting a king with replacement
   Event $B$: Selecting a black card
3. A shipment of 150 television sets contains 3 defective units. Determine how many ways a vending company can buy three of these units and receive the following:
   a. No defective units.
   b. All defective units.
   c. At least one good unit.

4. In problem 3, find the probability of the vending company receiving the following:
   a. No defective units.
   b. All defective units.
   c. At least one good unit.

5. The access code for a warehouse’s security system consists of six digits. The first digits cannot be 0 or 9, and the last digit must be even. How many different codes are available?

6. From a pool of 30 candidates, the offices of president, vice president, secretary, and treasurer will be filled. How many different ways can the offices be filled?
Lesson Eleven: Chapter 3 Test

**Objective:**
The student will be able to show their knowledge of permutations, combinations and the counting principles.

**Lesson Description:**
Announce to the students at least two days before the test of the test date. Hold a review session the day prior to the test, and have the students take the test.

**Level:** Statistics and Probability

**Pre-Learning:** Permutations, Combinations and Counting Principles
CHAPTER 3 TEST
Statistics and Probability

1. If one card is drawn from a standard deck of 52 playing cards, what is the probability of drawing an ace?

2. The distribution of blood types for 100 Americans is listed in the table. If one donor is selected at random, find the probability of not selecting a person with blood type B+.

<table>
<thead>
<tr>
<th>Blood Type</th>
<th>O+</th>
<th>O-</th>
<th>A+</th>
<th>A-</th>
<th>B+</th>
<th>B-</th>
<th>AB+</th>
<th>AB-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>37</td>
<td>6</td>
<td>34</td>
<td>6</td>
<td>10</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

3. Which of the following cannot be a probability?
   a. 1        b. \(\frac{4}{3}\)        c. 85%        d. 0.0002

4. A group of students were asked if they carry a credit card. The responses are listed in the table. Round your answers to three decimal places.

<table>
<thead>
<tr>
<th>Class</th>
<th>Credit Card Carrier</th>
<th>Not a Credit Card Carrier</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman</td>
<td>24</td>
<td>26</td>
<td>60</td>
</tr>
<tr>
<td>Sophomore</td>
<td>37</td>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>61</td>
<td>39</td>
<td>100</td>
</tr>
</tbody>
</table>

4. If a student is selected at random, find the probability that he or she owns a credit card given that the student is a freshman.

5. If a student is selected at random, find the probability that he or she is a sophomore given that the student owns a credit card.

6. If a student is selected at random, find the probability that he or she is a freshman given that the student owns a credit card.

7. A tourist in Ireland wants to visit six different cities. How many different routes are possible?

8. Find the probability of getting four consecutive aces when drawing four cards without replacement from a standard deck of 52 cards.
9. The probability it will rain is 40% each day over a three-day period. What is the probability it will rain at least one of the three days? (HINT: Make a tree diagram)

10. Decide if the events $A$ and $B$ are mutually exclusive or not mutually exclusive. A person is selected at random.
   Event $A$: Their birthday is in the fall.
   Event $B$: Their birthday is in October.

11. The distribution of Master’s degrees conferred by a university is listed in the table. (Assume that a student majors in only one subject)

<table>
<thead>
<tr>
<th>Major</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>230</td>
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<tr>
<td>English</td>
<td>206</td>
</tr>
<tr>
<td>Engineering</td>
<td>86</td>
</tr>
<tr>
<td>Business</td>
<td>176</td>
</tr>
<tr>
<td>Education</td>
<td>222</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected student with a Master’s degree majored in English or mathematics?

12. A baseball team consists of 15 players. How many different batting orders are possible? (Assume a nine-man line-up)

13. A warehouse employs 24 workers on first shift and 17 workers on second shift. Eight workers are chosen at random to be interviewed about the work environment. Find the following:
   a. How many ways can eight people are chosen?
   b. What is the probability of choosing all first-shift workers?
   c. What is the probability of choosing four second shift workers?
14. In the California State lottery, you must select six numbers from fifty-two numbers to win the big prize. The numbers do not have to be in a particular order. What is the probability you will win the big prize if you buy one ticket?

15. In California, each automobile license plate consists of a single digit followed by three letters, followed by three digits. How many distinct license plates can be formed if the first number cannot be zero and the three letters cannot spell "GOD"?

16. You are one of 20 students in OSCAR. What is the probability of you and three of your closest friends in OSCAR being one of the following: the president, vice president, secretary and treasurer?

17. What is the probability of a full house in poker?

18. How many distinguishable permutations of the letters in the word STATISTICS are there?

19. The events A and B are mutually exclusive. If P(A) = 0.2 and P(B) = 0.1, what is P(A and B)?

20. Four students drive to school in the same car. The students claim they were late to school and missed a test because of a flat tire. On the makeup test, the instructor asks the students to identify the tire that went flat; front driver’s side, front passenger’s side, rear driver’s side, or rear passenger’s side. If the students didn’t really have a flat tire and each randomly selects a tire, what is the probability that all four students select the same tire?
Bibliography


Appendix

Lesson A1: Types of Probability

Objective:
- The student will be able to distinguish among classical probability, empirical probability and subjective probability.
- The student will be able to define the Law of Large Numbers.
- The student will be able to describe the range of probabilities
- The student will be able to find the probability of the complement of an event.

Lesson Description:
This lesson will begin with a compare and contrast of the three types of probability. This comparison will lead us into the law of large numbers and how it relates to the different types. Then we must discuss the range of probabilities in order for the students to understand probability answers. Lastly, we will introduce the students to the complement of an event and the probability notation.

Level: Statistics and Probability
Types of Probability

1. What are the three types of probability?

2. How are they similar and how are they different?

3. Explain the Law of Large Numbers.

4. Complete the diagram below of the Range of Probabilities.

5. What is the complement of an event?

6. Complete the following probability notation for the complement of an event:
   a. \( P(G) + \text{_____} = 1 \)
   b. \( 1 - P(G') = \text{_____} \)
   c. \( 1 - \text{_____} = \text{_____} \)

7. If you had a 20 sided die, what is the probability of rolling at least a 7?

8. State the complement of problem 7, and the find the probability.
9. Using the given wheel, find the following:
   a. \( P(\text{red}) \)
   b. \( P(\text{white}) \)

10. Two dice are rolled, list all the possible outcomes.

11. From your outcomes above, what is the probability that the sum of the dice is 7?

12. An urn contains 30 marbles, 12 are blue, 6 are green, 2 are black, 5 are red, and 5 are white. Find the following:
   a. \( P(\text{blue marble}) \)
   b. \( P(\text{red marble}) \)
   c. \( P(\text{black and white marbles}) \)
   d. \( P(\text{blue or green marble}) \)
   e. \( P(\text{not green marble}) \)
Lesson A2: Conditional Probability

Objective:
The student will be able to find the probability of an event given that another event has occurred.
The student will be able to distinguish between independent and dependent events.

Lesson Description:
Begin a discussion of events happening in sequence that leads into conditional probabilities. Follow the discussion with guided practice examples of the probability of an event given another event has occurred. In your last example, have two independent events to begin that discussion. Finish with the steps to determine if two events are independent, including more guided practice examples.

Level: Statistics and Probability
Conditional Probability and Independent Events

1. What is conditional probability and its notation?

2. If you were to draw two cards in sequence, what is the probability of drawing a queen given you drew a king first?

3. If you were to draw two cards in sequence, what is the probability of drawing a king given you drew a king first?

4. The table below shows the results of a survey in which 146 parents were asked if they own a computer and if they will be taking a summer vacation this year.

<table>
<thead>
<tr>
<th>Summer Vacation</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>Own a Computer</td>
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<tr>
<td>Yes</td>
<td>46</td>
<td>11</td>
<td>57</td>
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<td>No</td>
<td>55</td>
<td>34</td>
<td>89</td>
</tr>
<tr>
<td>Total</td>
<td>101</td>
<td>45</td>
<td>146</td>
</tr>
</tbody>
</table>

a. Find the probability that a randomly selected parent is not taking a summer vacation this year.

b. Find the probability a randomly selected parent is taking a summer vacation given they do not own a computer.

c. Find the probability a randomly selected parent is not taking a summer vacation given they do not own a computer.

d. Are the events of owning a computer and taking a summer vacation this year independent or dependent events? Explain.
5. What are independent events?

6. Complete the steps to determine if two events are independent:
   - Find __________
   - Find __________
   - If __________
   - If __________

7. Classify the following events as independent or dependent:
   a. Selecting a king from a standard deck, replacing it, and then selecting a queen from the deck.
   
   b. Returning a rented movie after the due date and receiving a late fee.
   
   c. A numbered ball between 1 and 50 is selected from a bin, not replaced, and the second numbered ball is selected from the bin.
   
   d. Rolling a six-sided die and then rolling the die a second time so that the sum of the two rolls is seven.
Lesson A3: Multiplication Rule

Objective:
The student will be able to use the Multiplication Rule to find the probability of two events occurring in sequence.
The student will be able to use the Multiplication Rule to find conditional probabilities.

Lesson Description:
Start with the Multiplication Rule and how to distinguish between independent and dependent events and their connection to conditional probabilities. End with many guided practice examples. Emphasize that the word “and” indicates multiplication for two events.

Level: Statistics and Probability
Multiplication Rule

1. What is the Multiplication Rule?

2. Why are there two cases for the Multiplication Rule?

3. When should you use the specific cases?

4. Two cards are selected without replacement from a standard deck. Find the probability of selecting a 5 and then selecting an ace.

5. A die is rolled and a coin is tossed. Find the probability of rolling a 3 and getting a tail.

6. The probability of a particular knee surgery is successful is 0.85. Find the probability that three knee surgeries are successful.

7. Find the probability that none of the three surgeries are successful.

8. Find the probability that at least one of the three surgeries is successful.
Lesson A4: The Addition Rule

Objective:
The student will be able to determine if two events are mutually exclusive.
The student will be able to use the Addition Rule to find the probability of two events.

Lesson Description:
Begin with a Venn diagram in order to introduce and define mutually exclusive events. Follow the definition with guided practice examples. Remind students that the word “and” indicates multiplication. This will lead into the Addition Rule, with the two cases for mutually exclusive events and not mutually exclusive events. Emphasize that the word “or” indicates addition. Finish with many guided practice examples.

Level: Statistics and Probability
The Addition Rule

1. What are Mutually Exclusive events? (Use a Venn diagram in your explanation)

2. State whether the following events are mutually exclusive:
   a. Event $A$: Rolling a 3 on a die.
      Event $B$: Rolling a 4 on a die.
   b. Event $A$: Drawing a Jack from a standard deck.
      Event $B$: Drawing a face card from a standard deck.
   c. Event $A$: Randomly selecting a nursing major at UWRF.
      Event $B$: Randomly selecting a male at UWRF.

3. Explain the Addition Rule.

4. Why are there two cases for the Addition Rule?

5. When should you use each case?

6. Select one card from a standard deck. Find the probability that the card is a 6 or a Jack.

7. Roll a twelve-sided die. Find the probability of rolling at least a 9 or an odd number.
8. The table below shows the number of blood donors who gave each blood type. A donor is selected at random

<table>
<thead>
<tr>
<th>Rh-factoer</th>
<th>O</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>156</td>
<td>139</td>
<td>37</td>
<td>12</td>
<td>344</td>
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<td>Negative</td>
<td>28</td>
<td>25</td>
<td>8</td>
<td>4</td>
<td>65</td>
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<tr>
<td>Total</td>
<td>184</td>
<td>164</td>
<td>45</td>
<td>16</td>
<td>409</td>
</tr>
</tbody>
</table>

a. Find the probability a donor has type O or type A blood.

b. Find the probability a donor has type B blood or is Rh-negative.

c. Find the probability a donor has type AB blood or is Rh-positive.

d. Find the probability a donor has type B blood given that they are Rh-positive.

e. Are the events “Rh-positive” and “type A blood” mutually exclusive? Explain.

9. **Addition Rule for Three Events.** The Addition Rule for the probability that events \(A\) or \(B\) or \(C\) will occur, \(P(A\ or\ B\ or\ C)\), is given by

\[
P(A\ or\ B\ or\ C) = P(A) + P(B) + P(C) - P(A\ and\ B) - P(A\ and\ C) - P(B\ and\ C) + P(A\ and\ B\ and\ C),
\]

And is shown below in the Venn diagram

If \(P(A) = 0.40\), \(P(B) = 0.10\), \(P(C) = 0.50\), \(P(A\ and\ B) = 0.05\), \(P(A\ and\ C) = 0.25\), \(P(B\ and\ C) = 0.10\), and \(P(A\ and\ B\ and\ C) = 0.03\), find \(P(A\ or\ B\ or\ C)\).
Lesson A5: Quiz 1

Lesson Description:
Announce two days prior to this day there is a quiz (Possibly hold a review and/or study session of the topics covered). Have the students complete the quiz.

Level: Statistics and Probability
Quiz 1
Statistics and Probability

1. The table shows the number (in thousands) of earned degrees conferred in the United States in the year 2008 by level and gender.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
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<td>665</td>
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<tr>
<td>Bachelor’s</td>
<td>595</td>
<td>804</td>
<td>1339</td>
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<td>Master’s</td>
<td>230</td>
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<tr>
<td>Doctorate</td>
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<tr>
<td>Total</td>
<td>1110</td>
<td>1561</td>
<td>2671</td>
</tr>
</tbody>
</table>

A person who earned a degree is randomly selected. Find the probability of selecting someone who:

a. Earned a bachelor’s degree.

b. Earned a bachelor’s degree given that they are female.

c. Earned an associate degree or a bachelor’s degree.

d. Earned a master’s degree or is female.

e. Earned an associate degree and is male.

2. Decide if the events are mutually exclusive. Then decide if the events are independent or dependent. Explain your reasoning.

   Event $A$: Selecting a King without replacement
   Event $B$: Selecting a red card

3. The access code for a warehouse’s security system consists of six digits. The first digit cannot be 0 and the last digits must be even. How many different codes are available?
## Summary of Probability

<table>
<thead>
<tr>
<th>Type of Probability and Rules</th>
<th>In Words</th>
<th>In Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Classical Probability</strong></td>
<td>The number of outcomes in the sample space is known and each outcome is equally likely to occur.</td>
<td>$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Number of outcomes in sample space}}$</td>
</tr>
<tr>
<td><strong>Empirical Probability</strong></td>
<td>The frequency of outcomes in the sample space is estimated from experimentation</td>
<td>$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}}$</td>
</tr>
<tr>
<td><strong>Range of Probabilities</strong></td>
<td>The probability of an event is between 0 and 1, inclusive.</td>
<td>$0 \leq P(E) \leq 1$</td>
</tr>
<tr>
<td><strong>Complementary Events</strong></td>
<td>The complement of an event $E$ is the set of all outcomes not in the sample space of $E$, denoted by $E'$.</td>
<td>$P(E') = 1 - P(E)$</td>
</tr>
</tbody>
</table>
| **Multiplication Rule**      | The Multiplication Rule is used to find the probability of two events occurring in a sequence “AND” | **Dependent Events**
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

**Independent Events**
$P(A \text{ and } B) = P(A) \cdot P(B)$ |
| **Addition Rule**            | The Addition Rule is used to find the probability of at least one of two events occurring “OR” | **Not Mutually Exclusive Events**
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

**Mutually Exclusive Events**
$P(A \text{ or } B) = P(A) + P(B)$ |
### Osceola High School Statistics and Probability Chapter 3 Test Scores

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GM = Grouped Mean (the individual class scores added up)
TM = Total Mean (all the scores for that year added up)
GSD = Group Standard Deviation (the individual class scores per year)
TSD = Total Standard Deviation (of all the scores for that year)
A7: Survey Response Numbers

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<tr>
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<th>TM</th>
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<td>B</td>
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</table>

GM = Grouped Mean (the class scores added up)

TM = Total Mean (all the scores for that year added up)
A8: Counting Principles Curriculum Survey

Circle the number that best fits your answer to the question or statement.

1. My self-assessment of my knowledge of permutations and combinations is as follows:

   None  Poor  Below Average  Average  Good  Excellent  Mastery
   -3    -2     -1          0        1       2       3

2. I felt the counting principles worksheets were beneficial to my knowledge.

   Completely Disagree  Strongly Disagree  Somewhat Disagree  Neutral  Somewhat Agree  Strongly Agree  Completely Agree
   -3                 -2                         -1               0       1          2                  3

3. The worksheets gave me a deep understanding of permutations and combinations.

   Completely Disagree  Strongly Disagree  Somewhat Disagree  Neutral  Somewhat Agree  Strongly Agree  Completely Agree
   -3                 -2                         -1               0       1          2                  3

4. I found the counting principles worksheets were helpful.

   Completely Disagree  Strongly Disagree  Somewhat Disagree  Neutral  Somewhat Agree  Strongly Agree  Completely Agree
   -3                 -2                         -1               0       1          2                  3

5. I understand what a permutation \(_nP_r\) and a combination \(_nC_r\) mean and know how to calculate them.

   Completely Disagree  Strongly Disagree  Somewhat Disagree  Neutral  Somewhat Agree  Strongly Agree  Completely Agree
   -3                 -2                         -1               0       1          2                  3

6. I have a conceptual understanding of the \(_nP_r\) and \(_nC_r\) formulas.

   Completely Disagree  Strongly Disagree  Somewhat Disagree  Neutral  Somewhat Agree  Strongly Agree  Completely Agree
   -3                 -2                         -1               0       1          2                  3
Please only answer the next two questions if you have already finished a full year of pre-calculus.

A. I remember permutations and combinations from pre-calc.

<table>
<thead>
<tr>
<th>Completely Disagree</th>
<th>Strongly Disagree</th>
<th>Somewhat Disagree</th>
<th>Neutral</th>
<th>Somewhat Agree</th>
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<td>-1</td>
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<td>3</td>
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</tbody>
</table>

B. I felt the counting principles worksheets deepened my understanding of permutations and combinations.

<table>
<thead>
<tr>
<th>Completely Disagree</th>
<th>Strongly Disagree</th>
<th>Somewhat Disagree</th>
<th>Neutral</th>
<th>Somewhat Agree</th>
<th>Strongly Agree</th>
<th>Completely Agree</th>
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</table>

Name (optional) ______________________________________________

Math Class I was in last year ____________________________

Please use the space below to let me know what you enjoyed or disliked about the counting principles worksheets:

Additional comments/suggestions for these worksheets: