Application of an Accelerometer to a Rigid Pendulum

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Abstract
The costs and sizes of accelerometers and microcontrollers have dropped significantly in the last several years. We tested a microcontroller-accelerometer combination on a well-understood system, the rigid pendulum, in order to determine if this new generation of electronic devices allows in situ measurements of acceleration. Our data agreed closely with the theoretical predictions, and we conclude that the new accelerometers and microcontrollers are effective tools for studying the acceleration of dynamic systems in situ.

Experimental Procedure
- Assembled the circuit using MMA 7455 3-axis accelerometer, Basic Stamp microcontroller, and powered by 9 V battery
- Made pendulum apparatus of lab stands and a metal meter stick
- Programmed microcontroller to take measurements in the X and Y radial tangential direction
- Took 247 measurements over 13.74 seconds

Experimental Complications
- Recorded accelerometer measurements included the acceleration due to gravity – must subtract by components
- Axes of accelerometer difficult to align with pendulum – off by angle \( \beta \) (see diagram)

Sample Code
```
197 StoreDataLoop:
198     FREQUENT 4, 50, 4000
200     FREQUENT 4, 50, 4000
202     FREQUENT 4, 50, 4000
203     FREQUENT 4, 50, 4000
204     FREQUENT 4, 50, 4000
205     DEBUG CLS, "Recording..."
206     FOR eeIndex = 0 TO 760 STEP 3
207     GOSUB ReadDataLoop
208     WRITE eeIndex, XAccel
209     WRITE eeIndex + 1, YAccel
210     NEXT
```

Analysis of the Data
- Converted recorded accelerometer data to standard SI units
- Determined the ideal theoretical equation for X and Y directions to be:
  \[
  A_x^{\text{ideal}} = -a \omega^2 L \cos(\omega t + \varphi) + g \sin(\alpha \cdot \cos(\omega t + \varphi)) \\
  A_y^{\text{ideal}} = a^2 \omega^2 L \sin^2(\omega t + \varphi) + g \cos(\alpha \cdot \cos(\omega t + \varphi))
  \]
- Used rotation matrix to account for slight misalignment of accelerometer (see \( \beta \) above).
- Final theoretical equations for X and Y accelerations were found to be:
  \[
  A_x^{\text{Theoretical}} = \cos(\beta) \cdot A_x^{\text{ideal}} - \sin(\beta) \cdot A_y^{\text{ideal}} \\
  A_y^{\text{Theoretical}} = \sin(\beta) \cdot A_x^{\text{ideal}} + \cos(\beta) \cdot A_y^{\text{ideal}}
  \]
- Found \( \omega, \varphi, \) and \( \beta \) that optimized the fit using a spreadsheet and the method of least squares

Results
- Theoretical X Acceleration Values vs. Experimental X Acceleration Values
- Theoretical Y Acceleration Values vs. Experimental Y Acceleration Values

Conclusions
- Our data fit the theoretical prediction well
- We have demonstrated that microcontroller/accelerometer combination in situ can accurately measure a system’s motion.
- We are confident this apparatus could be used to measure more complex physical systems

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