Incorporating Interactive Demonstration Applets
into the Mathematics Classroom

By

Kathleen E. Hertz

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Abstract

Recognizing that the incorporation of technology into the modern mathematics classroom often falls far behind the ideal, the author outlines the history and reasons for the delay and lists some of the difficulties and concerns with the full implementation of technology in the classroom. She then conducts an exploration of the types of technology that are available for use in the classroom. This is followed up with a brief investigation of web-based technology types and a description of their strengths and weaknesses. The author then highlights research supporting the use of these technologies and the benefits for learning that they can provide. After identifying a digital library of dynamic visualization applets designed by teachers for use in the classroom, the Wolfram Demonstrations Project (Wolfram, 2012a), the author conducts an in-depth search of the website. The author uses this online digital library to construct a catalog of applets suitable for use in basic developmental college mathematics courses, as well as for intermediate and college algebra. The author gives descriptions of these applets, lists the courses that they might be suitable for, and rates the applets as to appropriateness, accuracy, interactivity, clarity, and ability to enhance understanding. She includes these findings along with a discussion as to the strengths and weaknesses of the different types of applications. Along with suggestions for uses of the applications in the classroom, the author also offers several lesson plans that incorporate inquiry-based and constructivist methods in order to utilize the applets in a classroom situation in a format recommended to optimize learning benefits for the students.
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Chapter 1
Introduction and Background

From interactive whiteboards to computers to handheld calculators, varying degrees of technology populate our classrooms. This is, however, the ideal and not the reality. The reality is that many school districts and colleges do not have the resources to invest in technology for the mathematics classroom alone. Rather than providing individual computers for student use in the math classroom, most schools have computer labs, which are primarily utilized for teaching computer literacy courses. As such, they are accessible for limited times by reservation for math or other general classes to use. I teach in a small, rural Midwestern college, with a full-time enrollment of just over 600 students. We have two computer labs, which can be reserved on an availability basis only. We do not have interactive whiteboards, “clickers” (Personal Response Systems (PRSs)), or computers that can be used for general math classes. There is no likelihood of acquiring the resources to purchase any of these items in the foreseeable future due to budget constraints.

In fact, colleges in general are behind public schools in embracing the use of technology (Madison, 2001; Risser, 2011). In 1985, I was a sophomore at the University of Minnesota-Twin Cities, taking trigonometry and then calculus. Our professor used an overhead projector and wrote on transparencies, rather than using a chalkboard. This was his technology choice since there were over a hundred students in the class and this medium could be enlarged for optimal viewing. We were allowed, although not required, to use graphing calculators on exams and for our homework. These were the only nods to technology I experienced in the math classroom at that time. Fifteen years later, I finished my degree in mathematics at the University of Wisconsin-Stevens Point. Here also, in the math department, there was a dearth of technology,
ameliorated only by the use of graphing calculators and the use of the Minitab or Excel programs for statistics classes. In fact, the most relevant use of technologies for math classes occurred in my methods classes, not courses in the math discipline itself. My experiences with lack of technology in the college math classroom were validated in an article by Christopher Knapper, which claimed that “a good many contemporary university students probably find themselves taught by methods largely unchanged from those experienced by their parents or grandparents” (Knapper, 1982, as quoted by Albright& Graf, 1992). Ten years later, Albright noted little improvement on the college campus (Albright& Graf, 1992). Now another ten years have passed, and at the college where I teach, UW-Baraboo/Sauk County, things remain unchanged in the mathematics department. Although I do, occasionally, utilize PowerPoint as an information delivery mechanism and I include a large amount of calculator and graphing calculator work into my classes, to me the need for more vibrant technology is apparent.

Among the high-tech classroom technologies obtainable today, there is one hardware tool that is available in most classrooms despite budget constraints and other obstacles. It is a single computer with a projector, often referred to as an LCD projector. The data projector has been described as the “visual backbone of the 21st century classroom,” (Sheskey, 2010) and is now widely available in schools throughout the United States, including UW-BSC. Unfortunately, these powerful tools are often under-utilized or ignored while more traditional presentation methods are favored. The LCD projector could be utilized with interactive software programs. There are numerous types of interactive software programs available for educational purposes; some of the more well-known include Geometer’s Sketchpad, Mathematica, and Maple. These require computer lab access, as well as the cost of purchasing and upgrading the software. However, various forms of digital media, such as web-based applications, Java applets, graphics,
digitized videos and online quiz and worksheet generators are available on web pages via the Internet. These are generally referred to as “pedagogic hypermedia” (Looms, 1999). Many of these sources can be found free of charge, although some require membership fees for access. Since nearly all of these can be utilized with a single computer and data projector, available in most classrooms today, they are of particular interest to those who wish to incorporate technology on a tight budget.

**Web-based technologies.**

Web-based technologies for use in mathematics follow seven general formats: game format, informational format, quiz format, virtual manipulatives, static representation format, math forum, and interactive object format. Each format has its own benefits and drawbacks which make it useful for fulfilling different goals and purposes (Bos, 2009).

- Game formats are varied and highly motivational because they are more engaging than serious, businesslike formats. Students like them because they require swift reactions and quick strategic thinking. These tools are useful for instant recall of facts and skill-building drill. The drawback is that the desired goal is to win, not to learn. Quick-time action drives the game format, and distracts from exploration necessary for deeper understanding. An example of game format is a basketball-themed game in which quickly and correctly answering questions wins the player the opportunity to attempt virtual free-throws to see how many baskets can be made (Popovici, 2007). The ensuing game play distracts from the learning process in favor of the “fun factor.” This format requires individual computers for each student in the classroom, or if necessary, students may work in pairs.
• Informational formats are used for presentation of information for direct instruction. They provide the facts, but lack “intuitive, logical connections and interactivity” (Bos, 2009) for making and testing conjectures. A PowerPoint presentation is an example of an informational format and can be visually appealing when time and effort are expended in its creation. Existing web-based examples, such as “A Maths Dictionary for Kids” (Eather, 2001), are also visually appealing, useful sources of information, but do not promote conceptual understanding. Most of the entries include interactive components in this dictionary, but they are generally so narrow that students are not provided the opportunity to make conjectures.

• The quiz format provides multiple-choice, matching, or true/false questions to check for understanding after an informational format has been utilized. This is often done with “clickers” (PRSs), which allow simple responses on an immediate basis. These can be ideal in a classroom situation, because answers can be anonymous, allowing students the freedom to participate without the anxiety of publicly giving an incorrect answer. Classrooms with more elaborate systems can allow students to input verbal answers using a computer keyboard. This is a good way to get instant feedback, but again does not allow for testing conjectures.

• Virtual manipulatives are visual representations of actual, physical manipulative learning tools. Algebra tiles, an example of a commonly used manipulative tool, are differently sized tiles that represent constants and variables in algebra equations. They are often used to teach factoring or multiplying polynomials. Different tile sizes correspond to units, $x$ or $x^2$. Students can move the tiles around to achieve the configuration that corresponds to the correct solution. Virtual algebra tiles are computer generated shapes
that can be clicked on and dragged to the same effect as the physical tiles. These manipulatives are building blocks for conceptual understanding, and demonstrate mathematical ideas well, but they require detailed instruction and guidance from the instructor in order to be productive (Bos, 2009; Moyer, Bolyard & Spikell, 2002).

- Static representation is a format which generally provides a calculation, either by handheld calculator or by a web-based application. A series of steps leads to a correct answer. In this format, the procedural process is emphasized, which leads students to the idea that rote memorization of the steps is crucial to success and the logic behind each step is less important. Only the formula is involved—the underlying concepts are not illustrated in any significant fashion. Static representation will determine an outcome from a given specific input. It is, therefore, more dynamic than an informational format, such as PowerPoint, which only provides information in a non-manipulative format.

- Math forum websites generally offer examples of homework problems, often with a video of an instructor or a student performing the operations. Some sites also offer sample problems for users to try, with steps for solving the problems offered as “hints” if required. Some commonly used sites are Purple Math (Staple, 2009), Math TV (McKeague, 2011) and The Kahn Academy (Kahn Academy, 2012).

- Interactive object formats are illustrations where multiple representations change with given input. Using these tools, patterns can be observed and manipulated, and conjectures can be formulated and tested. For example, shifting, stretching and reflecting of common functions, which is often taught using paper and pencil graphing or through graphing calculators, can be done using this format. It takes time to generate graphs by hand drawing, and also by punching them into a graphing calculator. An interactive
object format, however, can illustrate the difference that changing a coefficient or a constant has on the graph quickly and fluidly by dragging a slider. This type of application gives deeper understanding by allowing students to discover predictable math patterns, which is why it is specified by the NCTM (NCTM, 2011). The caveat is that the guided discovery approach, where activity and data collection is carefully orchestrated by the instructor, requires time for planning as well as for execution in the classroom. Students must expend the effort to develop understanding through trial and error. When used in a classroom situation, the instructor must give the time needed for students to explore and make conjectures. Excessive prompting can be counterproductive to students acquiring a personally developed understanding, rather than facilitating that development (Bos, 2009).

For extensive use in the classroom, applications of game format, quiz format and virtual manipulatives require individual or at least small group computer access. Although the interactive object format generally requires individual student computers to allow for exploration of concepts, the applications can be utilized in demonstration mode by the teacher when this is not an option. Since many instructors, like myself, cannot expect to have computers for each student in the classroom, using the demo form with a single computer and LCD projector is the optimal format when technology resources are slim.

The courses that I teach are generally developmental mathematics courses, which give “non-degree credit” only. Math 090, Basic Mathematics, is intended for students who have a minimum algebra background or who have been away from mathematics for several years. It includes arithmetic of whole numbers, fractions and percents and some basic algebra concepts.
Math 091, Elementary Algebra, assumes competence with arithmetic, and covers exponential laws, operations with polynomials, factoring, linear equations in one or two variables, and graphing. Math 105, Introduction to College Algebra, is an elective credit course, but is still considered to be a developmental mathematics course. It expands on algebraic techniques with polynomials, fractional expressions, exponents and radicals, covers linear and quadratic equations and inequalities, and introduces functions and their graphs and analytic geometry. Math 110, College Algebra, gives credits which can be applied to mathematical science requirements. The course expands on the concepts introduced in Math 105, and also covers functions (transformations, composition, inverse functions), theories of polynomial equations, logarithms, systems of linear equations, matrices, and conic sections.

The focus of this paper will be identifying and reviewing technologies that are appropriate to the courses that I teach, and including methods and lesson plans to demonstrate including them into the curriculum. These courses are taught at all the UW Colleges campuses, and Math 105 and Math 110, or their equivalents, are taught at most postsecondary institutions, making these suggestions broadly applicable.
Chapter 2

Review of Literature

Technology is an essential component of education in our modern world. The National Council of Teachers of Mathematics (NCTM) stresses that practitioners should “include the development of mathematics lessons that take advantage of technology-rich environments and the integration of digital tools in daily instruction.” It credits technology with supporting both the learning of mathematical procedures and skills as well as the development of advanced mathematical proficiencies, such as problem solving, reasoning and justifying.” The NCTM (2011) further states that “effective teachers optimize the potential of technology to develop students’ understanding, stimulate their interest, and increase their proficiency in mathematics.” The state of Wisconsin follows closely in advocating technology in the math classroom as well, stating that “the use of technology must be an integral part of teaching and learning mathematics. Such use should aim at enhancing conceptual understanding and problem solving skills” (WI DPI, 2009). While use of technology is highly encouraged by instructional authorities such as the NCTM and Wisconsin DPI, there are some concerns that require attention.

An overview of pedagogic metadata by Robson (2001), of Oregon State University, identifies three major concerns associated with incorporating technologies in the classroom, particularly the incorporation of pedagogic hypermedia. Some of these concerns involve the technology itself.

The first major problem with using these materials in a classroom is finding them in the first place. An internet search is likely to reveal large numbers of irrelevant sites that are not truly useful. It can sometimes take hours of weeding through the results to find a single useful
application. Although digital libraries are becoming increasingly abundant, they can be difficult to navigate and still yield small results for hours of searching (Robson, 2001).

A second issue with incorporation of technology is compatibility. For example, a program that is written for a UNIX platform is likely to be incompatible with Windows. In addition, older computers may not support newer programs, and older programs may not be supported by newer browsers. In all cases, the need for upgrading and replacing existing software is both discouraging and expensive (Robson, 2001).

The final issue with the technology, particularly web-based resources, is that applications found online can raise the question of content validity. It is understood that an article found in an academic publication has a tacit legitimacy conferred by the publishing company, especially if it is a refereed journal, but online sources generally have no claim to this kind of validity. Items found on the web must be carefully inspected by the user for accuracy to ensure that mathematical fidelity is preserved (Robson, 2001). With the recent increase in number of digital libraries found online, some of which have screening and review policies, this situation is beginning to change.

The major issue, however, surrounds not the technology itself, but the human component—the instructors and administrators who are responsible for obtaining and using the technology. There is often reluctance on the part of teachers and administrators to fully endorse the implementation of technology in the classroom. This is partly due to the belief that technology will not enhance student learning compared with other approaches, or that use of technology in the classroom will utilize time at the expense of covering all required topics. There is also a concern that the expense of software and equipment may place it out of the reach of some students, raising questions of equitable accessibility (Pierce & Ball, 2009). At the
college level, progress has been even slower than in secondary schools for various reasons. Mathematicians fear that technology will result in a neglect of basic skills in favor of electronic aids, leaving students unprepared for higher level mathematics (Risser, 2011). Many faculty members have a “commitment to traditional teaching methods,” and the assessment and reward systems are not set up to recognize efforts to improve teaching methods or test results (Albright & Graf, 1992). Before technology can be fully implemented at the college level, it must be fully supported by faculty and administration.

In addition to the expense of obtaining, maintaining and upgrading both the hardware and software utilities, costs of professional development often becomes an impediment to comprehensive incorporation of technology. Computer technology, like any tool, is more effective in the hands of a proficient user, yet the additional time needed to acquire “technological pedagogical content knowledge” and integrate that knowledge into practice can also be a barrier (Pierce, R., and Ball, L., 2009). Instructors may be concerned that training for proficiency with equipment and programs will take excessive amounts of their personal time. In addition, there is fear that equipment failure or operator error will cause disruption of the continuity of the class session (Ozel, Yetkiner & Capraro, 2008). These factors contribute to the delay in fully incorporating technology at the college level.

One recent study provides evidence to suggest that, for secondary school as well as college educators, there may be an unexpected advantage to including technology in the pedagogic repertoire, particularly in math and sciences. A survey of high school students in math, science, English and social studies, asked students to assess the frequency and types of technology used by their teachers. Students were asked for their perceptions of the teachers, as well. For teachers in the math and science fields, there was a significant correlation between
teachers who use technology in the classroom and students’ perception of teacher competence (Frye & Dornisch, 2008). Efforts to become proficient with pedagogic technology can therefore increase students’ estimation of their instructor’s professionalism and expertise.

Technology has been shown to affect students’ attitudes in the mathematics classroom. A study accomplished through extensive interviews with teachers of mathematics found that, in their views, regular access to technology and the resultant familiarity with the hardware and software resulted in greater student success, deeper understanding and increased motivation (Ruthven & Hennessy, 2002). In their analysis of research and articles concerning use of technology in mathematics classes, Guerrero, Walker & Dugdale concluded that use of technology can improve attitudes toward learning, enhance student engagement in math classes, and promote abstract understanding when correctly and proficiently utilized (Guerrero, Walker, & Dugdale, 2004). Incorporating technology in the mathematics classroom can improve students’ perceptions and impressions of learning mathematics.

A study of high school students in geometry and AP calculus classes showed that when students are the primary users of the technology, the results can be increased engagement as well as increased understanding. They posit that increased engagement follows because the technology makes the topic more interesting, and provides “variety and fun,” thereby increasing motivation and participation (Sheehan & Nillas, 2010).

Much research has been done on the effectiveness of technology on learning in mathematics education. Early studies suggested that while computer technology could significantly enhance achievement in the learning of mathematics, the mere presence of such technology did not guarantee better outcomes (Clark, 1983; ETS, 1997). This led to a flurry of research concerning the relative success of varying types of technology, from graphing
calculators to tools for drill and practice, to computer modeling and visualization software. Recently, Li and Ma compiled a meta-analysis of 46 different studies on the effects of computer technology on mathematics learning. Their re-examination of primary studies showed that “using technology in ways to help students [build] appropriate mental models of mathematics” did contribute to greater success in achievement (Li & Ma, 2010). This study suggested that modeling and visualization can be very effective in building conceptual knowledge.

Although many studies are finding that the effectiveness of technology is enhanced when students are the primary users, there is still a case for computers as a presentation tool. Wood, et al, claim that “a central purpose of lectures is to be a natural venue for the links between different representations of mathematics,” making lecture paramount to concept-building through the different media. Their ideas are developed through the dissection and analyses of lessons with multiple components, including lecture, board examples, and Mathematica files for visualization (Wood, Sadhbh, Petocz & Rodd, 2007). This suggests that interactive visualization tools can be utilized to great effectiveness as an instructional aide, particularly when used for linking multiple representations, such as symbolic representation, graphs, or spreadsheet data sources.

Numerous studies have suggested that a constructivist approach to teaching is associated with the more successful technology implementations. Constructivism has been described as discovery-based learning which builds on what the student already knows (Jacobson & Cheng, 1998). Li and Ma’s meta-analysis showed definitively that different approaches resulted in different performance levels and that use of computer technology did not always result in improved performance. They note that in addition to the technology tool itself, the teaching method employed with the technology is paramount to the success of the implementation. The
researchers suggest that in order to achieve maximum advantage, computers should not be used merely as an instructional delivery medium. The optimal usage employs a constructivist, rather than a traditional, approach to teaching. This method moves the emphasis in the classroom from the teacher to the student, using strategies that are oriented more toward discovery or inquiry-based learning. This is not to say that directed instruction is not useful as well, but that the technology itself should be delivered with constructivist methodologies in mind (Li& Ma, 2010). Methods of incorporation of technology, then, should emphasize student discovery rather than teacher exposition of mathematical concepts.

Extensive research on the subjects of multiple intelligence and learning styles has shown that students are very individualistic in the way they absorb and assimilate information. A visual tool can provide an additional angle of approach, thus enhancing the opportunity for all students to learn (Chen, 2006). The NCTM lists interactive presentation devices among technologies that are “vital components of a high-quality mathematics education” which can “support and extend mathematical reasoning and sense making, gain access to mathematical content and problem-solving contexts, and enhance computational fluency” (NCTM 2008).

Visualization tools are considered a valuable addition to curriculum because they can provide much detail in a short time by eliminating the need for exacting calculations (Liang & Sedig, 2009). This reduces the wait time and the tedium on the part of the students, giving them the results in a more expedient manner, and allowing them to see a bigger picture of the concept being explored. Richard Palais, a professor emeritus of mathematics at Brandeis University, and creator of the 3D Filmstrip software for Macintosh computers states that visualization tools not only complete the required calculations in an instant, they also “provide morphing animations that can bring the known mathematical landscape to life in unprecedented ways” (Palais, 1999).
Specific studies regarding interactive visualization tools suggest that these tools can increase the effort students expend thinking about the concepts and aspects, the reinforcement of reasoning processes, and the depth of student learning:

A study which compared a multimedia approach to traditional methods in teaching integrals showed “remarkably higher” scores on the part of the students exposed to the multimedia. In addition, an overwhelming 82% of the students in the study preferred the multimedia approach, finding it made concepts more interesting as well as easier to see, understand, and remember (Milovanovic, Takaci, & Milajic, 2011). Another study of college students in Calculus I, II and III showed that increasing the number of instructors who used technology and integrated technology and “lively application projects” into the courses significantly increased the performance of students in advanced courses following Calculus III (Schreyer-Bennethum & Albright, 2011).

Research involving students who used interactive virtual and physical balance scales showed that the multiple representations of algebraic relationships which these manipulatives provided allowed students to “translate among pictorial, manipulative, symbolic, and written representations and to develop representational fluency” (Suh & Moyer, 2007). A 2009 study explored the use of three different types of computer visualizations: 3D illustration (static representation), 3D animation, and 3D interactive animation. This research showed conclusively that both of the animation technologies increased interest and engagement in the topic (Korakakis, Pavlatou, Palyvos & Spyrellis, 2009).

Effective use of technology can help transform abstract concepts and processes into precise and objective visualizations that are more easily assimilated. Even the static visualization given by graphing calculators has been shown to strengthen students’
understanding of the relationship between graphs and their symbolic representations (Ruthven, 1990). While graphing and computer algebra system (CAS) calculators remain excellent tools for exploration of mathematical concepts, the dynamic nature of some computer applications can more clearly illustrate concepts. A 2006 study showed that visualization software can increase engineering students’ understanding of mathematical spatial skills, (Sorby, 2009). Another study showed that similar applications also increase the understanding of spatial skills of younger students in middle school and high school (Hungwe, Sorby, Drummer, & Molzan, 2007), suggesting that the benefits of such visualizations extend across multiple age-groups.

A study by Zhou, Brouwer & Nocente has shown that the use of applets as teaching demonstrations enhances students’ learning of physics concepts, but the effectiveness of these demonstrations was more profound when used to set the stage for new learning. The applets were less effective when used primarily as “examples of knowledge application,” without the opportunity for students to conjecture and make predictions. This article went further, suggesting that the use of applets at the beginning of class, before presentation of the related theory, optimized the effectiveness of the medium. They speculated that using the applets after the theory was presented eliminated curiosity, and therefore interest, from the students’ experience. The instructors who enjoyed the greatest success in the study used the applets as introduction to conceptual construction (Zhou, Brouwer, & Nocente, 2005).

The benefits of visualization tools can extend from enhancing interest, motivation and engagement to extending mathematical fluency across multiple representations and increasing students’ learning and performance. I believe that the incorporation of such visualization tools into classroom activities can only have a positive effect on improving teaching styles and students’ learning abilities.
Chapter 3

Method

Given that computers with data projectors are already present in most classrooms, including all classrooms at UW-BSC, I see key elements for the successful implementation of pedagogic hypermedia in the classroom as: 1) finding appropriate, valid instructional resources for classroom demonstration, 2) evaluating these resources for possible incorporation into courses, and 3) integrating this technology into the curriculum in a meaningful and effective manner. This paper will attempt to address these elements in a small way by exploring a technologically compatible, easily accessible source of dynamic visualization applets, evaluating the applets for accuracy and appropriateness, and determining methods for incorporating them into lesson plans for developmental mathematics courses, as well as intermediate and college algebra. These courses are offered at all two-year colleges in the UW Colleges system.

A Source for Mathematical Visualization Applets

Since its inception in the early 1990s, the World Wide Web has grown explosively in terms of size, but unequally in terms of usefulness. Much of the content available on the web is valuable and accurate, but even more of it is not. It remains to the user to critically evaluate content and determine its validity and appropriateness for use; this is even more critical when educational purposes are concerned. For use in my classes, I will be evaluating interactive demonstration applets from the Wolfram Demonstrations Project website (Wolfram, 2012a).

In 2007, Stephen Wolfram debuted a new digital library, “The Wolfram Demonstrations Project.” This is a collection of nearly 7,900 dynamic demonstrations that illustrate topic areas such as: Mathematics, Sciences, Life Sciences, Business, Arts, Computation, Our World, Kids
and Fun. Applets in this library are written using Wolfram’s Mathematica program, but the site offers a free download of the Computable Document Format (CDF) player so that the applets are accessible without the Mathematica program itself (which can be expensive). The site has an internal validation process in which all demonstrations undergo rigorous review by experts in relevant fields for quality, clarity, and accuracy, following standards similar to those of traditional academic publications, before being included (Wolfram, 2012a). Although thoroughly reviewed, the contents of the Wolfram site are widely varied in terms of mathematical level, complexity, and mathematical topic, and no index of specific applets is available at this time. Although the site is broken down by discipline and general topics (i.e. Mathematics-Algebra), applications are best located via keyword search. I propose to locate in this manner, all demonstrations which fit topics in the courses listed above.

**Review of Mathematical Visualization Applets**

Having identified a source of demonstration applets, evaluation and assessment of the applets for possible inclusion in the curriculum is the next step. There are numerous articles available which offer strategies and guidelines for assessing the value and quality of software programs for educational use in order to assist educators in making wise and cost-effective decisions for software purposes. Although I will not be purchasing software, the guidelines for choosing appropriate and educationally valuable materials still apply. Some of the recurring evaluative categories include quality, depth, ease of use, graphics and appeal, interactivity, appropriateness, and bias (Tate & Alexander, 1996; Comer & Geissler, 1998; Herring, Notar,& Wilson, 2005; Forster, 2006).

For my purposes, the criteria that I have identified as being most relevant to demonstration applets are: appropriateness to the course content; cognitive fidelity, which I
interpret as the ability to enhance learning; clarity, which encompasses ease of use and understanding with minimal explanation; interactivity; and feasibility for use, either in the classroom or as assignments outside the classroom. The rubric I have developed for evaluating the demonstrations, with a brief description of each item, is included as Appendix A. Here I include a detailed explanation of the evaluation categories and provide specific examples to contrast the different levels of ratings for each of the categories.

**Appropriateness.** When determining appropriateness of an applet for use in my classes, I looked for items that fit as closely as possible to the course content as to level of difficulty and essential detail for the specific topic, and rated the overall appropriateness of the demo as poor, inadequate, average, good or excellent. I found a good example of appropriateness in two applets on the topic of linear inequalities. In Elementary Algebra (Math 105) classes, graphing linear inequalities is introduced, as well as the intersections and unions of the graphs. The first applet I found relating to this topic, “Graphs of Inequalities” (Pegg, E.), shown in Figure 1, features a system of three inequalities. Since I am specifically looking for explorations involving two linear inequalities, this applet is too complex. The applet does not allow for the deletion of one of the lines, nor is it possible to move a line out of visual range. While an excellent
representation of systems of three inequalities, the extraneous third graph and shading might be difficult for students to disregard in their efforts to understand the intersections of the two graphs. I rated this as “average” for appropriateness for this topic. Had I been unable to locate another applet on the same subject, this option might have been better than none at all, so it would not have been classified as “poor” or “inadequate.” Fortunately, there is another applet that covers the same topic using only two lines, exactly as I wish to present it in the classroom: “Graphing Systems of Inequalities” (Thomas, T., and Glon, A.), Figure 2. Both applets allow for dotted lines or solid lines, and clearly show the intersections with shading, but the second applet adheres more closely to the course guideline, and allows students to “guess” where to shade, and test the hypothesis, earning it a score of “Excellent.” Other applets on the Wolfram site show inequalities in one or three variables, making them “poor” examples of appropriateness for this course topic.
Cognitive fidelity. Cognitive fidelity describes the ability of the demonstration to enhance understanding of the concept and whether the demo gives a good visual of the concept it is demonstrating. For example, I looked for a good representation of graphing trigonometric functions that shows phase shifts and amplitude changes. I found two applets on trigonometric functions, only one of which showed phase and amplitude changes. The drawback of the applet in Figure 3, “Comparison of Trigonometric Functions,” by Amir Amadhi, is that it does not show the equation of the trigonometric function at all, so the changes in the variables cannot be directly related from the equation to the changes in the graph. To use this demo for facilitating connections between representations, it would be necessary to show the function’s equation on the board, with the appropriate variables, and then explain to students which variable is being manipulated. On its own, for visualizing the concept of phase shift and amplitude changes, this applet is “inadequate.”

On the other hand, if I wanted to show the comparison of a sine graph to a cosine or tangent graph, this application works well. It will even show secant and cosecant graphs if you set both the sine and cosine function exponents to -1 (this gives a reciprocal, not an inverse function). Showing sine’s relation to cosecant is more problematic, since the function and the
inverse function are both represented by sine on the demo, but it can be done using a phase shift. I would rate it “good” for the purpose of comparing trigonometric functions.

Some of the demos rank low for cognitive fidelity because they are deficient in the area of interactivity. For example, the demo in Figure 4, Multiplying Binomials, by J. Bryant, allows you to change the values of the variables, but the entire calculation changes instantaneously. Yes, the demo works, is accurate, and there is a nod to interactivity, but there is no enhancement of understanding achieved merely by showing multiple calculations of FOIL in this manner.

**Accuracy.** Accuracy reflects how well the demo maintains mathematical fidelity, or adherence to mathematical accuracy, and is free from typographical or other errors. Due to Wolfram’s review process, I have only found one demo with an accuracy shortfall. It occurs in one of the proofs included in the demo “Adding and Subtracting Positive and Negative Integers,” by G. Beck. The proof claims that: \((-12) + 4 = (-12) + (4) = - (12 - 4) = -8.\) This is clearly a typographical error, but I would have expected Wolfram’s review board to have caught it.
Clarity. For the purposes of this rubric, I am defining clarity to be the ease or difficulty of the demonstration in being understood with a minimum of explanation. An example of an applet with good cognitive fidelity, but poor clarity, is the Order of Operations Tree, by S. Lichtblau (Figure 5). The applet uses a tree diagram to break the expression into component operations. It is necessary to read these trees from left to right and from the bottom to top, and not from left to right, top to bottom, as is intuitive for most students. Read properly, these trees give a good representation, but they require some explanation before the students can relate the tree to the order of operations. By contrast, the applet in Figure 6, Reference Angles (T. DeVries), is clear with minimal explanation. It shows the actual angle under investigation, the shaded reference angle, as well as the degree of each angle.

Interactivity. This criterion considers the interactivity of the applet against other Wolfram Demonstration applets as high, average or low. Figure 7, Multiplying Complex Figures (S. Wolfram), is rated as “low” interactivity. The toggles allow you to change the values, but it
is basically a static representation of a completed calculation. The applet, “Distance Between Two Points” (E. Schulz), Figure 8, is rated “average.” The user may manipulate the points, turn the two legs of the triangle on or off using the “hint” toggle, and then, when ready, show the solution. It would have a higher value if it allowed the user to enter a guess as to the distance, rather than simply giving the correct solution. An example of “high” interactivity is the applet, “Lines: Two points” (A. Brown), shown in Figure 9. This applet allows the user to position the points at grid intersections, half values, or free points, and then choose from two different line formulas. Because of the freedom given to the user by the design of this applet, it could be used for introducing the Cartesian coordinate system, plotting points, or demonstrating slope, as well as for graphing lines or finding equations of lines given a point and a slope. Again, it does not allow for the user to try again after an incorrect guess, but it offers enough other options to earn it the high interactivity rating.
Feasibility. The feasibility score describes how likely I am to use the demo in a particular class. Scores are listed as not feasible, unlikely, possibly, very likely and certainly. Some demonstrations are more suitable for some classes than others, depending upon the level of the course and the strengths and weaknesses of the demonstration. This category takes the other categories into consideration, but is not necessarily an average of the other categories. The ratings are based on the degree to which I feel the demonstration can add value to the lesson.
Chapter 4

Results

In my analysis of the applets available on the Wolfram Demonstrations Project site I identified 43 applets that pertained to topics in the developmental math courses, Introduction to College Algebra and College Algebra. Of these applets, I found 23 that I am very likely to use in my classes, based on my evaluation using the rubric (Appendix A). I deemed many of the demonstration applets to be of little value because they simply gave a number of calculations very quickly, and so they did not add depth of understanding to a lesson. The applets I consider to have most potential value were those that gave a visual representation of the concept they illustrate, either a graphic or a pictorial representation. These are even better when the pictorial representation is linked to an algebraic representation, so that the connection can easily be seen. These may better help students develop a connection between the different representations of the concept. The complete analyses of these applets follow.
Reviews of Demonstration Applets

1. **Title of Demo:** A Negative Times a Negative is Positive (G. Beck)

**Description of Demo:** This demo uses number lines to show that multiplying by $-1$ rotates the number line by $180^\circ$. It goes further to give examples of multiplying by other negative numbers. This demo is relatively static—after a multiplier is selected, you choose a value to be multiplied, and then follow it down to a parallel number line for the solution.

![Number line visualization](image)

**Course(s) it might be appropriate for:** Basic Mathematics

**Curriculum topic the demo is relevant to:** Multiplication/division of whole numbers and integers

**Appropriateness:** Average.

**Clarity:** This demo does a good job of illustrating the patterns of multiplying positive and negative numbers. The red for negative and blue for positive shows the sign changes clearly.

**Cognitive fidelity:** Average.

**Accuracy:** No discrepancies noted.

**Interactivity:** Average

**Feasibility of use in class:** Very likely. This would be a real quick demo to show that when you multiply by a negative number, the sign of the multiplicand will always change (unless it is zero).
**How this demo might be improved:** If the multiplier also was color-coded red if negative or blue if positive, the concept would be more clearly illustrated.
2. **Title of Demo**: Adding and Subtracting Positive and Negative Integers (G. Beck)

**Course(s) it might be appropriate for**: Basic Mathematics, Beginning Algebra

**Curriculum topic it is relevant to**: Addition/Subtraction of Whole Numbers and Integers

**Description of Demo**: This demo uses a number line to give a visual interpretation of addition and subtraction of integers. The user sets an initial value, either positive or negative, and either adds or subtracts a second value. The actual equation is displayed, arrows appear on the number line for the two values, and a third arrow shows the final sum or difference.

![Number line demo](image)

**Appropriateness**: Poor. I would not recommend this demo to my 090/091 students for exploration on their own. Although the use of absolute values is mentioned, the arithmetic calculations that accompany the demo involve factoring out negatives: \(4 - 12 = (-12) - (-4) = -(12 - 4) = -8\). This makes a relatively simple calculation too complex for students who have not yet had factoring of greatest common factors. For in-class demonstration, an instructor’s explanation using the absolute value method would be clearer and more effective than the explanation given in the demo’s notes.

**Cognitive Fidelity**: Inadequate.

**Clarity**: Average. The demo could be used (without the cumbersome calculations) to show visually how adding two positives always gets larger, adding two negatives always gets smaller.
(more negative), and adding numbers with two different signs will end up with the sign of the number with the largest absolute value. This visualization is not as clear as the one by Lichtblau on the same topic.

Accuracy: There is an error in one of the proofs in this demo. It claims:

For example: \((-12) + (4) = (-12) + (-4) = -(12 - 4) = -8\).

This is clearly a typographical error, but I would have expected Wolfram’s review board to have caught it.

Interactivity: Average

Feasibility of use in class: Not feasible. Since there is a clearer representation of this topic on the site, I would probably never use this one.
3. **Title of Demo:** Addition and Subtraction of Integers (S. Lichtblau)

**Course(s) it might be appropriate for:** Basic Mathematics, Beginning Algebra

**Curriculum topic it is relevant to:** Addition/Subtraction of Whole Numbers and Integers

**Description of Demo:** This demo uses bar graphs to represent positive and negative numbers, and is color-coded so that negative numbers appear pink and positive numbers appear blue. The user can toggle between addition and subtraction, and adjust the value of the numbers up or down. The solution is also represented on a bar graph coded pink or blue as necessary.

**Appropriateness:** Good.

**Cognitive fidelity:** Good. Adding and subtracting of signed numbers often presents a great challenge to students at the pre-algebra level. This visual is an additional representation, and might be helpful to understanding.

**Clarity:** Good. The colors show that the sum of two negatives will always be negative, the sum of two positives will always be positive, and the sum of two numbers with different signs can be positive or negative depending on which number had the greater absolute value. The concept of subtracting a negative number is clear only when considered as adding a positive number.

**Accuracy:** No discrepancies noted.
**Interactivity:** Average

**Feasibility of use in class:** Very likely. I might try this demo in a Math 090 class in combination with “An Example of Subtraction of Negative Numbers” for multiple representations of the concept. This demo might be improved if, rather than showing negative values below the midline and positive values above the midline, it showed them side by side with only the color to show positive/negative. This way it could be clearly seen which number has the greater absolute value—the one with the bigger stack. It would also be clearer that the solution is the difference between the two stacks, with the color (sign) of the larger stack. Also, if the solution bar graph had the same scale, the solution would be clearer.
4. **Title of Demo**: Annotated Quadratic Polynomial (S. Wolfram)

**Description of demo**: This demo shows quadratic equations and their graphs, and gives the values of the x-intercepts, if any exist, as they would be calculated with the quadratic formula.

**Course(s) the demo might be appropriate for**: Math 105, Math 110

**Curriculum topic the demo is relevant to**: Solving quadratic equations. Graphing equations, intercepts.

**Appropriateness**: Good.

**Cognitive fidelity**: Good. Gives a good visual of intercepts of quadratic equations what solutions obtained with the quadratic formula look like.

**Clarity**: Good.

**Accuracy**: No discernible errors.

**Interactivity**: Average

**Feasibility of use in class**: Possibly.
5. **Title of Demo:** Area of a triangle as half a rectangle (S. Wolfram)

**Description of Demo:** The demo partitions a triangle into squares, representing the area of the triangle. The toggles allow you to increase or decrease the resolution (size of the squares), and adjust the slant of the triangle.

![Diagram of triangle partitioned into squares]

**Course(s) it might be appropriate for:** Basic Mathematics, Beginning Algebra

**Curriculum topic the demo is relevant to:** Geometry: Area and perimeter; Literal equations and formulas

** Appropriateness:** Good.

**Clarity:** Very good. This demo clearly illustrates that the area of a triangle depends only on the base length and the height of the triangle—the slant is not relevant.

**Cognitive fidelity:** Very good.

**Accuracy:** No discrepancies noted.

**Interactivity:** Average

**Feasibility of use in class:** Very likely. This demo, along with the “Areas of Parallelograms and Trapezoids” demo, will make a nice addition to a unit on perimeters and areas of geometric figures.
6. **Title of Demo:** Areas of parallelograms and trapezoids (H. Papadopoulos)

**Description of Demo:** This demo illustrates the transformation of a parallelogram or a trapezoid into a rectangle. This illustrates the relationship of the area formula of the quadrilaterals to the area formula of a rectangle.

![Diagram of parallelogram transformation](image)

**Course(s) it might be appropriate for:** Basic Mathematics; Beginning Algebra

**Curriculum topic the demo is relevant to:** Geometry: Area and perimeter; Literal equations and formulas

**Appropriateness:** Good.

**Clarity:** Very good. This simple illustration can help students understand the relationship between a parallelograms and trapezoids to a rectangle, and how the formulas for their areas are similar and how they are different.

**Cognitive fidelity:** Good.

**Accuracy:** No discrepancies noted.

**Interactivity:** Average

**Feasibility of use in class:** Very likely. This is a good demo for illuminating applications that students sometimes memorized by rote, rather than really understanding the concepts.
7. **Title of Demo:** Circumference of a Circle (S. Wolfram)

**Description of demo:** This demo uses a circle with radius 1 unit, and rolls it along a line marked with units the length of the radius to show that the circumference of a circle of radius 1 is $2\pi$.

![Image of the demo](image)

**Course(s) the demo might be appropriate for:** Beginning Algebra

**Curriculum topic the demo is relevant to:** Math 090, geometric terms, perimeter; Math 091, simple literal equations and formulas;

** Appropriateness:** Good. This demo gives a clear visual on circumference of a circle that might fit in well with a section on unit conversions, perimeter and area.

**Cognitive fidelity:** Good. I have often found myself describing circumference in terms of tying a sock around a bicycle tire and rolling it along the ground. This demo gives an actual picture of that operation.

**Clarity:** Excellent.

**Accuracy:** No errors noted.

**Interactivity:** Average

**Feasibility of use in class:** Very likely. I would be very likely to use this demo in the section on perimeter and area, especially in conjunction with other demos that fit in with the unit.
8. Title of Demo: Converting Fractions (J. Nochella)

Description of Demo: The demo gives two rational numbers, one with a missing numerator or denominator. The user can calculate the missing element and then click the box to check the actual solution.

Course(s) it might be appropriate for: Basic Mathematics, Beginning Algebra

Curriculum topic the demo is relevant to: Rational numbers: reducing to lowest terms, adding and subtracting (fundamental property of fractions)

Appropriateness: Good. Students in both Basic Mathematics and Beginning Algebra struggle with fractions.

Cognitive fidelity: Good, however this applet is more for drill than for enhancement of learning.

Clarity: Good.

Interactivity: Average

Feasibility of use in class: Possibly. This demo might be useful for providing multiple examples for a quick check of understanding during class, or students might use it on their own for a quick drill or practice.
9. **Title of Demo:** Distance between two points (E. Schulz)

**Description of demo:** This demo computes the distance between two points using the distance formula while illustrating how the formula is essentially finding the hypotenuse of a right triangle using Pythagorean Theorem.

**Course(s) the demo might be appropriate for:** Intermediate Algebra

**Curriculum topic the demo is relevant to:** Analytic geometry, the distance formula

**Appropriateness:** Excellent. The demo allows you to hide the solution until students have had the opportunity to attempt it themselves.

**Cognitive fidelity:** Excellent. This demo draws the line and then adds the vertical and horizontal distances for you, showing the relationship of the line to a right triangle.

**Clarity:** Excellent.

**Accuracy:** No discrepancies noted.

**Interactivity:** Average

**Feasibility of use in class:** Certainly. Having the perfect graph drawn for you to illustrate this concept is a very valuable classroom tool.
10. **Title of Demo:** Do Not Divide by Zero (G. Beck)

**Description of Demo:** The demo shows division as repeated subtraction of the divisor until what is left is smaller than the divisor. Once students are comfortable with this illustration of division, they can be shown that if the divisor is zero, repeated subtractions of zero never diminishes the dividend, so there can be no quotient or remainder.

![Diagram of division](image)

**Course(s) it might be appropriate for:** Basic Mathematics, Beginning Algebra

**Curriculum topic the demo is relevant to:** Whole numbers and integers, multiplication/division

**Appropriateness:** Average.

**Cognitive fidelity:** Average. The concept behind the demo is very good for illustrating the topic, but the demo itself is lacking.

**Clarity:** Inadequate. The concept is good, but the visualization is not clear. The concept can be demonstrated better visually with manipulatives, rather than pure mathematical means, for pre-algebra students. It might be good for more advanced students who are interested in the reason behind a concept they have already accepted.

**Accuracy:** No errors
Interactivity: Low

Feasibility of use in class: Unlikely. This demo might be more effective if it used virtual manipulatives to remove multiples of the divisor from the given quantity. A more visual approach might give more dimension to understanding than multiple numerical calculations alone can give.
11. **Title of Demo:**  Equation of a line game, (S. H. Mejia)

**Description of demo:** This demo is an activity designed to help students practice finding the equation of a line in slope-intercept form. A line appears on the graph, and several possible equations for the line. User must choose the correct line by quickly finding the slope and intercept of the graph and determining which of the given formulas is a match. Points are awarded for matches found in a certain time frame.

![Image of the demo](image.png)

**Course(s) the demo might be appropriate for:** Beginning Algebra, Intermediate Algebra

**Curriculum topic the demo is relevant to:** Equations of graphs in two variables; finding intercepts; finding slopes; finding equations of lines.

**Appropriateness:** Good. This activity is good for individual student practice, or can be done in a classroom competitively with two teams.

**Cognitive fidelity:** Good. It gives the equation in various forms of $y = mx + b$.

**Clarity:** Excellent. The demo gives good instructions for the activity.

**Accuracy:** No noted errors.

**Interactivity:** High
**Feasibility of use in class:** Certainly. I would try to reserve the computer room for this lesson, and let students “play” with the applet. It might appeal to students to use on their own for drill because of its game format.
12. **Title of Demo**: An example of subtraction of negative numbers

**Course(s) it might be appropriate for**: Basic Mathematics, Beginning Algebra

**Curriculum topic it is relevant to**: Addition/Subtraction of Whole Numbers and Integers

**Description of Demo**: This demo uses a submarine to give a visual interpretation of subtraction of integers.

![Submarine Image]

**Appropriateness**: Excellent. This is a perfect example of subtraction of negative numbers in the real world that can be explored in a very visual way. The subtraction of a negative number is shown by removing depth from a submerged submarine, making it rise.

**Cognitive Fidelity**: Average.

**Clarity**: Poor. The submarine in the demo starts at the “final” depth, and moves to the “initial” depth. The equation given is $f - i = \text{change in depth}$. While this is accurate, students who are struggling with simply adding and subtracting integers might be able to see this more clearly if the submarine began at the initial depth and subtracted some “depth” to give a final depth ($i - d = f$ where $d$ is a negative value).

**Accuracy**: No noted discrepancies.
**Interactivity:** Average, but this demo does have the rare inclusion of a real picture, not just a graph.

**Feasibility of use in class:** Very likely. I would try using the applet as it is and see how well it is received. By phrasing the question as, “The submarine needs to go to a depth of $-10$ feet in order to use the periscope. If we start out at -85 feet, how much depth do we need to subtract?” students might be able to assimilate it.
13. Title of Demo: Exponents (S. Lichtblau)

Description of Demo: The demo does expansions of exponents and gives their numerical value.

Course(s) it might be appropriate for: Basic Mathematics, Beginning Algebra

Curriculum topic the demo is relevant to: Whole Numbers and Integers, Exponents

Appropriateness: Good. The demo is accurate, and shows that exponents are a way to annotate repeated multiplication.

Cognitive Fidelity: Average. Students can do the same thing on a calculator and be more hands-on about it. The demo removes the experience from the students’ own hands and makes it a passive viewing, which does little to enhance learning.

Clarity: Good. The demo clearly shows how to expand and evaluate exponentials, and gives a visual that exponents are a way of writing repeated multiplication of a single factor.

Accuracy: No errors.

Interactivity: Low. The demo simply shows various exponential expressions with their expanded form and evaluation.

Feasibility of use in class: Not feasible. I think the benefits of this demo are not worth the effort to show it in class.
14. **Title of Demo:** Finding an Inverse (E. Pegg)

**Description of demo:** The demo shows graphs of functions, and their inverses (where inverses exist). When a function is not one-to-one, the graph shows each section of the original and its corresponding inverse in matching colors.

![Graph of functions and their inverses](image)

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Inverse functions

**Appropriateness:** Excellent. This demo can be used to illustrate the reflection of a function and its inverse across the line $y = x$.

**Cognitive fidelity:** Very good. It gives an excellent visual of functions that are not one-to-one, and the limits that must be placed on their domain in order to find an inverse of the limited function.

**Clarity:** Very good. The only difficulty is that it takes a little practice to become familiar with how to arrange the points to get the exact function you wish to illustrate.

**Accuracy:** No discrepancies noted.

**Interactivity:** High

**Feasibility of use in class:** Certainly. This is an excellent way to get multiple representations of this concept in a short amount of time.
15. **Title of Demo:** Integer Polynomials (E. Weisstein)

**Description of demo:** This demo gives polynomials with integer coefficients between -10 and 10, and the factored form of the polynomial, if it factors.

**Course(s) the demo might be appropriate for:** Beginning Algebra, Intermediate Algebra

**Curriculum topic the demo is relevant to:** Exponents and polynomials

**Appropriateness:** Poor. Although this applet is related to polynomials and factoring, it does nothing except show polynomials and their factored form.

**Cognitive fidelity:** Poor. This applet does not enhance learning of any curricular topics.

**Clarity:** Good.

**Accuracy:** No discrepancies noted.

**Interactivity:** Low

**Feasibility of use in class:** Possibly. This applet might be used to demonstrate how few polynomials with integer coefficients can actually be factored. Students are aware that all quadratics cannot be factored, but are often unaware that the ones which can be factored are such a small proportion of the set of all polynomials.
16. **Title of Demo:** Interval Notation (G. Beck)

**Description of demo:** A real interval is shown on a number line graph and described in words, as a set, and in interval notation.

**Course(s) the demo might be appropriate for:** Beginning Algebra, Intermediate Algebra.

**Curriculum topic the demo is relevant to:** Interval notation and the graphs of intervals.

**Appropriateness:** Good. This demo gives descriptions, such as “bounded” and “half-open” that are valuable, but not always included in some texts.

**Cognitive fidelity:** Excellent. I prefer having the interval notation located directly below the graph, so that students can see the relationship between the notation and the graph, but this is still a good visualization.

**Clarity:** Good.

**Accuracy:** No discernible errors.

**Interactivity:** Average

**Feasibility of use in class:** Certainly.
17. **Title of Demo:** Laws of Exponents (G. Beck)

**Description of Demo:** The demo gives exponential laws and algebraic expansions to illustrate them. User can shift the values of the exponents up or down.

![Exponential Laws Demonstration](image)

**Course(s) it might be appropriate for:** Basic Mathematics, Beginning Algebra, Intermediate Algebra

**Curriculum topics the demo is relevant to:** Exponent rules; multiplying and dividing with exponents; using laws of integer exponents.

**Appropriateness:** Good.

**Cognitive fidelity:** Inadequate for enhancing understanding.

**Clarity:** Good.

**Accuracy:** No noted errors.

**Interactivity:** Low.

**Feasibility of use in class:** Possibly. This might be a convenient way to review exponential rules for a Mat 105 course. Students come in to MAT 105 with previous knowledge of the rules, and rules are briefly reviewed before students are expected to apply them. In MAT 091, exponential rules are introduced a few at a time, making this format less appropriate. This demo
might be improved if the quotient rule \( \frac{a^m}{a^n} = a^{m-n} \) examples were shown vertically like we show the rule itself: \( \frac{a^5}{a^3} = \frac{a \times a \times a \times a \times a}{a \times a \times a} \). Instead, they are shown as \( a^5 \div a^3 = a \times a \times a \times a \div a \times a \times a \), which is less clear.
18. **Title of Demo:** A Library of Functions with Transformations (E. Zaborowski)

**Description of demo:** This demo explores the different types of transformations that can be performed on the eight "parent" functions (linear, quadratic, cubic, reciprocal, square root, semicircle, absolute value, and exponential).

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Translating and stretching graphs

** Appropriateness:** Good. This demo fits in well with the topic of translating graphs

**Cognitive fidelity:** Good. The demo shows clearly what a vertical or horizontal shift looks like, and includes a standard form for equations: \( y = b + a f(x - c) \). The drawback is that although the toggles are labeled “vertical shift,” “horizontal shift,” etc, equations for the functions are not given. There is no explanation in the demo of the correlations between the variables in the parent function and the changes in the graph, i.e. a vertical shift changes “b” in the equation.

**Clarity:** Average.

**Accuracy:** No errors noted.

**Interactivity:** High
**Feasibility of use in class:** Certainly. This demo would be good for in-class demonstration, where the teacher can elucidate the connection between translations in the graphs and changes in the constants in the equations.
19. **Title of Demo:** Lines: Point-slope (A. Brown)

**Description of Demo:** This demo creates a line from a point and a slope. The toggle allows the user to change the slope, and the point can be dragged to new locations. The user can hide the equation, or show point-slope form or slope-intercept form. The grid can be set for integers, half-values or free points.

![Diagram of line with point-slope form](image)

**Course(s) it might be appropriate for:** Basic Mathematics, Beginning Algebra

**Curriculum topic the demo is relevant to:** Basic linear equations

**Appropriateness:** Good.

**Clarity:** This demo gives a good representation of point-slope and slope intercept forms of the line.

**Cognitive fidelity:** Average. The toggle that controls slope is difficult to maneuver precisely. I was unable to set it for exactly m=1.

**Accuracy:** No discrepancies noted.

**Interactivity:** Good

**Feasibility of use in class:** Very likely. This is one of several similar applets by the same contributor. Each one concentrates on a specific aspect of lines. These applets can be used alone, or several in the same lesson to illustrate specific points.
20. Title of Demo: Lines: Slope-Intercept (A. Brown)

Description of demo: The demo allows you to graph a line by adjusting the slope and the y-intercept. Values may be given in fractions or decimals.

Course(s) the demo might be appropriate for: Basic Mathematics, Beginning Algebra

Curriculum topic the demo is relevant to: Math 090, line graphs; Math 091, graphing linear equations

Appropriateness: Good. This demo would be appropriate for the two classes listed, but would be too basic for a Math 105 class.

Cognitive fidelity: Good

Clarity: Good. The only drawback to this demo is that there is no ability to snap the line to a grid with integer values. You may draw a line with the equation hidden, but if you are trying to draw \( y = 2x - 1 \), you are likely to get \( y = \frac{25}{12}x - \frac{13}{12} \).

Accuracy: No discrepancies noted.

Interactivity: High

Feasibility of use in class: Very likely. This is a good way to draw accurate graphs to illustrate slope as well as to graph using the slope-intercept method.
21. **Title of Demo:** Lines: Two points (A. Brown)

**Description of demo:** The applet gives the graph of a line and its equation in either point-slope or slope-intercept form. Points can be chosen as integers, half values, or in fractional form.

![Graph of a line](image)

**Course(s) the demo might be appropriate for:** Basic Mathematics, Beginning Algebra, Intermediate Algebra

**Curriculum topic the demo is relevant to:** Basic Mathematics: Line graphs; Beginning Algebra: Graphing linear equations, x and y intercepts, slope and its interpretation; Intermediate Algebra: Slope, equations of lines.

**Appropriateness:** Excellent. This applet has the advantage of allowing the user to toggle between two different forms of the line.

**Cognitive fidelity:** Excellent. This applet can be used for illustrating various things: calculating slope from a graph, graphing a line from a point and a slope, and slope-intercept and point-slope forms of a line.

**Clarity:** Good.

**Accuracy:** No discernible errors.

**Interactivity:** High.

**Feasibility of use in class:** Certainly. This demo provides an excellent opportunity for using computer-generated graphs over hand-drawn illustrations.
22. **Title of Demo:** Matrix Multiplication (A. Brown)

**Description of demo:** This demo shows how to multiply matrices. The user can toggle the dimensions of the matrices to be multiplied.

![Matrix Multiplication Diagram](image)

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Matrix algebra

** Appropriateness:** Excellent

**Cognitive fidelity:** Excellent. Although this demo is basically a static representation of the multiplication process, the toggling illustrates which row and column are being multiplied, and where the product fits into the new matrix. The actual multiplication and addition is shown below, making it easy to follow.

**Clarity:** Excellent.

**Accuracy:** No discrepancies noted.

**Interactivity:** Average

**Feasibility of use in class:** Very likely. This representation is very clear, accurate, and orderly, as opposed to a representation on a white board. It does have the drawback of allowing only random generated examples, rather than specific ones.
23. **Title of Demo**: Multiplying Binomials (J. Bryant)

**Description of demo**: The demo uses the FOIL method to multiply two binomials. The sliders are used to adjust values of the variables through positive integers less than ten.

\[(a \times b)(c \times d) = \]
\[(3 \times 8)(3 \times 7) = \]
\[(3 \times 3) + (3 \times 7) + (8 \times 3) + (8 \times 7) = \]
\[9x^2 + 21x + 24x + 56 = \]
\[9x^2 + 45x + 56 \]

**Course(s) the demo might be appropriate for**: Basic Mathematics, Beginning Algebra

**Curriculum topic the demo is relevant to**: Special polynomial products; multiplying two binomials.

**Appropriateness**: Average. The demo would fit the topic better if it allowed for negative values.

**Cognitive fidelity**: Average. There is a learning disadvantage to having the entire calculation immediately appear. There is no time for students to anticipate or calculate mentally for themselves. The demo is not interactive enough, so it gives the feeling of reading a textbook example.

**Clarity**: Average.

**Accuracy**: No discernible errors.

**Interactivity**: Low
**Feasibility of use in class:** Unlikely. This demo adds no value above what can be done with colored markers on a white board. There is also a recent movement against using the acronym “FOIL” in college math classes, even developmental classes, because its scope extends only to multiplying two binomials.
24. **Title of Demo:** Multiplying Complex Number (S. Wolfram)

**Description of demo:** The demo multiplies two complex numbers. The user may toggle the coefficients in both numbers.

![Complex Number Multiplication Demo](image)

\[(1 + 15i)(10 + 15i) = -215 + 165i\]

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Complex numbers

**Appropriateness:** Good.

**Cognitive fidelity:** Inadequate. This applet gives a static representation of the factors and the product.

**Clarity:** Average. It is assumed that the user knows that binomials must be distributed when multiplied.

**Accuracy:** No discrepancies noted.

**Interactivity:** Low.

**Feasibility of use in class:** Unlikely. There is no real valued added in this applet. The same calculations, given on the white board, give students the opportunity to calculate and anticipate the solution, and then affirm their calculations when the teacher does them.
25. **Title of Demo:** Number line solutions to absolute value equations and inequalities (Eric Schulz)

**Description of demo:** This applet shows how the graphs of absolute value equations and inequalities change as the user moves the slide to change values of the right side of the equation and the constant inside the absolute value.

![Graphs of absolute value equations and inequalities](image)

**Course(s) the demo might be appropriate for:** Beginning Algebra

**Curriculum topic the demo is relevant to:** Solving equations and inequalities involving absolute value.

** Appropriateness:** Excellent.

**Cognitive fidelity:** Excellent. This demo clearly shows how changing various parts of an absolute value equation will affect the graph on a number line. Through guided explorations, students can discover the characteristics of absolute value equations and inequalities.

**Clarity:** Good.

**Accuracy:** No discrepancies noted.

**Interactivity:** High

**Feasibility of use in class:** Certainly. I would use this demo with an inquiry-based guided discovery lesson plan (Appendix B).
26. Title of Demo: Order of Operations Tree (S. Lichtblau)

Course(s) it might be appropriate for: Basic Mathematics, Beginning Algebra, Intermediate Algebra

Curriculum topic the demo is relevant to: Order of Operations

Description of Demo: The demo breaks down random algebraic expressions according to the order of operations using a tree diagram.

Appropriateness: Good. This method of analyzing an expression is very different from the way it is presented in most texts, but it adds a new perspective to the topic.

Clarity: Poor. This demo is not self-explanatory to students who are just learning order of operations for simplifying algebraic expressions. It will require the instructor to explain the tree break-down and how the breakdown relates to order of operations. In addition, some of the items on one level (like “46” and “Times” in the above thumbnail) are too close together, giving the impression that they are connected. It is not “46 times (-4 times a),” it is “46 plus (-4 times a).

Cognitive Fidelity: Average. Once the tree diagram is understood, it might be beneficial for some students.

Accuracy: No discrepancies noted.
Feasibility of use in class: Possibly. This can be a very confusing topic for some students, but once they understand the way this breakdown works, the applet might be useful for individual practice.
27. **Title of Demo:** Polynomial Long Multiplication (S. Blake)

**Description of demo:** This demo shows the multiplication of two randomly generated polynomials.

![Image of polynomial multiplication](image)

**Course(s) the demo might be appropriate for:** Beginning Algebra, Intermediate Algebra

**Curriculum topic the demo is relevant to:** Operations on polynomials

** Appropriateness:** Average. I find this demo more appropriate for illustrating that the order of the product is the sum of the orders of the two factors.

**Cognitive fidelity:** Good if used as outlined above. If used for showing method of long division, it loses its value because it is instantaneously delivered.

**Clarity:** Good.

**Accuracy:** No discrepancies noted.

**Interactivity:** Low

**Feasibility of use in class:** Unlikely. I might use this demo in class to show clearly that the sum of the orders of two polynomial factors will equal the order of their product, or to emphasize that \((x + y)^2\) will not give you the binomial \(x^2 + y^2\). I do not think that the demo enhances learning of the algorithmic process of long division in a meaningful way.
28. **Title of Demo:** Polynomial Roots (E. Pegg)

**Description of demo:** The demo shows a polynomial with real roots in factored form. It gives the graph of the polynomial, showing that the graph touches or crosses the x-axis at the zeros of the polynomial.

![Graph of a polynomial](image)

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Rational roots of polynomial equations.

**Appropriateness:** Excellent. This demo will be a good way to show several accurate graphs of polynomials with single roots or multiplicities of roots in a short time.

**Cognitive fidelity:** Excellent.

**Clarity:** Excellent.

**Accuracy:** No discrepancies noted.

**Interactivity:** High.

**Feasibility of use in class:** Very likely. It would be better if it included factors like “$x^2 + 1$” so that students could see what complex solutions look like on the graph.
29. **Title of Demo:** Polynomial Long Division (S. Blake)

**Description of demo:** The demo shows the long division of two randomly generated polynomials.

**Course(s) the demo might be appropriate for:** Intermediate Algebra, College Algebra

**Curriculum topic the demo is relevant to:** Intermediate Algebra: dividing polynomial expressions; College Algebra: remainder and factor theorems, rational roots of polynomial equations, complex numbers and the Fundamental Theorem of Algebra.

**Appropriateness:** Very good. This demo shows the operation clearly, with an appropriate amount of work shown.

**Cognitive fidelity:** Average. This demo lacks the interactivity that adds value to a lecture. Although the sliders allow you to toggle the order of the numerator and denominator, and generate multiple random examples, each example is merely a static display of the completed problem.

**Clarity:** Excellent

**Accuracy:** No errors noted.

**Interactivity:** Low
**Feasibility of use in class:** Possibly. I might use this demo to generate multiple examples to show that the order of the quotient is the difference of the orders of the dividend and the divisor—quotient rule of exponents. The demo’s usefulness for enhancing learning of polynomial division is limited because it shows the entire solution at once, leaving no time for the student to anticipate the next step.
30. Title of Demo: Quadratic in vertex form (or turning point form)

Description of demo: This demo gives a quadratic equation in vertex form (or turning point form) and graphs it as a dilation and/or translation of \( y = x^2 \).

Course(s) the demo might be appropriate for: College Algebra

Curriculum topic the demo is relevant to: Analytic geometry, functions and their graphs, conic sections

Appropriateness: Good.

Cognitive fidelity: Excellent. This demo shows how the graph changes as the variables of the equation in vertex form are manipulated.

Clarity: Excellent. Very clear representation of the connections between the equation and the graph.

Accuracy: No discrepancies noted.

Interactivity: Average.

Feasibility of use in class: Very likely. This demo is good for showing the vertex formula and its relation to the graph, and the demo can also be used generically when graphing a parabola is necessary.
31. **Title of Demo:** Rectangles: Perimeter and area (S. Lichtblau)

**Description of Demo:** This gives the area and the perimeter of a rectangle. Toggles allow the user to manipulate the length and width of the rectangle.

**Course(s) it might be appropriate for:** Basic Mathematics

**Curriculum topic the demo is relevant to:** Geometry: Areas and perimeter

**Appropriateness:** Average.

**Clarity:** Good.

**Cognitive fidelity:** Average. This demo simply evaluates the area and perimeter as the length and width are adjusted. It would more clearly illustrate area if the rectangle were marked with a grid of squares. Perimeter might be more clearly shown if the edges were marked with some gradation markings so we can get the idea of what we are counting.

**Accuracy:** Accuracy would be improved if area was given in square units and perimeter in linear units. The demo gives numerical values without any labels.

**Interactivity:** Average
Feasibility of use in class: Possibly. The demo might be used to show contrast between area and perimeter calculations.
32. **Title of demo:** Rules for Logarithms (G. Beck)

**Description of demo:** This demo gives the basic rules for logarithms with the exponential rules and basic algebra to explain them.

![Diagram of logarithmic rules]

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Exponential and Logarithmic Functions: basic properties, applications, equations.

**Appropriateness:** Excellent. This is a good list of logarithmic rules and the explanation of why they are true.

**Cognitive fidelity:** Average. Although the representation of the rules is excellent, the demo is only a static representation of data in the text.

**Clarity:** Very good.

**Accuracy:** No discrepancies noted.

**Interactivity:** Low

**Feasibility of use in class:** Unlikely. Using this as an introduction to the logarithmic rules would ensure accuracy in presentation of the rules, but that value is outweighed by the very static representation.
33. **Title of Demo:** Simple Rational Functions (E. Pegg)

**Description of demo:** This demo graphs simple rational functions in the form \( \frac{(x-a)(x-b)}{(x-c)(x-d)} \) with the appropriate horizontal and vertical asymptotes.

![Graph of simple rational function](image)

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Graphing rational functions

** Appropriateness:** Average. This demo is limited to a single type of rational expression, with a horizontal asymptote of \( y = 1 \). It would be inappropriate for showing multiple representations of rational functions, because it limits itself to only one type.

**Cognitive fidelity:** Good. Although it shows only one type of rational function, and is limited to horizontal asymptotes of \( y = 1 \), it does a very good job of showing these graphs and their major characteristics.

**Clarity:** Although limited in scope, this demo shows what it is intended to show in a very clear and precise fashion.

**Accuracy:** No discrepancies noted.

**Interactivity:** High.

**Feasibility of use in class:** Very likely. This applet does show critical points and their graphic representations very clearly.
34. **Title of Demo:** Slope and equations of lines through points (T. Falcone)

**Description of Demo:** This demo allows the user to construct a line from two points. Students can plot an additional point through which they can find the equation of a line parallel or perpendicular to the first line. All three points can be dragged and repositioned.

**Course(s) it might be appropriate for:** Intermediate Algebra

**Curriculum topic the demo is relevant to:** Equations of Lines

**Appropriateness:** Good.

**Clarity:** Very good.

**Cognitive fidelity:** Good. This visual representation might be helpful to give students a better idea of writing a second line that is parallel or perpendicular to the first. They often confuse the two lines.

**Accuracy:** No discrepancies noted.

**Interactivity:** Good.

**Feasibility of use in class:** Very likely. This might be good to assign as an assistance tool when students are working on homework for this topic. Assessing whether they actually used the site would be difficult, because it does not allow you to copy the applet they are working on and share it via email.
35. **Title of Demo:** Solve Quadratic Equations with Integer Coefficients, (R. Aufmann)

**Description of demo:** This application allows you to toggle values for a, b, and c in a quadratic equation, and then plugs them into the quadratic formula and simplifies it, giving rational, irrational and complex number solutions to quadratic equations.

**Course(s) the demo might be appropriate for:** Intermediate Algebra, College Algebra

**Curriculum topic the demo is relevant to:** Solving quadratic equations via the quadratic formula.

** Appropriateness:** Good. This application uses color coding for a, b and c values so it is easy to discern where each value comes from, and which signs are part of the formula as opposed to part of the value of a, b or c.

**Cognitive fidelity:** Good. This demonstration shows the quadratic formula with all of the steps for simplification required to obtain the final solution.

**Clarity:** Good.

**Accuracy:** No discernible errors.

**Interactivity:** Low
Feasibility of use in class: Unlikely. This application might be used for toggling values to show the relationship between a, b and c to real or complex answers, or to show what the discriminant tells about the solution(s). I think it has limited value for teaching the quadratic formula itself, because it presents the problem, the formula application and the solution all instantaneously. I feel that the program gives too much information too quickly to be used for learning to use the quadratic formula.
36. **Title of Demo:** Solving Quadratic Equations (A. J. I. Rivas)

**Description of demo:** This demo allows the user to create and solve quadratic equations in the form \( ax^2 + bx + c = 0 \), and solve them using the quadratic formula.

![Image of quadratic equation demo]

**Course(s) the demo might be appropriate for:** Intermediate Algebra, College Algebra

**Curriculum topic the demo is relevant to:** Solving quadratic equations via the quadratic formula.

** Appropriateness:** Poor. This applet lacks the capacity to include any negative values for a, b and c; its ability to solve specific examples is limited.

**Cognitive fidelity:** Poor. The applet does not give the opportunity to see how negative values calculate in the formula.

**Clarity:** Good.

**Accuracy:** No discernible errors.

**Interactivity:** Low
**Feasibility of use in class:** Not feasible. There are two applets that show the quadratic formula in similar fashion. This one does not show the detail in the simplification of the radical, and it is also limited to positive values for the coefficients, so I would be more likely to use the other one.
37. **Title of Demo:** Standard Form of the Equation of a Circle (G. Vargas)

**Description of demo:** This demo gives a visual representation of the effect that changing values in the standard equation of a circle has on the graph of the circle.

![Graph of a circle](image)

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Graphing equations: Circles

**Appropriateness:** Excellent.

**Cognitive fidelity:** This demo gives a very good visual of how changing r, h, or k will change the graph of the circle.

**Clarity:** Excellent. This demo is simple, but covers all necessary aspects of graphing from the standard form of a circle.

**Accuracy:** No errors noted.

**Interactivity:** High

**Feasibility of use in class:** Very likely. This might be used in class or for student exploration outside the classroom.
38. **Title of Demo**: The Mixture Problem (V. Oussa)

**Description of demo**: This demo gives a visual representation of mixture problems typically given in algebra courses.

**Course(s) the demo might be appropriate for**: Beginning Algebra, Intermediate Algebra, College Algebra

**Curriculum topic the demo is relevant to**: Formulate simple real world applications in one or more variables and solve them algebraically.

** Appropriateness**: Poor. Although this demo represents mixture problems, I cannot see how it might be used in the classroom for visualization or for exploration to enhance understanding of mixture problems. It might be intended to generate mixture problems, but there is no statement to that effect on the demo.

**Cognitive fidelity**: Poor. The demo does not show the equations which represent the mixture problems. It merely shows that as concentration goes up, quantity must go down.

**Clarity**: Poor. I had a difficult time determining the purpose of this demo.

**Interactivity**: Average.
Accuracy: The demo maintains mathematical fidelity, or adherence to mathematical accuracy, and is free from typographical or other errors.

Feasibility of use in class: Not feasible for use in class, or for student use. May be used to generate mixture problems, but concentrations and quantities will generally involve decimal percentages, which does not conform to real-life situations.
39. **Title of Demo:** Transformation of functions (E. Schulz)

**Description of demo:** This demonstration explores the transformation of the graph of a function $f(x)$ to a new function $af(b(x - c) + d)$. The demo allows you to toggle the values of $a$, $b$, $c$ and $d$, and shows the graph of the original as well as the transformed graph. It also plots a single point and the corresponding point in the transformed graph.

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Translating and stretching graphs

** Appropriateness:** Excellent. This demonstration gives a better visualization of transformation of functions than the “Library of Functions with Transformations” applet by Zaborowski, in that it is very clear what variable in the equation is being manipulated.

**Cognitive fidelity:** Good. This demonstration shows a piecewise function as one of its representations, which is a good addition. Its drawback is that it is difficult to select integer values for the variables, which makes the actual translations more difficult to see.

**Clarity:** Good

**Accuracy:** No discrepancies noted.

**Interactivity:** High.
Feasibility of use in class: Very likely. This demo allows the use of many different models for transformation.
40. **Title of Demo:** Transforming Parent Functions (J. Wysocki)

**Description of demo:** This demo explores the different types of transformations that can be performed on the eight "parent" functions (linear, quadratic, cubic, reciprocal, square root, semicircle, absolute value, and exponential).

![Graph of parent functions with transformations]

**Course(s) the demo might be appropriate for:** College Algebra

**Curriculum topic the demo is relevant to:** Translating and stretching graphs

**Appropriateness:** Good. This demo fits in well with the topic of translating graphs.

**Cognitive fidelity:** Average. The demo shows clearly what a vertical or horizontal shift looks like; however the demo would be greatly improved if it included the equation of the graph in a standard form $y = af(x + h) + k$ so that students could explore which variables result in vertical shifts, horizontal shifts or stretches.

**Clarity:** Good.

**Accuracy:** Free from typographical or other errors.

**Interactivity:** High

**Feasibility of use in class:** Unlikely. I am disappointed that this demo does not include the actual equations of the graphs. The multiple representations given by the equation and the
graphs as the variables are manipulated would enhance the concept greatly. It would be beneficial to be able to assign exploration on this topic, but this demo is not adequate for this activity.
41. **Title of Demo:** Two points determine a line (G. Brown)

**Description of demo:** This demo graphs a line, given any two points, and lists the equation of the line in slope-intercept form. The grid can utilize rational numbers, decimals, or can be set to snap to integer values.

**Course(s) the demo might be appropriate for:** Basic Mathematics, Beginning Algebra, Intermediate Algebra


**Appropriateness:** Excellent.

**Cognitive fidelity:** Excellent. This demo represents a vast improvement over hand-drawn graphs.

**Clarity:** Excellent.

**Accuracy:** No discernible errors.

**Interactivity:** High
Feasibility of use in class: Certainly. This applet can be used for illustrating various things: calculating slope from a graph, graphing a line from a point and a slope, and slope-intercept form of the line.
42. Title of Demo: Two-Step Equations (S. Lichtblau)

Description of Demo: The demo allows you to choose \(a, b, c < 20\) for simple linear equations in the form \(ax + b = c\), along with the step-by-step solution.

Course(s) it might be appropriate for: Basic Mathematics, Beginning Algebra

Curriculum topic the demo is relevant to: Basic linear equations.

Appropriateness: This demo does a good job of showing simple linear equations. However, there is no variation of form, which limits its value.

Cognitive fidelity: The demo gives a good illustration of equations in the form \(ax + b = c\). It does not allow for different forms or equations which require more simplification prior to solving. This limits its effectiveness.

Accuracy: No errors noted

Clarity: Good. No illustration, just equations.

Interactivity: Low. The interactivity is limited to merely providing a new equation with its solution.

Feasibility of use in class: Unlikely. This demo is extremely limited in form. It might be improved for developmental level mathematics students if it actually showed the last step in its
calculation: the use of the multiplication property to remove the coefficient on x. It would be particularly clear if division by $-2$ was shown in red on both sides, the way use of the addition property is shown for the first step.
43. **Title of Demo:** A Variable Can Be Anything (J. Nochella)

**Description of demo:** The demo gives several mathematical operations and gives you the option of choosing a variable and assigning it a value.

![Image of the demo](image)

\[
\begin{align*}
r &= 25 \\
\sqrt{5} r &= \sqrt{5 \times 25} \\
&\approx 11.180
\end{align*}
\]

**Course(s) the demo might be appropriate for:** Basic Mathematics

**Curriculum topic the demo is relevant to:** Introduction to Algebra; variables, evaluation.

** Appropriateness:** Poor. This demo only allows positive integer values for its variables. Even for very basic pre-algebra, this is overly simplistic.

**Cognitive fidelity:** Average. The demo does allow evaluation of some fairly complex operations, but wastes an excellent opportunity to show evaluation of \(x^2\) for positive and negative numbers.

**Clarity:** Excellent.

**Accuracy:** The title “A variable can be anything,” is misleading if positive integers less than or equal to 25 are the only values allowed.

**Interactivity:** Low

**Feasibility of use in class:** Not feasible.
Application of Demonstration Applets

Now that evaluations of the demonstration applets have been completed, some discussion on uses of the applets in the classroom is in order. The Wolfram Demonstration Project was designed as a collection of teaching tools to be used in the classroom by the teacher to illustrate and give visualizations of concepts. For this reason, the majority of the applications are most useful as presentation tools or in-class group exploration, but others do allow for exploration by individual students. Although assignments can be given for exploration outside the classroom, there is no tracking on the website to allow an instructor to assess whether students have visited the site. However, questions can be formulated and placed on D2L or other course website to assess the performance of the explorations.

To integrate the applets into my curriculum in a manner that is both meaningful and effective, I have identified three different formats for using the applets, depending upon how each applet lends itself to the cognitive task at hand: 1) inquiry-based exploration of the topic facilitated by the teacher in demo-mode, 2) simple use of the applets alone or in combination as illustrations of a concept, and 3) student activity requiring the advance reservation of a computer room for a particular lesson to be given on a particular day.

A good example of a demo that lends itself to in-class exploration led by the teacher in an inquiry-based format is “Equations and Inequalities with Absolute Values” (E. Schulz). I considered having students perform this exploration on their own with a guided study worksheet, but concluded that students might not take the time to make conjectures of their own before they move the slider to discover the answer. There is also no way to assess whether students actually visited the website, or just opted to go straight to the exercises in the text. A sample lesson plan for using this applet is included as Appendix B. For use in the classroom itself, this is an
inquiry-based lesson plan, intended to draw students out and encourage verbal participation, as well as to promote conceptual thinking. In addition, it will give students the opportunity to make conjectures based on their own observations. We can then manipulate the applet to see if students’ conjectures hold true, and formulate our own set of rules for these inequalities.

Since it has been shown that when the students are the primary users of the technology, the effectiveness of the technology is enhanced (Li and Ma, 2010), I wish to put the students in the driver’s seat whenever possible. In order to facilitate students’ primary use of the demos, I can reserve a computer lab for my class on occasion. In keeping with the desire to use a more constructivist method to enhance benefits of technology in the classroom (Jacobson & Cheng, 1998), I have developed a guided discovery approach lesson plan, included as Appendix C. A short, in-class worksheet is included in the plan and can be used for assessment purposes.

Some of the applets are not in-depth enough that an entire lesson can be based around them, but they have value nevertheless. For a unit on area, perimeter and volume of geometric figures, there are several applets that give good visuals of concepts that have been hitherto difficult for students to grasp, such as why the areas of triangles and trapezoids are calculated as they are. I will likely use several of these applets to illustrate these concepts in a single lesson. A sample lesson plan is included as Appendix D.

In addition to the above formats, recent upgrades to the Wolfram programming now enable the demos to be embedded on web pages or electronic documents. It is possible to place the demos on the course website so that students can access them easily on their own for additional experience or exploration, although the student must download the CDF player in order to manipulate the applets. In order to embed an applet, the destination web page must allow e-frames. At this time my own class website does not have this capability, but links can be
included on the course website to facilitate student access to particular applets. For assessment purposes, some of the applets allow for copying of a completed frame, and “sharing” it, or sending it to the instructor, to confirm that the application has been utilized by the student. This has the added advantage of being “paperless.”
Chapter 5

Conclusions

Teaching is a highly subjective field, and differences between individual instructors and their preferred or effectual methods are myriad. I have identified in this paper many technology applications that I believe will be useful for incorporating relevant technology into my classes in an effective and appropriate manner. I recognize that some applications that I consider to be excellent might be viewed in a different light by other instructors. I have, therefore, included in my paper analyses of applications that I consider very good as well as some that I find less useful, in the hopes that this analysis may help others to locate items that may be helpful to them.

Many of the demonstrations I evaluated for this paper were impaired by the fact that too much information was given all at once, which makes the applets less valuable as teaching tools. An example of this is “Do Not Divide by Zero (Beck). Although there are no inaccuracies in the method Beck uses to illustrate his point, it is purely mathematical, and a single manipulation brings the entire calculation into view.

I use basically the same method in my Basic Mathematics classes, but I use physical manipulatives to demonstrate. I get a student to volunteer, and hand him or her 20 pencils. I ask the volunteer to divide by separating the pencils into piles. We discuss divisions that give a remainder, as well as divisions by fractions, but each time the division is finished when all the pencils are moved into piles. Students discover that when you must place all the pencils into piles of zero pencils, you will never be finished. This method gives the same information, but it is more visual and more “hands-on” than a mere screen full of calculations.

More benefit is derived from students making the discoveries themselves than when premature exposition by the teacher (or even by a computer application) removes the opportunity
for students to make those connections themselves. Many of the more useful applications that I identified are the more dynamic ones. These have the capability of revealing enough information that students have the opportunity to make conjectures without giving all the concepts at once. Some are clear and in-depth enough to base an entire lesson around. The lesson plan I developed on absolute value inequalities is a good illustration of this (Appendix B). By manipulating the applet carefully to expose only certain parts at a time, the teacher can give students the opportunity to anticipate and to speculate on the nature of absolute value inequalities and discover the rules that govern them.

Some of the applications are excellent tools for visualization of an abstract concept. The applet, “Areas of parallelograms and trapezoids” (H. Papadopoulos) is ideal for showing students the physical reality of the area calculations. This can give a new vitality to a concept that is most often learned by rote due to lack of a visual correlation.

I have limited the scope of this paper to one collection of electronic applications, the Wolfram Demonstrations Project, due to its accessibility and ease of use. Given the immense number of applications on the website, I was disappointed to find relatively few that were usable or pertinent to my courses. I have now personally experienced Robson’s concern that sometimes little reward is found after hours of searching for viable resources (Robson, 2001). This is, however, a single source, and given time and patience, many more resources are available. For myself, to prevent stagnation and complacency in my teaching style, and in the interest of variety and appeal for my students, I will continue to seek out and use a multiplicity of resources with the goal of having a technological innovation for every class session.

Dynamic illustrations are now being included into e-texts for mathematics education. Wolfram is already sponsoring several e-texts which include interactive capability for all
illustrations. Their calculus book, with over 650 interactive figures, is already in use in over 140 universities, and additional texts for geometry and algebra are expected to be unveiled shortly (Wolfram, 2012b). Computer interactivity in mathematics teaching and learning is no longer the wave of the future—it exists in the present. Incorporating these interactive illustrations into daily class activity now will enhance the learning experience of my students, hopefully providing clarity and interest. It will increase my teaching effectiveness as well, and help prepare me for advances in teaching technology in the future.
Chapter 6

References


www.grsc.k12.ar.us/mathresources/Instruction/Manipulatives/Virtual%20Manipulatives.pdf


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Appendix A

Rubric for Evaluation of Demonstration Applets

Title of Demo: The exact title from the Wolfram Demonstration Project website. Using this title facilitates returning to the demo on the Wolfram site.

Description of demo: Brief description of the demo, how it works and what its interaction displays. How dynamic or static is the representation? When possible, a thumbnail diagram from the website is included.

Course(s) the demo might be appropriate for: From the list of classes I teach or expect to teach (Mat 090, Basic Mathematics; Mat 091, Beginning Algebra; Mat 105, Intermediate Algebra; Mat 110, College Algebra)

Curriculum topic the demo is relevant to: Lists the topic from UW Colleges Department of Mathematics Course Guidelines for the above listed classes that the demo addresses, as well as the key elements illustrated by the demo.

Appropriateness: Appropriateness describes how well the demo adheres and conforms to a topic from the course description. The topic should be covered in a manner that is neither too difficult nor too simplistic for its intended audience. The overall appropriateness of the demo is rated as poor, inadequate, average, good or excellent.

Cognitive fidelity: The ability of the demonstration to enhance understanding of the concept. Does the demo give a good visual of the concept it is demonstrating? The overall cognitive fidelity of the demo is rated as poor, inadequate, average, good or excellent.

Interactivity: This rates the interactivity of the applet among all other Wolfram Demonstration applets as high, average or low.

Clarity: The ease or difficulty of the demonstration in being understood with a minimum of explanation. The overall clarity of the demo is rated as poor, inadequate, average, good or excellent.

Accuracy: The demo maintains mathematical fidelity, or adherence to mathematical accuracy, and is free from typographical or other errors. No scale is used for this category; errors will be noted specifically.

Feasibility of use in class: What are the chances I might use this demo in a specific class? Feasibility will be the determination not feasible, unlikely, possibly, very likely, certainly.
Appendix B

Sample Lesson Plan for Inquiry-Based Instruction (Math 105)

Equations and Inequalities with Absolute Values

Give a short review of absolute values: \(|-3|, |5|, -|8|\)

Go to Wolfram demo “Number line solutions to absolute value equations and inequalities” (Eric Schulz)

A. Use the equality setting to begin with. (|x| = n)

1. Move the slider through positive right hand side values.
   Ask what changing \(n\) does to the graph.
   Ask what will happen if \(n = 0\), and why. Move the slider to \(n = 0\).
   Ask what will happen if \(n < 0\), and why. Move the slider through negative values of \(n\).

2. Leave the right hand side slider (\(n\)) on some positive value, and move the “middle value” (\(a\)) slider through positive values (slider moves left for positive values).
   Ask what changing the value of \(a\) does to the graph.
   Ask why it moves left if you add a constant to \(x\).
   Confirm that it will move right if you subtract a constant from \(x\).
   Note that the distance between the solutions remains unchanged, only their location on the graph changes.

Remember: \(|x| = 5\) (or any positive constant) has two solutions
\(|x| = 0\) has only one answer, because 0 is neither positive nor negative.
\(|x| = -7\) (or any negative constant) is Inconsistent. No absolute value can ever be negative.

Examples: Solving absolute value equations

\[ |8 - x| = 2 \quad |5x - 3| = 7 \quad \left| \frac{1}{2}x + 3 \right| = 0 \quad |2x - 8| = -8 \]

Two absolute values
\[ |2x + 5| = |5x + 2| \]

Inequalities with absolute values

B. Use the equality setting to begin with. (|x| = n)

1. Watch what happens when = is changed to <.
   Ask students in what way(s) this differs from the equation graphs.
   Ask what will happen if n gets larger, smaller, 0 or less than zero.

2. Move the slider to positive right hand side values.
   Ask what changing n does to the graph.
   Ask what will happen if n = 0, and why.
   Ask what will happen if n < 0, and why.

3. Leave the right hand side slider (n) on some positive value, and move the “middle value” (a) slider through positive values (slider moves left for positive values).
   Change the inequality to >.
   Ask students in what way(s) this differs from the equation graphs.
   Ask what will happen when n gets larger, smaller, 0 or less than zero.

4. Move the slider to positive right hand side values.
   Ask what changing n does to the graph. Ask why this happens.
   Ask what will happen if n = 0, and why this will happen.
   Ask what will happen if n < 0, and why this will happen.

5. Leave the right hand side slider (n) on some positive value, and move the “middle value” (a) slider through positive values (slider moves left for positive values).

Key Concepts (Assist students in developing these rules)

\(|x| < \text{any positive number}\) will have one “in-between” equation, one part graph and one interval.
$|x| > \text{any positive number}$ will have two inequality solutions, two parts to the graph, and two intervals (union).

Examples:  
\[ |3 + 2x| > 7 \]
\[ \left| \frac{1}{3}x - 8 \right| < 11 \]
\[ -2|4 - x| \geq -4 \]
\[ |3x - 12| - 4 \leq 0 \]

WATCH FOR:  
\[ |8x - 15| > -2 \quad \text{This is ALWAYS TRUE} \]
\[ |-3x + 10| < -1 \quad \text{This is NEVER TRUE} \]

Last thing: Remember, before you decide whether your problem is a greater than or a less than problem or always true or never true, you MUST isolate the absolute value on one side of the =.

Examples:  
\[ \frac{1}{3}|2x - 1| + 1 > 2 \]
\[ -3|6 - x| \leq -3 \]
Appendix C

Sample Lesson Plan with Interactive Student Activity for Computer Lab

Slope-Intercept Form of a Line

Finding the Slope of a line from the equation. So far we can find the slope of a line using two points, or from the graph. What happens if all we are given is a line? Wait to see if anyone volunteers a solution that uses the tools we have. If not, suggest or lead them to the idea that we could find two points from the equation and get the slope from the points using the slope formula.

Use this method to find the slope of the line $2x + 3y = 6$

Yes, this works, but it is a rather cumbersome method.

Show the applet “Lines: Two Points” by A. Brown. Place two points so that the intercept is visible on the graph. Have students determine the slope and the intercept from the graph (if intercept is a fraction, have them estimate the value). Compare with the equation on the applet, and what do you notice? Show a couple of examples this way, until they get the idea that “$m$” is the slope and “$b$” is the y-intercept.

Hand out the worksheet on slope-intercept form and have students complete sections A and B (practice writing linear equations in slope-intercept form).

Graphing a line using slope intercept method. Have students use the same three equations from the worksheet and graph using the applet “Lines: Two Points.”

Steps: First, plot the y-intercept, then count the rise and run using the slope to find the next point.

Parallel and perpendicular lines. Parallel and perpendicular lines are ALL ABOUT SLOPE! You cannot determine if lines are exactly perpendicular from a graph!! In order to
determine whether two lines are perpendicular, parallel or neither, you must find the slopes of both lines, and compare.

Two lines are parallel if they have exactly the same slope.

Two lines are perpendicular if they have opposite reciprocal slopes.

Examples: \( \frac{1}{4} \) and 4 ,

\( -\frac{2}{3} \) and \( \frac{3}{2} \)

Have students complete parts C and D of the in-class worksheet.

**Activity.** Have students go to the “Equation of a Line Game” (S.H. Mejia). Spend the rest of the hour playing with the game. (Do not check the advanced box.) Help students who have difficulty. Everyone needs to get at least ten lines correctly. Have students record their highest game scores. (Don’t expect to get all ten in one game!)

The link to the Wolfram site is on the course website. Suggest that students visit the site and play the game for practice.
In-class Worksheet on Slope Intercept Form of a Line

A. Slope intercept form is: _______________________________

Any linear equation can be arranged in slope intercept form by solving for y:

B. Solve for y. List the slope and the y-intercept for each equation:

Slope:  \( m = \frac{x}{2} + 4 \)

\[ y = 8 \]

y-intercept: (0, __)

Slope:  \( m = 2 \)

\[ 2x + 5y = 5 \]

y-intercept:

Slope:  \( m = 3 \)

\[ 3x + 2y = 9 \]

y-intercept:

C. Use the slope and y-intercepts of the linear equations in B to graph the lines on the computer with the applet “Lines: Two Points.”

D. Find slopes parallel and perpendicular to the given slopes:

<table>
<thead>
<tr>
<th>( m )</th>
<th>Parallel</th>
<th>Perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = -2 )</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>( m = \frac{4}{3} )</td>
<td>______</td>
<td>______</td>
</tr>
</tbody>
</table>
E. Are the following lines parallel, perpendicular or neither? Show how you determine this!

$L_1$: Goes through (1, -3) and (4, 0)

$L_2$: Goes through (2, -4) and (-1, -7)

and

$L_1$: $5x - 3y = 11$

$L_2$: $3x + 5y = 8$
Appendix D

Using several small demonstration applets in a single lesson plan

Areas and perimeters of geometric figures

**Perimeters of geometric figures**

Steps: 1) **Make sure all units are the same;** convert if necessary.

2) Add all side lengths or curve lengths together or use formulas.

Triangle: Just add up the sides

Square: 4s (s = length of side)

Rectangle: 2L + 2W (L and W are Length and Width)

Circle: 2πr (r = radius)

Examples: Try a rectangle that is 2 yards by 4 feet

**Areas of geometric figures:**

**Perimeter** is a line, one dimensional (can be measured by a string), and can be represented by fencing around a field, trim work around a window, baseboards around a room, etc. Keyword is “around.” It is measured in linear units: feet, inches, miles, meters, etc. Calculated by *adding* lengths together

**Area** is two dimensional (can be covered with tiles, or carpet), and is often represented as the size of a field, floor or patio. Area is represented in square units: square feet, square inches, square meters. Keyword: cover. Often read as “meters squared” or inches squared. Can be abbreviated as ft², etc. Calculated by *multiplying* two linear dimensions (length x width, radius squared, base x height).

Example: How much carpet will it take to cover the floor of a room that is 4 yds wide by 6 yds long? Draw squares.
Formulas: Be certain all measurements are in correct units before calculating areas! If your answer needs to be in square feet, convert all lengths to feet first. Converting from square feet to square inches is not simple. It is much easier to do the conversions first!!

Square: \( A = S^2 \)  
2 m x 2m

Rectangle: \( A = LW \) (bh)  
3 ft x 5ft

Show the applet “Areas of triangles” to give the visual of why area of a triangle is half the area of a rectangle

Triangle \( A = \frac{1}{2}bh \)  
1.5 ft by 9” (in inches)

Show the applet, “Areas of Parallelograms and Trapezoids” to give a visual of transforming parallelograms and trapezoids into rectangles and then multiplying base times height to get the area.

Parallelogram: \( A = bh \)  
b = 16”  h = 4”

Trapezoid: \( A = \frac{1}{2} (b_1 + b_2)h \)  
b_1 = 7  b_2 = 9  h = 5

Circle: \( A = \pi r^2 \)  
r = 8”

Half-sheet worksheets for in-class group work:

Finding areas and perimeters of odd-shaped figures.

Work in pairs to calculate the following areas and perimeters. Discuss in your group which parts of the figures are important for finding the area, and which parts are included in the perimeter

Find area and perimeter of figure:

Find area and perimeter of figure: