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ABSTRACT

We present a graph-based hierarchical clustering method for chaining edges together into boundaries. The notion of edge employed is quite general, viz., directed line segment with associated probability. Edges are treated as vertices in an arc-weighted directed graph, EG. The weight of any arc, $e_i \to e_j$, in EG is a value between 1 and ∞ representing the degree to which the edge e_j continues e_i, lower values corresponding to better continuations. The measure of continuation from one edge to another is a function of their probabilities, lengths, and locations. EG is initially partitioned into simple chains by deleting any arc $e_i \to e_j$ unless it is both the lowest weighted arc leaving e_i and the lowest weighted arc terminating at e_j. Further subdivision of these chains leads to a tree structure of subgraphs, each of which corresponds directly to a boundary. A probability for each subgraph/boundary is then computed, based on the number of its vertices (edges) and the weight of its highest weighted arc (weakest edge continuation) relative to values of the same characteristics for its ancestors and descendants in the tree.

1. Introduction

This paper describes a parallel model for the formation of boundaries from individual edges. For our purposes (and as described more fully in Lester [1]) an edge is defined to be a directed line segment with associated probability; its intented interpretation is not as a 'divider' between two regions, but rather as a fragment of the outline or contour of a single region. Informally, boundaries are sequences of edges, as suggested in Figure 1, which also illustrates the figure-ground convention for edges and boundaries. Boundaries inherit the 'semantics' of their component edges: that is, they are descriptions of the shape of a single region.

The method used is a clustering technique based on a graph-theoretical model of the fuzzy set [2] of edges for a scene. An arc-weighted directed graph is derived from this set using a measure of continuation between pairs of edges.

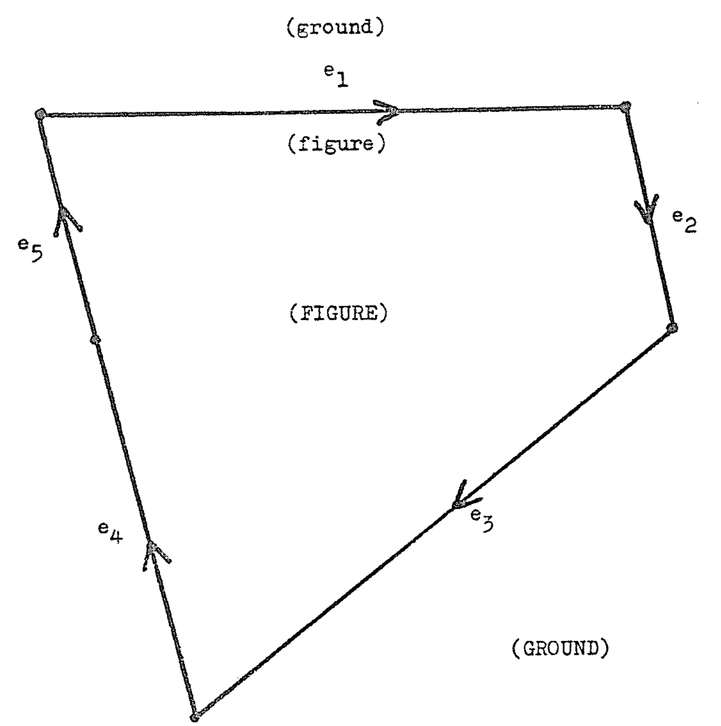

Fig. 1. An edge is a directed line segment (with associated probability and descriptive information, not shown) with "figure" side to the right and "ground" to the left facing in the direction of the edge, as shown for e_1. A sequence of edges (more precisely, edges alternating with connecting segments; see section 2.1) forms a boundary, for example the closed boundary e_1,e_2,e_3,e_4,e_5,e_1. A point P is on the figure side of a boundary when a straight line can be drawn from P to the figure side of any edge in the boundary without intersecting any other component edge.

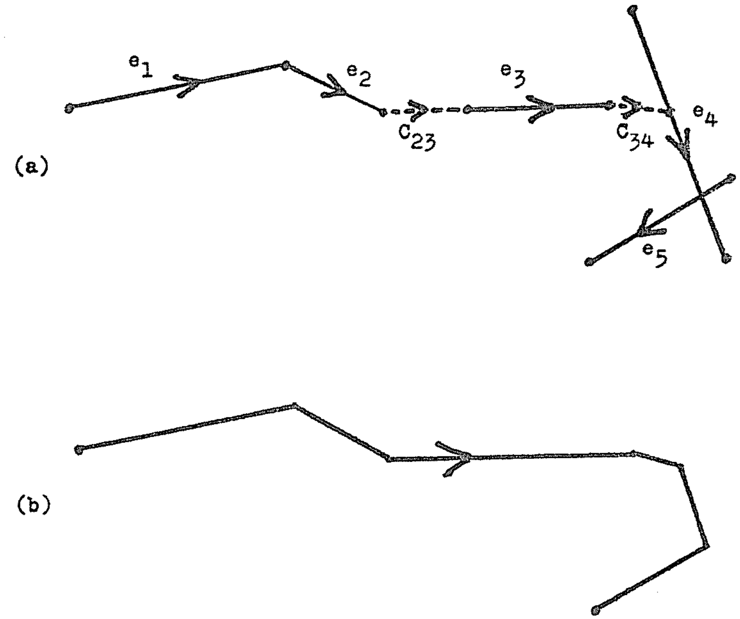

Fig. 2. (a)A set of edges and connectors (dotted lines). The sequence $e_1,e_2,C_{23},e_3,C_{34},e_4,e_5$ (the connectors C_{12} and C_{45} are of zero length and not represented -- in this case we assume that they begin and end at the point of intersection of the edges concerned) is a boundary whose interpretation as a contour or outline is given in (b). Note that sections of e_4 and e_5 that extend beyond the intersection with the preceding or following segment have been discarded.

Local properties of the graph are then used to delete arcs. The pruned graph determines a tree structure of subgraphs -- each corresponding to a boundary -- for which probabilities are computed.

The perception of form has been a problem of longstanding interest in the fields of psychology and automatic scene analysis, and the approach described here to this problem has been influenced by research in both fields. Thus, in broad outline this model resembles that of O'Gorman and Clowes [3], who use clustering of edges to find straight lines in a scene. Zahn [4] has made effective use of a graph-theoretical clustering technique for the perception of scenes of identical disks (as in Fig. 6(a)) directly, that is, without the intervening process of edge detection. Wertheimer's ideas on the basic organizing principles of perception [5] have also influenced the ideas (and terminology) of this paper.

A brief outline of the paper follows:

In section 2 the graph model is defined and the pruning technique described.

In section 3 the transformation from graph to hierarchically structured set of clusters is described.

In section 4 several complete examples of boundary formation are presented. These are based on output from BNDRY, a program embodying the principles set forth below.

2. Edge Graphs

2.1 Continuation of edges

In this section we define COND, a numerical measure of continuation between two edges. Intuitively, this measure should meet the following requirements:

(a) If e_1 and e_2 are edges, then e_2 continues e_1 "perfectly" if the start of e_2 coincides with the end of e_1, both edges point in the same direction, and both are of high probability.

(b) As the edges move apart the continuation becomes weaker.

(c) As the edges point more in different directions the continuation becomes weaker.

(d) As e_2 moves away from the area ahead of e_1 and e_1 moves away from the area behind e_2 the continuation becomes weaker.

(e) The more improbable the edges, the weaker the continuation.

(f) The measure must be scale independent.

The definition involves the construction of a connecting segment or connector, C , between points of each edge. In addition to its function in the definition of COND, this connector supplements the interpretation of a sequence of edges as a boundary by, in effect, redefining the edge length and filling possible gaps between edges, as shown in Figure 2. With the help of this notion we can now define boundary more formally:

(1) A directed line segment beginning at a point of boundary b_j and terminating at a point of boundary b_k (possibly of zero length if b_j and b_k have a non-null intersection) is a connector, C_{jk}, from b_j to b_k.

An edge is a boundary.

A sequence of boundaries alternating with connectors, $b = b_1, C_{12}, b_2, C_{23}, \ldots, C_{n-1\,n}, b_n$, is a boundary. This sequence may not contain any repetitions -- with the exception of $b_1 = b_n$, in which case b is said to be closed.

As a step towards defining COND we introduce an auxiliary function, CD, which for a given e_1, e_2, and C_{12} takes on a value between 1 and ∞, smaller values corresponding to better continuations; the value of COND for e_1 and e_2 is the least of these values for all possible C_{12}. CD is composed of a number of factors, each making precise one of (b), (c), (d), or (e) above (see Figure 3):

The distance between edges:

(2) $DI_C(e_1,e_2) = 1.\emptyset + [2.\emptyset \times len(C)]/[len(e_1') + len(e_2')]$ where len gives the length of its segment argument, e_1' is the segment beginning at the start of e_1 and ending at the start of C, and e_2' is the segment beginning at the terminus of C and ending at the terminus of e_2.

The angle:

(3) $ANG_C(e_1,e_2) = \cos^2([\theta_1+\theta_2]/2.\emptyset)$ if $len(C) \neq \emptyset$

$= \cos^2(\theta/2.\emptyset)$ if $len(C) = \emptyset$

where θ_1 is the angle made by C with e_1, θ_2 is the angle made by e_2 with C , and θ is the angle made by e_2 with e_1.

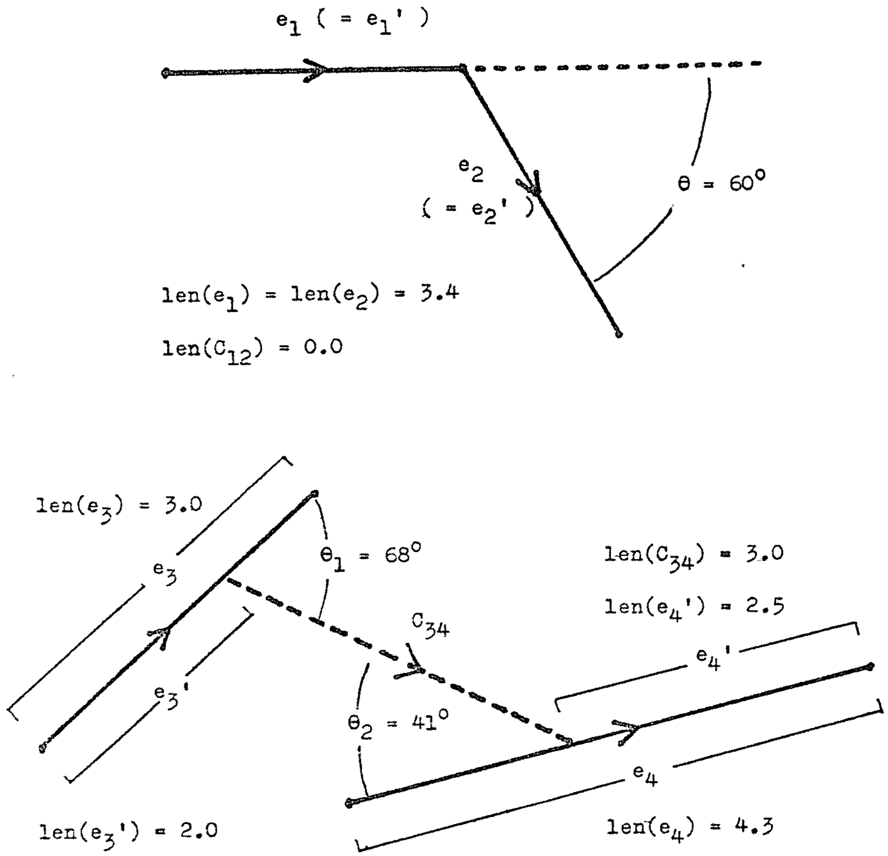

Fig. 3. Two edge pairs, and one possible connector for each. The values of (2) – (4) and (7) for these pairs, assuming PROB(e_i) = 1.0 for i = 1,...,4 (and, therefore, PRB(e_1,e_2) = PRB(e_3,e_4) = 1.0):

<u>DI</u>(e_1,e_2) = <u>1.0</u>	<u>DI</u>(e_3,e_4) = 1.+(2.+3.)/(2.+2.5) = <u>2.3</u>
<u>ANG</u>(e_1,e_2) = $\cos^2(60^\circ/2.0)$ = <u>.75</u>	<u>ANG</u>(e_3,e_4) = $\cos^2((68^\circ+41^\circ)/2.)$ = <u>.34</u>
<u>SAC</u>(e_1,e_2) = <u>1.0</u>	<u>SAC</u>(e_3,e_4) = 1.-(1.+1.8)/3.0 = <u>.067</u>
<u>CD</u>(e_1,e_2) = 1.0/.75 = <u>1.3</u>	<u>CD</u>(e_3,e_4) = 2.3/(.34*.067) = <u>102.</u>

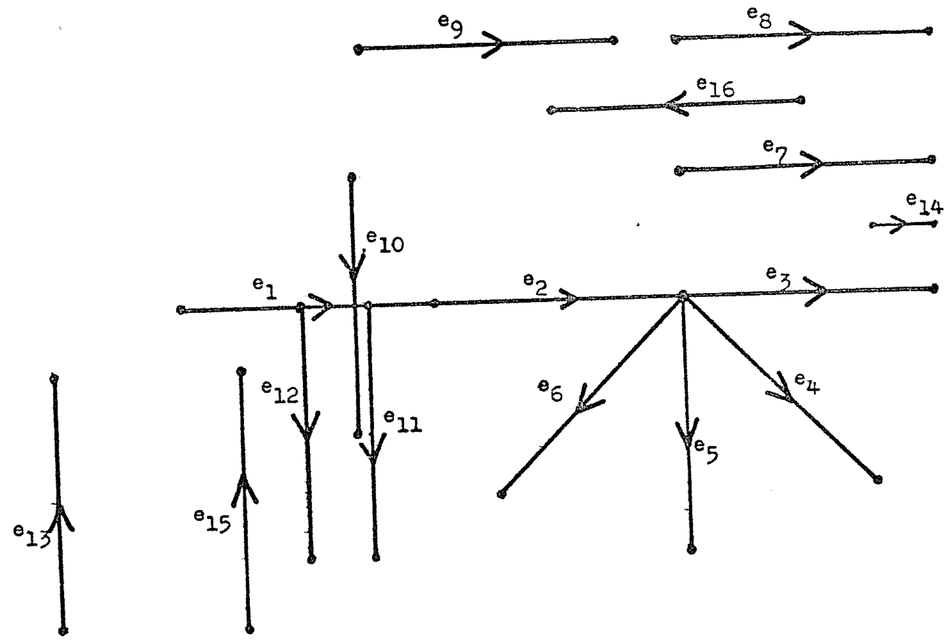

Fig. 4. **Values of COND for all (e_1,e_i) above:**

COND(e_1,e_2) = 1.0	COND(e_1,e_{10}) = 8.0
COND(e_1,e_3) = 2.0	COND(e_1,e_{11}) = 2.7
COND(e_1,e_4) = 2.3	COND(e_1,e_{12}) = 4.0
COND(e_1,e_5) = 4.0	COND(e_1,e_{13}) = ∞
COND(e_1,e_6) = 14.	COND(e_1,e_{14}) = 3.9
COND(e_1,e_7) = 2.7	COND(e_1,e_{15}) = ∞
COND(e_1,e_8) = 4.8	COND(e_1,e_{16}) = ∞
COND(e_1,e_9) = 53.	

The <u>sacrifice</u> of part of one or of both edges:

(4) $SAC_C(e_1,e_2) = \max(\emptyset, 1.\emptyset - [(PROB(e_1)\times[len(e_1)-len(e_1')] + PROB(e_2)\times[len(e_2)-len(e_2')])/ \min(PROB(e_1)\times len(e_1), PROB(e_2)\times len(e_2))])$

where PROB is the probability of an edge.

(SAC is zero when the total probability-weighted length sacrificed is equal to or exceeds the probability-weighted length of the shorter edge.)

The <u>combined probabilities</u>:

(5) $PRB(e_1,e_2) = [2.\emptyset\times PROB(e_1)\times PROB(e_2)]/[PROB(e_1)+PROB(e_2)]$

An additional factor, the <u>affinity</u>, is highly dependent on the particular manner of generating edges. It may, for example, have values close to 1.∅ for two edges which border regions of similar intensity, color, and/or texture, and lower values for edges which border regions differing in these qualities, provided that the edge detecting procedure produces such descriptive information. Although we later (in section 4) present an example of boundary formation in this "richer" edge environment, for the sake of clarity we temporarily assume:

(6) $AFF(e_1,e_2) = 1.0$

COND and CD (for a given C) can be thought of as the inverse probability of one edge continuing another:

(7) $CD_C(e_1,e_2) = DI_C(e_1,e_2)/[ANG_C(e_1,e_2)\times SAC_C(e_1,e_2)\times PRB(e_1,e_2)\times AFF(e_1,e_2)]$

$= \infty$ if above expression undefined (by division by zero)

(8) $COND(e_1,e_2) = \min(CD_{C_{12}}(e_1,e_2))$

$=$ for all C_{12}

Figure 4 gives values of COND (approximate in some cases) for a variety of edge pairs.

2.2 <u>Definition of the edge graph</u>

A set of edges and their continuations can be described as an arc-weighted directed graph, EG, in which each vertex corresponds to an edge, and each arc from vertex e_i to e_j (for convenience we name vertices as we do their corresponding edges) is labelled with the value $COND(e_i,e_j)$. Figure 5 (a)

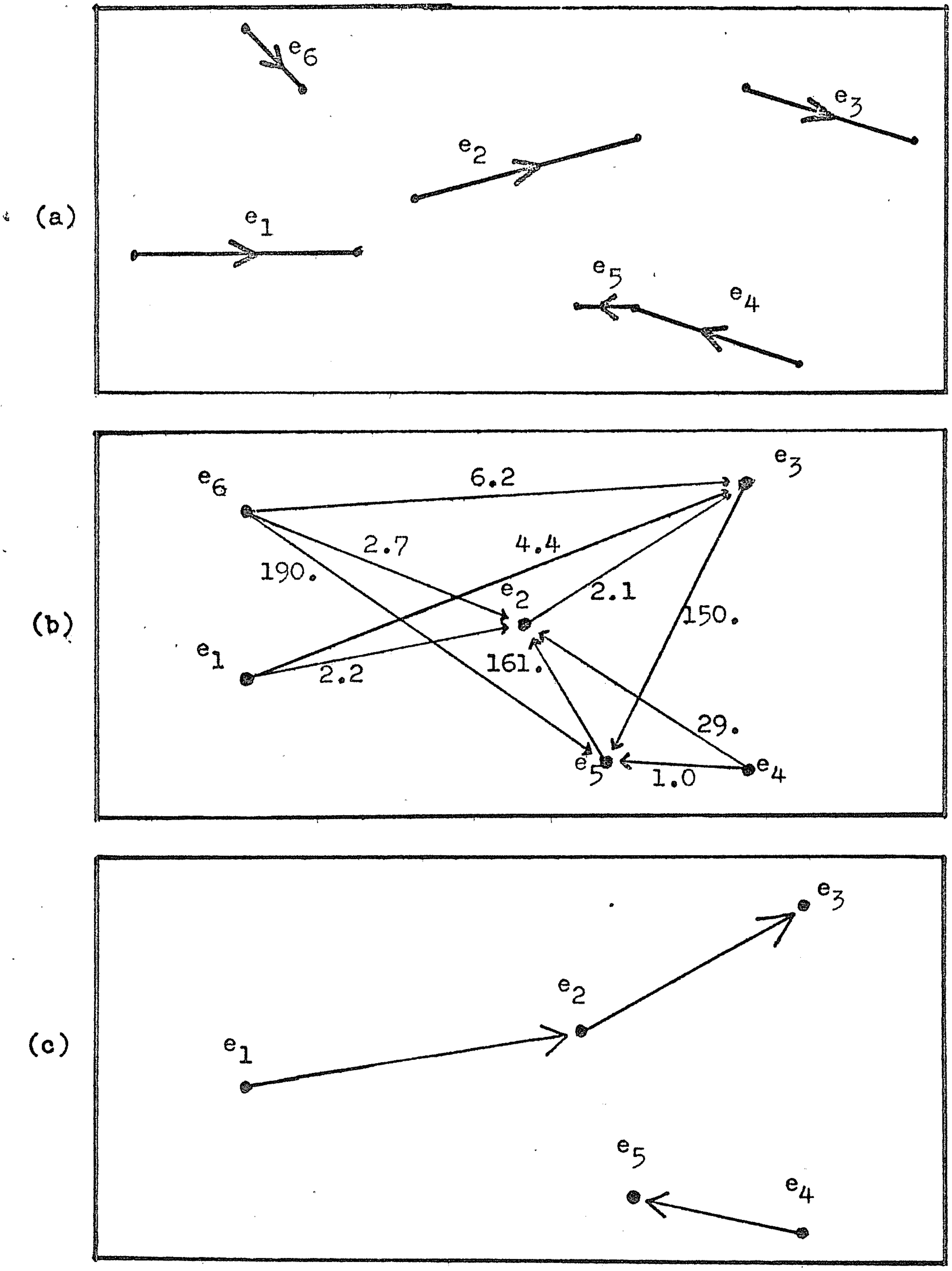

Fig. 5. (a)A set of edges (all probabilities assumed equal to unity). (b)The edge graph for these edges. Every arc $e_i \rightarrow e_j$ is labelled with COND(e_i,e_j); arcs with weights of ∞ (for example $e_1 \rightarrow e_5$) are not shown. (c)The graph pruned according to (9).

and (b) display a small set of edges along with its edge graph.

In terms of the edge graph, boundaries correspond to simple paths or chains (i.e., without repetitions, with the obvious exception made for closed boundaries). The connectors associated with the arcs of EG* are not represented in the graph because they do not enter into the determination and assessment of significant boundaries as described below. However, we note that in order to reinterpret any path as a boundary the associated connector must be known (and, therefore, as a practical matter not discarded after the calculation of COND).

2.3 Pruning the edge graph

The strategy for pruning EG is based on the following simple rules:

(a) An edge can be a part of only one boundary; equivalently, two boundaries cannot have any edges in common.

(b) If e_j immediately follows e_i in a boundary, then of all edges in the scene e_j must be the best continuation (in the sense of having a minimal value of COND) for e_i, and e_i the edge having the best continuation to e_j.

The first of these is justified directly by the 'semantics' of edges and boundaries, that is, their intended interpretation as (partial) outlines of shapes: any such outline must be a simple curve, without branches. On the other hand, (b) (which, of course, implies (a)) is a heuristic designed to eliminate as many spurious boundaries as possible, including those made up of very improbable edges or a mixture of 'good' and 'bad' edges, and those made up of good edges from more than one actual shape or from a single contour with edges missing or improperly sequenced. The effect of (b), then, is to retain boundaries generally faithful to the actual contours, but in practice, for scenes of some complexity and with edges generated on the basis of local evidence -- composed of relatively few edges and falling short of 'ideal' closed boundaries. (A partial solution to the problem of completing boundaries is described below in section 4.2)

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*The connector yielding the lowest value of COND for some edge pair is not necessarily unique. We assume, in this event, some uniform method for choosing a single optimal connector.

The pruning procedure:

(9) For every arc, $e_i \to e_j$, in EG, prune it if and only if it is not both the lowest weighted arc leaving e_i and the lowest weighted arc terminating at e_j. Arcs with weights of ∞ are also deleted. All vertices isolated by these deletions are discarded.

Figure 5(c) shows the result of applying (9) to the graph of Figure 5(b).

In Figure 6 we present a scene composed of identical disks, the edges produced by an edge detecting program (EDGE [1]), and the graph produced by (9). This simple example makes it clear that more remains to be done. First, some of the chains are perceptually inconsequential; second, in spite of the conservatism of the pruning strategy, some sub-chains correspond to more reasonable boundaries than the complete chains of which they are parts. In the next section we introduce a method for segmenting and evaluating chains that resolves these problems.

3. Hierarchical Clustering of Edges

3.1 Structuring chains

In the previous section we used a graph to represent a set of edges and their continuations. In this section we derive a tree representation for the chains formed by (9) with the following procedure:

(10) (a) Create a tree node for each chain, and attach them as sons to a root node, R.

(b) Divide each chain into sub-chains by deleting from each the highest-weighted arc(s).

(c) Attach to each chain node one son node for each sub-chain formed in (b).

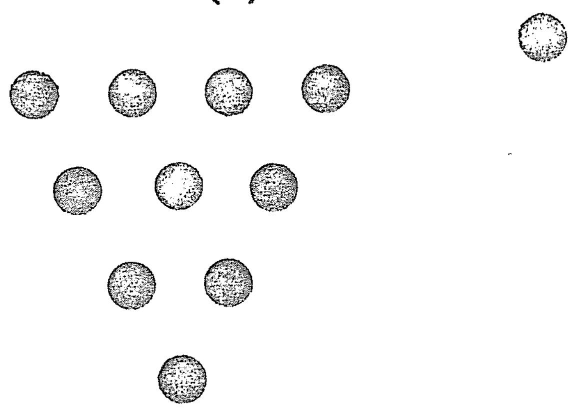
(d) Continue (b) and (c) until no further subdivision is possible.

Every non-terminal in the tree except R corresponds to a boundary; every terminal corresponds to an edge. Figure 7 shows the hierarchical (tree) structure imposed on the chains of Figure 6 by this procedure.

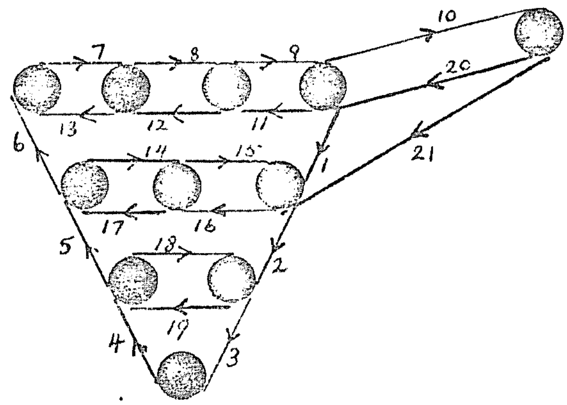
3.2 Evaluation of subtrees

Any non-terminal S , except R , can be characterized by the largest weight on any arc in its corresponding path, DF , and the number of vertices in same (equal to the number of terminals in the subtree whose root is S),

(a)



(b)



PROB( )=		PROB( )=	
e_1	.16	e_{10}	.11
e_2	.20	e_{11}	.026
e_3	.21	e_{12}	.026
e_4	.22	e_{13}	.026
e_5	.22	e_{14}	.026
e_6	.22	e_{15}	.026
e_7	.25	e_{16}	.026
e_8	.25	e_{17}	.026
e_9	.24	e_{18}	.026
		e_{19}	.026
		e_{20}	.036
		e_{21}	.054

(c)

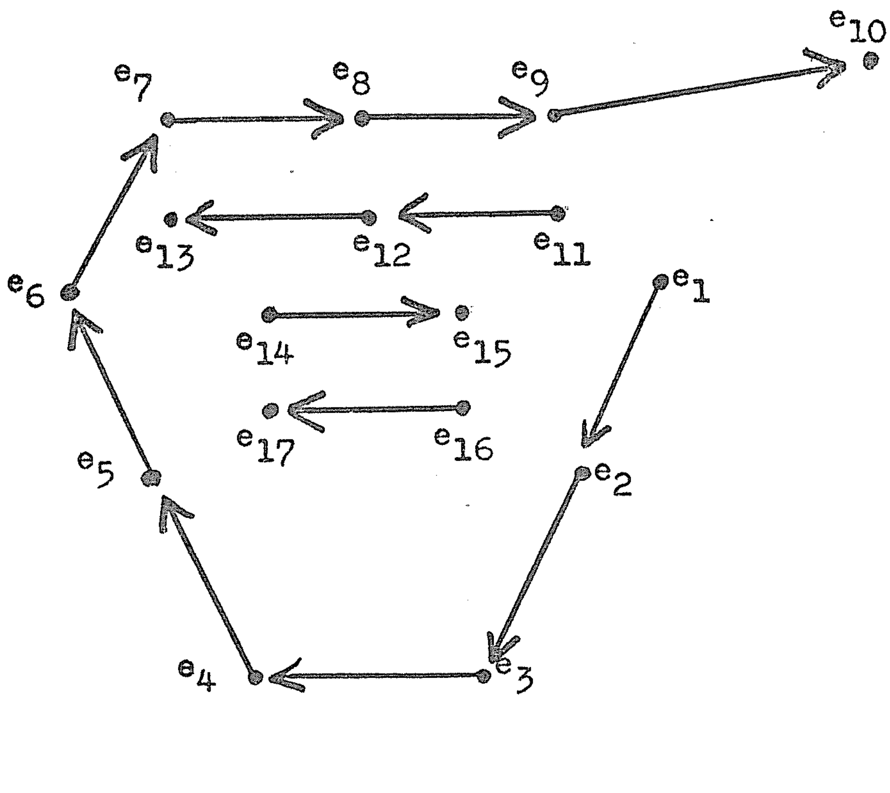


Fig. 6. (a)Scene of identical disks. (b)The edges and probabilities for this scene (produced by EDGE[1]). (c)The pruned graph — consisting of four chains — for the edges of (b). Note that four of the original twenty-one edges have been isolated and deleted.

CNT. Any S will have strictly lower values of both DF and CNT than any ancestor of S.

We now assign probability values to each non-terminal S (except R) of the tree based entirely on CNT and DF values of the node and their relationship to values of these characteristics for ancestors and descendants of S. Thus in this evaluation the directed or ordered nature of the original edge sequences has been lost, and boundaries are now treated simply as clusters or groups of edges with particular CNT and DF values.

Probabilities are computed from three interrelated measures, all with values in the range Ø.Ø to 1.Ø:

The absolute strength of a cluster:

(11) $ABS(S) = 1.0/[1.0+(DF(S)-1.0)/\sqrt{(CNT(S)-1.0)}]$

The strength based on descendants:

(12) $DESC(S) = \min(1.0-[CNT(D)/CNT(S)]\times[(DF(S)-DF(D))/DF(S)])$ for all D that are non-terminal descendants of S

$= 1.0$ if there are no descendant non-terminals

The strength based on ancestry:

(13) $ANC(S) = [DF(F)-DF(S)]/DF(F)$

$= 1.0$ if $F = R$

where F is the father of S

ABS takes on values closest to 1.Ø for groups consisting of a large number of edges close together; DESC scores low only if there is a large and compact sub-cluster; ANC scores low only if the father of the cluster has a DF value close to that of S itself.

The complete probability function, Q , combines these measures multiplicatively:

(14) $Q(S) = ABS(S)\times DESC(S)\times ANC(S)$

Figure 8 gives values of (11) - (14) for the non-terminals of Figure 7.

4. Experimental Results

4.1 Two examples

We now present two complete examples of boundary formation. In both, the initial scene is far more complex than that of Figure 6, and,

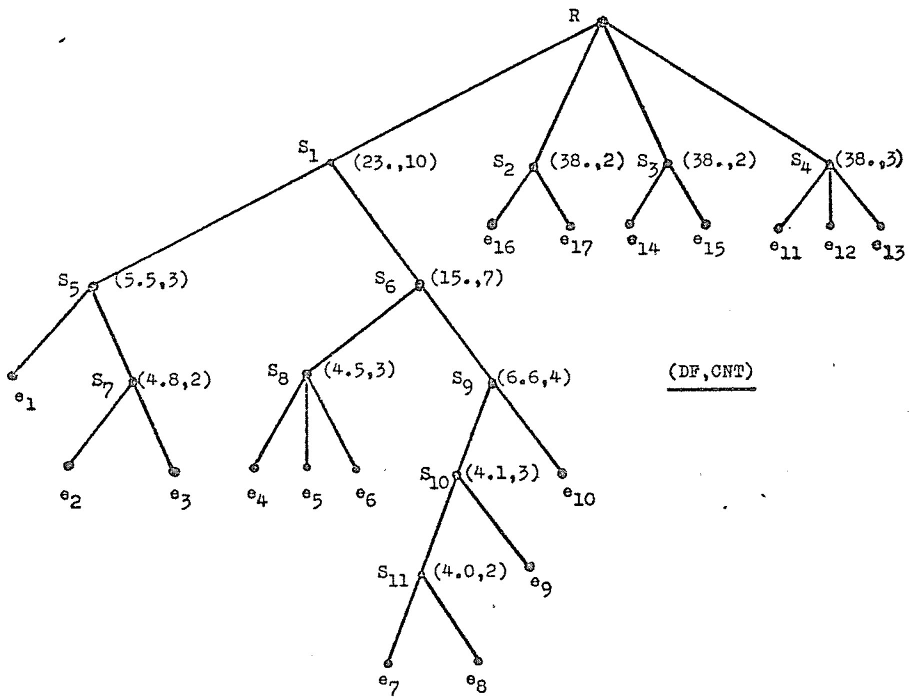


Fig. 7. Hierarchical clusters produced by (10) for the chains shown in the preceding figure. Values of DF and CNT are given beside each of the eleven non-terminal nodes corresponding to possible boundaries.

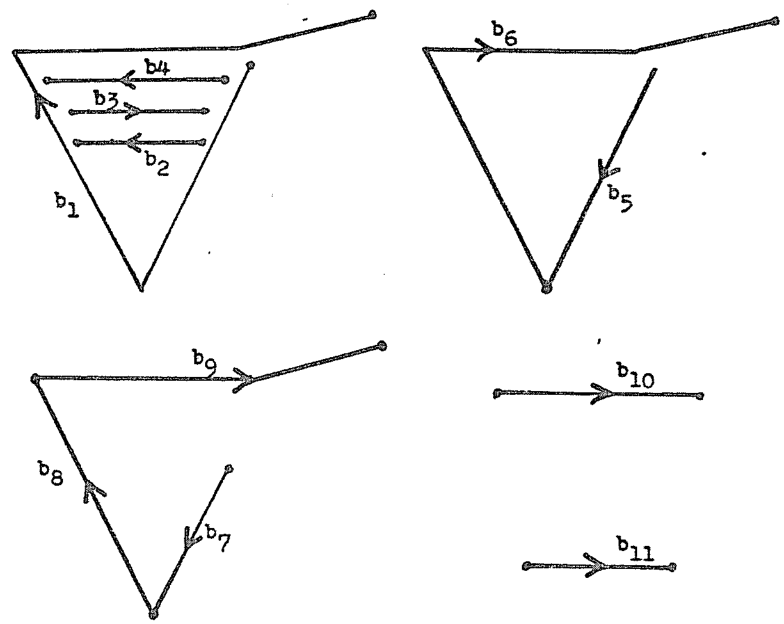


Fig. 8. Boundaries corresponding to the non-terminals/clusters of Fig. 7 (with the convention that b_i corresponds to S_i). The values of Q — given below along with ABS, DESC, and ANC — show b_5, b_8, and b_{10} to be the most probable of these.

$ABS(S_1) = .12$
$DESC(S_1) = .72$
$Q(S_1) = .086$

$ABS(S_2) = .026$
$Q(S_2) = .026$

$ABS(S_3) = .026$
$Q(S_3) = .026$

$ABS(S_4) = .037$
$Q(S_4) = .037$

$ABS(S_5) = .24$
$DESC(S_5) = .92$
$ANC(S_5) = .76$
$Q(S_5) = .17$

$ABS(S_6) = .14$
$DESC(S_6) = .61$
$ANC(S_6) = .35$
$Q(S_6) = .032$

$ABS(S_7) = .21$
$ANC(S_7) = .13$
$Q(S_7) = .027$

$ABS(S_8) = .30$
$ANC(S_8) = .69$
$Q(S_8) = .21$

$ABS(S_9) = .24$
$ANC(S_9) = .56$
$DESC(S_9) = .725$
$Q(S_9) = .097$

$ABS(S_{10}) = .315$
$ANC(S_{10}) = .38$
$DESC(S_{10}) = .99$
$Q(S_{10}) = .12$

$ABS(S_{11}) = .25$
$ANC(S_{11}) = .0$
$Q(S_{11}) = .0026$

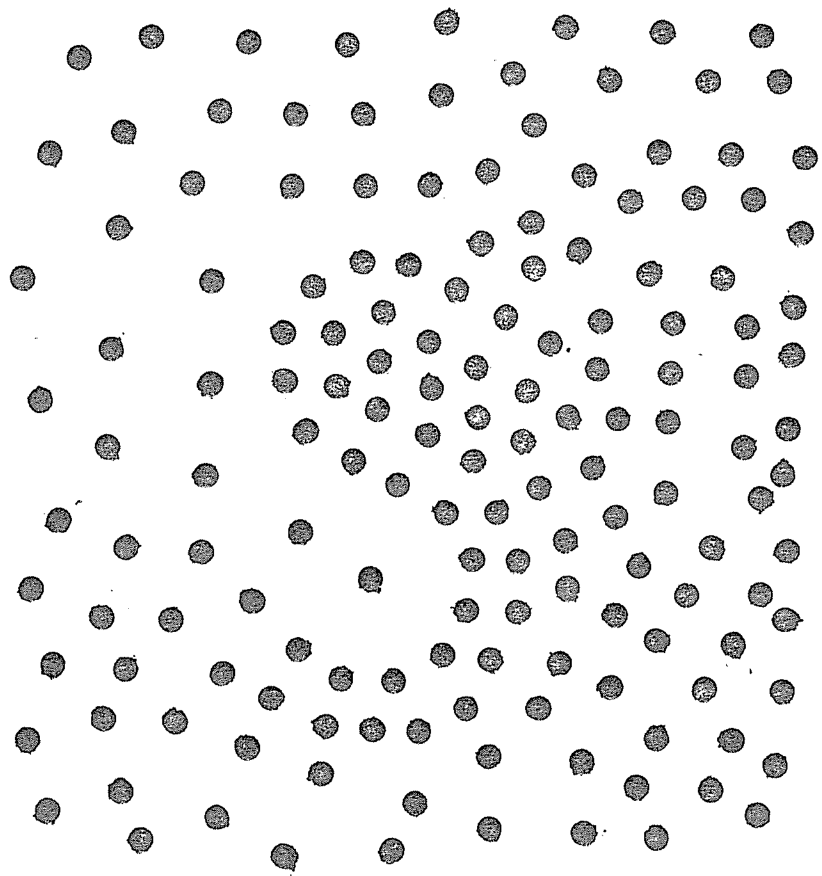
consequently, the sets of edges for these scenes -- produced in each case by the same program, EDGE (described in Lester [1]) -- are much larger and must less neat in comparison. Although the results have been transcribed from line printer output for the sake of clarity, the entire process of boundary formation, from the calculation of continuations through the assessment of probabilities, has been carried out automatically by BNDRY, a FORTRAN program running on the University of Wisconsin's Univac 1110 computer.

The first example, Figure 9, is a scene of identical disks, roughly square in overall shape and with a clear "S" showing in the center. Figure 9(c) demonstrates the appropriate perception of this scene, in spite of the large number of spurious edges and a very intricate edge graph (and the continued restriction that AFF = 1.0; we do use a non-constant AFF for Figure 10, which could have been employed here as well to some advantage).

This example was originally devised to buttress an argument for an edge/boundary -- as opposed to a regional (for example, Brice and Fennema [6], Zahn [4] or Tomita [7]) -- approach to figure extraction. Briefly, the argument is that boundaries are more general, in the sense that every region has a corresponding closed boundary, but boundaries which are not closed have no corresponding regions. This observation would be trivial were it not for the fact that non-closed boundaries are quite definitely perceivable and potentially significant, as shown by Figure 9.

Figure 10 (a) is a line printer representation of a digitized T.V. image, produced in the Systems Laboratory of the Computer Sciences Department, University of Wisconsin. Such images are normally characterized by a good deal of noise, a fact which is obscured somewhat by the "thresholding" effect of this representation (seven levels of gray are used). A cross section through the major (disk) contour of the edges produced from the raw data -- displayed in Figure 10 (b) --suggests more accurately the difficulties faced by BNDRY: principally, this thick band of edges -- but with an additional complication presented by edges scattered about the two apparently homogeneous regions (not shown).

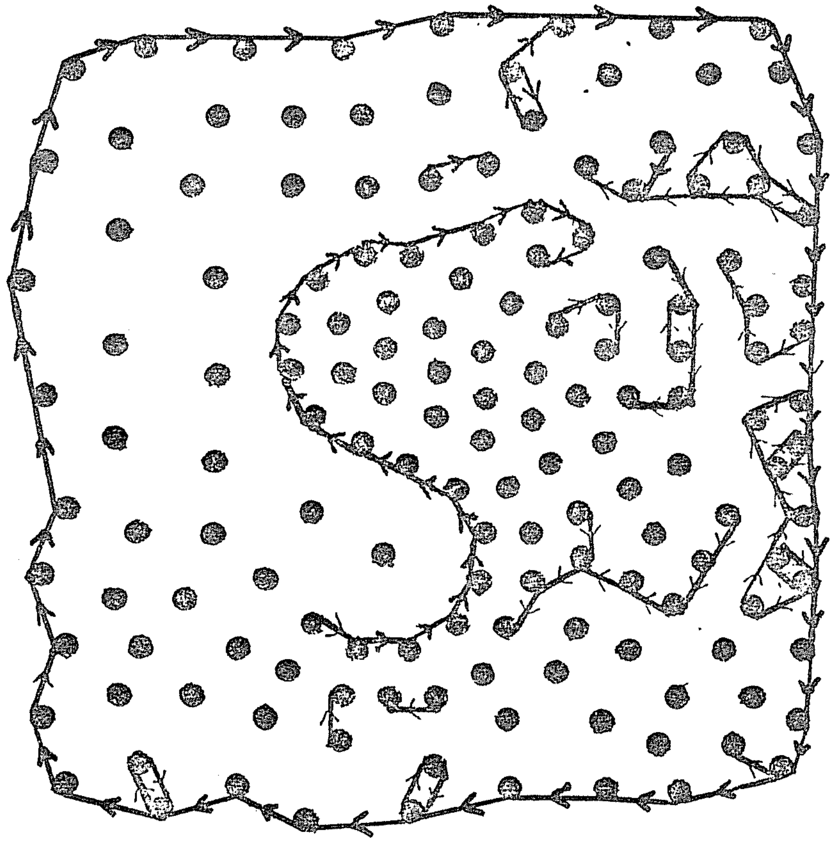
In this example, the following function was used to measure the affinity between edges:



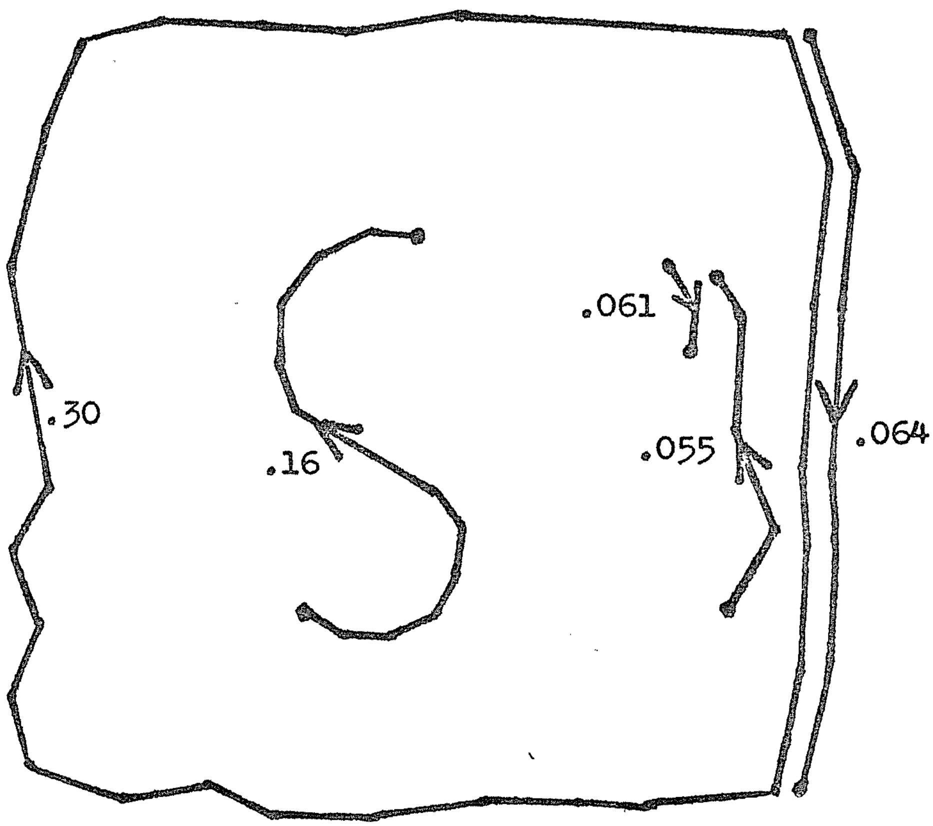
(a)

Fig. 9. (a)Scene of identical disks.

(b)



(c)



(b)The one hundred most probable edges (produced by EDGE [1]) for this scene. (c)The five most probable boundaries found by BNDRY (based on these edges). The values of Q given alongside each boundary clearly indicate two major contours: a closed outer "square" and a non-closed "S". (The vertical boundary along the right hand margin of the scene is a sub-boundary of the "square" boundary.)

(15) $AFF(e_1,e_2) = \min[len(e_1),len(e_2)]/(\max[len(e_1),len(e_2)] \times [1.0+|AVINT(e_1)-AVINT(e_1)|])$

where $AVINT(e)$ is an estimate of the average intensity of the area on the figure side of and close to the edge e.

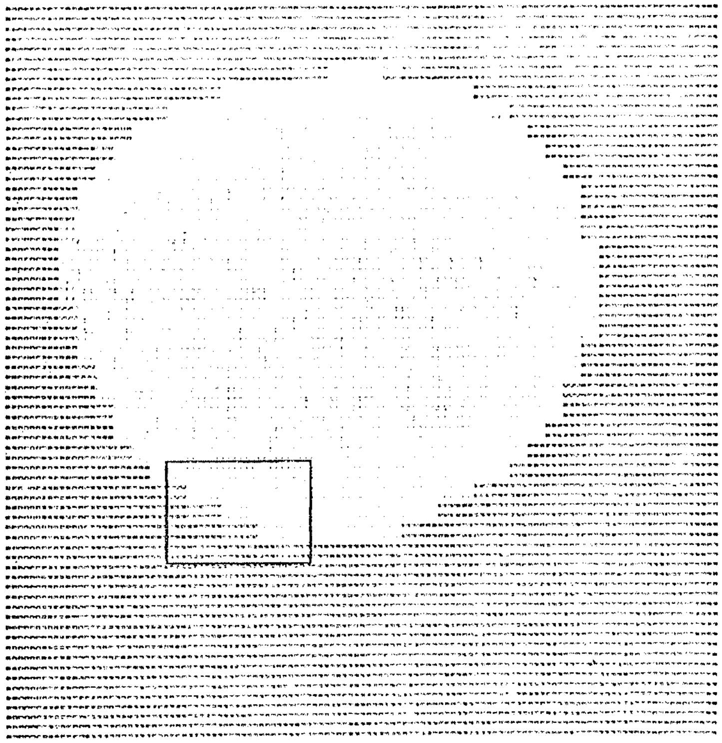
A thorough explanation and justification of this particular function is impossible here since so much depends on the mechanisms of EDGE (again the reader is referred to [1]). The net effect for this example, however, is to produce large values of COND between edges bordering regions differing greatly in intensity.

It follows from the semantics of boundaries (as we use this concept) that, ideally, we would like to find two closed boundaries for this scene: a clockwise circular boundary for the light disk, and a counterclockwise circular boundary for the darker background. The actual set of boundaries found, the best of which are shown in Figure 10(c), are indeed fragments of these ideal boundaries.

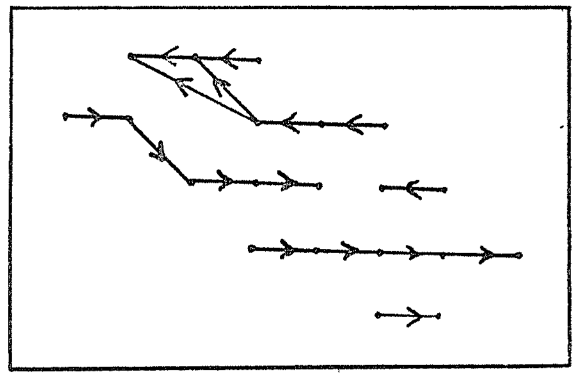
4.2 An extension and a conclusion

It seems natural to suggest that the boundaries of Figure 10(c) be treated as edges and grouped into larger clusters until closed boundaries are formed. In fact, this suggestion is implicit in the recursive definition of boundary, (1), given previously. To apply the methods already developed for edges is actually straightforward, with one exception: the measure of continuation between edges, COND, must be generalized to handle boundaries that are not straight. Work is now in progress on this generalization and its realization in program form.

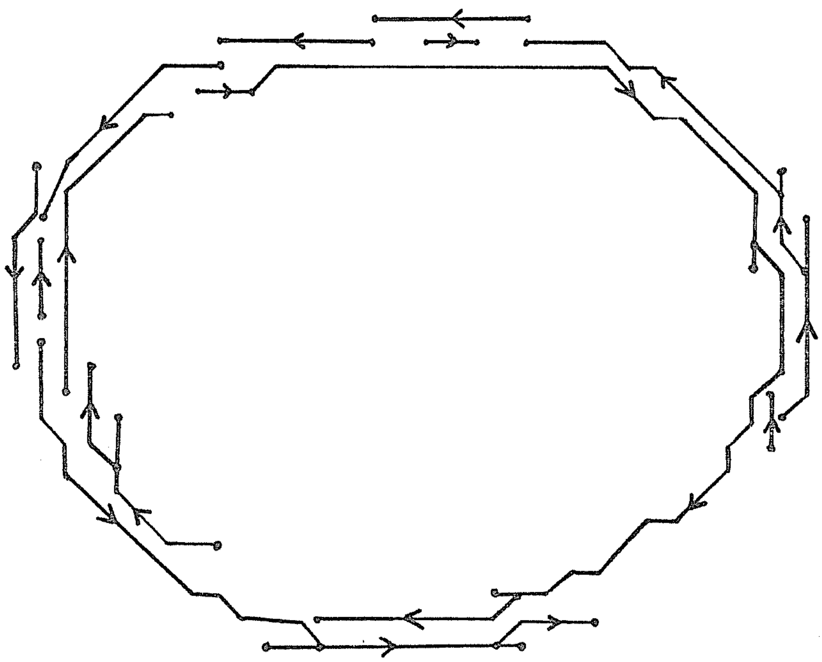
In summary of this paper and present accomplishments: We have described a model of boundary formation for visual perception suitable for highly parallel implementation and employing a graphical representation of the (fuzzy or probabilistic) set of edges derived from a scene and a hierarchical clustering technique. Boundaries are seen as following the outlines of particular figures -- not as separating two regions -- and may be either straight or curved (i.e., jagged), closed or not closed. The model is embodied in a program which provides examples illustrating the capabilities and potential of the model in complex and noisy environments.



(a)



(b)



(c)

Fig. 10. (a)Line printer representation of a digitized T.V. picture. (b)The edges (produced by EDGE[1]) in the outlined area of (a). (c)The twenty most probable boundaries as determined by BNDRY (based on the two hundred fifty most probable edges in the scene).

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