

# An International Study of Mathematics in the Middle Grades: China, Russia, and the United States

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## Introduction

- Motivation:** Mathematics not only applies to daily life, but is the foundation of all sciences. A large part of global competition revolves about the growing economy and the advances in technology. To maintain a leading position requires a well-educated work force and talented people to do research.  
**How does the American mathematics education program stand up to the challenge?**
- Research Team:** The team includes three students— Chelsey, a mathematics education major from the United States; Alice, a mathematics major from Russia; and , Ying, a mathematics major from China. Alice and Ying received middle grades education in their respective countries. Two professors are also involved: Dr. Simei Tong is a mathematician in functional analysis, and Dr. Kate Masarik is a mathematics educator. The team has met weekly since the summer of 2010.
- Project Work:** Using mathematics textbooks from each of their respective countries, the team compared two selected topics. The manner in which the topics were introduced and explained, the level of difficulty of examples and homework exercises, and the type of assessment questions that were used were examined and discussed. The team also shared knowledge of their respective schools and their middle school experiences.
- Recommendation:** Based on their findings, the research team has developed a list of recommendations that could enhance the mathematics education in the middle grades in the US.

## Layout of Pre-College School Years

China	Russia	United States
<i>Primary (Elementary)</i> # of years: 6 Ages: 6 – 11	<i>Primary</i> # of years: 4 Ages: 7 – 10	<i>Elementary K – 5</i> # of years: 6 Ages: 6 – 11
<i>Junior Level Middle School</i> # of years: 3 Ages: 12 – 14 Length of school day: ~ 8.5 hrs <sup>1</sup> # of hrs of math/week: ~5.25	<i>Basic General Education (lower secondary)</i> # of years: 5 Ages: 11 - 15 Length of school day: ~ 8 hrs <sup>2</sup> # of hrs. of math/week: ~5.25	<i>Middle School</i> Grades 6 – 8 # of years: 3 Ages : 12 – 14 Length of school day: ~6.5 hrs <sup>3</sup> # of hrs of math/week: ~4.2 <sup>4</sup>
<i>Senior Level Middle School</i> # of years: 3 Ages: 15 – 17 Test required for entry into Senior Level Middle School	<i>Secondary (complete) General Education</i> # of years: 2 years Ages: 16 – 17 Test required for entry into Secondary General Education	<i>Secondary (High School)</i> Grades 9 – 12 # of years: 4 Ages: 15 - 18

## Introduction to Inequalities

### China

**Question:** At 11:20 a car moving at a constant speed is 50 kilometer away from station A. If the car would pass station A before 12:00 a.m., what is the car's speed?  
Let  $x$  be the car's speed. Based on the time and distance conditions;  
a) The car will pass station A before 12:00am, with this speed the 50 kilometer drive will take less than  $\frac{2}{3}$  hours, so  $\frac{50}{x} < \frac{2}{3}$  ①  
b) The car will pass station A before 12:00am, with this speed while driving  $\frac{2}{3}$  hours, the distance must over 50 kilometers, so  $\frac{2}{3}x > 50$  ②  
① and ② express the car's speed based on different conditions. These are called inequalities.

### Russia

**Rule:** A given number  $a$  is bigger than a given number  $b$ , if the difference  $a-b$  is positive number;  $a$  given number  $a$  is smaller than a given number  $b$ , if the difference  $a-b$  is negative number.  
**Example:** Prove that with any given  $a$  the following inequality holds:  $(a-3)(a-5) < (a-4)^2$   
**Proof:** Let's put it on one side and simplify the difference:  
 $(a-3)(a-5) - (a-4)^2 = a^2 - 3a - 5a + 15 - a^2 + 8a - 16 = -1$   
With any given  $a$  the difference is negative, therefore  $(a-3)(a-5) < (a-4)^2$ .

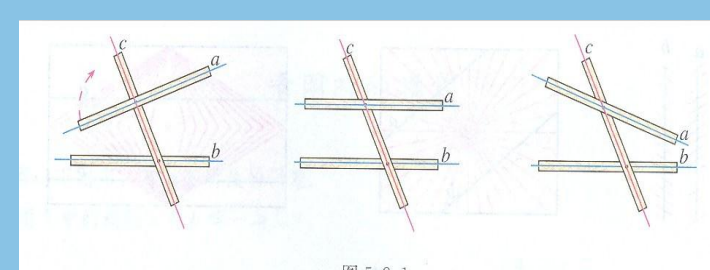
### United States

An **inequality** states that two quantities either are not equal or may not be equal.  
**Write an inequality for each situation.**  
There are at least 25 students in the auditorium.  
number of students  $\geq 25$   
"At least" means greater than or equal to.

## Introduction to Parallel Lines

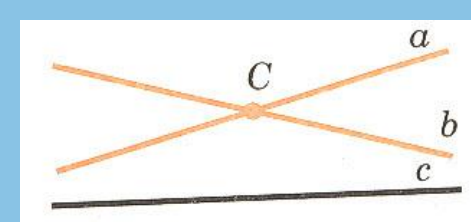
### China

**Thinking:** In the figure, nail sticks  $a$ ,  $b$ , and  $c$  together, and imagine all of them can be extended infinitely at both ends. Have line  $a$  intersect line  $b$  on the left side, then rotate line  $a$  to intersect line  $b$  on the right side of line  $c$ . During this rotation, is there a position that line  $a$  does not intersect with line  $b$ ?  
During the stick rotation, there exists a position that line  $a$  does not intersect with line  $b$ , and the lines are parallel to each other, notation  $a \parallel b$ .



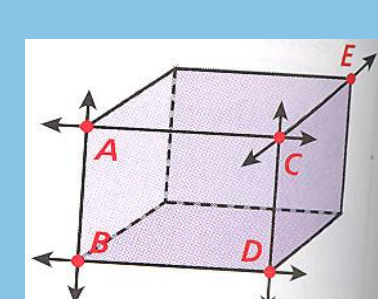
### Russia

**Theorem:** Two lines that are parallel to a third line is also parallel to each other.  
**Proof:** Let two lines  $a$  and  $b$  be parallel to a line  $c$ . Assume that lines  $a$  and  $b$  are not parallel, which means that they will cross each other at the point  $C$ . Thus, those lines that are parallel to a line  $c$  But that is a contradiction because we cannot draw more than one line through this point that is parallel to the given line.



### United States

Some lines in the same plane do not intersect at all. These lines are **parallel lines**. Segments and rays that are parts of parallel lines are also parallel.  
**Tell whether the lines  $\leftrightarrow$  and  $\leftrightarrow$  appear parallel, perpendicular, or skew.**  
 $\leftrightarrow \perp \leftrightarrow$   
 $\leftrightarrow \rightarrow \leftrightarrow$   
The lines appear to intersect to form right angles.



## Assessment

### Inequalities

**Homework**  
Solve for positive integers:  $\frac{x-3}{2} \geq \frac{2x-5}{3}$   
**Test**  
If the solution of the equation  $3m(x+1)+1=m(3-x)-5x$  is a negative number then the range for  $m$  is  
a)  $m > -5/4$  b)  $m > 5/4$   
c)  $m < -5/4$  d)  $m < 5/4$

### Homework

Approximate the length of the side of a square if its perimeter is  $P$  cm, where  $15.6 \leq P \leq 15.8$ .

Knowing that  $a$  is a negative number, put the following in increasing order:  
 $2a, a\sqrt{3}, a(\sqrt{3} - \sqrt{2}), -a\sqrt{7}, 3a$

### Homework

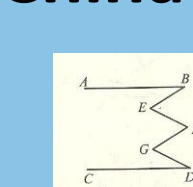
A student scored 92, 87, and 85 on three tests. She wants her average score for five tests to be at least 90. What is the lowest score the student can get, on average, on her fourth and fifth tests?

### Test

Solve the inequality:  $-4.8x - 24 \leq 28$

### Relationships Between Lines

#### China



**Homework**  
Given  $\angle E + \angle G = \angle B + \angle F + \angle D$ , explain  $AB \parallel CD$ .

#### Test

In the figure,  $EF \parallel AD$ ,  $\angle 1 = \angle 2$ ,  $\angle BAC = 70^\circ$ . Fill out the following blanks of finding  $\angle AGD$ . Because  $EF \parallel AD$ , so  $\angle 2 = \underline{\hspace{1cm}}$ . Also,  $\angle 1 = \angle 2$ , so  $\angle 1 = \angle 3$ , so  $AB \parallel \underline{\hspace{1cm}}$ . So  $\angle BAC + \underline{\hspace{1cm}} = 180^\circ$ , because  $\angle BAC = 70^\circ$ , so  $\angle AGD = \underline{\hspace{1cm}}$ .

#### Russia

#### Homework

Segments  $AB$  and  $CD$  cross each other in the middle at point  $E$ . Prove that lines  $AC$  and  $BD$  are parallel.

One of the angles that were formed when two parallel lines was crossed by another line is equal 30 degrees. Could one of the other seven angles equal 70 degrees? Explain.

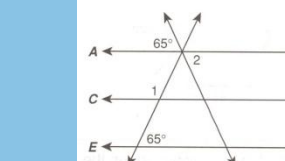
#### United States

#### Homework

Lines  $x$  and  $y$  are  $\underline{\hspace{1cm}}$ .  
 $\angle 3$  and  $\angle 4$  are  $\underline{\hspace{1cm}}$ . They are also  $\underline{\hspace{1cm}}$ .

#### Test

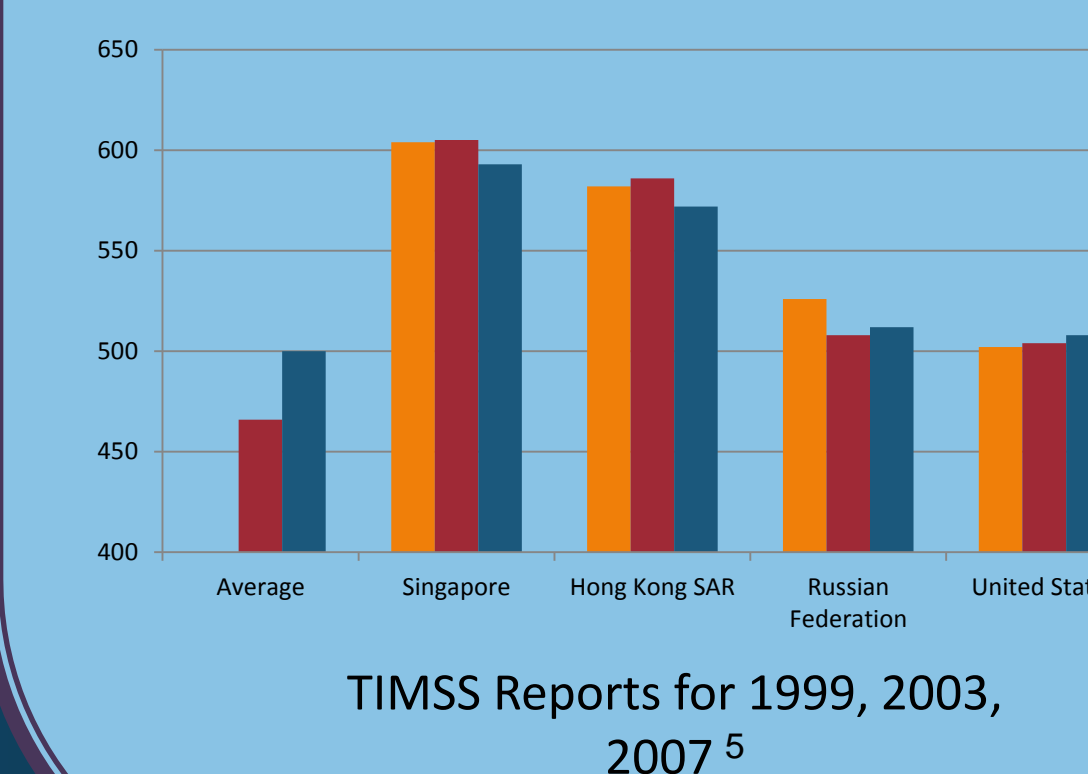
If  $CD$  and  $DF$  are parallel  
What is the measure of  $\angle 1$ ?



## Conclusions

- First impressions of the textbooks:
  - There are a greater number of visuals in the textbooks from China and the United States.
  - The textbooks from China and the United States use real-world examples, while the textbook from Russia focuses on abstract thinking.
  - The content in the textbook from the United States is more basic than the other two textbooks.

- After further research:
  - Including difficult examples and problems using abstract thinking provides students with opportunities to meet challenges.
  - Students are able to understand logical proofs, as shown in the textbooks from China and Russia, yet are not introduced until higher level classes in the United States.
  - Content with mathematical value, including solving systems of equations, is included in the textbooks from China and Russia and excluded at the same level in the textbook from the United States.



- Consequences of this discrepancy when considering the TIMSS studies over the past 10 years:
  - Students in the United States, although consistently improving, still perform significantly lower than students from Asia.
- Importance of collaboration:
  - By studying the educational systems in other countries, all can improve their own systems.

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## Acknowledgements

Department of Mathematics, University of Wisconsin-Eau Claire  
Office of Research and Supported Programs, University of Wisconsin-Eau Claire  
Graph Created in Microsoft Excel