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Chapter 1

Introduction

1.1 The Importance of Transport in Fusion Plasmas

Heat and particle losses in magnetically confined fusion devices are dominated by anomalous losses that are thought to be driven by micro-turbulence. Micro-turbulence contributes to the inverse cascade of energy from small-scale fluctuations to large-scale turbulent eddies, which serves to increase the level of particle and thermal loss observed in experiments. The transport of heat and particles from fusion devices is one of the greatest hurdles to magnetically confined fusion as an energy source, and understanding anomalous transport would contribute significantly to developing a fusion reactor.

Transport in plasma physics is commonly characterized as classical/neoclassical or anomalous. Classical diffusive transport is an irreducible floor to transport caused by collisions in homogeneous fields, and neoclassical transport is the enhancement over classical transport by magnetic field inhomogeneity. Anomalous transport is the transport not described by neoclassical theory. The sourcing of fuel and the removal of impurities in a fusion reactor depends on particle transport, while the size of the reactor is primarily driven by thermal transport [1] making accurate predictions of transport necessary for fusion experiments. There are two fusion experiments under construction in Europe that will be greatly affected by anomalous transport: ITER and W7-X. These two experiments are thought to be the intermediary step necessary for
a demonstration fusion reactor of their respective configurations. ITER represents the tokamak configuration, and W7-X represents an optimized stellarator configuration.

These two forms of magnetic confinement devices are differentiated by their confining magnetic field. Each is topologically a torus, but the confining magnetic field of the tokamak is partially generated by an inductively driven current, while the confining field of the stellarator is entirely generated by external coils. Transport in each configuration is dominated by anomalous transport, but their fundamental differences allow that transport to be studied [2].

Heat pulse propagation experiments can be used to measure thermal transport. 1D simulations of modulated heating and the resulting heat pulses have been completed for the Helically Symmetric eXperiment (HSX). A spatially localized, modulated plasma heating source and an electron temperature diagnostic with sufficient temporal resolution are necessary to perform heat pulse propagation experiments. Simulation results are reported in Chapter 2, and the interpretation of heat pulse propagation simulations is described in Sections 2.1 and 2.1.2. The installation of a modulatable Electron Cyclotron Resonant Heating (ECRH) system is described in Section 3.1, and the upgrade and analysis of the Electron Cyclotron Emission (ECE) diagnostic on HSX is described in Sections 3.2 and 3.3.

1.2 Thermal Transport

Transport theory is used to generate a closed set of equations for the evolution of particle densities, pressures and currents in a plasma [3]. The conservation of particles, $n_s$, and pressure, $p_s$, for an arbitrary species, $s$, may be expressed as:

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_s = \sum S$$

$$\frac{3 \partial p_s}{2 \partial t} + \vec{\nabla} \cdot \vec{q}_s = \sum Q$$

Where $\vec{\Gamma}_s, \vec{q}_s$ represent the particle and heat flux respectively, and the right is a sum over sources and sinks of particles ($S$) and heat ($Q$). Thermodynamic forces drive these fluxes and currents through interactions with the plasma, which in general can be represented as a matrix of transport coefficients.
\[
\begin{pmatrix}
\vec{\Gamma}_{\perp} \\
\vec{q}_{e,\perp}/n_e \\
\vec{q}_{i,\perp}/n_i \\
\vec{j}_{\parallel}
\end{pmatrix}
= \begin{pmatrix}
D & ? & ? & W \\
? & \chi_e & ? & ? \\
? & ? & \chi_i & ? \\
B & ? & ? & \sigma
\end{pmatrix}
\begin{pmatrix}
\nabla_{\perp} n \\
\nabla_{\perp} T_e \\
\nabla_{\perp} T_i \\
E_{\parallel}
\end{pmatrix}
\] (1.3)

\(\vec{\Gamma}_{\perp}, \vec{q}_{e,\perp},\) and \(\vec{q}_{i,\perp}\) represent the cross-field particle and heat fluxes, while \(\vec{j}_{\parallel}\) and \(\vec{E}_{\parallel}\) represent respectively the current and electric field parallel to the magnetic field, and the gradients are taken perpendicular to the flux surfaces [1]. Dynamic transport experiments include the temporal dynamics of the system, while static transport experiments do not. Experimental estimates of transport coefficients are commonly pulled from static measurements with a diffusive assumption in which the transport matrix is assumed to be diagonal. These effective diffusion coefficients (i.e., \(\vec{q}_{i,\perp} = n_i \chi_{i,\text{eff}} \nabla T_i\)) contain all the parametric dependencies of the transport matrix. Dynamic plasma transport experiments allow greater access to off-diagonal terms that contribute to “convective” transport than static transport experiments alone; this will be specifically discussed in the context of power-balance thermal analysis vs heat pulse propagation analysis in section 2.1.

### 1.2.1 Classical and Neoclassical Transport

Classically, the characteristic step size for diffusion is one gyroradius, \(\rho_L\), and the characteristic collision frequency, \(\nu\), is that for the cumulative effect of many small-angle scattering events. The characteristic collision frequency for classical particle and electron thermal transport is the electron-ion collision frequency, while the characteristic collision frequency for ion thermal transport is the ion-ion collision frequency. The net effects are classical diffusion rates that are greatly reduced by increases in confining magnetic field and temperature [4], with most of the collisional thermal transport coming from the ions at a similar scaling.
The inverse temperature dependence to the collision frequency 1.4 causes the lowest collisionality regime to be the most relevant to fusion reactors. Particles mirror between regions of varying magnetic field densities when their velocities parallel to the field are insufficient to pass through regions of increasing magnetic field strength. At low collisionality, these trapped particles have their orbits altered by the varying magnetic field and undergo banana orbits, named for the characteristic shape, where step length is larger than a gyroradius in toroidal geometry [5].

In this regime diffusion in a stellarator and a tokamak scale differently. Tokamaks are axisymmetric; consequently, particles undergoing banana orbits have no net drift radially unless they undergo a collision and diffusion decreases as collisionality is decreased. Stellarators do not generally possess a direction of symmetry, and particles drift radially outward while undergoing banana orbits due to asymmetry in the magnetic field. These radial drifts lead to direct losses as particles attempt to undergo super-banana orbits and hit the wall of the machine. This transport increases as collisionality decreases, resulting in the $1/\nu$ regime for stellarators which is illustrated in Figure 1.1. One form of stellarator optimization concentrates on minimizing, or eliminating entirely, the $1/\nu$ regime of neoclassical transport which increases drastically with temperature (1.6).

Asymmetries in magnetic field strength cause transport that is not intrinsically ambipolar, meaning that the loss of electrons and ions is not necessarily equal [7], and a radial electric fields arises as ambipolar transport develops in steady-state that further suppresses neoclassical transport in stellarators (Figure 1.1) [8] [6]. Non-ambipolarity adds terms to the transport matrix, considerably complicating neoclassical transport calculations in stellarators. Large radial electron flux driven by ECRH [9] [10] or plasma transport may result in a large positive

$$D_\perp \approx \frac{\Delta x^2}{\Delta t} \approx \rho_{L,e}^2 \nu_{i,e} \sim \rho_{L,e}^2 T^{-3/2} \sim B^{-2} T^{-1/2}$$

(1.4)

$$\chi_\perp \approx \chi_i = \left(\frac{m_i}{m_e}\right)^{1/2} \chi_e \approx \left(\frac{m_i}{m_e}\right)^{1/2} D_\perp$$

(1.5)
Figure 1.1: Neoclassical diffusion coefficient scaling vs normalized collision frequency. Adapted from [5] to include relevant (unoptimized) stellarator transport scaling at low collisionality (left). Stellarator neoclassical transport with radial electric field of optimized HSX stellarator (right). Reproduced from [6].

radial electric fields [8]. A transition between a region of small, sometimes negative, electric-field (ion-root) to large positive radial electric field (electron-root) results in significant radial electric field shear. This shear, specifically the resulting ExB flow shear, can suppress anomalous transport by decreasing the radial correlation length of the turbulence [11] [12] [13] or in some cases eliminate it entirely [11] [14].

1.2.2 Anomalous Transport

In tokamaks and stellarators, neoclassical theory adequately describes the cross-field ion thermal diffusivity and parallel transport, but the cross-field particle diffusion, electron thermal diffusivity, and impurity diffusion remain anomalous [15] [16]. The radial electric field, $E_r$, also plays an important role in determining the neoclassical level of transport in stellarators making the determination of anomalous transport difficult when measurements of the radial electric field are unavailable. Additionally, the convective transport caused by the off-diagonal
elements of the transport matrix (1.3) adds complexity to analyzing effective diffusion coefficients in stellarators [16]. These difficulties make static analysis of anomalous transport in stellarator configurations insufficient. Pairing static transport experiments with dynamic transport experiments of particles and heat provides the necessary information to effectively determine the transport coefficients as explained in section 2.1.

Drift wave type instabilities are believed to drive fluctuations in the 10-500 kHz range with radial correlation length of order 1-5 cm [17] [18]. Drift wave driven anomalous transport ($\chi \sim T_e^{3/2}$) is characterized by critical temperature gradients. Above these critical gradients thermal transport is observed to increase from some residual (not necessarily neoclassical) background level to a large anomalous level. The theoretical critical gradient for this transition is different for electron temperature gradient (ETG), ion temperature gradient (ITG), and trapped electron modes (TEMs) [19] which allows experimental study of drift wave turbulence through heat pulse propagation experiments in conjunction with static analysis [20] as is discussed in Section 2.1.

The axisymmetric Weiland model for quasilinear transport [21] [22] was used to calculate linear growth rates for TEM turbulence in the QHS configuration of HSX in [23]. These calculations were benchmarked for the QHS configuration against linear 3D gyrokinetic calculations from the GS2 code at three effective radii ($\rho = 0.24, 0.51, \text{ and } 0.86$). Figure 1.2 shows the calculated linear growth rates for TEM turbulence from GS2 (left) and the axisymmetric Weiland model (right) vs normalized temperature gradient at the $\rho = 0.86$ surface of HSX.

These results show that for a flat or nearly flat density (small $a/L_{n_e}$) profile the critical gradient for TEM turbulence in the HSX stellarator at the $\rho = 0.86$ surface is approximately 0.5 [24] [Figure 1.2 (left)]. Although the axisymmetric Weiland model reproduces the gyrokinetic results for large $a/L_{T_e}$, it does not reproduce the gyrokinetic results for small $a/L_{n_e}$ [Figure 1.2 (right)] and predicts a critical gradient of 2 for small $a/L_{n_e}$ in the QHS configuration.

Nevertheless, the Weiland model has successfully reproduced results from modulation experiments on other machines and includes its own critical gradient model that is compared to
Figure 1.2: TEM linear growth rates $\gamma$ at $\rho = 0.86$ normalized radius of HSX vs normalized temperature gradient $a/L_{Te}$ for multiple normalized density gradients $a/L_{ne}$ from the GS2 code (left) and the Weiland model (right). Reproduced from [23], where $a/L_{Te} = -a \nabla \ln(T_s)$ (2.2) and described in depth in [25]. Other transport models include a critical gradient dependence as well [25], and it is possible that a gyrokinetic-based model, such as GLF23 [26], will reproduce HSX results. Future gyrokinetic calculations using the GS2 code may be necessary.

ETG-driven turbulence is comprised of short wavelength fluctuations ($k_\theta \rho_s \approx 10$), while ITG- and TEM-driven turbulence is characterized by long wavelength fluctuations ($k_\theta \rho_s \approx 0.3$). $k_\theta$ is the poloidal wave number of the unstable mode and $\rho_s = c_s/\omega_c = \sqrt{(m_iT_e)/(eB)}$ is the sound speed divided by the ion cyclotron frequency. TEMs are stabilized by increasing collisionality ($\nu_{eff} \sim \nu_{ci}/\omega_{De}$) due to detrapping, and its critical gradient is dependent on normalized temperature ($R/L_{Te}$) and density ($R/L_{ne}$) gradient, rotational transform ($\iota$), and magnetic shear ($\hat{s}$) [27]. ITG turbulence is independent of collisionality while exhibiting a critical gradient in $R/L_{Te}$. ETG is isomorphic to ITG turbulence but simulations predict that ETG develops radially elongated streamers that allow non-local plasma radial transport to occur [20]. HSX has been shown to be linearly stable to ITG turbulence but linearly unstable to TEM turbulence and ETG turbulence [23] [28] [29] [30].
1.3 Thermal Transport in the HSX Stellarator

HSX is a four field period quasi-symmetric stellarator with 48 non-planar modular coils [31] [32]. The machine has an average major radius of 1.20 m, average plasma minor radius of 0.15 m, and plasma volume of 0.44 m$^3$ with additional planar auxiliary coils for configurational flexibility. The plasma is heated to a core temperature between 500-2500 eV by first harmonic O-mode electron cyclotron waves during 1 T or second harmonic X-mode electron cyclotron waves at 0.5 T operation with density ranging from 0.1-0.5 $10^{19}$ m$^{-3}$. The plasma duration is 50 ms with a maximum heating power of 100 kW at a power density of approximately 0.23 W cm$^{-3}$ and an energy confinement time of 2 ms.

The Thomson Scattering System on HSX has 10 spatial channels for measurement of electron density and temperature profiles. The density measurement is verified by a 9-channel microwave interferometer. The Charge eXchange Recombination Spectroscopy diagnostic indicates impurity ion temperatures of 20-70 eV [33].

1.3.1 Optimization for Neoclassical Transport in the HSX Stellarator

Neoclassical transport in conventional stellarators increases in the low collisionality regime, making them an unattractive fusion reactor concept. The direct loss orbits that result in the $1/\nu$ super-banana transport can be minimized through stellarator optimization [2]. One method of stellarator optimization is to restore symmetry to the magnetic configuration. HSX exhibits quasihelical symmetry (QHS) by maintaining a helical direction of nearly constant magnetic field strength that minimizes direct loss orbits. The magnetic field strength is most easily represented by a cosine series in Boozer coordinates where the toroidal ($\phi$) and poloidal ($\theta$) angles are related by the rotational transform, $\theta = \nu \phi$.

$$B = B_0(1 + \sum_{m,n} \varepsilon_{n,m} \cos(n - \nu m) \phi)$$ (1.7)

In a perfectly axisymmetric device, the only variation in the magnetic field strength is due to toroidicity and the $\varepsilon_{0,1} = -\varepsilon_t \approx r/R_o$ term adequately describes the magnetic field spectrum.
Similarly, a QHS device can be represented by a single term in the helical direction. The \( \varepsilon_{4,1} \) term dominates the QHS configuration of HSX with symmetry-breaking terms that are less than 1%.

Figure 1.3: Mode amplitude of the magnetic spectrum as a function of normalized minor radius for HSX in the QHS configuration. The helical mode, \( \varepsilon_{4,1} \) dominates the spectrum.

Although the physical transform of HSX is approximately 1, the effective transform of HSX is approximately 3 as is shown in equation 1.8. This results in smaller drifts off of flux-surfaces and lower neoclassical transport.

\[
B = B_o (1 - \varepsilon_t \cos(t\phi)) \quad \text{Ideal Tokamak Field}
\]
\[
B = B_o (1 - \varepsilon_h \cos(n - \omega m)\phi) \quad \text{Ideal Quasi-helical Field (general } n, m)\]
\[
\iota_{\text{eff}} = |n - \omega m| (n = 4, m = 1, \iota \approx 1) \approx 3 \quad \text{Effective } \iota \text{ of HSX} \quad (1.8)
\]

1.3.2 Thermal Transport in the HSX Stellarator

Configurational flexibility allows the quasi-helical symmetry of HSX to be intentionally spoiled by adding symmetry-breaking terms to the magnetic field spectrum of Figure 1.3. This Mirror configuration mimics the fields of a conventional stellarator while maintaining the fidelity of major plasma parameters of the QHS configuration. Planar coils are energized to add the \( \varepsilon_{4,0} \) and \( \varepsilon_{8,0} \) spectral components to the field spectrum of (1.8).

During 0.5 T operation, at similar heating powers, the temperature profile of QHS is substantially higher than that of Mirror, and a density hole appears in the core of Mirror plasmas which is driven by an outward convective particle flux. Comparisons between QHS and Mirror
transport are made by varying the heating power until the temperature profiles match as closely as possible [34].

Figure 1.4: Neoclassical and experimental thermal diffusivity from power-balance analysis of 0.5 T QHS and Mirror Configurations (left). Reproduced from [34]. Corresponding anomalous thermal diffusivity: \( \chi_{e,anom} = \chi_{e,exp} - \chi_{e,neo} \). Reproduced from [35].

Figure 1.5: Neoclassical and experimental thermal diffusivity from power balance analysis of 1 T QHS and Mirror Configurations. Reproduced from [6]. Corresponding anomalous transport during 1 T operation, including results for electron- and ion-root plasmas. Reproduced from [36].
Figure 1.4 illustrates the neoclassical and experimental thermal diffusivities for each configuration during 0.5 T operation. Power-balance analysis using the diffusive assumption for electron heat transport, \( Q_e = -n_e \chi_e \nabla T_e \), yields thermal diffusivities that are dominated by anomalous transport outside of 40% of the minor radius in both configurations (1.4). In the central 20-40% of the plasma minor radius, the experimental thermal transport in QHS (\( \chi_e \approx 2 \, \text{m}^2/\text{s} \)) is reduced in comparison to the Mirror configuration (\( \chi_e \approx 4 \, \text{m}^2/\text{s} \)) commensurate with the decrease in neoclassical transport (\( \approx 2.5 \, \text{m}^2/\text{s} \)) [34].

Figure 1.5 illustrates the neoclassical and experimental thermal diffusivities for the QHS and Mirror configurations. The decrease in experimental transport while operating at 1 T field strength is of the same order as the decrease during 0.5 T operation. Peaked electron temperatures are observed during 1 T operation. Figure 1.5 shows the estimated anomalous transport for Mirror and QHS at 1 T with electron- and ion-root plasmas. Large radial electric field shear resulting from the transition between ion- and electron-root fields may be responsible for the decrease in thermal transport at 1 T [6]. This may be due to a decrease in anomalous transport driven by TEM turbulence [23].
Chapter 2

Experimental Methods and Simulation

2.1 Power Balance vs Incremental Measurements

Static measurements are inherently steady-state measurements that yield information about transport at specific operating points. They have been extremely useful in the development of transport scaling laws, such as those used in the design of W7-X and ITER [37]. Dynamic measurements are inherently non-steady-state measurements that are capable of identifying the cause of specific types of transport and have led to a greater understanding of plasma transport interdependencies and plasma turbulence in general. Together, static and dynamic measurements offer the most information about plasma transport.

Figure 2.1: Heat flux vs temperature gradient (cartoon A) heat flux (cartoon B) and corresponding thermal diffusivity (cartoon C) vs normalized temperature gradient. The difference between the static ($\chi^{PB}$) measurement and the dynamic ($\chi^{pert}$) thermal diffusivity measurement is graphically illustrated. The threshold in $R/L_{Te}$ is interpreted differently by static and dynamic transport measurements. Adapted from [20].
Figure 2.1 illustrates the difference between a static and a dynamic measurement of thermal diffusivity using the diffusive assumption. The power balance thermal diffusivity is typically smaller than the incremental thermal diffusivity, which is indicative of a non-linear relationship between heat flux and $n \nabla T$. Incremental analysis applies the diffusive assumption locally by measuring the change in transport relative to a local change in temperature gradient, while power balance analysis applies the diffusive assumption globally by measuring the total transport relative to a temperature gradient. A critical gradient threshold is characterized by a sudden change in heat flux and $\chi_{pert}$.

\[
\chi^{PB} = -\frac{q}{n \nabla T}, \quad \chi^{HP} = -\frac{\partial q}{\partial (n \nabla T)} = \chi + \frac{\partial \chi}{\partial \nabla T}
\]  

Figure 2.2: Heat flux vs temperature gradient cartoons that illustrate multiple scenarios. The slope of the line to the operating point from the origin (red-dashed line) is the power balance thermal diffusivity. The local derivative is the incremental diffusivity. A) Perturbation drives plasma unstable, B) perturbation induces transition to a bifurcated state, C) incremental diffusivity is infinite, D) thermal diffusivity similar to that of the critical gradient model for drift waves.

Figure 2.2(D) is a cartoon of the critical gradient model for drift-wave induced transport. A generalized critical gradient model (2.2) was developed for ASDEX Upgrade ECRH heated plasmas based on linear GS2 calculations and the empirical existence of a threshold in normalized temperature gradient ($R/L_{Te,crit}$) [20]. The model includes the local gyro-Bohm ($T_{eB} \rho_s / R$) magnetic field and temperature scaling and enforces the empirically-observed dependence on
plasma current in tokamaks \(q^{3/2}\). One method of determining the critical gradient is to use two ECRH sources to locally change the temperature gradient, while holding total power and local temperature constant by varying the ratio of heating power from each source, and use model 2.2 to determine the critical gradient \([20]\). Commonly \(\alpha\) is taken equal to unity \([38]\), although the Weiland critical gradient model mentioned in Section 1.2.2 calls for \(\alpha = 0.5\) \([25]\).

Then \(\chi_s, \chi_0,\) and \(R/L_{Te, crit}\) are fit to experimental data from scanning \(R/L_{Te}\).

\[
\chi_e = \chi_s q^{3/2} \frac{T_e}{eB} \frac{R}{L_{Te}} \left( R \frac{R}{L_{Te, crit}} \right)^\alpha H \left( \frac{R}{L_{Te}} - \frac{R}{L_{Te, crit}} \right) + \chi_0 \frac{T_e}{eB} \frac{R}{L_{Te}} \tag{2.2}
\]

With concurrent power modulation, both power balance and heat pulse propagation analysis can be used to provide the most information about \(\chi_e\) and \(R/L_{Te, crit}\). The critical gradient model has been applied successfully in conjunction with transient experiments on several machines, including the ASDEX Upgrade, TCV, JET, FTU, Tore Supra and DIII-D tokamaks, as well as the W7-AS stellarator \([38]\), and is an attractive model for empirical studies of turbulence.

### 2.1.1 Power Balance Analysis

Power balance analysis of the static thermal diffusivity requires knowledge of the zero-order sources and sinks, as well as any convective contributions. Commonly the particle flux is unknown, and the power balance thermal diffusivity is an effective thermal diffusivity that includes off-diagonal contributions from the transport matrix. With the diffusive assumption, \(\vec{q}_e = -n_e \chi_e \nabla T_e\), the energy equation (equation A.2) may be linearized, then integrated to yield a power balance thermal diffusivity \(\chi_e^{pb}\) at zero order in \(\varepsilon\) and an incremental thermal diffusivity at first order in \(\varepsilon\), as is done in Appendix A.

\[
\begin{align*}
\varepsilon^0 : \quad & \frac{1}{V'} \frac{\partial}{\partial \rho} \langle q_e \rangle = \langle S_e \rangle \quad \chi_e^{PB} = -\frac{1}{V' \langle (|\nabla \rho|^2) n_e \frac{\partial T_e}{\partial \rho} \rangle} \int_0^\rho' \langle \tilde{S}_0 \rangle V' d\rho' \\
\varepsilon^1 : \quad & \frac{1}{V'} \frac{\partial}{\partial \rho} \tilde{q}_e = \tilde{S}_e - \frac{3}{2} n_e \frac{\partial \tilde{T}_e}{\partial t} \quad \chi_e^{INC} = -\frac{1}{V' \langle (|\nabla \rho|^2) n_e \frac{\partial T_e}{\partial \rho} \rangle} \int_0^\rho' \left[ \frac{3}{2} n_e \frac{\partial \tilde{T}_e}{\partial t} - \tilde{S}_e \right] V' d\rho' \tag{2.4}
\end{align*}
\]

Evaluation of the power balance thermal diffusivity (2.3) from Section 1.3.2, indicated in Figures 1.4 and 1.5, used a power deposition profile from ray-tracing calculations, with total
absorbed power from analysis of the change of integrated Thomson scattering profiles after ECRH turn-off. The decay of plasma-stored energy from the diamagnetic loop is affected by suprathermal electron populations during 0.5 T operation [34] and during 1 T low density operation. Consequently, the integrated Thomson profiles are used. During 1 T operation the integrated Thomson profiles and diamagnetic loop stored energy converge at the “higher” line averaged densities of $4 \times 10^{18}$ m$^{-3}$.

![Figure 2.3: Ray-tracing calculations of power deposition in the QHS and Mirror configurations during 0.5 T operation (left [35]) and 1 T operation (right [6]).](image)

The power deposition profiles from ray-tracing calculations for the QHS and Mirror configurations are similar and shown in Figure 2.3 [35] [6]. Equation (2.3) is solved by assuming an analytic (polynomial) form of thermal diffusivity and performing a least squares fit of the electron temperature resulting from integrating $\frac{\partial T_e}{\partial \rho}$ and the electron temperature from the Thomson diagnostic. $\langle |\nabla \rho| \rangle$ and $\langle |\nabla \rho|^2 \rangle$ are nearly constant across the plasma radius ($\rho = 0$ to $a$, the minor radius) and equal to $1.2/a$ and $1.6/a^2$ respectively [6]. The resulting thermal diffusivity profiles are those shown in Section 1.3.
2.1.2 Heat Pulse Propagation

Incremental measurement of the thermal diffusivity is slightly more complicated than static measurements. Equation (2.4) can be solved similarly to equation (2.3) under identical assumptions, provided a temperature diagnostic with sufficient temporal resolution is available and the sources and sinks at each order are known. An alternative method of determining the incremental thermal diffusivity is through Fourier analysis. A modulated heat source causes harmonic modulations of the plasma temperature and other parameters. If the modulation depth is small and density perturbations are neglected, the incremental effective thermal diffusivity can be solved for, as is done in Appendix A, Section A.2.

Experimentally, the upper limit of the modulation frequency is the inverse energy confinement time \( f_{\text{mod}} \tau_e > 1 \) [39]. Above this frequency, the analysis no longer yields information about thermal transport but provides information about the power deposition. In the limit of high frequency modulations, where transport occurs on a much longer time-scale than the time-scale of analysis, the energy decay of a modulation yields the power deposition profile (2.5).

\[
P_{\text{abs}}(r) = \frac{3}{2} n_e \left( \frac{\partial T_e}{\partial t} \right|_{t-} - \left. \frac{\partial T_e}{\partial t} \right|_{t+}
\]

(2.5)

Outside of the power deposition region, the incremental effective thermal diffusivity in a cylindrical geometry is given by equation (2.6). The frequency \( \omega \), normalized derivative of the amplitude of the perturbation \( \tilde{T} / T_e \), and the radial derivative of the phase \( \phi' \) of the perturbation, along with the scale length of the thermal diffusivity \( \chi_e \) and density \( \chi_{\phi} \), determine the thermal diffusivity.

\[
\chi_e = \frac{3}{4} \omega \frac{\chi_A}{-\phi' \left( \frac{T_e}{T_e} + \frac{1}{2r} - \frac{1}{2r_x} - \frac{1}{2r_n} \right)}
\]

(2.6)

defining \( \chi_A = \frac{3}{4} \omega \left( \frac{T_e}{T_e} + \frac{1}{2r} - \frac{1}{2r_x} \right)^2 \) and \( \chi_{\phi} = \frac{3}{4} \omega \left( \phi' \right)^2 \)

then \( \chi_e = \sqrt{\chi_A \chi_{\phi}} \)

(2.7)

This is the reduced model of [40]. Reference [20], among others, finds it convenient to neglect the scale length of the thermal diffusivity and define \( \chi_A \) and \( \chi_{\phi} \) so that \( \chi_e \) is the geometric
mean of the two. $\chi^A$ and $\chi^\phi$ are determined purely from the amplitude decay and purely from the phase delay between radially separated temperature measurements.

Adding the possibility of a heat pinch, $U$, back into the problem adds significant experimental complexity but is mathematically trivial in the reduced model of [40]. Multiple frequency measurements are then required to deduce the heat pinch velocity and thermal diffusivity.

\[
\chi_g = \frac{\frac{3}{4} \omega}{\phi' \left( \frac{T_e}{\chi_e} + \frac{1}{2r} - \frac{1}{4r_x} - \frac{1}{2r_e} + \frac{U}{2\chi_e} \right)}
\]

where \[ \frac{1}{\chi_g} = \frac{1}{\chi} (1 + \frac{\beta}{\nu_\phi}) \] (2.8)

\[ v_\phi = -\frac{3\omega}{2\phi'} \quad \text{and} \quad \beta = U + \frac{\chi_\prime}{2} \]

In the limit of zero $\beta$ (not the normalized plasma plasma pressure), the thermal diffusivity from Fourier analysis is the actual thermal diffusivity. Analysis at multiple modulation frequencies allows the thermal diffusivity to be determined by projecting to the zero $\beta$ (infinite modulation frequency) limit. Once the thermal diffusivity is known, the experimentally measured thermal velocity can be determined. This is done graphically in figure 2.4.

![Figure 2.4](image)

Figure 2.4: Simulated data at several modulation frequencies and radial positions. The $1/\chi_g$ intercept yields the thermal diffusivity, while the slope yields the heat velocity. Reproduced from [40].

The addition of the convective particle velocity $\vec{\Gamma}_e/n_e$ can be pushed through the derivation or can be included in the definition of the heat pinch velocity $\vec{U}_{eff} = \vec{U} + \frac{5}{2} \vec{V}$ as long as density perturbations are small. Density and heat transport are coupled through the transport
matrix, and consequently, heat pulse propagation experiments alone cannot discern between heat and particle velocities. However, combined with results from density pulse propagation experiments made through laser blow-off or gas-puff modulation experiments, the two velocities may be determined independently, making dynamic transport measurements powerful tools for experimentalists.

2.2 Thermal Pulse Simulation

1D thermal transport simulations have been performed for experimentally relevant profiles in the QHS configuration of HSX. These simulations show that for source modulations between the inverse plasma duration and the inverse energy confinement time (25 Hz ≤ f_{mod} ≤ 500 Hz) a wide range of thermal diffusivity profiles may be reconstructed upon Fourier analysis of the resulting heat pulses.

The 1D simulations are performed in a cylindrical geometry with a circular cross-section for simplicity. Matlab’s built-in function “pdepe” was used to numerically solve the perturbed heat equation given by:

\[
\frac{3}{2} n_{e,o} \frac{\partial \tilde{T}_e}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r}(r n_{e,o} \chi_e \frac{\partial \tilde{T}_e}{\partial r}) = \tilde{S}_e. \tag{2.9}
\]

The thermal diffusivity profiles modeled are necessarily effective profiles due to the azimuthal symmetry in the simulation. The power deposition profile used in these simulations is a least squares fit to ray-tracing calculations.

Two thermal diffusivity profiles have been examined. The first is the HSX neoclassical thermal diffusivity given in [34] and the second is the HSX power balance thermal diffusivity given in [23] for 1 T operation, as in Figure 2.5, along with the power deposition profile used in the simulations. The injected power was tailored to produce a 50 eV temperature perturbation in the deposition region. These results are for 100 Hz and 120 Hz modulation frequency and 15% modulation depth. The model faithfully reproduces the input thermal diffusivity outside of the power deposition region, but as stated earlier simulations with off-axis heating are required to reproduce the full profile.
2.2.1 Simulation Results

For a power source located on the axis and cycled at a variable frequency, a heat pulse is generated that diffuses radially outward. The same source located at $\rho = 0.6$ produces a heat pulse that diffuses in both directions. Since the power is square-wave modulated, pulses occur at odd harmonics of the modulation frequency. The first three odd harmonics are below the upper limit of the inverse confinement time and provide data that describe the heat pulse as it moves. The higher-frequency pulses move more quickly than the lower-frequency pulses and also disperse more quickly. The fundamental has the highest signal-to-noise ratio when noise is added to the system, and it provides the best reconstruction of the input thermal diffusivity profile. The boundary conditions are evident in this solution, with zero heat flux across the axis and zero temperature at $\rho = 1$.

The decay of the cross-spectral density, defined as the Fourier transform of the cross-covariance between two locations, for each position is used to determine the phase-lag between the power deposition region and each spatial channel for each harmonic. The power spectral density for each position is used to determine the amplitude of each harmonic. A hybrid finite difference scheme (forward/backward at end-points, centered in core) is used to determine the slope of the cross-phase and the amplitude across the minor radius, then equation (2.7) is used to reconstruct the thermal diffusivity.
Figure 2.6: Simulated temperature perturbations for a neoclassical thermal diffusivity. On-axis power deposition ($\rho = 0$ left), off-axis power deposition ($\rho = 0.6$, right).

Figure 2.7 shows the reconstructed thermal diffusivity profile for power deposition at $\rho = 0.3$ from the first three modulation harmonics, including the contribution from the decay of the amplitude (upper-right pane) and the phase (lower-right pane).

The reconstructed thermal diffusivity agrees well with the input thermal diffusivity across a wide portion of the minor radius, but the reconstruction requires multiple deposition radii to fill the gap caused by the power deposition profile. The neoclassical electron thermal diffusivity from [34] is reconstructed faithfully across a wide range of the minor radius in Figure 2.8. The region of power deposition causes the “hole” that appears near $\rho = 0.6$.

The simulation was repeated for the experimental thermal diffusivity and the results for deposition at $\rho = 0.0$ and 0.6 are shown below in Figure 2.9.

The first three harmonics of the modulation frequency are combined into a “net” thermal diffusivity ($\chi_{\text{net}}$), and the analysis is repeated for power deposition at $\rho = 0.0, 0.3, \text{ and } 0.6$ to generate the full neoclassical thermal diffusivity in Figure 2.5, as in the left pane of figure 2.10. This analysis was then repeated for the experimental thermal diffusivity in Figure 2.5 to obtain the reconstructed experimental thermal diffusivity in the right pane of Figure 2.10.
Figure 2.7: Reconstructed neoclassical thermal diffusivity from the first 3 harmonics of the modulation frequency for power deposition at $\rho = 0.3$. The contributions from $\chi^A$ and $\chi^\phi$ are also shown.

Figure 2.8: HSX neoclassical thermal diffusivity from [34]. The reconstructed diffusivity agrees across a wide range of the minor radius.

The Fourier analysis works well outside the power deposition region. However, within the power deposition region pulses run into each other, and the above analysis is not meaningful.
Figure 2.9: Simulated temperature perturbations for the experimental thermal diffusivity. On-axis power deposition ($\rho = 0$ left), off-axis power deposition ($\rho = 0.6$, right).

Figure 2.10: Reconstructed neoclassical (left) and experimental (right) thermal diffusivities from three power deposition simulations $\rho = 0.0, 0.3$, and 0.6. $\chi_{net}$ represents the mean of the results from the first three modulation harmonics.

These holes in the inferred thermal diffusivity profile must be filled by repeating the experiment with the power deposition region shifted.
The reduced model for analysis of heat pulse propagation overestimates the thermal diffusivity profile in the outer half of the experimental thermal diffusivity simulation of Figure 2.10, because the analysis neglects the scale length of the thermal diffusivity. As the normalized gradient of the thermal diffusivity increases, it decreases the denominator of equation (2.8) increasing the estimated $\chi$. This will be resolved by iterating upon the solution until $\chi$ is self consistent.

A purely diffusive profile can be reproduced by multiple deposition region measurements, but the HSX thermal transport includes a convective portion that grows in significance toward the outer half of the profile. The convective heat flux was calculated in [34] from DEGAS modeling for 0.5 T Mirror and QHS plasmas and is reproduced in Figure 2.11. The convective velocity and thermal diffusivity profiles can be recovered by using multiple modulation frequency measurements as described in Section 2.1.2.

Figure 2.11: Calculated total heat flux (left) and convective heat flux (right) from [34] for QHS and Mirror plasmas. The upper and lower dashed lines in the left pane indicate the range for the convective heat flux resulting from systematic errors. The right pane includes the upper-bound of an estimate for the convective heat flux for the QHS and Mirror configurations.

2.3 Possible Difficulties with Heat Pulse Propagation Experiments in HSX

Heat pulse propagation experiments in HSX will require an electron temperature diagnostic with sufficient temporal resolution to observe the electron temperature perturbations. The
Electron Cyclotron Emission diagnostic is ideal for performing thermal transport analysis, but several factors can complicate the interpretation of ECE on HSX. The low operating density (0.1-0.5 \(10^{19}\) m\(^{-3}\) line-average) causes low optical depth (\(\tau_{X_2} \sim n_e T_e \sim O(1)\)), which leads to re-absorption and emission of the ECE along the line of site of the diagnostic. It also allows the ECE to collect additional reflected radiation that has traversed the length of the minor radius many times, developing an approximately isotropic background radiation [41]. Additionally, fast particles are well-confined in HSX [42], and non-thermal electron populations emit cyclotron radiation strongly into the low-field side channels of the ECE diagnostic when present.

The absorption of ECRH by trapped particles can also lead to broadening of the effective power deposition profile [43], which would cause the applicable regime of analysis to shrink, as discussed in section 2.2.1. Another candidate for deposition profile broadening and pulse confusion is multi-pass absorption of the ECRH. Ray-tracing results (courtesy of Konstantin Likin, shown in Figure 2.12) indicate that ECRH is weakly absorbed in its first pass through the plasma during 1 T operation, with absorption as low as 33.5% in cold plasmas. These results will be verified by power deposition profile measurements as discussed in section 2.1.2.

The state of ECE modeling on HSX will be discussed in section 3.3.1.
Figure 2.12: Ray-tracing calculations (courtesy of Konstantin Likin) show that the first pass absorption of first harmonic O-mode ECRH during 1 T operation drops from 62.7% to 33.5% when the central temperature drops from 1 keV to 0.4 keV with 50 kW of injected power in the QHS configuration. The drop at $\rho=0$ is due to poor numerical resolution of the core.
Chapter 3

ECRH Upgrades and the ECE Diagnostic

Several projects must be completed before heat pulse propagation experiments may be performed on HSX. Thanks to significant efforts by Simon Anderson, Konstantin Likin, Paul Probert, Jerahmie Radder, and myself, a second source of ECRH has been installed and is currently undergoing testing. This is necessary so that a plasma may be maintained by one source while perturbations are induced by small modulations of the second source.

The electron cyclotron emission diagnostic must also be upgraded and calibrated before it may be used to record electron temperatures. Modeling of finite reflectivity and optical depth has been completed, but an absolute calibration of the radiometer with new components is necessary before implementation.

3.1 ECRH Upgrade

A second 200 kW, 28 GHz ECRH system has been installed and is currently undergoing testing. The Varian gyrotron VGA-8050M has a multimode output of which the TE02 mode is dominant and carries about 90% of the total power in the spectrum. To maximize the RF power that can be delivered to the torus over a pure wave guide transmission line, a hybrid transmission line has been implemented to deliver power to the plasma with correct polarization (O1/X2) and as a well-focused beam. The line consists of a Vlasov mode converter with a set of focusing mirrors, a polarizer, 4” and 2.5” dual-mode waveguides, and a steerable launch antenna.
An approximately 12′ (2.5” ID) aluminum waveguide (broken by a flexible bellows, and arc-detector) connects the gyrotron to the mode converter, which is composed of a waveguide cut and a parabolic reflector that converts the TE02 mode output from the gyrotron into an astigmatic (approximately Gaussian (3.1)) beam. The power density distribution of the TE02 waveguide mode is illustrated in the left pane of Figure 3.1, and the electric field distribution of a Gaussian beam with the same total power as the accompanying TE02 wave are shown in the right pane of Figure 3.1.

\[ E(r, z) = \sqrt{\frac{2}{\pi w^2}} \exp \left( -\frac{r^2}{w^2} - jkz - \frac{j\pi r^2}{\lambda R} + j\phi_o \right) \]  \hspace{1cm} (3.1)

Where \( E(r, z) \) is the electric field distribution, \( w \) is the beam waist, \( z \) is the length along the beam-axis, \( R \) is the radius of curvature of the beam front, and \( \phi_o \) is the phase shift of the beam. The quasi-optical techniques used in the design of this beam line, as well as in the analysis of the ECE diagnostic, are partially described in Appendix B. For more information reference [44] for quasi-optical methods and [45] for hybrid transmission line design on the HSX stellarator.

Figure 3.1: Normalized power density distribution of the TE02 waveguide mode (left). Normalized electric field strength of the TE02 waveguide mode and a Gaussian with equal total power (right).
The mode converter is followed by an ellipsoidal-focusing mirror, cylindrical steering mirror, and a grooved polarizer plate for first harmonic O-mode operation at 1 T magnetic field strength. The polarizer plate can be switched out for a plane mirror for second harmonic X-mode heating at 0.5 T magnetic field strength. The Vlasov mode converter assembly model is imaged below in the left pane of Figure 3.2.

![Figure 3.2: 3D model of the Vlasov Mode Converter Assembly that converts the TE02 mode output from the gyrotron to an approximately Gaussian beam (3.1) (left). Model of the mirror assembly that steers the beam toward HSX (right).](image)

The beam then enters an approximately 17’ (4” ID) copper waveguide that transmits the Gaussian beam to a quasi-optical steering mirror assembly, which is modeled in the right pane of Figure 3.2. There are three ellipsoidal mirrors and an approximately 2’ (2.5” ID) aluminum waveguide within the steering mirror assembly that refocus and steer the Gaussian beam toward the antenna within HSX. The steering mirrors are necessary to guide the beam around permanent structures in the HSX laboratory.

The final section of the beam line is a 5’ (2.5” ID) aluminum waveguide (broken by a flexible bellows, arc-detector, quartz window, and the HSX vacuum vessel) incident on a steerable
Figure 3.3: 3D model of the ECRH Antenna that changes ECRH resonance location based on mirror tilt-angle and is capable of rotating through 24.8 degrees with 0.6” extension of the linear actuator depositing heat from roughly 4” above the magnetic axis to 7” below the magnetic axis.

ellipsoidal mirror, which acts as the ECRH antenna (illustrated in Figure 3.3). The final mirror assembly is capable of rotating from an incident angle 57.7 degrees to an incident angle of 32.9 degrees. The as-manufactured calibration is shown in the left pane of Figure 3.4.

The resonance locations in the right pane of Figure 3.4 are estimates based on the dimensions of the assembly and the magnetic field of HSX. As the launch angle moves off-axis, wave
Figure 3.4: Calibrated mirror angle vs linear actuator position (left). Estimated resonance location vs reflected angle (right) based on the HSX magnetic field and designed assembly. The right pane does not include the effect of diffraction from the plasma, which is important for off-axis deposition. Ray-tracing calculations are required.

diffraction becomes important, and the wave will resonate at a different location than is illustrated in the right pane of Figure 3.4. Ray-tracing calculations are required to determine the actual heating location.

Testing remains in progress, including calorimetry of the output power from the gyrotron, which is expected to deliver a maximum of 200 kW and be capable of full-depth square-wave modulations at over 6 kHz. The beam will then be imaged with an infrared camera to observe the alignment of the beam line and determine the coarse mode structure of the beam. Finally, calorimetry of the beam just above HSX will be performed to determine the power entering the HSX vacuum vessel (and presumably the plasma), which is expected to be roughly 100 kW at full-power based on results from previous ECRH experiments on HSX.

The steerable mirror allows power deposition studies, and one particularly interesting application is the study of power deposition on electron Internal Transport Barrier (eITB) formation in the HSX stellarator as discussed in [6]. The eITB is believed to be the result of a transition from core electron-root to edge ion-root radial electric field that results in strong shear. This
shear was used to explain the peaking of electron temperature that is observed in plasmas with significant carbon content, which occurs during methane fueling or carbon wall conditioning in HSX [46]. Off-axis heating will be used to increase the electron temperature until the electron-root radial electric field appears and a shear-layer develops [6]. This method could also be used to move the shear layer that appears in plasmas with significant carbon content.

3.2 ECE Diagnostic Upgrade

The current electron cyclotron emission diagnostic is a 16-channel heterodyne radiometer that observes second harmonic X-mode radiation (50.8 to 60.8 GHz). The diagnostic collects plasma radiation incident upon an ellipsoidal-focusing mirror with a horn antenna. During 1 T operation, the emission is high-pass filtered to remove gyrotron signal and the lower side-band to which the radiometer is sensitive by a WR-75 waveguide and two WR-28/75 transitions (40 GHz cut-off). The mixing element is a square-law diode that combines the incident RF fields with a 42.5 GHz local oscillator (LO) into an intermediate frequency (IF) between 8.3 and 18.3 GHz. A pin switch diode protects the mixer diode from incident radiation while engaged with 40 dB of insertion loss. The power is then split into 16 channels that are passed through the band-pass IF filters before the RF signal is rectified and amplified by a set of crystal detectors and video-amplifiers.

At high density, the 8 channels on the low-field side (LFS) of the magnetic axis are used. Channels 9-16 represent the HFS and exhibit low-sensitivity, while the band-pass intermediate frequency (IF) filters for channel 8 (core) and channels 12-15 (HFS) have high insertion-loss. Gyrotron signal is also observed at the IF of two HFS channels with respect to the local oscillator (LO). The layout of the ECE system is shown in the block diagram for the diagnostic in Figure 3.5.

These problems are being resolved by replacing the crystal detectors on these channels, replacing the IF filters on these channels, and if necessary, inserting a 56 GHz notch filter. These upgrades are funded by the American Reinvestment and Recovery Act.
3.3 The ECE Diagnostic

The intensity of radiation from the plasma is a function of frequency. Each channel of the ECE radiometer observes a specific frequency, which resonates along a contour of constant magnetic flux density. $\omega_{c,m} = n \frac{q |B|}{\gamma_m}$ where $|B|$ is a function of space and is nearly vertical in the boxports of HSX, $n$ is the harmonic of the cyclotron frequency, and $\gamma$ is the relativistic factor from the mass dependence. The bandwidth of each channel, as well as particle-specific contributions to the resonance condition, lead to a finite thickness of the resonance layer.

The resonance layer can be considered a slab of plasma (within the bulk) that emits radiation from each point along the ray. This radiation, as well as the radiation that is incident on the back of the slab, is partially absorbed during transit. Across this slab we measure a bundle-average of all the emitting rays or a volume-averaged radiant intensity.
The radiation from the plasma is governed by the radiation transport equation (3.2) [47], where $I_\omega(inc)$ represents the radiant intensity incident on the outside of the resonance layer, and the optical depth ($\tau$) is the integral of the absorption ($\tau = \int_o^s 2\alpha(s')ds'$) along the line of sight ($s$) of the diagnostic.

$$n^2_r \frac{\partial}{\partial s} \left(I_\omega \frac{n^2_r}{n^2_r} \right) = j_\omega - \alpha_\omega I_\omega \quad \text{or} \quad \frac{\partial}{\partial \tau} \left(I_\omega \frac{n^2_r}{n^2_r} \right) = \frac{I_\omega}{n^2_r} - S_\omega$$  \hspace{1cm} (3.2)

where

$$S_\omega = \frac{1}{n^2_r} \frac{j_\omega}{\alpha_\omega} = \frac{\omega^2}{8\pi^2c^2} k_BT_{rad} \quad \text{and} \quad d\tau = -\alpha_\omega ds$$

then

$$I_\omega = I_\omega(inc)e^{-\tau_o} + \int_o^{\tau_o} S_\omega e^{-\tau} d\tau$$ \hspace{1cm} (3.3)

### 3.3.1 Slab Model for Finite Reflectivity and Optical Depth

The generation term in equation (3.3) can be written in terms of the blackbody radiation emitted from the resonance layer, $I_{bb,\omega}$, and optical depth as $\int_o^{\tau_o} S_\omega e^{-\tau} d\tau = I_{bb,\omega}(1 - e^{-\tau})$. If this radiation undergoes reflections and traverses the plasma many times before reaching the ECE diagnostic, then in the limit of infinite reflections between parallel planes the collected radiation can be written as an infinite series:

$$I_{rad} = I_{bb}(1 - e^{-\tau}) \sum_{n=0}^{\infty} \rho^n e^{-n\tau},$$

$$I_{rad} = I_{bb} \frac{1 - e^{-\tau}}{1 - \rho e^{-\tau}}.$$ \hspace{1cm} (3.4)

Where $\rho$ represents the reflectivity of the planes. Equation (3.4) is a solution to the radiation transport equation for a slab (3.3) when the incident radiant intensity of (3.3) is replaced by an isotropic background intensity, $I_{o,\omega}$, that in general includes contributions from polarizations and harmonics other than the target mode and polarization (in this case X2) [41] as in (3.5):

$$I_\omega(R, z, \theta, \phi) = I_{bb,\omega}[1 - e^{-\tau}] + I_{o,\omega}e^{-\tau}.$$ \hspace{1cm} (3.5)

Equations (3.4) and (3.5) are equivalent when the isotropic emission incident on the back of the slab is $I_{o,\omega} = I_{bb,\omega} \rho \frac{1 - e^{-\tau}}{1 - \rho e^{-\tau}}$ which explicitly includes the final reflection.
Equation (3.4) is the relevant solution to the radiation transfer equation for a slab geometry in terms of black-body emission $I_{bb}$. It describes the intensity of emission from a slab plasma, including the effects of finite optical depth and finite wall reflectivity ($\rho$).

In the DIII-D tokamak, the isotropic background of (3.5) is replaced by radiation from overlapping optically-thick second harmonic, while observing optically-thin third harmonic radiation [48], which demonstrates the utility of the slab model.

There are four important limits of equation (3.4). In the zero-reflectivity limit, $\rho = 0$, the generation term is recovered and $I_{rad} = I_{bb}(1 - e^{-\tau})$. In the total reflection limit, $\rho = 1$, the plasma behaves as a blackbody, $I_{rad} = I_{bb}$, and the intensity of emission is proportional to the local plasma temperature: $I_{rad} \propto T_e(R,z)$. In the optically thick limit with finite reflections, where reabsorption is large and $\tau \gg 1$ (or $e^{-\tau} \ll 1$), equation (3.4) reduces to $I_{rad} \approx I_{bb}$ and the plasma is approximately blackbody. In the optically thin limit with finite reflections, where reabsorption is negligible and $\tau \ll 1$, equation (3.4) reduces to $I_{rad} \propto \tau I_{bb}$. The optical depth of second harmonic X-mode radiation is proportional to density and temperature, so that $I_{rad} \propto n_e(R,z)T_e(R,z)^2$ for optically thin plasmas.

### 3.3.2 Effect of Finite Reflectivity and Optical Depth on $T_{rad}$

Using $T_{rad}(R,Z) = \frac{8\pi^3c^2I_\omega}{\omega^2}$, which contains an implicit correspondence between frequency and position, to write equation (3.4) in terms of radiation temperature and inverting gives a relationship for the electron temperature in terms of the measured radiation temperature from ECE on HSX.

$$T_{rad}(R,Z) = T_{e,bb}(R,Z) \frac{1 - e^{-\tau}}{1 - \rho e^{-\tau}}$$

$$T_{e,bb}(R,Z) = T_{rad}(R,Z) \frac{1 - \rho e^{-\tau}}{1 - e^{-\tau}}$$  \hspace{1cm} (3.6)

The correction term ($\frac{T_{e,bb}}{T_{rad}}$) from equation (3.6) is plotted against line average density at constant reflection coefficient for several temperatures in Figure 3.6 and at constant temperature for several reflection coefficients in Figure 3.7.
The correction for finite optical depth and reflectivity is most important below a line average density of $3.5 \times 10^{18} \text{ m}^{-3}$ for warm plasmas. At a constant reflection coefficient of 0.9, the correction factor spans from 1.10 to 1.24 for temperatures between 400 and 800 eV at a line average density of $3.5 \times 10^{18} \text{ m}^{-3}$ and becomes less important as temperature increases. Similarly, at a constant electron temperature of 400 eV, the correction factor spans from approximately 1.02 to 1.48 as the reflectivity of the walls is varied between 0.8 and 0.99, and it becomes less important as reflection coefficient is increased.
3.3.3 Absorption and Spatial Localization of the Radiation

The frequency of radiation corresponds to a specific plasma location through knowledge of the distribution of magnetic field strength in HSX and the resonance condition for second harmonic ECE \( \omega_{c,n} = 2 \frac{q|B|}{m} \). Large magnetic field strength corresponds with high resonant frequencies and small magnetic field strength corresponds with low resonant frequencies. In this way, high frequency radiation is assumed to be emitted from the high-field side (HFS) of the magnetic-axis of HSX, and low-frequency radiation is assumed to be emitted from the low-field side (LFS) of the magnetic-axis of HSX. The resonance location for each channel is calculated by propagating a quasi-optical beam from the ECE horn to the mirror and through the plasma, then calculating its intersection with the resonant vacuum field of HSX. This cold-plasma resonance has finite breadth due to the bandwidth of each ECE channel, which is between 200
and 400 MHz. The cold-plasma resonance is used as the starting position for calculations of the plasma temperature for each ECE channel and is displayed in Figure 3.8.

Figure 3.8: Cold-plasma resonance for each ECE channel in the helical-cut of the boxport in HSX (horizontal is the major radial direction).

The resonance condition for each particle is dependent upon the the velocity of that particle due to the relativistic effect on the mass \((m = \gamma m)\). High-energy, hot, particles emit radiation at lower frequencies than low-energy, cold, particles at the same magnetic field strength. This energy dependence broadens the frequency range in which plasma particles emit radiation, causing the spatial localization of the radiation to decrease when hot particle populations are present. When a population of very hot, suprathermal, particles is present in the plasma they emit into even lower frequencies. When suprathermal particles are present in HSX, they are assumed to be generated within the ECRH resonance region or on the magnetic axis. These particles rapidly circulate the machine and are assumed to be present on both the LFS and HFS of the magnetic axis. Populations located in the ECRH resonance or on the axis may
emit into frequencies that are observed by LFS ECE channels causing the observed radiation temperature to be asymmetric across the magnetic axis. This effect can be used to diagnose the presence of suprathermal particles by comparing the magnitudes of the ECE from the HFS and LFS channels, although a model for the asymmetric emission and the distribution function is necessary to determine the suprathermal populations.

The absorption is calculated for each channel using formula (28) from [49] for second harmonic X-mode radiation observed perpendicular to the magnetic field. It also requires that the emitted radiation be from a Maxwellian distribution of plasma particles at non-relativistic temperatures, where the index of refraction is determined by the Appleton-Hartree dispersion relation.

Absorption shapes along the midplane of the boxport of HSX are shown in the left pane of Figure 3.9 for 1 T operation and carbon wall conditioning in the QHS configuration of HSX. Significant plasma carbon content leads to peaked electron temperature profiles at 1 T in HSX. The central ECE channels are broadened by the higher energy radiation in comparison to the edge channels, which show very good localization but small absorption. The optical depth of HSX plasmas with carbonized walls is shown in the right pane of Figure 3.9. The same figures are reproduced for boron wall conditioning in Figure 3.10, which has been observed to lower the core temperature of HSX plasmas (but increases the density control and reproducibility of the plasma).

The resonance of each channel must be recalculated when plasma is present because of particle-specific energy contributions to the absorption. Presently, the resonance location is taken as the maximum of the absorption, but a more appropriate measure of the location would be the average of the half-max points of the absorption shapes. Figure 3.12 shows an ensemble average of the absorption shapes intersected by the ECE beam in the 2D helical cut of the boxport under boron wall conditioning (at left) that is used in a relative calibration of the ECE diagnostic on Thomson Scattering.

The optical depth in the 2D case is given by $\tau_\omega = \frac{1}{\delta Z} \int \int 2\alpha_\omega dRdZ$, where $\delta Z$ is the beam-width at the resonance location and the integral is over the absorption region for that channel.
Figure 3.9: ECE Absorption Shapes (left) and optical depth (right) during 1 T, 50 kW operation with carbon wall conditioning along the mid-plane of the HSX boxport. The optical depth is 1.4 in the core and falls to 0.2 at $\rho = 0.4$.

Figure 3.10: ECE Absorption Shapes (left) and optical depth (right) during 1 T, 50 kW operation with boron wall conditioning along the mid-plane of the HSX boxport. The optical depth is 1 in the core and falls to 0.2 at $\rho = 0.4$.

The optical depth corresponding to the above absorption is plotted as a function of effective radius in the right pane of Figure 3.12.
3.3.4 Implementation of the Model

The electron temperature equation (3.6) is implemented by iterating upon the solution until the optical depth and the electron temperature are self-consistent. First the ECE radiation temperature from the HFS of the plasma and the Abel inverted interferometer density profile are used to calculate a preliminary optical depth. Then the resulting HFS electron temperature is used to recalculate the optical depth. The HFS radiation temperatures are used to avoid contamination from suprathermal populations. The interferometer is used rather than the Thomson diagnostic because the Thomson requires ensemble averaging over multiple shots due to low photon statistics, while the interferometer provides a density profile on a shot-to-shot basis. The line-average density from the interferometer is typically about 130% that of the Thomson density due to non-zero density in the plasma scrape-off layer but the effect on the ECE temperature is small.

The converged ECE temperature is insensitive to the density profile. When the density profile was doubled and then halved, the converged temperature at an effective radius of 0.24
changed from 977 eV to 917 eV, a difference of only 60 eV (a relative change of 6%). When the density profile increases, the ECE correction factor $\left(\frac{T_{\text{ece}}}{T_{\text{rad}}}\right)$ decreases. The converse is also true. This indicates that the effective optical depth due to reflections is higher than that calculated above, which leads to an observed electron temperature closer to the black body level.

The electron temperature is considered self-consistent when the residual between iterations is less than 20 eV. The routine used to implement equation (3.6) typically requires 3-7 iterations to converge. In those cases where the routine does not converge to a self-consistent electron temperature profile it is commonly due to a single outlier and the remainder of the profile is self-consistent, as in the Figure 3.15. Although not the case in Figure 3.15, asymmetry in the ECE also occurs when fast particles are present in HSX plasmas. An absolute calibration of the radiometer is necessary to complete the analysis of the ECE diagnostic, but at high densities...
where suprathermal populations are suppressed, a relative calibration based on the Thomson diagnostic can be used.

The above procedure may be applied for each time slice of the 50 ms plasma discharge, resulting in 6250 time slices. This method can be used to calculate the electron temperature on a shot-by-shot basis, but the results require approximately 3 seconds to reconstruct a single time-slice. Reconstructing 15 ms of plasma time (roughly 1200 time-slices at the full temporal resolution of the diagnostic) in the full 2D geometry takes approximately an hour, which is too long to be useful in HSX run day operations. This problem can be resolved with some simplifying assumptions that do not decrease the temporal resolution of the diagnostic.

The weak dependence on optical depth allows the slowly varying background of the ECE to be used to calculate the optical depth. This speeds up the computation significantly and has little effect on the result. 1D calculations along the axis of the beam, with optical depth defined as $\tau_\omega = \int \alpha dR_{bp}$, where $dR_{bp} = \sqrt{dR^2 + dZ_{hel,cut}^2}$, speed up the calculation by orders of magnitude and also have a small effect on the calculated ECE temperatures. A comparison
of results from the 2D analysis and the 1D analysis is shown in Figure 3.14. The small effect on calculated ECE temperatures is due to the Gaussian beam carrying most of its power along its axis of propagation. The 2D model can still be improved by applying a weighting to each point in the helical cut of the boxport based on the power density of the beam at that location. The cold plasma resonance for 1T operation occurs for a main coil current of 10726 A. HSX typically operates at 11000 A main coil current because it yields the greatest stored energy during operation. This difference of 275 A would lead to a resonance shift of 20% of the minor radius. Due this ambiguity the following results are reported for 1T on-axis magnetic field.

![Figure 3.14: The converged ECE temperature from shot 43 on January 5, 2011, with 100 kW launched power. 2D results (green HFS, blue LFS) and 1D results (magenta) are similar across the minor radius.](image)

These alternatives provide the capability to use the above procedure on a shot-by-shot basis, provided that a trustworthy absolute calibration is available. An absolute calibration is pending the completion of the diagnostic upgrade; consequently, a relative calibration from the Thomson Scattering diagnostic is used. The relative calibration is completed by treating the Thomson
Scattering measurement as the blackbody ECE temperature and reversing the analysis applied above. First the optical depth is calculated across the resonance, then the blackbody temperature is divided by the calculated correction factor to produce a Thomson radiation temperature. This radiation temperature is then used to calibrate the ECE radiometer.

Figure 3.15: At left, ensemble and time-averaged ECE (2D results) and Thomson temperature for 7 shots on October 8, 2010 (carbon walls). The relative calibration is from 8 shots on December 7, 2010 (boron walls). At right, Thomson temperature from the two days.

A calibration based on 8 shots from December 7, 2010 (boron walls), in which the plasmas had 52.3 J of ensemble average stored energy and $4.1 \times 10^{18}$ m$^{-3}$ ensemble line-averaged density, was used to calculate the ECE temperature for October 8, 2010 (carbon walls), in which the plasmas had 54.1 J of ensemble average stored energy and $4.8 \times 10^{18}$ m$^{-3}$ ensemble line-averaged density.

For easy comparison, the calculated ECE temperature was ensemble averaged over the same 7 shots that the Thomson Scattering signal was ensemble averaged. The ECE signal was also averaged for a 2 ms time window over the Thomson laser time. Figure 3.15 shows the results. The core ECE channel agrees well with the Thomson result, but the rest of the channels underestimate the Thomson temperature by a varying margin. It should be noted that
the gas-puffer opposite the ECRH antenna (A’ puffer) was used on December 7, 2010, while the gas-puffer next to the ECRH antenna (C’ puffer) was used on October 8, 2010.

Figure 3.16: At left, 2D results for the calibrated ECE temperature from shot 35 on December 7, 2010 (not in the calibration set). The single-shot Thomson profile is overlaid. In shot 35 the magnetic field is in the clockwise direction (CW); the calibration set was made for a counterclockwise field (CCW). At right is the ensemble average profiles from December 7, 2010, for CW shots [32,33,35:39] and CCW shots [42,44:47,49:51].

When the relative calibration is applied to shot 35 from December 7, 2010 (not in the calibration set), the ECE temperature agrees well with the Thomson signal (Figure 3.16). This indicates that there are many factors in selecting an appropriate calibration set when performing a reference calibration on the HSX stellarator. One significant example that has already been discussed is the electron distribution function. Electron cyclotron emission is known to be extremely sensitive to the distribution function and non-thermal populations [43], and it is suspected that suprathermal populations are present in low density HSX discharges [42] [35] [6].
Figure 3.17: Ensemble and time-averaged ECE (2D results) and Thomson temperature for CW shots [32,33,35:39] on December 7, 2010. These are not part of the calibration set. At right, the comparison of the Thomson temperature profiles for CW and CCW are reproduced.

An ensemble and time-averaged set of CW shots from December 7, 2010, is compared to the CCW shots that comprise the calibration set from the same day in Figure 3.17. The results agree well with the Thomson Scattering diagnostic.

The calibration set was also applied to shots [43:54] from January 5, 2011, in which the launched power was twice that of the calibration set (100 kW), and the results are very good. Figure 3.18 shows the electron temperature profile from the converged ECE diagnostic and Thomson Scattering. The result agrees with Thomson Scattering across the minor radius but the three core channels are much larger than the Thomson core channel. As core temperatures approach 3 keV in HSX plasma discharges at line-average densities of $4 \times 10^{18}$ m$^{-3}$, the optical depth approaches 3, which is close to the blackbody level for perturbation experiments [50]. It is likely that the ECE core temperature in Figure 3.18 is driven up by a suprathermal population of particles.
HSX plasmas are optically semi-transparent, or optically gray, meaning that they lie between the limits of large reabsorption of the emitted radiation ($\tau \gg 1$) and negligible reabsorption of the emitted radiation ($\tau \ll 1$). Reflections serve to increase the effective optical depth by allowing radiation to take multiple passes through the plasma [50]; however, this may lead to difficulty in interpreting heat pulse propagation experiments without a viewing dump to suppress the effects of reflections.

The reflectivity of the HSX vacuum vessel, which is made of stainless steel, approaches 1 for cyclotron frequencies and a reflectivity of 0.9 is assumed for the ECE analysis [51], based on a reflectivity of 0.76 from graphite tiles in the DIII-D tokamak [52]. The estimated change in the correction factor of Figure 3.6 is about 7% between a reflectivity of 0.9 to 0.8 or 0.9 to 0.99.
During modulation experiments in optically-thin plasmas, the electron cyclotron emission diagnostic observes both temperature and density perturbations as a consequence of their finite optical depth. Peters, et al., [50] shows that for modulation experiments neglecting density perturbations, the minimum optical depth for observation of temperature modulations accepting a 15% relative error is $\tau > 1.0$ for a reflectivity of 0.8 and $\tau > 3.0$ for zero reflectivity. HSX plasmas reside near the lower limit for optical depth given by [50], making it difficult to estimate whether heat pulse propagation analysis with ECE is possible without experimental testing. The interferometer diagnostic will detect line-average perturbations to the plasma density and reflectometry can be used to observe localized density fluctuations. These two diagnostics will be used to determine the level of density perturbations during ECRH modulation.

The low number of ECE channels across each side of the magnetic axis leads to poor radial resolution. No channels are located in the outer 30% of the effective radius during 1 T operation, and several methods are being investigated to increase coverage. One option is to reconfigure the diagnostic layout so that all of the channels observe the HFS. This has the advantage of decreasing suprathermal contamination from the diagnostic and doubling the radial resolution. Another option is to slightly change the magnetic field strength of HSX by altering the coil-currents. This will move the ECE resonance positions. Unfortunately, this will lead to off-axis heating by the stationary ECRH system in boxport C.

### 3.4 Conclusions

1D simulations of heat pulse propagation experiments have been completed for the HSX stellarator and results show that an electron temperature diagnostic with sufficient temporal resolution, like the ECE diagnostic, will be able to reconstruct experimental thermal diffusivity profiles using Fourier analysis of the pulses. The motivation for heat pulse propagation on HSX is to investigate the effects of QHS on thermal transport. Heat pulse propagation will also be used to investigate the existence of critical gradients in thermal transport that may be used to identify suspected drift wave-driven turbulence and possible electron temperature profile resilience.
The existence of an eITB will be investigated using the steerable mirror of the newly implemented ECRH system to perform off-axis heating experiments. This system increases the nominal heating capacity of HSX from 200 kW to 400 kW while providing the ability to modulate the source at over 6 kHz. The additional heating capacity will allow a power-per-particle scan to investigate non-thermal populations and to further investigate eITB formation in HSX. The ability to perform high-frequency modulation experiments will allow the experimental power-deposition profile of HSX to be determined and to estimate the total absorption of ECRH using the ECE diagnostic.

Finally, a method of analyzing ECE in HSX has been developed that takes into account the finite reflectivity and optical depth of the experiment. After the planned upgrades to the ECE diagnostic are completed, the radiometer will be absolutely calibrated allowing estimates of the non-thermal population and giving HSX access to an electron temperature diagnostic with good temporal resolution.
LIST OF REFERENCES


Appendix A: Power Balance and Incremental Thermal Diffusivity

A.1 The Heat Transport Equation

Thermal transport is governed by the energy conservation equation (1.2). Degeneralizing (1.2) to include known sources and sinks and splitting the divergence of the pressure tensor into convective ($\vec{u}_s \cdot \nabla p_s$) and compressible ($\nabla \cdot \vec{u}_s$) flows, as well as generation due to the Reynolds stress (non-diagonal pressure components ($\pi : \nabla \vec{u}_s$)). Then including electron-ion drag ($Q_{e,i}$) and heating power ($Q_e$) yields the pressure evolution equation (A.1). Defining the scalar electron pressure as $p_e = n_e T_e$ and the convective flow as $\vec{u}_e = \frac{\vec{u}}{n_e}$ and lumping the small contribution due to the Reynolds stress with the sum yields an electron energy transport equation for a general geometry fusion device (A.2) with heat flux $\vec{q}_e$ and convective particle flux $\vec{\Gamma}$.

$$\frac{3}{2} \frac{\partial p_e}{\partial t} + \frac{3}{2} \vec{u}_e \cdot \nabla p_e + \frac{5}{2} p_e \vec{v} \cdot \vec{u}_e = -\vec{\nabla} \cdot \vec{q}_e + \pi : \nabla \vec{u}_e - Q_{e,i} + Q_e - \sum_{\text{sinks}} Q \quad (A.1)$$

$$\frac{3}{2} \vec{u}_e \cdot \nabla p_e + \frac{5}{2} p_e \vec{v} \cdot \vec{u}_e = -\vec{\Gamma}_e \cdot \nabla p_e + \vec{\nabla} \cdot \frac{5}{2} T_e \vec{\Gamma}_e$$

$$\frac{3}{2} \frac{\partial (n_e T_e)}{\partial t} + \vec{\nabla} \cdot [\vec{q}_e + \frac{5}{2} T_e \vec{\Gamma}_e] = P_{ECRH}(r,t) + \vec{\nabla} \cdot \frac{\nabla (\frac{5}{2} T_e)}{n_e} + \sum Q \quad (A.2)$$

Where $\sum Q$ represents all other energy sources and sinks, and $P_{ECRH}$ represents the HSX specific contribution from Electron Cyclotron Resonance Heating (ECRH).

The density evolution may be included by using the continuity equation (A.3) and substituting for $\frac{\partial n_e}{\partial t}$ in (A.2) to yield the temperature evolution equation for arbitrary sources, sinks, heat flux and particle flux (A.4).

$$\frac{\partial n_e}{\partial t} = \sum_{\text{part.}} S_e - \vec{\nabla} \cdot \vec{\Gamma}_e \quad (A.3)$$

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} + \vec{\nabla} \cdot (\vec{q}_e + \vec{\Gamma}_e T_e) = \sum_{\text{heat}} Q_e - \frac{3}{2} T_e \sum_{\text{part.}} S_e + \vec{\Gamma}_e \cdot \vec{\nabla} \left( \frac{\nabla (\frac{5}{2} T_e)}{n_e} \right) + T_e \frac{\nabla n_e}{n_e} \quad (A.4)$$
Lumping the RHS of (A.4) into an effective source term $\hat{S}$ and heat flux $\hat{q}_e$, then expanding in the ECRH modulation depth $\varepsilon = \tilde{S}_e/S_e$ while assuming the source drives heat flux perturbations and density perturbations that are second order in $\varepsilon$ in the cylindrical geometry then solving for the effective thermal diffusivity within the ordered expansion:

$$\frac{3}{2} \frac{\partial}{\partial t} (n_e T_e) + \nabla \cdot \vec{q}_e = \hat{S}$$

$$q_e = q_{e,o}(r) + \tilde{q}_e(r,t)$$

$$\varepsilon^0 : \quad \nabla \cdot \tilde{q}_{e,o} = S_{e,o}$$

$$\varepsilon^1 : \quad \nabla \cdot \tilde{q}_e = \tilde{S}_e - \frac{3}{2} n_{e,o} \frac{\partial \tilde{T}_e}{\partial t}$$

This calculation can be reproduced in flux coordinates to yield the appropriate geometry factors where $\rho$ is the square root of normalized toroidal flux enclosed, $V'$ is the derivative of the plasma volume with respect to $\rho$, and $\langle \rangle$ represents flux surface averaging as in [53]:

$$\varepsilon^0 : \quad \frac{1}{V'} \frac{\partial}{\partial \rho} (q_e) = \langle S_e \rangle$$

$$\varepsilon^1 : \quad \frac{1}{V'} \frac{\partial}{\partial \rho} \tilde{q}_e = \tilde{S}_e - \frac{3}{2} n_{e,o} \frac{\partial \tilde{T}_e}{\partial t}$$

A.2 Fourier Analysis of Heat Pulse Propagation

Alternatively, the thermal transport equation can be solved through Fourier analysis. Outside of the power-deposition region, where $r_v = -v/\nabla v$ represents the scale length of $v$, $\alpha$ represents the damping rate of the under-damped system, $k$ the wave-number of the perturbation, and $\phi = \int k dr$ the phase delay.

$$n(r,t) \approx n_{e,o}(r)$$

$$T_e(r,t) = T_{e,o}(r) + \tilde{T}_e e^{i(k \cdot \vec{x} - \omega t)}$$

$$\tilde{T}_e \sim e^{-\alpha x} \quad \text{yielding} \quad \alpha = -\frac{\tilde{T}_e'}{\tilde{T}_e} \quad \text{and} \quad |k| = \phi'$$

$$\frac{\partial T_e}{\partial t} = -j \omega \tilde{T}_e$$

$$\frac{\partial T_e}{\partial r} = \frac{\partial T_{e,o}}{\partial r} - (\alpha - jk) \tilde{T}_e$$

$$\frac{\partial^2 T_e}{\partial r^2} = \frac{\partial^2 T_{e,o}}{\partial r^2} + (\alpha - jk)^2 \tilde{T}_e$$
\[
\frac{3}{2} n_{e,o} \frac{\partial \tilde{T}_e}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( r n_{e,o} \chi_e \frac{\partial \tilde{T}_e}{\partial r} \right) = \tilde{S}_e \quad \text{and} \quad \frac{1}{r_l} = \frac{1}{r \chi} + \frac{1}{r_n} - \frac{1}{r}
\]

\[
\frac{3}{2} j \omega n_{e,o} \tilde{T}_e + n_{e,o} \chi_e \frac{\partial^2 \tilde{T}_e}{\partial r^2} + \frac{1}{r_l} \frac{\partial \tilde{T}_e}{\partial r} = -\tilde{S}_e
\]

Expanding and solving the imaginary part yields an experimentally useful form that matches the reduced model of [40]. \( \chi^A \) and \( \chi^\phi \) are derived purely from the amplitude decay and phase delay of a pulse, and their geometric mean is the thermal diffusivity as in [20]:

\[
\text{imag:} \quad \frac{3 \omega}{2 \chi_e} - 2 \alpha k + \frac{1}{r_l} k = 0
\]

\[
\frac{3 \omega}{2 \chi_e} = (2 \alpha - \frac{1}{r_l}) k
\]

\[
\chi_e = \frac{\frac{3}{2} \omega}{k(2 \alpha - \frac{1}{r_l})} = -\phi'(\frac{T_e'}{T_e} + \frac{1}{2r} - \frac{1}{2r_n} - \frac{1}{2r_n})
\]

(A.5)

Defining \( \chi^A = \frac{\frac{3}{2} \omega}{(\frac{T_e'}{T_e} + \frac{1}{2r} - \frac{1}{2r_n})^2} \) and \( \chi^\phi = \frac{\frac{3}{4} \omega}{(\phi')^2} \)

then \( \chi_e = \sqrt{\chi^A \chi^\phi} \) \hspace{1cm} (A.6)

Adding the possibility of a heat pinch, \( U \), back into the problem adds significant experimental complexity, but is mathematically trivial in the reduced model of [40]. Multiple frequency measurements are then required to deduce the heat pinch velocity and thermal diffusivity as is discussed in section 2.1.2.

\[
\chi_g = \frac{\frac{3}{2} \omega}{\phi'(\frac{T_e'}{T_e} + \frac{1}{2r} - \frac{1}{2r_n} - \frac{1}{2r_n} + \frac{U}{2 \chi_e})}
\]

where \( \frac{1}{\chi_g} = \frac{1}{\chi} (1 + \frac{\beta}{v^*)} \) \hspace{1cm} (A.7)

\[
v^* = -\frac{3 \omega}{2 \phi'} \quad \text{and} \quad \beta = U + \frac{v^*}{2}
\]
Appendix B: Quasi-Optical Beam Propagation

Quasi-optical (QO) techniques were used in the design (by Konstantin Likin and Jerahmie Radder) of the beam-line that transmits power from the gyrotron to the plasma. QO makes use of the paraxial approximation to model the propagation of approximately Gaussian beams in the limit of finite wavelength radiation [44]. Then the ray transfer matrix formalism may be used to analyze beam propagation by modeling reflectors as thin lenses. QO techniques are also used in the modeling of the ECE diagnostic.

B.1 The Gaussian Solution to the Paraxial Wave Equation

The paraxial wave equation describes the propagation of a wave with a well-defined direction of propagation but some transverse variation. Paraxial optics assumes the variation in the amplitude of the wave, $u$ due to diffraction along the direction of propagation is small over a wavelength $\lambda$, and that the axial variation of the wave is small in comparison to it’s transverse variation. The paraxial approximation is approximately valid as long as the angular divergence of the beam is confined to 0.5 radians (30 degrees) [44].

In cylindrical coordinates $(r, \varphi, z)$, a wave propagating along the $z$-direction with axial symmetry follows the axially symmetric paraxial wave equation whose fundamental solution is a Gaussian beam mode. The electric field solution is (B.1), where $q$ is the complex beam parameter, $w$ is the beam-waist radius ($w_o = w[z = 0]$), $R$ is the radius of curvature of the beam front and $\phi(r)$ is the phase variation from a fixed plane along $z$ to the beam front. Higher-order
solutions are in terms of Laguerre polynomials.

\begin{align*}
\text{Tansverse Field} & \quad E(r, z) = \left[ \frac{2}{\pi w^2(z)} \right]^{0.5} \exp\left[-\frac{r^2}{w^2} - jkz - \frac{j\pi r^2}{\lambda R(z)} + j\phi_o\right] & \text{(B.1)} \\
\text{Beam Radius} & \quad w(z) = w_o \left[1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2 \right]^{0.5} & \text{(B.2)} \\
\text{Radius of Curvature} & \quad R(z) = z + \left(\frac{\pi w_o^2}{\lambda} \right)^2 \frac{z}{z} & \text{(B.3)} \\
\text{Beam Phase Shift} & \quad \phi_o(z) = \tan^{-1}\left(\frac{\lambda z}{\pi w_o^2}\right) & \text{(B.4)} \\
\text{Beam Parameter} & \quad q = z + j\pi w_o^2 \left(\frac{1}{\lambda} \right) & \text{(B.5)} \\
\text{Alternatively:} & \quad \frac{1}{q} = \frac{1}{R} - \frac{j\lambda}{\pi w^2} & \text{(B.5)} \\
\text{Beam Power Distribution} & \quad \frac{P(r)}{P(0)} = \exp\left[-2\left(\frac{r}{w(z)}\right)^2\right] & \text{(B.6)} \\
\text{Far-field Divergence Angle} & \quad \theta_0 = \frac{\lambda}{\pi w_o} & \text{(B.4)} \\
\text{Power FWHM in Far-field} & \quad \theta_{fwhm} = 1.18\theta_0
\end{align*}

A graphical illustration of some of these parameters is provided in figure B.1 where the Rayleigh range \( z_R = \frac{\pi w_o^2}{\lambda} \) occurs at \( w(z_R) = w_o\sqrt{2} \).

![Figure B.1: Quasi-optical Gaussian Beam waist and divergence (left pane). Radius of curvature and equiphase surfaces (beam fronts) reproduced from [54] (right pane)](image-url)
B.2 Ray Transfer Matrices

In the paraxial approximation, the angle of propagation with respect to the beam-axis is assumed to be small such that $\sin(\theta) \approx \theta$. Then the wave propagation can be described by a matrix equation for the ray position and ray slope (B.7). Where the 2x2 matrix is called the ABCD matrix. Then a system can be described as being composed of a set of elements, each of which possess their own ABCD matrix. Propagation through the system results in a single ABCD matrix that is the multiplication of the system elements ray transfer matrices.

$$
\begin{pmatrix}
  r_{out} \\
  r'_{out}
\end{pmatrix}
= 
\begin{pmatrix}
  A & B \\
  C & D
\end{pmatrix}
\begin{pmatrix}
  r_{in} \\
  r'_{in}
\end{pmatrix}
$$

(B.7)

For quasi-optical transmission, the ray transfer matrices are the same as for the geometrical optical system element, with the output beam parameter related to the input beam parameter through the ABCD law (B.8).

$$
q_{out} = \frac{Aq_{in} + B}{Cq_{in} + D}
$$

(B.8)

A table of ray transformation matrices is available from many sources, including [54] and [44]. Some fundamental RTMs used in the analysis of the ECE diagnostic are:

Distance L in uniform media

$$
\begin{pmatrix}
  1 & L \\
  0 & 1
\end{pmatrix}
$$

Thin lens of focal length $f$

$$
\begin{pmatrix}
  1 & 0 \\
  -\frac{1}{f} & 1
\end{pmatrix}
$$

Transformation by a thin lens

$$
\begin{pmatrix}
  1 - \frac{d_2}{f} & d_{in} + d_2(1 - \frac{d_{in}}{f}) \\
  -\frac{1}{f} & 1 - \frac{d_{in}}{f}
\end{pmatrix}
$$

(B.9)

Where $d_2$ represents the distance following the thin lens of focal length $f$, and $d_{in}$ represents the distance to the next element. For a spherical mirror of radius of curvature $R$, the focal length is given by $\frac{1}{f} = \frac{2}{R}$. For an ellipsoidal mirror, where $d_1$ and $d_2$ are the distances from the section center of the ellipsoid to the respective foci, the focal length is given by $\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2}$. 