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Baroclinic and Barotropic Edge Waves
on a Continental Shelf

by

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BAROCLINIC AND BAROTROPIC EDGE WAVES
ON A CONTINENTAL SHELF

1. INTRODUCTION

Certain coastlines of the world are favored with winds which transport surface water offshore and bring cool, nutrient-rich water up into the sunlit layers. This phenomena, called coastal upwelling, generates planktonic blooms which eventually produce some of the world's richest commercial fisheries. Because of the economic and social importance of these fisheries, upwelling has become the central theme for a considerable amount of oceanographic research.

Although the most significant output of the upwelling process is biological in nature, in order to understand and perhaps predict the biology one must come to understand the physics of the situation -- how nutrients and the living organisms themselves are moved about and mixed with the coastal waters. Due to the complexity of coastal phenomena, there is no obvious way to chart a single, most efficient course toward a useful understanding of advection and diffusion in an upwelling zone. The approach which has evolved within IDOE-CUEA is rather to employ a variety of theoretical and observational studies to understand the physical processes of the upwelling system and provide some insights into its underlying structure. One of the lines of approach is the interesting notion that transient alterations in the upwelling currents and even the mixing rates across the shelf may be governed by stable and unstable, low-frequency wave dynamics. It is the focus of this paper to investigate the properties of these waves as they are influenced by the stratification, frontal upwarping, and mean longshore currents found in the Oregon coastal upwelling zone.
2. BACKGROUND

In the absence of any stratification, it is known that the oscillations or waves trapped over a sloping continental shelf in a rotating system constitute two distinct species. The higher frequency species are the familiar Stokesian edge waves, which depend upon gravity and are modified by the earth's rotation. The lower frequency species are the quasi-geostrophic edge waves whose existence depends on a nonvanishing gradient of the potential vorticity in the mean state. On a rotating earth, such a gradient can be provided by either meridional variations of the planetary vorticity, a current shear, or a bottom slope.

For the homogeneous fluid, edge waves were investigated by Reid (1958) and Ball (1967). In these investigations, the shelf width normal to the coast was assumed to be unbounded. Robinson (1964) and Mysak (1968a) considered a finite width shelf bounded by a deep ocean, which has an effect of trapping these waves more strongly on the shelf.

Properties of the quasi-geostrophic species, often called "shelf waves", have been more thoroughly investigated in subsequent studies such as those by Mysak (1967 and 1968b), Buchwald and Adams (1968), and Adams and Buchwald (1969) from a theoretical point of view. Properties of shelf waves were studied from an observational point of view by Mooers and Smith (1968), Cutchin and Smith (1973), and Cartwright (1969), and from an experimental point of view by Caldwell, Cutchin and Longuet-Higgins (1970).

In the theoretical studies mentioned above, the stratification aspect on the shelf has been ignored (as well as any mean flow on the shelf). If the vertical stratification is discretised by two homogeneous layers, one obtains, in addition to the barotropic edge waves and shelf waves, a baroclinic spectrum of these waves. The case of a two-layer system, with each layer possessing a constant mean flow, was considered by Iida (1970). The mathematical problem was simplified by assuming that the continental shelf has a constant slope and extends to infinity.
Orlanski (1968) considered a similar problem but took into consideration the topography of the shelf and the horizontal variation in the upper layer flow. The lower layer was assumed to have zero mean flow and the upper surface was bounded by a rigid lid. The primary emphasis in that study was to examine possible instabilities of the free modes and their growth rates. Orlanski found that for the Gulf Stream unstable modes exist having wave lengths of the order of 500 km and growth rates of the order of 5 to 7 days. The time and space scales of these unstable waves appear to be related to the eddies of the Gulf Stream. Orlanski used a numerical "shooting" method to find the unstable modes. The same method has been used by Wang (1975) to examine the properties of stable shelf waves in a two-layer system without mean flow. Allen (1975) has treated the same case by an analytical perturbation technique valid for small values of the ratio of the baroclinic radius of deformation to the horizontal scale length.

In the edgewave analysis which follows, we also assume that the density stratification of the coastal sea is resolved into two discrete, homogeneous layers (Fig. 1). Each layer contains a time-averaged mean current which is shore parallel. The magnitude of the current is uniform over the depth of its layer and uniform along shore but arbitrarily variable with distance offshore. The mean position of the interface between the two layers and the depth of the bottom are also arbitrary functions of the offshore coordinate and independent of the longshore coordinate. The interface is assumed to intersect the surface exactly at the beach and the surface is not confined by a rigid lid.

The governing equations (as derived in Section 3) are solved in Section 4 by discretising the continuous differential equations and solving the resulting system of algebraic equations through a direct eigenvalue approach. In Section 6 this technique is applied to the case of a mean hydrographic and topographic profile characteristic of an Oregon coastal upwelling zone. In Section 10 the technique is applied to several variations on the original profile.
Figure 1. Two-layer shelf model with coordinate system.
3. DYNAMICAL EQUATIONS

Consider a two-layer system on a rotating earth. Let the density of the upper layer be $\rho_1$ and that of the lower layer be $\rho_2$. Take a coordinate system with x-axis directed normal to the coast and towards the deep sea, y-axis parallel to the coast with the coast on the left and z-axis (axis of rotation) directed upwards. We consider a basic state characterized by mean flows $V_1(x)$ and $V_2(x)$ parallel to the coast in the upper and lower layers. These flows are assumed to be in geostrophic balance with the pressure gradients generated by variations in the equilibrium depths $h_1$ and $h_2$ of the upper and lower layers. The equations governing the dynamics of hydrostatic perturbations on such a two-layer system are:

\[ \frac{\partial M_j}{\partial t} = -ik\nu_j M_j + f N_j - \rho_j g h_j \left( \frac{\partial \zeta_2}{\partial x} + \epsilon_j \frac{\partial \zeta_1}{\partial x} \right) \]  

\[ \frac{\partial N_j}{\partial t} = -ik\nu_j N_j \left[ \frac{d\nu_j}{dx} \right] + f M_j - ikgh_j \left( \zeta_2 + \epsilon_j \zeta_1 \right) \]  

\[ \frac{\partial \zeta_j}{\partial t} = -ik\nu_j \zeta_j - \frac{\partial M_j}{\partial x} - ikN_j \]  

In the above equations, $j = 1$ represents upper layer and $j = 2$, lower layer. If $\zeta$ represents the deviation of the free surface from its equilibrium position and $\zeta_2$ that of the interface, $\zeta_1 = \zeta - \zeta_2$. $M_j$ and $N_j$ are the transports (velocity X mean depth) in x and y directions; $f$ is the coriolis force and $g$ is the constant of apparent gravitational attraction. The parameter $\epsilon_j$ is given by

\[ \epsilon_j = \begin{cases} 
1 \text{ if } j = 1 \\
\rho_1/\rho_2 \text{ if } j = 2 
\end{cases} \]
Finally, in equations [1-3], it has been assumed that all perturbations have a long shore variation of the form \( \exp(iky) \) where \( k \) is a wave number. The system of equations [1-3] are normally solved subject to the inshore boundary conditions:

\[
M_j = 0 \text{ at the coast } x = 0;
\]

and for trapped waves, the offshore boundary conditions

\[
\zeta_j \to 0 \text{ as } x \to \infty.
\]  

4. METHOD OF SOLUTION

One of the conventional methods for solving the system of equations [1-3] is to reduce the number of equations by elimination of the variables, for example, \( M_1, N_1 \) and \( M_2, N_2 \). Such a procedure leads to two coupled differential equations involving the variables \( \zeta_1 \) and \( \zeta_2 \). Even though it may be convenient to deal with two equations in two unknowns, the problem when formulated in this manner becomes an irregular eigenvalue problem since the frequency of oscillation explicitly enters the boundary conditions. From a computational point of view, it is more advantageous to formulate the problem so that it is in a regular eigenvalue form. This may be accomplished by directly considering the dynamical equations [1-3] without carrying out any elimination of variables.

In an attempt to solve the equations for arbitrary topography and mean currents, the continuum equations are discretised on a grid shown in Figure 2. Let \( x_n = n\Delta x \) where \( n = 1, 2, \ldots \) is an integer and \( \Delta x \) is the grid interval. The dependent variables \( M, N \) and \( \zeta \) are arranged such that \( N \) and \( \zeta \) are evaluated at the same grid points and \( M \) is evaluated at points between \((N, \zeta)\)-points. The derivatives with respect to \( x \) in the equations [1-3] are replaced by central differences. The resulting equations may now be written as:
Figure 2. Finite difference numerical grid with grouping of the dependent variables.
\[
\frac{d\hat{N}^n_j}{dt} = -ik \left[ \bar{V}_j^n \hat{N}^n_j + \frac{1}{2k\Delta x} \left( \bar{v}_{j+1/2}^{n+1/2} - \bar{v}_{j-1/2}^{n-1/2} + f \Delta x \right) \right] \\
\left( M_j^{n-1/2} + M_j^{n+1/2} \right) + gh_j^n \left( \hat{\zeta}_2^n + \epsilon_j \hat{\zeta}_1^n \right) \]  

[5]

\[
\frac{d\hat{\zeta}_j^n}{dt} = -ik \left[ \bar{V}_j^n \hat{\zeta}_j^n - \frac{1}{k\Delta x} \left( M_j^{n+1/2} - M_j^{n-1/2} \right) + \hat{N}_j^n \right] 
\]  

[6]

\[
\frac{dM_j^{n-1/2}}{dt} = -ik \left[ \bar{V}_j^n \frac{M_j^{n+1/2} + f}{2k} \left( \hat{N}_j^n + \hat{N}_j^{n+1} \right) + \right. \\
\left. gh_j^{n+1/2} \left( \hat{\zeta}_2^n - \hat{\zeta}_2^{n+1} + \epsilon_j \hat{\zeta}_1^n - \epsilon_j \hat{\zeta}_1^{n+1} \right) \right] 
\]  

[7]

In the above equations \( n = 1, 2, \ldots \) \( (n_L - 1) \) where \( n_L \) is the last grid point. \( \hat{N} \) and \( \hat{\zeta} \) are defined as:

\[ iN = \hat{N}, \quad i\zeta = \hat{\zeta}. \]

This definition makes all the coefficients inside parentheses of the equations [5-7] real. The \( M \)-equation is not applied at the coast since \( M = 0 \) there. The coriolis terms are approximated as the arithmetic means of the appropriate quantities on either side of a grid point.

The outer boundary condition given in equation [4] implies that both \( \zeta \) and \( \zeta_2 \to 0 \) at large distances from the coast. If \( n_L \) indicates the last grid point, the length of the shelf plus deep-sea region is then given by \( n_L \Delta x - 1/2 \Delta x \). If we assume that this length is sufficiently long, then the boundary condition that \( \hat{\zeta}_1 \) and \( \hat{\zeta}_2 = 0 \) might be applied at \( n = n_L \). However, it was found that imposition of such a condition at any reasonable distance from the coast led to a distortion of the external Kelvin-Stokes wave solutions.
Consequently, an alternative boundary condition was utilized. This condition allows \( \hat{\xi}_1 \) and \( \hat{\xi}_2 \) to remain different from zero but specifies that the gradient of \( M \) remains constant as the outer boundary is approached. This constant value was set equal to the value of the gradient at the point immediately in the interior of the outer boundary. That is,

\[
\left( \frac{\partial M}{\partial x} \right)_n = n_L = \left( \frac{\partial M}{\partial x} \right)_n = n_L - 1
\]

or

\[
\begin{align*}
M_{n_L + 1/2} - M_{n_L - 3/2} &= M \\
M_{n_L - 1/2} - M_{n_L - 3/2} &= M \\
\end{align*}
\]

This condition is now applied to equations [5, 6] at the outer grid point, \( n = n_L \), resulting in the following equations:

\[
\frac{d\hat{N}}{dt}^{n_L} = -ik \left[ \frac{n_L}{\bar{V}_j} \hat{N}_j + \frac{1}{2k\Delta x} \begin{pmatrix} n_L + 1/2 & n_L - 1/2 \\ n_L - 1/2 & n_L - 3/2 \end{pmatrix} \left( \bar{V}_j - \bar{V}_j \right) + f\Delta x \right]
\]

\[
\left( 3M_j \hat{\xi}_2 \right) + gh_j n_L \left( \hat{\xi}_2 + \epsilon_j \hat{\xi}_1 \right)
\]

\[
\frac{d\hat{\xi}_j}{dt}^{n_L} = -ik \left[ \frac{n_L}{\bar{V}_j} \hat{\xi}_j - \frac{1}{k\Delta x} \begin{pmatrix} n_L - 1/2 & n_L - 3/2 \\ M_j & M_j \end{pmatrix} \right] + \hat{N}_j n_L
\]

Equations [5-7] and [8-9] can be cast into a matrix form:

\[
\frac{\vec{d}A}{dt} = -ik\vec{\alpha}A
\]
where the column vector $A$ consists of an arrangement of the dependent variables as follows:

$$
\vec{A} = \text{col}(\hat{N}_1, \hat{N}_2, \hat{\zeta}, \hat{\zeta}_1, M_1^{11/2}, M_2^{11/2}, \hat{N}_1^2, \ldots, \hat{N}_n, \hat{N}_L, \hat{\zeta}, \hat{\zeta}_L)
$$

The elements of the coefficient matrix $\alpha$ are obtained from equations [5-7] and [8-9]. Matrix $\alpha$ is a banded matrix of size $(6n_L - 2)$.

Since equation [10] is simply an ordinary differential equation with constant coefficients, it has a solution of the form:

$$
\vec{A} \sim e^{-ikct} = e^{-i\omega t}
$$

where $\omega = kc$.

Substitution of the above solution into equation [10] results in:

$$
(\alpha - Ic)\vec{A} = 0
$$

where $I$ is the identity matrix. It is clear from equation [12] that $c$'s are the characteristic values of the real matrix $\alpha$. These characteristic values and the associated characteristic vectors may be determined by standard eigenvalue-eigenvector routines.

5. **Verification**

Computations based on the numerical method described in Section 4 were verified by comparing the results with some of those obtained by Iida (1970), and by Cutchin and Smith (1973). Table 1 shows the dimensionless eigen-frequencies obtained by Iida (his tables 3-1 and 3-2) for his Case (I) ambient conditions together with wave number $k = 0.01025 \text{ km}^{-1}$, and upper and lower mean velocities, $\bar{V}_1 = 0.125 \text{ m/sec}$ and $\bar{V}_2 = -0.125 \text{ m/sec}$, respectively. The fractional differences between Iida's results and our results

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Table 1. Trapped wave, nondimensional eigenfrequencies obtained for Iida's Case (I) ambient hydrographic and topographic conditions and with wavenumber \( k = 0.01025 \text{ km}^{-1} \), and \( \bar{V}_1 = 0.125 \text{ m/sec} \), \( \bar{V}_2 = -0.125 \text{ m/sec} \).

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<td></td>
<td></td>
<td>( \omega )</td>
<td>( \Delta \chi )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 ( \frac{\omega}{T} )</td>
<td>13.1</td>
<td>21.9</td>
</tr>
<tr>
<td>Internal Stokesian edgewaves</td>
<td>1</td>
<td>+2.1</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-1.03</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>Internal shelf waves</td>
<td>1</td>
<td>-0.0137</td>
<td>13.1</td>
<td>0.139</td>
</tr>
<tr>
<td>Internal Kelvin-Stokes wave</td>
<td>0</td>
<td>-0.0150</td>
<td>13.1</td>
<td>0.153</td>
</tr>
<tr>
<td>External shelf waves</td>
<td>1</td>
<td>-0.343</td>
<td>13.1</td>
<td>0.380</td>
</tr>
<tr>
<td>Internal Stokesian edgewaves</td>
<td>1</td>
<td>-1.02</td>
<td>13.1</td>
<td>1.01</td>
</tr>
<tr>
<td>External Kelvin-Stokes wave</td>
<td>0</td>
<td>-12.2</td>
<td>13.1</td>
<td>-12.1</td>
</tr>
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</table>
for $n_L = 33$ and $\Delta X = 7.5$ km, are small except in the case of the external shelf wave mode. For uniformly sloping geometry the eigenfunction for the shelf wave extends so far into the deep sea that our arbitrary imposition of a boundary condition a finite distance (247.5 km) from the coast must perturb the solution.

Nevertheless, the convergence of the eigenvalues with decreasing values of $\Delta X$ seems to be satisfactory for all modes shown. As discussed later, numerical convergence, especially for wave modes with sharply curved eigenfunctions, is sometimes a problem.

Cutchin and Smith (1973) calculated the eigenfrequencies and eigenvectors of barotropic shelf waves propagating through still, homogeneous water. Their numerical technique was a shooting method previously described by Caldwell, Cutchin and Longuet-Higgins (1972). The out-to-sea boundary condition for trapped modes was the appropriate exponential decay governed by the frequency. For external shelf wave modes propagating in still water over the same shelf topography the results of Cutchin and Smith (1973) and the results of this paper are in very close agreement.

6. APPLICATION TO THE COAST OF OREGON

Familiar references for studies in upwelling and other coastal processes are the extensive data sets obtained in the waters offshore from Depoe Bay, Oregon (e.g. Smith 1974, Huyer 1974, Huyer and Pattullo 1972, etc.) The Depoe Bay region was also examined by Mooers and Smith (1968) and Cutchin and Smith (1973) in their comparison of theoretically predicted continental shelf wave characteristics and real ocean measurements. Therefore, we have chosen as parameters for the numerical edgewave model described above the bathymetric and hydrographic X-Z profiles typical of those observed along a line radiating out to sea from Depoe Bay and crossing the depth contours at generally right angles. This profile set will subsequently be referred to as the standard "DB Line" profile. It is roughly representative of most other profiles along the Pacific Northwest Coastline.
As indicated in Section 4 above, the construction of the coefficient matrix $\alpha$ requires $\bar{V}_1$, $\bar{V}_2$, $\bar{h}_1$ and $\bar{h}_2$, at a series of equally-spaced grid points across the continental shelf and a short distance into the deep sea. The particular algorithm used allowed for 1) arbitrary $H(=\bar{h}_1 + \bar{h}_2) \bar{V}_1$, and $\bar{h}_1$ profiles; and 2) calculation of a geostrophically balanced $\bar{V}_2$ from local values of $\bar{V}_1$, and $\Delta \bar{h}_1/\Delta x \approx \frac{d\bar{h}_1}{dx}$. In the cases shown here, however, profiles for $\bar{V}_1$, $\bar{h}_1$, and $H$ were separately generated through sets of simple analytical functions. These functions were originally chosen as (intuitive) best fits to average Depoe Bay Line observations made during the summer upwelling season.

The bottom profile along the Depoe Bay Line was approximated by:

$$
\begin{cases}
H = S x & 0 < x < 45 \text{ km} \\
= H_0 \exp \left(-3 \times \frac{120-x}{120}\right) & 45 < x < 120 \text{ km} \\
= H_0 & x > 120 \text{ km}
\end{cases}
$$

where $S = 9.711 \times 10^{-3}$, and $H_0 = 2.85 \text{ km}$. This bottom profile is shown in Figure 3 as a solid line with hatching.

Kundu, Allen, and Smith (1975) have recently observed that almost all of the low-frequency variations in Oregon coastal currents during upwelling can be attributed to barotropic or first vertical mode baroclinic oscillations. This indicates that a discretised two-layer model is sufficiently complex to reproduce the gross features of the important wave modes. However, the fact that the real ocean is not divided into two homogeneous layers still makes it very difficult to determine appropriate values for $\bar{h}_1 (x)$ and $\Delta \rho$. In the summer the Oregon coastal regime is characterized by a strong (1 or 2 $\sigma$), but shallow (10 to 20 m), seasonal pycnocline and a more diffuse permanent
pycnocline at about mid-depth. The seasonal pycnocline is transitory on relatively short time and space scales and was, therefore, excluded from consideration in determining the appropriate values for the two-layer model. The permanent pycnocline off Oregon has been traditionally delineated by the 25.5 and 26.0 $\sigma_t$ surfaces since Collins (1964) and Collins et al. (1968). During upwelling this band is 15 to 30 m thick and has a density contrast of 1 or 2 $\sigma_t$. It warps up from an open ocean depth of about 100 m to form a surface front in shallow water within (5-10 km) from the beach. Mooers, Collins, and Smith (1976) place the mean position of the front offshore 5-10 km, but the available data leave room for other interpretations. As stated above, we assume an interface which intersects the surface exactly at the beach.

For the standard DB Line profile the density contrast between the two discrete layers is assumed to be

$$\frac{\Delta \rho}{\rho} = 10^{-3}. $$

This is at the lower end of the "reasonable" range. The selection of the low value is based on an estimate of how much the maximum density difference in the water column actually influences the two phenomena of interest here, i.e., the geostrophically balanced shear flow and the propagation of first vertical mode internal waves. Most of the density difference from top to bottom, (2.5 $\sigma_t$), is associated with horizontal isopycnals and, therefore, does not contribute to the $V_1 - V_2$ shear.

The mean velocity in the upper layer, $V_1(x)$, was chosen to be a constant = +10 cm/sec (southward along the Oregon coast). This is less an estimate of the "mean" flow than an estimate of the steady "background" flow upon which are superimposed (usually positive) surges due to isolated storms. A sidewall, frictional boundary layer has not been imposed as a device to bring $V_1$ to zero at the beach. While such boundary layers may have an influence on slow moving wave modes with eigenfunctions packed
tightly against the coast, such as the internal Kelvin wave, not much can be done to explore the question because the characteristics of frictional coastal boundary layers are presently not well defined.

The function $h_1(x)$, describing the mean depth of the interface, was chosen to coincide approximately with the average observed depth of the 25.75 $\sigma_t$ isopycnal during upwelling. The function $h_1(x)$ and the resulting lower layer mean velocity, $\bar{V}_2(x)$, as given by geostrophic condition, are also shown in Figure 3. The lower layer flow exhibits approximately (-10 cm/sec) nearshore counter-current often observed off Oregon (e.g. Smith 1974). $\bar{V}_2(x)$ increases smoothly with distance offshore and finally assumes the same value as $\bar{V}_1$ under the horizontal section of the pycnocline.

7. RESULTS FOR DB LINE PROFILE

The properties of the free modes of oscillation for trapped waves are presented in two basic formats: 1) the dispersion curves for the non-dimensional eigenfrequencies of selected wave modes vs. the dimensional wave number, and 2) eigenfunctions for the x dependent part of selected wave modes at selected wave numbers. Figure 4 shows the dispersion curves corresponding to the DB Line profile explained immediately above. Following Iida (1970), the dispersion curves have been compressed vertically by displaying $\frac{\omega}{f}^{1/4}$ rather than $\omega/f$. Some such device is necessary to resolve individual mode curves at the lower frequencies. One disadvantage of this nonlinear frequency axis is that constant phase velocities, $C_p = \frac{\omega}{k}$, are no longer simply represented by straight lines.
Figure 4. Dispersion curves for standard Depoe Bay Line profile. Circled number is the e-folding time for an unstable mode.
radiating away from the origin. In order to reorient the reader, curves corresponding to \( C_p = +10 \text{ cm/sec} \) and \( C_p = 4.4 \text{ m/sec} \) have been included in the figure. Waves propagating in the +y direction, i.e., opposite to a Kelvin wave in the northern hemisphere, occupy the upper half of Figure 4, and waves propagating in the -y direction occupy the lower half.

Some of the distinct eigenvalues and dispersion curves allowed by the smooth boundary condition correspond to wave modes which are not coastally trapped. At the deep sea boundary their associated eigenfunctions appear to be either increasing exponentially or oscillating with undiminished amplitude. In Figure 4 the dispersion curves for these modes have been presented as dashed rather than solid lines. It may be noted that some of the dashed curves cross the high frequency, low wavenumber regions:

\[
\left| \frac{\omega}{f} \right| > \sqrt{\beta + 1} \quad \left( \beta = k \cdot \frac{c}{f}; c = \sqrt{gh} \right)
\]

usually reserved for the leaky mode continuum (e.g. Munk, Snodgrass and Wimbush 1970). That numerical routine converges on certain eigenvalues in these regions does not necessarily mean that other, intermediate frequencies are excluded.

This numerical approach produces \((6n_L - 2)\) eigenvalues corresponding to each value of the wavenumber \( k \). \( 6n_L - b \) of these eigenvalues are divided into six groups or bundles corresponding to the six possible trapped wave species: negatively and positively propagating external (barotropic) Stokesian Edgewaves (ESEW), negatively and positively propagating internal Stokesian Edgewaves (ISEW), and the internal ICSW and external (ECSW) versions of the continental shelf wave modes. Two of the remaining four eigenvalues, which do not exactly fit into any one bundle, are the internal
(IKW) and external (EKW) versions of the fundamental Kelvin-Stokes edge-wave mode. They might be considered as either the zeroth modes for the internal and external shelf waves or the zeroth modes for the negatively propagating edgewaves. The final eigenvalues, which occur for $0 < \frac{\omega}{f} < 1$, corresponds to external and internal Kelvin-wavelike modes which ride an outer "wall" provided by the smooth boundary condition. Figure 4 shows only five distinct bundles of dispersion curves. The high frequency bundles at the top and bottom are the external Stokesian edgewaves. The shaded regions above and below these bundles indicate regions occupied by higher mode ESEW dispersion curves. The associated eigenfunctions for these high modes are structurally complicated and violate the trapped wave assumption, and therefore, their curves were not included in Figure 4.

The shaded bands around $\omega/f = \pm 1$ represent the very tightly packed dispersion curves corresponding to the ISEW modes. Only one or two of the lowest modes are coastally trapped. The lowest modes for both the ISEW and ESEW are on the low-frequency side of their respective bundles. For the internal waves the dispersion curves corresponding to the lowest modes are simply parallel to the abcissa and separated by so small a frequency interval that they cannot be resolved on a figure of this relative size.

The dispersion curves corresponding to the ECSW and ICSW modes are located in a double bundle around $\frac{\omega}{f} = 0$. Most of this double bundle is shaded and represents either very high modal number external waves or low mode internal shelf waves overlapping with the highest mode external waves. The central bundle has an upper limit for $+\frac{\omega}{f}$ which closely coincides with the curve $C_p = +10$ cm/sec. This would indicate that the highest mode shelf waves are being advected backwards at a velocity equal to $\overline{V}_1$ ($= \overline{V}_2$ beyond the upwelling zone). The advective effects of the slow mean currents on the dispersion curve diagram are not otherwise noticeable.

The lowest mode ECSW curves appear on the underside (high frequency edge) of the central bundle. The first and second mode curves agree almost exactly with those determined through the homogeneous model of
Cutchin and Smith (1973). At wavenumber $= 0.002 \text{ km}^{-1}$ the phase velocity of the first mode wave is $-4.4 \text{ m/sec}$. The $-4.4 \text{ m/sec}$ curve in Figure 4 shows how the first mode phase velocity diminishes at higher wavenumbers. Above $k \approx 0.03$ the frequency of first mode wave also gradually diminishes and the group velocity reverses.

Of particular interest is the internal Kelvin-Stokes wave mode. Its dispersion curve is shown, together with the closely adjacent curve of the sixth order external shelf wave mode, as a heavy line within the shelf wave bundle. Iida's (1970) analytically derived expressions for $k = 0.01 \text{ km}^{-1}$ and a linearized approximation of the DB Line profile over the very uppermost reaches of the shelf, yield a phase velocity of about $C_p = -0.07 \text{ m/sec}$ or

$$\left(\frac{\omega}{f}\right)^{1/4} = -0.29.$$  For $k = 0.01 \text{ km}^{-1}$ our values are $C_p = -0.15 \text{ m/sec}$ and

$$\left(\frac{\omega}{f}\right)^{1/4} = -0.35.$$  The prediction by Iida includes an advective effect of about $+0.075 \text{ m/sec}$ due mostly to the steady $+0.10 \text{ m/sec}$ flow in the upper layer. Our results steadily approach those of Iida as the value of the grid interval $\Delta x$ is reduced. This sensitivity to the size of the grid interval is not nearly as strong in the case of the predicted eigenfrequencies for other wave modes. The IKW mode has special problems because it is trapped so closely to the coastline. The baroclinic radius of curvature for the wave eigenfunction (Fig. 10) is only slightly greater than the grid interval.

The lack of complete numerical convergence for the IKW mode is inconvenient, but the indicated direction of convergence -- toward lower frequencies and phase velocities -- is still an interesting result. It means that for a wide range of wavelengths, $\approx$ to about 150 km, the resonant frequencies for internal Kelvin wave activity are substantially below the typical period of the atmospheric forcing function.
8. INSTABILITIES

According to Orlanski (1968), Kasahara and Rao (1972) and others, the potential for instabilities in an inviscid, rotating, two-layered shear flow exists for a wide variety of geometries and initial (mean) flow conditions. One can expect to find instabilities in any situation where mean kinetic or potential energy is available for transfer to time-dependent perturbations. The existing literature, however, does not allow one to readily determine if the shear flow in a particular coastal upwelling zone is stable or unstable or what is the growth rate of the unstable perturbations. Pedlosky (1964) gives some necessary, but not sufficient, conditions for the onset of baroclinic instability in a quasi-geostrophic two-layer model. His first condition requires that the gradients of the mean potential vorticities in the upper and lower layers either be of opposite sign or change sign across the shear flow region. The first two curves in Figure 5 are the mean potential vorticities for the upper and lower layers of the standard DB Line profile. The gradients are always negative so baroclinic quasi-geostrophic instabilities are necessarily excluded. Rao and Simons (1970) have clearly noted that the quasi-geostrophic assumption, while allowing rotational modes, filters out the gravity wave modes and thereby misses the possibility of "mixed mode" or hybrid instabilities. In the Rao and Simons model these hybrid instabilities result from the coupling of gravity and rotational mode waves whose dispersion curves intersect. Characteristically, unstable perturbations produced by rotational modes alone have long wave lengths and perturbations produced by gravity wave modes alone (Kelvin-Helmholtz type instabilities) have short wave lengths. The hybrid instabilities have wave lengths on intermediate scales.

Unstable wave modes correspond to complex values of $C$, the solution to the eigenvalue problem (12). The complex solutions occur in conjugate pairs and the magnitude of the imaginary parts $C_I$ are proportional to the growth (or decay) rate of the unstable mode; i.e. $\tau_e = 1/(\kappa C_I)$ where $\tau_e$ is the e-folding time.
Figure 5. Potential vorticity in upper and lower layers for profiles shown in Figure 3.
Over the wavenumber range shown in Figure 4, the eigenfrequencies for the standard DB Line profile were mostly real. Consistent with Pedlosky's (1964) potential vorticity condition there were no instabilities derived exclusively from the rotational mode spectrum. In Figure 4 there are several places where dispersion curves for different waves intersect or closely approach and provide an opportunity for hybrid-type instabilities. The most interesting opportunity occurs as a result of the proximity of the IKW mode with the sixth ECSW mode. However, for the standard DB Line profile the curves remain separate and produce no instabilities. Another opportunity exists where in a very small region around k = 0.0005 and $\omega \tau = -1$ the EKW mode curve passes through the ISEW bundle. With respect to the ISEW modes convergence of the numerical method was rather poor and nothing definite can be determined about this opportunity. A third opportunity exists at the low wave number ends of the ESEW mode curves. In these regions of $(\omega, k)$ space the ESEW curves do coalesce with dashed curves (nontrapped wave modes) over a short range of wave numbers. The eigenvalues are complex and a sample e-folding time, 2.6 days, is noted next to one of the intersections at the top center of Figure 4. These instabilities were not investigated in detail because they appeared to be weak, limited to small ranges of $\omega$ and $k$, and dependent upon the nontrapped wave modes.

9. **EIGENFUNCTIONS**

Figures 6, 7, and 8 show, for the standard DB Line profile, some features of the $x$-dependent parts of selected eigenfunctions for three values of the longshore wavenumber, $k$. In each of the individual function plots within Figures 6 - 8, the coastline is at the left and the deep sea at the right. The square wave superimposed upon each plot indicates the sense of rotation of the current vector in the lower layer; positive for clockwise (anticyclonic) and negative for counterclockwise (cyclonic) rotation. The solid curve with vertical hatching represents the normalized amplitude of the free surface deformation and, where they appear, the open dots represent the separately normalized amplitude of the interface deformations.
Figure 6. Standard DB Line eigenfunctions for wavenumber $k = 0.01$ for surface (solid line with vertical hatching) and interface (dots) deformations. Integer to upper left is a sequence number for the mode, number to the lower left is $\left(\omega/f\right)^{1/4}$, and, where both interface and surface are shown, number immediately to left of function is ratio of normalizing constants.
Figure 7. Standard DB Line eigenfunctions for wavenumber $k = 0.03$ for surface (solid line with vertical hatching) and interface (dots) deformations. Integer to upper left is a sequence number for the mode, number to the lower left is $(\omega/f)^{1/2}$, and, where both interface and surface are shown, number immediately to left of function is ratio of normalizing constants.
Figure 8. Standard DB Line eigenfunctions for wavenumber $k = 0.05$ for surface (solid line with vertical hatching) and interface (dots) deformations. Integer to upper left is a sequence number for the mode, number to the lower left is $(\omega/f)^{1/4}$, and, where both interface and surface are shown, number immediately to left of function is ratio of normalizing constants.
Where both interface and surface are shown, the number immediately to the left of the plot is the ratio of the normalization constants. Large values of the ratio correspond to relatively large interface deformations. The integers to the left and above the individual eigenfunctions are the sequence number of the eigenvalues in descending order according to frequency. The number below the sequence number is \(\left(\frac{\omega}{f}\right)^{1/4}\). Positive values of \(\omega/f\) indicate waves propagation in the positive y-direction (coast to the left) and negative values in the negative y-direction.

With respect to the lowest order continental shelf wave mode (#131), it might be noted that the interface deformations, especially over the continental slope, are relatively higher in amplitude than would be expected for a purely barotropic mode. The third, fourth, fifth, and sixth order shelf wave modes also display some very high amplitude interface deformations. This is especially true close to the coastline within a baroclinic radius of deformation (\(\approx 10\ km\)). For increasing wave number, the gradual increase in the amplitude of the interface deformation for low mode shelf waves is in qualitative accord with Iida (1970).

What appear as the sixth (#126) and seventh (#125) mode shelf wave eigenfunctions also display a relatively simple form which decreases almost monotonically with distance from the coastline. This is in contrast to the situation for both higher and lower order shelf wave modes where the eigenfunctions are more complicated. This anomalous behavior is apparently caused by hybridization between some shelf wave modes and the internal Kelvin-Stokes wave mode. This interpretation is supported by numerical experiments described in an earlier section. Characteristically, the eigenfunctions associated with two closely adjacent eigenfrequencies, such as #125 and #126, are almost identical. This makes positive identification of the mode very difficult if not impossible.

Figures 9 and 10 show more detailed eigenfunctions for the three lowest continental shelf wave modes and the internal Kelvin-Stokes wave.
Figure 9. Eigenfunction details for three lowest continental shelf wave modes at $k = 0.01$ (DB Line profile). Sequence numbers for 1, 2 and 3 are 131, 130 and 129.
Figure 10. Detail of internal Kelvin-Stokes wave mode (#125) for standard DB Line and $k = 0.01$. 
For the shelf waves $u_1 \approx u_2$ and $v_1 \approx v_2$ for simplicity, only the lower layer velocities have been plotted. The free surface deformation $\zeta$ and the two velocity components agree quite well qualitatively and quantitatively with the predictions of Cutchin and Smith (1973) for the homogeneous case. For the IKW most most of the kinetic energy is trapped in the top layer, and therefore, only the top layer velocity components are shown. The eigenfunctions for $u_2$ and $v_2$ are lower amplitude and much more complicated. The baroclinic radius for this mode is 0 (10 km) and the ratio of the interface deformation to the surface deformation is 1000:1. This mode might therefore play a part in the dynamics of upwelling events with similar spatial dimensions. The one substantial drawback to this interpretation is the temporal dimension of the mode, approximately 50 days. This is much longer than the week-long time scales of upwelling events observed, for example, by Halpern (1972). The eigenfunctions (not shown here) for shelf wave modes at frequencies lower than that of the internal Kelvin-Stokes mode exhibit the bottom trapping of kinetic energy as discussed, for example, by Wang (1975).

10. ALTERNATIVE PROFILES

One drawback to this numerical method of solving the wave equations is that a considerable amount of computation and interpretation is necessary to obtain the dispersion curves and the modal structures corresponding to just one bathymetric, hydrographic profile. In spite of the difficulties involved it is still desirable to be able to examine the dispersion curves and eigenfunctions for a variety of current and bathymetric profiles. A compromise is to select a particular wavenumber and examine the variation of the eigenfrequencies and eigenfunctions at selected wavenumbers as the standard Depoe Bay profile is gradually deformed into a new profile. One possible technique for changing the profile is to form different weighted averages of the two profiles. This is the technique followed here.
Two idealized ocean boundary profiles which have been of considerable interest to theoretical oceanographers are:

1) the linear, constant slope profile with zero depth at the beach (e.g., Iida 1970, Reid 1958); and

2) the constant depth profile with a vertical beach (e.g., Allen 1973).

A linear approximation (Fig. 3) to the Depoe Bay profile was generated with $V_1 = 10$ cm/sec, interface slope $= 10^{-3}$, bottom slope $= 1.267 \times 10^{-2}$ and $V_2 = 0$. The right edge of Figure 11 shows the eigenvalues corresponding to this linear profile for $k = 0.01$ km$^{-1}$. The center of Figure 11 shows the eigenvalues corresponding to the Depoe Bay profile for $k = 0.01$ km$^{-1}$. These exact eigenvalues also appear in Figure 4. Between the center of Figure 11 and the right edge are shown the locus of the changing eigenvalues as the Depoe Bay profile is progressively averaged with the linear profile. The weight $T$ associated with the Depoe Bay profile is indicated along the abcissa. The weight associated with the linear profile is simply $1-T$.

Figure 11 shows that the transition to a linear profile ($T = 1 \rightarrow T = 0$) has little effect upon the high frequency internal or external edge-waves. On the other hand, the external shelf wave eigenvalues are distorted by what first appears to be the migration of the internal Kelvin-Stokes wave mode "identity" towards higher frequencies and phase velocities. The eigenfrequency curves never intersect in this region but approach one another in succession and then veer away. Inspection of the eigenfunctions (Figs. 12 and 13), however, indicates that the first few modes in the shelf wave bundle retain the characteristic shape of the first, second, third, etc., shelf waves as determined for the DB Line profile.

The typical IKW mode shape for $0 < T < 1$ is exhibited by eigenfunction #122 (not shown) which has an eigenfrequency of $\omega T = -0.009$. The phase velocity is $-9$ cm/sec and the period 78 days. This result is apparently numerically convergent and does not change with diminishing $\Delta x$. Iida (1970) predicts, for
Figure 11. Transition of eigenvalues, at $k = 0.01$, array from positions for standard DB Line profile as profile is progressively warped into flat profile (on left) and linear profile (on right). D and T are weighting factors for the warping.
Figure 12. Surface and interface eigenfunctions corresponding to Figure 11 in the range $0 < D < 1$. Interpretation of symbols and numbers same as for Figure 6.
Figure 13. Surface and interface eigenfunctions corresponding to Figure 11 for $T = 0$ and 0.5. Interpretation of symbols and numbers same as for Figure 6.
this exact geometrical situation, \( \frac{\omega}{T} = +0.17 \times 10^{-3} \) and therefore, a very small phase velocity. This prediction includes an advective effect which retards the normal (negative y) progress of the wave by almost exactly +9 cm/sec. In contrast to Iida, these results do not show this retardation.

Figures 12 and 13 are companion figures to Figure 11 in the same sense Figures 6, 7, and 8 are to Figure 4. They show some of the properties of selected eigenfunctions at \( k = 0.01 \text{ km}^{-1} \). The center column (duplicated for the most part in Figure 4) corresponds to the Depoe Bay profile \( (T = 1) \), and the right-most column corresponds to the pure linear profile \( (T = 0) \). The column between these two corresponds to \( (T = 0.5) \). The symbols and numbers in Figures 12 and 13 have the same meanings as in Figures 6, 7, and 8.

For linear profiles, Iida (1970) has noted the resolution of the central, low frequency band of dispersion curves into two separate groups. The relatively higher (negative) frequency group corresponds to an external wave species and the relatively lower frequency group, an internal wave species. For a two-layer system they might be called the external and internal forms of continental shelf waves. For the linearized Depoe Bay profile this clear separation of the two wave species does not occur. The highest mode external waves overlap and interfere with the lowest mode internal waves. The numerically determined eigenfunctions in the overlap region exhibit complicated forms. Since the internal shelf wave eigenfunctions are probably packed closely to the coastline -- even more closely than the internal Kelvin wave mode -- their form may be severely distorted by the numerical grid spacing in that area. The external/internal shelf wave species separation, however, clearly did occur (not shown here) when Iida's high slope, high shear profiles were used in the numerical model. The lowest mode internal shelf wave eigenfrequency for a high slope profile is noted in Table I. It compares quite well with Iida's analytical result. Certain irregular details in the complete dispersion curves are complete eigenfunctions for the Depoe Bay profile give some indication of the position of the dispersion curve relating to the first mode internal shelf wave. The characteristic periods seem to be so long for this species that they probably do not contribute significantly to the dynamics of upwelling events.
Bay profile give some indication of the position of the dispersion curve relating to the first mode internal shelf wave. The characteristic periods seem to be so long for this species that they probably do not contribute significantly to the dynamics of upwelling events.

(a) **Constant depth profile**

The second idealized ocean boundary profile (Fig. 3) involves a constant total depth, \( H \), equal to 2.8 km, a constant mean interface depth, \( \bar{h}_1 \), equal to 136 m, and constant mean velocities in the upper and lower layers, \( \bar{V}_1 \) and \( \bar{V}_2 \), equal to +10 cm/sec. The left edge of Figure 11 shows the eigenvalues corresponding to this "flat" profile with \( k = 0.01 \text{ km}^{-1} \).

Between the center of Figure 11 and the left edge are shown the locus of the changing eigenvalues as the Depoe Bay profile is progressively averaged with the flat profile. The weight associated with the Depoe Bay profile is indicated along the abcissa. Figure 12 shows the variation in the form of selected eigenfunctions for \( D = 1., 0.5, 0. \)

The most interesting result of transforming to the flat profile is the clear emergence of the internal Kelvin-Stokes wave from amongst the external shelf wave modes. The shelf waves, as expected, converge towards zero phase velocity relative to the mean flow, \( \bar{V}_1 = \bar{V}_2 = +10 \text{ cm/sec} \). At \( D = 0.0 \) the internal Kelvin wave mode has a phase velocity, relative to the mean flow, of -1.14 m/sec. The phase velocity \( C_p \) for this situation can be calculated directly via

\[
C_p = \sqrt{g' h'}
\]

where

\[
g' = g \frac{\Delta \rho}{\rho} \quad \text{and} \quad h' = \frac{\bar{h}_1 \bar{h}_2}{\bar{h}_1 + \bar{h}_2}.
\]

The result is -1.13 m/sec.
Mooers et al. (1976) and others have postulated the existence of a surface "jet" current off Oregon during upwelling. This jet is located seaward of the surface front, 10-20 km wide, 25-50 m thick, and has a strength of 0 (25-50 cm/sec). Since strong lateral shears could generate dynamic instabilities, it was decided to use another alternative profile incorporating this coastal jet. For this profile (Jet-1):

\[ \bar{V}_1(x) = A_1 + A_2 \cdot \frac{x}{b} \cdot e^{(1 - \frac{x}{b})} \]

where \( A_1 = 10 \text{ cm/sec} \), \( b = 20 \text{ km} \), and \( A_2 = 40 \text{ cm/sec} \). This function produces a smooth jet with \( \bar{V}_1 = +10 \text{ cm/sec} \) at the coast, a maximum amplitude of 50 cm/sec at \( x = 20 \text{ km} \), and a gentle decay towards +10 cm/sec in the deep sea (Fig. 3). The mean interface depth is described by:

\[ h_1(x) = A_3 \cdot \left( h_1'(x) - h_1'(0) \right) \]

where

\[ h_1'(x) = \tan^{-1}\left( \frac{x - d}{a} \right) \]

\( A_3 \) is set to 55 m, \( d = 17 \text{ km} \) and \( a = 21 \text{ km} \). This produces a smooth interface profile with \( h_1(0) = 0 \), a maximum slope near the centerline of the surface jet, and a gentle leveling off at about 125 m in the deep sea. The lower layer mean velocity, \( \bar{V}_2(x) \), computed from \( \bar{V}_1(x) \) and \( h_1(x) \) has a nearshore counter-current of strength 0 (-10 cm/sec) out to about 40 km. The density contrast for the jet profile was increased to \( \frac{\Delta \rho}{\rho} = 2 \times 10^{-3} \) in order to produce a more realistic profile for the geostrophically balanced interface slope and bottom layer velocity.

The mean potential vorticity curves for the Jet-1 profile are the third pair of curves in Figure 5. They have almost the same shape as the curves for the DB Line and linear profiles, and do not exhibit a reversal of the gradient of the potential vorticity. This means that quasi-geostrophic baroclinic instabilities are still impossible.
The Jet-1 profile is similar to some of the Gulf Stream profiles examined by Orlanski (1969). Using Orlanski's definitions, for Jet-1 the Rossby number \( R_o = \frac{\bar{V}_1 \text{max}}{L \cdot f} \) is about 0.21 and the Richardson number 
\[
R_i = g \frac{\Delta \rho}{\rho} \cdot \frac{H}{(\bar{V}_1 - \bar{V}_2)^2 \text{max}}
\]
about 350. (If the Richardson number is scaled by the depth under the maximum shear, rather than the depth of the deep sea it is reduced to about 7.7.) Orlanski's Gulf Stream profiles covered the range \( 0.2 \leq R \leq 0.5 \) and \( 20 \leq R_i \leq 80 \). In a manner similar to Jet-1 the potential vorticity for the upper layers decayed monotonically away from a maximum at the beach. Orlanski, however, also assumed large lower layer depths at the beach and was thereby able to produce some interesting potential vorticity profiles in the lower layer. Specifically, the lower layer potential vorticity gradient has both negative and positive values. This satisfies the necessary condition for baroclinic, quasigeostrophic instability. Further, by judicious selection of the shape of the bottom, Orlanski was able to both stabilize and destabilize the flow by increasing the amplitude of the topography.

Figure 14 is similar to Figure 11 in that it shows the changes in the eigenvalues, at a fixed \( k = 0.01 \text{ km}^{-1} \), as the standard DB Line profile is progressively averaged with the Jet-1 profile. The left edge, \( R = 1.0 \), corresponds to the pure DB Line and the right edge, \( R = 0.0 \), to the pure Jet-1 profile. Figure 14 shows, for the slower waves, a slight advective shift toward the positive as the velocity of the positively directed jet is increased. Most of the waves are traveling too fast for this advective effect to be noticeable.

Over the range \( 0.05 < R < 0.45 \) the sixth ECSW mode and the IKW mode couple together to give a weakly unstable hybrid wave mode (boxed region in Fig. 14). The e-folding times for the instability are rather long-- on the order of 200 days. Table 2 outlines a more extensive investigation of this instability as a function of \( k \) as well as the transition parameter \( R \).
Figure 14. Transition of eigenvalues as standard DB Line profile (left) is warped into Jet-1 profile (right). $R$ is the transition parameter. Boxed section of curves around $(\omega/\tau)^{1/4} = 0.3$ marks an unstable, hybrid mode.
For long wavelengths the instabilities never become very strong. The shortest e-folding times appear to be on the order of 80-100 days. There are stable regions in (k, R) space between successive unstable regions. The second and third unstable regions, at intermediate and very short wavelengths, show e-folding times as short as 20-30 days. The three instability regions or "bands" shown as a function of k in Table 2 probably represent the coupling of the IKW mode successively with the sixth, fifth, and fourth ECSW modes.

Table 2, and unless otherwise noted, all of the computations were done with n_L = 33 and a grid spacing of \( \Delta x = 7.5 \) km. Table 3 shows the variation in the real period and the e-folding time for the unstable IKW/ECSW hybrid mode as a function of n_L and k for a fixed value of R = 0.0. As noted previously and as is evident from Table 2, the numerical computations are not quite convergent for the IKW mode, probably because of the very short baroclinic radius of curvature. The results in Tables 2 and 3 are still interesting, however, in that they point the way towards the possibility of dynamic instability with e-folding times not much greater than the time scales for perturbations in the upwelling regime. It appears also that the strongest instabilities should occur on length scales as short as 50-150 km.

(c) The Jet-2 (finite depth) Profile

The standard DB Line profile, the linear profile, and the Jet-1 profile all involve an upper and lower layer whose depths taper toward zero at the beach. On the other hand, the current shear profiles assumed by Orlanski (1969) and the zero mean flow profiles assumed by Allen (1975) and Wang (1975) involved substantial upper and lower layer depths at the beach. The depth close to the beach is probably not an extremely significant factor when solving for wave modes with eigenfunctions which stretch over a large fraction of the width of the continental margin. It should have a substantial influence over wave modes, such as the IKW, which are trapped tightly within a few baroclinic radii of the beach. Specifically, increasing the depth
Table 2. Jet-1 IKW coupling instabilities.

<table>
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<th>$1(km^{-1})$</th>
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<th>.015</th>
<th>.02</th>
<th>.025</th>
<th>.03</th>
<th>.035</th>
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<td>-</td>
<td>x</td>
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<td>19.8 .45</td>
<td>-</td>
<td>66.1(517)</td>
<td>-</td>
<td>36.4(249)</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>-</td>
<td>x</td>
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<tr>
<td>.13</td>
<td>17.3 .40</td>
<td>-</td>
<td>-</td>
<td>38.3(108)</td>
<td>-</td>
<td>-</td>
<td>x</td>
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<td>.14</td>
<td>15.1 .35</td>
<td>-</td>
<td>48.5(111)</td>
<td>38.3(109)</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
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<td>.15</td>
<td>13.5 .30</td>
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<td>80.8(200)</td>
<td>40.4(96.3)</td>
<td>-</td>
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<td>.16</td>
<td>12.2 .25</td>
<td>145.4(374)</td>
<td>80.8(190)</td>
<td>42.8(93)</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>.17</td>
<td>11.0 .20</td>
<td>-</td>
<td>80.8(180)</td>
<td>58.2(113)</td>
<td>42.8(86.8)</td>
<td>34.6(80.2)</td>
<td>29.1(237)</td>
<td>x</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>.18</td>
<td>10.0 .15</td>
<td>181.8(349)</td>
<td>90.9(176)</td>
<td>42.8(92)</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>.19</td>
<td>9.1 .10</td>
<td>181.8(357)</td>
<td>90.9(197)</td>
<td>48.5(103)</td>
<td>38.3(80)</td>
<td>31.6(97)</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>.20</td>
<td>8.3 .05</td>
<td>181.8(419)</td>
<td>103.9(264)</td>
<td>51.9(255)</td>
<td>40.4(87.9)</td>
<td>34.6(84.8)</td>
<td>30.3(125)</td>
<td>x</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>.21</td>
<td>7.7 .00</td>
<td>-</td>
<td>42.8(114)</td>
<td>36.4(58.9)</td>
<td>31.6(101)</td>
<td>28.0(209)</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

For the standard DB Line -- Jet-1 profile transition are shown periods and, in parentheses, e-folding times for the unstable hybrid IKW/ECSW mode as a function of the wave number and the transition parameter $R$. Units are days. The symbol "x" indicates only separate, stable wave modes exist. The "-" indicates no computations for that parameter combination.
Table 3. Variation in real period and e-folding time for unstable IKW/ECSW hybrid mode as a function of $n_L$ and $k$ for a fixed value of $R = 0.0$.

<table>
<thead>
<tr>
<th>$n_L$, $k$ (km$^{-1}$)</th>
<th>0.005</th>
<th>0.010</th>
<th>0.015</th>
<th>0.020</th>
<th>0.025</th>
<th>0.030</th>
<th>0.035</th>
<th>0.040</th>
<th>0.050</th>
<th>0.060</th>
<th>0.070</th>
<th>0.080</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1257</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>18.6(29.3)</td>
<td>16.5(19.7)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>628</td>
<td>145(366)</td>
<td>94(552)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>15.5(30.9)</td>
<td>14.3(22.4)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>419</td>
<td>242(501)</td>
<td>121(233)</td>
<td>81(162)</td>
<td>56(170)</td>
<td>46(273)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>13.5(31.1)</td>
<td>12.8(33.0)</td>
<td>9.7(18.1)</td>
<td>x</td>
</tr>
<tr>
<td>314</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>43(114)</td>
<td>36(98.9)</td>
<td>32(101)</td>
<td>28(209)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>12.1(35.0)</td>
<td>9.3(29.4)</td>
<td></td>
</tr>
<tr>
<td>251</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>26(90.6)</td>
<td>22.0(97.3)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>209</td>
<td>182(406)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

For the pure Jet-1 profile periods and, in parentheses, e-folding times for the unstable hybrid IKW/ECSW mode as a function of wave number $k$ and the number of numerical grid intervals $n_L$. Units are days. The symbol "x" indicates that the wave modes were separate and stable.
should speed up the IKW mode. From the Jet-1 results it might be suspected that increasing the IKW speed would cause it to couple with lower ECSW modes at higher frequencies and produce stronger instabilities.

The assumption of finite depth at the beach is not entirely unreasonable in the context of Oregon coastal upwelling. Observations of mean hydrographic profiles from an upwelling zone may be interpreted to show an interface or frontal zone which steeply rises and intersects the surface some 5 to 10 km from the shore in 50 to 100 m of water. If one neglects the shallow single layer wedge inshore of this front then the finite depth models are a reasonable approximation to the real upwelling situation. A finite depth profile (Jet-2) was therefore constructed by imposing a vertical wall in 100 m of water 10 km from the coastline on the Depoe Bay bathymetric profile and allowing the interface to come up to meet the wall at a depth of 20 m. The interface depth and the mean velocity in the upper layer are given by

\[
\bar{h}_1(x) = A_1 (1 - e^{-a_1 x}) + h_o
\]

and

\[
\bar{V}_1(x) = A_2 + A_3 \cdot e^{-a_2 x}
\]

where \( x \) is measured from the wall, \( A_1 = 0.2 \text{ km}, a_1 = 0.02 \text{ km}^{-1} \), \( h_o = 0.02 \text{ km}, A_2 = 0.05 \text{ m/sec}, A_3 = 0.30 \text{ m/sec} \) and \( a_2 = 0.025 \text{ km}^{-1} \). This produces jet in the upper layer of maximum amplitude equal to +0.35 m/sec and, for \( \frac{\Delta \rho}{\rho} = 10^{-3} \), a shore-bound lower layer counter-current of maximum amplitude = -0.05 m/sec. Both the upper layer and lower layer currents decay monotonically away from the inshore wall.

Figure 15 shows the dispersion curves (solid lines) for some of the low frequency wave modes corresponding to the Jet-2 profile. They have been plotted on a negative linear frequency scale in order to facilitate comparison with the results of Wang (1975) and Allen (1975). The short dashed curves correspond to zero mean flow (ZMF), an interface level at
Figure 15. Dispersion curves for Jet-2 profile (solid) and for a zero mean flow, flat interface version (dashed lines). Symbol I indicates that eigenfunction is clearly that of the IKW mode.
30 m depth and the Jet-2 bathymetry. The ZMF curves are qualitatively quite similar to those of Wang (1975) and Allen (1975). The IKW dispersion curve successively approaches the second and first mode ECSW curves but veers away without intersecting. The characteristic IKW-like eigenfunction is passed from one curve to another as the near-intersections occur. The segments of dashed curves lying along a constant phase velocity \( C \approx 0.48 \) m/sec all represent modes which are strongly internal and decay monotonically within about 15 km of the beach.

The effects exerted by the nearshore jet and the interface curvature are relatively minor in the case of the low ECSW modes. The ZMF dashed curves are only slightly below the curves for the full-strength Jet-2. The tightly trapped IKW mode, however, is strongly influenced by the presence of the jet. It is advected backward (toward +y) by roughly 20 cm/sec. Also the points where the IKW curve impinges upon the ECSW curves are no longer easily definable. As in the case of the ZMF profile the characteristic IKW shape is passed to higher curves as the wavenumber increases. The symbol (I) marking some points on the solid curves represents modes which could be identified from their eigenfunctions (not shown) as looking much more like an IKW than any nearby mode. The IKW shape is shared between adjacent modes when the transfer is taking place.

The average phase velocity for the IKW mode in the case of the Jet-2 profile is very roughly \(-25\) cm/sec. The period, therefore, corresponding to a wavelength of 125 km is about 5 days and corresponding to 250 km is about 10 days. These time scales are more appropriate to coastal upwelling events than those quoted for the other profiles but the length scales are still quite short.

As a double result of a deepening the layers at the beach and simplifying the shape for the longshore jet (i.e. no extrema), 1) the hybrid mode instabilities found with Jet-1 have disappeared; and 2) the numerical computations converged for \( n_L = 33 \).
Table 4 contains some of the pertinent characteristics of the eigenfunctions for the Jet-2 profile. The ratios of the mean squares of $V_1$ and $V_2$ together with the ratios of the root mean squares of $Z_1$ and $Z_2$ are given for six of the ECSW and IKW modes of Figure 15. The table shows that the interface deformations increase relative to surface deformations as the modal number increases (frequency decreases) and as the wavenumber increases. Figures 6-8 for the standard DB Line profile show the same general trends but the interface deformations are not as pronounced as they are for the finite depth profile. This suggests that detection of upwelling events through surface level changes, as suggested by Gill and Clarke (1974), would be more feasible in the case of a finite depth profile like Jet-2.

With respect to fluctuations in $V_1$ and $V_2$, Table 4 may be roughly compared with Figure 6 of Wang (1975). At a fixed wavenumber Wang follows the ratio of the mean square of $V_1$ to the mean square of $V_2$ as a function of the density stratification. An increase in the density increases the frequency of the IKW mode as does an increase in $k$. Figure 6 of Wang and, to a greater extent, our Table 4 shows a "surface trapping" of the longshore current fluctuations for the IKW mode. A surface trapping effect is significant from an observational point of view because it would allow the IKW to pass over current meters moored beneath the pycnocline without leaving a strong signature. The "bottom trapping" effect predicted by Wang for modes below the IKW is not so readily noticeable in Table 4. Indeed, strong bottom trapping occurs for modes just above the IKW and weak trapping occurs for modes below the IKW. The discrepancies between our results and those of Wang are undoubtedly influenced by the mean conditions, especially the shear flow and the tapering upper layer.
Table 4. Jet-2 eigenfunction characteristics.

<table>
<thead>
<tr>
<th>Mode</th>
<th>k</th>
<th>0.01</th>
<th>RMS $Z_1$</th>
<th>$\frac{V_1^2}{V_2}$</th>
<th>RMS $Z_1$</th>
<th>$\frac{V_1^2}{V_2}$</th>
<th>RMS $Z_1$</th>
<th>$\frac{V_1^2}{V_2}$</th>
<th>RMS $Z_1$</th>
<th>$\frac{V_1^2}{V_2}$</th>
<th>RMS $Z_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.93</td>
<td>12.3x10^{-3}</td>
<td>0.819</td>
<td>3.35x10^{-3}</td>
<td>0.209</td>
<td>1.39x10^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>0.62</td>
<td>5.08x10^{-3}</td>
<td>0.389</td>
<td>2.75x10^{-3}</td>
<td>0.460</td>
<td>1.21x10^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>0.146</td>
<td>1.80x10^{-3}</td>
<td>0.228</td>
<td>1.32x10^{-3}</td>
<td>0.95</td>
<td>0.969x10^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>5.285</td>
<td>1.004x10^{-3}</td>
<td>0.942x10^{-3}</td>
<td>12.311</td>
<td>0.793x10^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>9.078</td>
<td>0.806x10^{-3}</td>
<td>0.769x10^{-3}</td>
<td>1.686</td>
<td>0.589x10^{-3}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>0.941</td>
<td>0.624x10^{-3}</td>
<td>0.726</td>
<td>0.568x10^{-3}</td>
<td>0.457</td>
<td>0.488x10^{-3}</td>
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<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Eigenfunction characteristics at selected wavenumbers for Jet-2 modes graphed in Figure 15. Given are the ratios of the mean squared values of the perturbation velocities in the upper and lower layers, and the ratios of the root mean squared values of the surface and interface level perturbations. Means are computed over x.
11. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The eigenvalue/eigenvector approach to the solution of the primitive equations of motion is conceptually simple and enables one to find all of the important modes, both stable and unstable, in a straightforward manner. Resolution and numerical convergence problems exist when eigenfunctions have sections with radii of curvature on the order of the grid spacing. Boundary conditions requiring zero mean transport normal to the coastline are naturally incorporated. Deep-water conditions requiring the decay of wave activity away from the coast cannot be exactly incorporated without destroying the standard eigenvalue problem form of the finite differenced equations. The substitute "smooth" boundary condition used here retains the essential form of the trapped wave solutions.

Attempts to represent the Oregon upwelling zone by two discrete layers of different density separated by a reasonably contoured interface have pointed up some possible inconsistencies in this idealized model. If the density contrast is increased much beyond $\Delta \sigma_t = 1.25$, the vertical shears, subsurface and counter current velocities become unreasonably large. On the other hand, top to bottom contrasts in excess of $\Delta \sigma_t = 2.0$ are observed off the Oregon coast. Horizontal isopycnals above and below the inclined "front" apparently do not contribute to the balanced shear. The total density contrast, however, may contribute substantially to increasing the phase speed and frequency of internal modes. It may be that the idealized two-layer model should be augmented by a third layer to give the density contrast without (necessarily) increasing the shear. The shear/frontal slope/density contrast consistency problem is neatly avoided in the level interface and zero mean flow models of Allen (1975) and Wang (1975). The most interesting features of those papers are accentuated by increasing the stratification to some fairly high values.

With the flows and $\Delta \rho/\rho$ values assumed here we find that the IKW mode propagates at relatively low phase velocities [(10 cm/sec)] and, for the wavenumbers less than and close to the longshore scale of Oregon's
topographic irregularities [0(100 km)], it appears at very low frequencies 
\( \frac{\omega}{T} < 0.2 \). The period of the IKW mode is therefore generally below the 
period usually required to set up an upwelling event. As another consequence 
of the slow phase velocity of the IKW mode, the "resonant" interactions 
with the external shelf waves occur at relatively short wavelengths and low 
frequencies.

Kinematically speaking the most noticeable new feature pro-
duced by the introduction of over-the-shelf stratification in a shelf model 
would seem to be a "top trapping" effect connected with the IKW mode. The 
amplitude of the velocity fluctuations in the upper layer are substantially 
increased. This means that IKW modes could pass over an array of current 
meters moored beneath the pycnocline without leaving a strong signature 
on the records. Wang (1975) notes the top trapping effect but apparently 
considers it less significant than "bottom trapping" at lower frequencies.

With respect to the interpretation of observations, it should be 
noted that both bottom and top trapping of current fluctuations takes place at 
the same frequencies. Spectral analysis alone cannot sort out from a single 
current meter record that part of the signal which should be trapped in 
either layer. It is further necessary to dissect spatial current patterns on 
the basis of their longshore wavenumber. Unfortunately, to sample in 
space at anywhere near the resolution used to sample in time is quite 
difficult.

The potential vorticity for the sample Oregon coastal profiles 
tested here decays monotonically away from a maximum at the beach. The 
sharp, tapering of the depth of the upper and lower layers is sufficient to 
mask any extrema that might have been introduced by variations in the hori-
shears of the mean flow. As a consequence, dynamic baroclinic instabilities 
in the mean flow are not possible. Mixed type instabilities are still possible 
especially with respect to the interaction between the IKW and the ECSW 
modes. For large wavenumbers such instabilities do occur in the case of a
strong jet-like profile (Jet-1), but the e-folding times for growth are relatively long. The numerical solutions for these unstable modes are not completely convergent and the subject deserves more detailed study. A fine gridded shooting method, such as that described by Orlanski (1968), would seem, in this instance, to be more suitable than the eigenvalue/eigenvector method. The only foreseeable problem in using the shooting method is in being able to pick out the unstable IKW mode from among a number of closely adjacent modes.

All of the Oregon coastal profiles examined here have an interface which, at the beach, surfaces or intersects a vertical wall. There are two layers over the entire profile. Mooers, Collins and Smith (1975), on the other hand, see the interface in a two-layer model as surfacing about 10 km from the beach. A natural extension of this work would be to study the influence of this type of a profile modification on the frequencies and kinematics of the trapped waves. Along these lines John Baines (Florida State University, personal communication) has recently modified the uniform slope, two-layer model of Iida to incorporate a surfacing interface and has solved for the trapped wave modes. With respect to possible instabilities, Kasahara and Rao (1972) have found that separation of a frontal zone from a solid boundary wall had the effect of destabilizing their sheared flow model.

12. ACKNOWLEDGMENTS

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REFERENCES CITED


