Optimal Proxy-Limited Lines for Representing Voltage Constraints in a DC Optimal Powerflow

by

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A thesis submitted in fulfillment of the

requirements for the degree of

Master of Science

(Electrical Engineering)

at the

UNIVERSITY OF WISCONSIN-MADISON

2009

Abstract

With the recent restructuring of the electric industry away from the vertically integrated monopolies, electricity markets have emerged in various parts of the country. Independent System Operators (ISOs) and Regional Transmission Organizations (RTOs) have been set up to oversee these markets.

While prices for the consumer have remained regulated, on the wholesale side companies are competing everyday to buy and sell power. These electricity markets require a way to value the energy at different locations. One of the most widely used methods today is called the Locational Marginal Price (LMP) or nodal price and can be used to settle markets. It is defined as the cost of supplying one more megawatt of power while abiding to the constraints of the system.

There are two commonly-used models for calculating LMPs in a given system, the “AC” optimal power flow (ACOPF) and the “DC” optimal power flow (DCOPF). Although both methods result in finding LMPs of the system while abiding to constraints, loads, and costs, they differ in significant ways. The ACOPF and DCOPF use dissimilar methods to arrive at a solution, and that solution can be very different in some cases. The ACOPF is considered by many to be the correct result, with nothing in this method being assumed or approximated. However, because of its nonlinear approach, the solution can take much longer to solve if it even reaches a solution at all. Many real-time applications use a form of the DCOPF. It neglects resistance, reactive power, and voltage magnitudes, but results in a linear function that solves much quicker and more reliably than the ACOPF.

Since the DCOPF doesn’t take into account voltage magnitudes there is no direct way of having voltage constraints in a system. The method in this paper describes a way of mapping a voltage limit in the ACOPF to a flow limit in the DCOPF. Constraining particular lines in the DCOPF model results in a solution of LMPs and dispatch that acts like it has a voltage constraint at a particular bus. These mappings of one limit to another are called proxy limits or the nomogram approach [1]. The method described in this paper will use a Mixed Integer Programming (MIP) technique to optimally choose lines needed to minimized the dispatch error, LMP error, or a combination of both.

The technique was carried out with the modified 30 bus system supplied by Matpower [2]. Results of minimizing the LMP error, dispatch error, and different combinations are shown below. Also shown, are how the lines chosen may achieve the voltage constraint mapped back to the ACOPF. Multiple constraints in the system and whether choosing from different subsets of lines has any effect on the system are also shown. Finally, results of how this method scales up or down with larger or smaller systems is examined.

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1. Introduction

1.1 Introduction

The electric industry has accomplished numerous engineering feats throughout the last one hundred years. None is more powerful or influential than our electricity grid, which is probably the largest and most complex industry in the world. Today, the industry is moving away from the vertically integrated utilities that have dominated for decades [3]. The industry in many areas has seen increased competition in the generation sector with the Federal Energy Regulatory Commission (FERC) Order Number 888, which mandated open access for generators to the nation's transmission system [4]. While prices for distribution and transmission remain regulated in most areas, many centralized electricity markets have sprung up around the nation. These electricity markets have been managed by Independent System Operators (ISOs) and Regional Transmission Organizations (RTOs) in areas like the Midwest, New England, and California [4].

As of 2005, over 50% of the nation's generating capacity operates within an ISO or RTO area [5]. The ISOs and RTOs are required to be independent of the individual market participants, such as transmission owners, generators, distribution companies, and end-users [3]. It is the ISO's and RTO's responsibility to commit and dispatch some or all system resources and to curtail resources or load for maintaining the system security, but also must ensure that proper economic signals are sent to all market participants. This is said to encourage efficient use of the grid and to motivate investment in resources capable of alleviating constraints [3].

It is within these models that the underlying wholesale electricity markets exist. The idea behind the wholesale electricity market is a bid-based security constrained dispatch framework with Locational Marginal Price or "Nodal" pricing [5]. The LMP method has been used or is being used at a number of ISO's such as PJM, New York ISO, ISO-New England, California ISO, Midwest ISO, and ERCOT [6].

The LMP is a cost of optimally supplying an increment of load at a particular location while satisfying all operational constraints [7]. Usually the LMPs are produced as a result of economic dispatch, which is the least expensive way of supplying load in the system. Dispatchable generators are used to keep the system in balance, and in the case of ISO New England, are optimized every 5 minutes to help meet the load at a minimum cost [7].

There are two other conditions that can alter the LMPs at a specific node: the first being losses and the second being congestion. If there are no losses and no congestion in the system, the LMPs will be the same at each node [7]. Congestion in lines can be a binding constraint, which will alter the flow of power, creating differences in LMPs.

Once the LMPs are known, they can be used for a variety of purposes. LMPs themselves are used to settle the markets on the wholesale side [8]. In a simplified version, a buyer and seller of electricity can both bid in with how much electricity to consume or produce and at what price, and the price that is paid or received is determined by the LMPs at each node [8]. They can also be used for establishing price differentials among the nodes in order to settle transmission property rights settlements to the holders of Financial Transmission Rights (FTRs) [9]. Even more importantly, they can be used to influence those in market, by taking advantage of the natural business sense of all market participants [9].

1.2 Power Flows

The basic methods on how to calculate LMPs for a given set of costs, constraints, loads, and generators is the Optimal Power Flow (OPF). The OPF has two commonly used methods for solving the system. The first is the AC Optimal Power Flow (ACOPF). This method uses the most widely used Newton-Raphson technique, solving a system of equations of the form $Ax = b$ where the square matrix A is the Jacobian (section 2) and the vector b contains mismatches [10]. However, the system is nonlinear, and therefore several iterations are needed to converge to a solution and in some cases may have difficulty converging.

The other widely accepted method is the DC Optimal Power Flow (DCOPF). This method uses linearizations around certain base points to avoid iterations [7]. This method allows users to calculate LMPs in a reasonable time frame for real time calculations. Typically, the DCOPF is utilized for LMP simulation or forecasting based on production cost model via linear programming (LP) [11]. Various third party LP programs are readily available to solve the DCOPF model [11]. However, since linearizations are made there is error associated with this model. The DCOPF model neglects resistance, voltage magnitude, and reactive power.

The standard "correct" model is often the ACOPF solution; however because of complexities and computational time, the ACOPF model is often simplified into the DCOPF solution. The DCOPF is used widely due its robustness and quickness, yet because of its lack of losses in the system the LMPs solved are not as accurate. If more precise LMPs are needed, various methods for further reading are shown in the references, such as iterating the DCOPF model with losses back in [6] [11] [12].

The method below will use another common technique that when applied to the DCOPF the LMPs act much more like the "correct" ACOPF solution.

1.3 Proxy Limits

A proxy limit is the mapping of one constraint or limit to another. ISOs use proxy limits for a number of different reasons. For example, NYISO uses proxy limits to transfer voltage limits to power flow limits on the lines for contingency purposes [13]. However, they provide no details on how the lines are selected, and there is no mention of LMP or dispatch error being minimized in practice [13].

In the case of this paper, binding AC voltage limits will be transferred to binding DC active power limits. When a DC power flow is used to settle markets, the use of proxy limits can introduce real world errors in LMPs, dispatch, or both. This has also been called the nomogram approach and can be hard to find a solution that minimizes this error for both prices and dispatch [1] [14]. Although it will be shown below that for n generators $n - 1$ lines are needed (in most cases) to completely minimize dispatch error, the nodal prices will not be as easy. It is very important to note that the prices obtained can differ substantially from the correct nodal prices depending on which lines are chosen [1].

The method introduced in this paper will optimally pick the first several lines for both dispatch matching, as well as price matching (thus correctly selecting the lines to minimize error). Also shown, will be weighted averages of dispatch as well as LMPs throughout the system. Third, multiple constraints in the same system will be shown and how it effects the lines that are chosen. Next, choosing from a subset of lines in the same system will be shown and compared to the original results. Finally, the system will be compared to a 14 bus system as well as a 118 bus system. Results from all three will point to how well the system scales up or down along with the computation time required for each case.

2. Weak Power to Voltage Link

2.1 Weak PV Link

In order to understand what this paper is trying to accomplish, one must first understand the limitations as well as the overall idea of the model that will be presented. As stated in the introduction, there are advantages and disadvantages to both AC and DC optimal power flows. In large scale systems, the DCOPF is very useful because it is linear and therefore converges to a solution much faster than its counterpart. However, since the DC power flow includes no voltage magnitude calculations, it cannot be used as a tool for estimating voltages. Also, solving a DCOPF is truly just a linear approximation of the actual solution. This means everything from LMP's, real power, and angles will be solved as an approximation to the actual solution.

Because the purpose of this paper will be to discuss optimal proxy limits on DCOPFs to both match AC dispatch and prices, it is important to understand that while it might make sense to use proxy limits in the DCOPF, in real world simulations it isn't as clear. The overall goal here is to limit a specific set of lines' active power in the DCOPF, to act like a binding voltage constraint in the ACOPF. However, the link between active power flow through a line (P) and voltage magnitude at a bus (V) is weak. In most cases the most assertive factor in determining the voltage magnitude at a bus is actually the reactive power flowing through a line (Q), which is unused in the DCOPF. The section below describes why this is so.

2.2 Power Flows Formulation of Problem – Fast Decoupled Power Flow

We first formulate the Newton-Raphson matrix representation of the AC power-flow problem where the matrix J1-J4 is the n by n Jacobian [4] [10].

$$\begin{bmatrix} J1(i) & J2(i) \\ J3(i) & J4(i) \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P(i) \\ \Delta Q(i) \end{bmatrix} \tag{1}$$

Where

$n \neq k$

$$J1_{kn} = \frac{\partial P_k}{\partial \delta_n} = V_k V_n [G_{kn} \sin(\delta_k - \delta_n) - B_{kn} \cos(\delta_k - \delta_n)]$$

$$J2_{kn} = \frac{\partial P_k}{\partial V_n} = V_k [G_{kn} \cos(\delta_k - \delta_n) + B_{kn} \sin(\delta_k - \delta_n)]$$

$$J3_{kn} = \frac{\partial Q_k}{\partial \delta_n} = -V_k V_n [G_{kn} \cos(\delta_k - \delta_n) + B_{kn} \sin(\delta_k - \delta_n)]$$

$$J4_{kn} = \frac{\partial Q_k}{\partial V_n} = V_k [G_{kn} \sin(\delta_k - \delta_n) - B_{kn} \cos(\delta_k - \delta_n)]$$

and for

$n = k$

$$J1_{kk} = \frac{\partial P_k}{\partial \delta_k} = -Q_k - B_{kk} V_k^2$$

$$J2_{kk} = \frac{\partial P_k}{\partial V_k} = \frac{P_k}{V_k} + G_{kk} V_k$$

$$J3_{kk} = \frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk} V_k^2$$

$$J4_{kk} = \frac{\partial Q_k}{\partial V_k} = \frac{Q_k}{V_k} - B_{kk} V_k$$

where P_k and Q_k are given as follows.

$$P_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\delta_k - \delta_i) + B_{ki} \sin(\delta_k - \delta_i)]$$

$$Q_k = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\delta_k - \delta_i) + B_{ki} \cos(\delta_k - \delta_i)]$$

Equation 1 and those that follow state the general relationships relating real power to both voltage magnitude and angle, as well as reactive power to the voltage magnitude and angle. In most real world applications however, other assumptions can be made. Two relationships show up in most cases of solved power flows [15].

$$\frac{X}{R} \gg 1 \text{ which implies } |B| \gg |G|$$

$$\text{and also } |\delta_k - \delta_n| < 20°$$

Both of these relationships prove to have small values in J2(i) and J4(i) above. This shows a very weak link between P and V, and also between δ and Q. Therefore, in cases where computing time is a factor, the J2(i) and J3(i) sections of equation 1 above can be dropped, resulting in the widely used Fast Decoupled Load Flow [16].

$$\begin{bmatrix} J1(i) \\ J4(i) \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P(i) \\ \Delta Q(i) \end{bmatrix} \tag{2}$$

This is essential evidence against the notion that the power has a direct link to voltage. However, the voltage can still be adjusted using power flow limits in lines to a lesser extent [1].

2.3 Shift Factors

Another way of showing the weak link between P and V is via a method of shift factors. By taking a specific solved optimal power flow, we can input another MW or MVAR at a specific bus and look at the effect it has on voltages. The example shown below is using the modified 30-bus system with parameters obtained in the Matpower distribution [2]. After solving the optimal power flow model, we then increase by one megawatt the power output from the generator at bus 23 and solve the power flow. We repeat this method, by increasing the reactive power by one MVAR at the same bus. From this, we can see how the voltages around the bus were affected. This result was done at several buses to show that the results were not unusual. The following table describes how the voltages were affected.

Table 1. Increasing at Bus 22

Bus #	MW Voltage Change	MVAR Voltage Change
10	8.7583e-5	1.1437e-3
21	2.5249e-5	3.2886e-4
24	1.6199e-5	2.2042e-4

Table 2. Increasing at Bus 23

Bus #	MW Voltage Change	MVAR Voltage Change
15	3.8063e-5	5.0611e-4
24	1.6199e-5	2.2042e-4

In the first case, the MVAR injection at bus 22 had over 8.5 times the effect that the MW injection had on voltages. In the second case, the MVAR injection at bus 23 had over 13 times the effect than the MW injection. This suggests that while active power limits can and do have an effect on voltages, it would be much more effective in real world applications to limit the reactive power through the lines.

Reiterating the trend here, usually in practice, voltage limits are adjusted generally by the injection of reactive power [1]. They can, however be indirectly adjusted, although less effectively, by changes in active power and power flows [1]. The focus of this thesis is in principle to adjust this voltage via active power flows in the lines while minimizing the LMP difference, the dispatch difference or both.

3. DC Optimal Power Flow Proxy Limits

3.1 Objective

In this section we develop the models needed to choose optimal proxy limits. We pose the problem as a mixed integer problem (MIP):

$$\min_{\substack{P_{PLi}, S_{PL} \\ i \epsilon S_{PL}}} \left(\alpha|\lambda_{AC} - \lambda_{DC}(P_{PLi})| + \beta|P_{AC} - P_{DC}(P_{PLi})|\right) \tag{3}$$

where P_{Pli} are the values for the proxy limits and S_{PL} is the set of lines that have proxy limits. The values of α and β weigh the differences between LMP and dispatch errors where α ranges from 0 to 1 and β is 1 – α. λ_{AC} and P_{AC} are the known ACOPF LMPs and dispatch. Likewise, λ_{DC} and P_{DC} are the LMPs and dispatches corresponding to the proxy-limited constrained DCOPF.

In order to identify which lines to bind the proxy limits and at what values we must first develop the DCOPF model. We then can use sensitivities from the model to best select a line to optimally match either LMPs or dispatches from the AC model.

The DC optimal power flow model voltage fluctuations, reactive power, and losses are all neglected. We can describe the power flowing in the branch elements using these assumptions by

$$P_{branch} = diag(b)A^T\theta \tag{4}$$

where diag(b) is a diagonal matrix of branch susesptances, θ is a vector of bus voltage angles, and A is the node-branch incidence matrix [17]. Each column of the incidence matrix serves as an indicator vector describing the connection of a line in the system. The columns contain both a single ‘+1’ entry and a single ‘-1’ entry in the rows corresponding to the terminal buses.

The power ‘injected’ into the network at each bus is given by:

$$P_{inj} = Adiag(b)A^T\theta \tag{5}$$

Therefore the DCOPF problem can be described as:

$$\min_{P_g} \sum_i C_i(P_{gi})$$

subject to

$$Adiag(b)A^T\theta - P_{inj} = 0 \tag{6}$$

$$P_{min} \leq \ diag(b)A^T\theta \leq P_{max}$$

We then replace the inequality constraints with chosen proxy limit constraints.

$$diag(b_{PL})A_{PL}^T\theta - P_{PL} = 0 \tag{7}$$

where ‘PL’ denotes a set of proxy-limited lines.

In our simulations we will only consider generators having quadratic cost functions while ignoring generator limits.

$$C(P_{gi}) = C_{0i} + C_{1i}P_{gi} + C_{2i}P_{gi}^2 \tag{8}$$

Therefore, the Lagrangian formulation for this problem can be set up.

$$L = \sum_i C_i\left(P_{gi}\right) + \lambda^T\left(Adiag(b)A^T\theta - P_{inj}\right) + \mu^T(diag(b)A^T\theta - P_{PL}) \tag{9}$$

By taking derivatives of equation (9) with respect to P_g, θ, λ, and μ and setting them equal to zero we can formulate the first-order conditions. We will substitute P_{inj} for $P_{inj} = P_g + P_D$ so it separates the injected powers into generator and load (demand) portions. Also note that E_g is a matrix, similar to the incidence matrix, that indicates generator buses. Each row has a single nonzero '+1' entry in the column corresponding to the generator bus.

$$\frac{\partial L}{\partial P_g} = 2diag(C_2)P_g + C_1 - E_g\lambda \tag{10}$$

$$\frac{\partial L}{\partial \theta} = \lambda A diag(b)A^T + A_{PL}diag(b_{PL})\mu \tag{11}$$

$$\frac{\partial L}{\partial \lambda} = Adiag(b)A^T\lambda - E_g^T P_g - P_D \tag{12}$$

$$\frac{\partial L}{\partial \mu} = diag(b_{PL})A_{PL}^T\theta - P_{PL} \tag{13}$$

Then we formulate this into a four by four matrix which we will call M.

$$\begin{bmatrix} 2diag(C_2) & 0 & -E_g & 0 \\ 0 & 0 & Adiag(b)A^T & A_{PL}diag(b_{PL}) \\ -E_g^T & Adiag(b)A^T & 0 & 0 \\ 0 & diag(b_{PL})A_{PL}^T & 0 & 0 \end{bmatrix} \begin{bmatrix} P_g \\ \theta \\ \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} -C_1 \\ 0 \\ P_D \\ P_{PL} \end{bmatrix} \tag{14}$$

Our task is to find lines and line limits that move the solution to minimize our objective described in (3), a weighted norm of dispatch and LMPs. To accomplish this we find it convenient to calculate sensitivities of dispatch, angle, prices, and line flows to values of the Lagrange multipliers, μ, associated with line constraints. We set up the following matrix, which we will call N.

$$\begin{bmatrix} 2diag(C_2) & 0 & -E_g & 0 \\ 0 & 0 & Adiag(b)A^T & 0 \\ -E_g^T & Adiag(b)A^T & 0 & 0 \\ 0 & diag(b)A^T & 0 & -I \end{bmatrix} \begin{bmatrix} \Delta P_g \\ \Delta\theta \\ \Delta\lambda \\ \Delta P_{branch} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -Adiag(b) \\ 0 \end{bmatrix} \Delta\mu \tag{15}$$

The solution of these equations is a matrix of sensitivities, each column corresponding to a different line. In (15), and in contrast to (14), we include all lines. Those lines that are not considered for proxy limits are eliminated by setting the corresponding value of $\Delta\mu = 0$. Let the sensitivity solution be described by

$$\begin{bmatrix} \Delta P_g \\ \Delta\theta \\ \Delta\lambda \\ \Delta P_{branch} \end{bmatrix} = \begin{bmatrix} & N & \end{bmatrix} \Delta\mu \qquad (16)$$

Because we have specified quadratic cost curves, (16) is an exact incremental model of the DCOPF. For specified values of $\Delta\mu$, the DCOPF solution is given by

$$\begin{bmatrix} \mathrm{P_{gDC}} \\ \theta_{\mathrm{DC}} \\ \lambda_{\mathrm{DC}} \\ \mathrm{P_{branch}} \end{bmatrix} = \begin{bmatrix} P_{g0} \\ \theta_0 \\ \lambda_0 \\ P_{branch0} \end{bmatrix} + \begin{bmatrix} & N & \end{bmatrix} \Delta\mu \qquad (17)$$

where the subscript ‘0’ is used to denote the solution to the unconstrained DCOPF.

In terms of $\Delta\mu$ it is straitforward to set up and solve a weighted least squares problem:

$$\min_{\substack{\Delta\mu_i \\ i\epsilon S_{PL}}} (\alpha|\lambda_{AC} - \lambda_{DC}| + \beta|P_{AC} - P_{DC}|) = \min_{\substack{\Delta\mu_i \\ i\epsilon S_{PL}}} (\alpha|\lambda_{AC} - \lambda_0 - \Delta\lambda| + \beta|P_{AC} - P_{g0} - \Delta P_g|) \quad (18)$$

Here a 2 norm is used to allow least squares analysis, and all variable quantities in (18) are functions of $\Delta\mu$ as described in (17).

3.2 Initial Results

In this section we apply the method described in the previous section to the task of assigning proxy limits for voltage constraints in a power system. We will first test our method with the 30-bus model with parameters and cost functions obtained in the Matpower distribution [2], an open source Matlab based power system solver developed at Cornell University. Aside from the ease of developing in Matlab, Matpower is useful because it enforces voltage limits.

Refer to Matpower distribution for the model parameters. The Appendix will show line and bus connections.

The 30-bus case provided in Matpower will be modified in two ways. First, the maximum voltage limit at bus 1 is decreased to 1.0 per unit. Second, the voltage limit at bus 19 was increased to 1.0 per unit. This situation creates two binding voltage constraints. It is important to note also that any lossy network solved in an optimal power flow will have at least one binding maximum voltage constraint since uniformly raising voltage will reduce losses. Since the Matpower 30 bus system is lossy, the losses in the DCOPF are considered by distributing them equally to loads at the terminals of the lines. This is a common technique [18].

The following tables provide results for the one-line, two-line, three-line, four-line, and five-line proxy limits. For each of these, we calculate the proxy limits for 11 different weightings, varying α between 0 and 1 in increments of 0.1 (correspondingly β varies between 1 and 0 in 0.1 increments). At the extremes of $\alpha = 0$, or $\beta = 0$, the optimal answers may not be unique, therefore we perturb them by 10^{-4} to seek a solution in line with the non-zero weighted answers. For example, there are many five-line solutions that match the ACOPF dispatch exactly (which will be discussed later). By perturbing the weights, we find a result in the direction of minimizing LMP errors.

Table 1 shows below the results for the best weighted solution for proxy limits for various values of α and β. The top row corresponds to a strongly LMP weighted solutions and the last row to a strongly weighted dispatch solution. The error described is the weighted 2-norm between the proxy limited solution and that of the original ACOPF.

In tables 2, 3, 4, and 5 similar results are discussed for their respective 2, 3, 4, and 5-line proxy limit solutions. Note that while identical lines may be listed for different weights, the errors may differ (compare rows 2-8 in Table 2). This is due to different values for line limits.

Table 3. Results for one-line proxy limits at bus 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.31	7.14	6
0.9	0.1	0.42	1.12	12
0.8	0.2	0.49	0.85	15
0.7	0.3	0.49	0.85	15
0.6	0.4	0.49	0.85	15
0.5	0.5	0.49	0.85	15
0.4	0.6	0.49	0.85	15
0.3	0.7	0.49	0.85	15
0.2	0.8	0.49	0.85	15
0.1	0.9	0.49	0.85	15
10^{-4}	1.0	0.49	0.85	15

Table 4. Results for two-line proxy limits at bus 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.25	6.15	6, 30
0.9	0.1	0.33	0.94	2, 35
0.8	0.2	0.34	0.69	2, 35
0.7	0.3	0.35	0.65	2, 35
0.6	0.4	0.35	0.65	2, 35
0.5	0.5	0.35	0.65	2, 35
0.4	0.6	0.35	0.65	2, 35
0.3	0.7	0.35	0.65	2, 35
0.2	0.8	0.47	0.60	1, 15
0.1	0.9	0.47	0.60	1, 15
10^{-4}	1.0	0.47	0.60	1, 15

Table 5. Results for three-lined proxy limits at bus 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.22	2.04	6, 22, 30
0.9	0.1	0.27	0.78	2, 22, 35
0.8	0.2	0.30	0.41	1, 6, 36
0.7	0.3	0.31	0.37	1, 6, 36
0.6	0.4	0.31	0.36	1, 6, 36
0.5	0.5	0.51	0.11	7, 14, 35
0.4	0.6	0.51	0.10	7, 14, 35
0.3	0.7	0.51	0.10	7, 14, 35
0.2	0.8	0.51	0.10	7, 14, 35
0.1	0.9	0.51	0.10	7, 14, 35
10^{-4}	1.0	0.51	0.10	7, 14, 35

Table 6. Results for four-lined proxy limits at bus 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.17	2.04	6, 25, 36, 38
0.9	0.1	0.20	0.73	2, 22, 35, 38
0.8	0.2	0.26	0.17	1, 3, 22, 36
0.7	0.3	0.29	0.04	1, 6, 22, 36
0.6	0.4	0.29	0.02	1, 6, 22, 36
0.5	0.5	0.30	0.01	1, 6, 22, 36
0.4	0.6	0.30	0.01	1, 6, 22, 36
0.3	0.7	0.30	0.004	1, 6, 22, 36
0.2	0.8	0.30	0.004	1, 6, 22, 36
0.1	0.9	0.30	0.004	1, 6, 22, 36
10^{-4}	1.0	0.30	0.0002	7, 16, 35, 41

Table 7. Results for five-lined proxy limits at bus 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.13	1.93	6, 25, 35, 36, 38
0.9	0.1	0.17	0.28	1, 3, 22, 36, 38
0.8	0.2	0.17	0.16	1, 3, 22, 36, 38
0.7	0.3	0.17	0.14	1, 3, 22, 36, 38
0.6	0.4	0.22	0.02	1, 6, 32, 36, 38
0.5	0.5	0.22	0.01	1, 6, 32, 36, 38
0.4	0.6	0.22	0.01	1, 6, 32, 36, 38
0.3	0.7	0.22	0.004	1, 6, 32, 36, 38
0.2	0.8	0.22	0.004	1, 6, 32, 36, 38
0.1	0.9	0.25	0.0002	1, 3, 22, 30, 35
10^{-4}	1.0	0.25	0.0000	1, 3, 22, 30, 35

Several features of these results to highlight.

- The lines that best match the dispatch are not generally the best lines to match prices. Compare the first and last rows in Tables 3-7. There is only one line in common (Table 7).
- A pattern of best lines is not apparent. It seems that some lines appear near generator buses and some appear near the constraint.

- Choosing the lines to match LMPs does improve dispatch as more lines are added, yet the error remains large. See the first rows of Tables 3-7.
- Choosing lines to match dispatch has no impact on bettering LMPs. The last rows in Tables 1-4 show no decrease in LMP error.
- Choosing five lines, there are enough degrees of freedom to exactly match the generator dispatch.

3.3 Voltage Constraint Matching

Another step in determining the proxy limited lines effectiveness in real world applications is to check whether the original voltage constraint has been met in the ACOPF. One way to do this is to set up the original ACOPF case without the voltage constraints. Instead, the voltage constraints will be converted to their proxy-limited lines found in the previous section. We set up cases to check if the voltage constraints have been met for 1, 2, and 3 proxy limited line solutions at values of $\alpha = 10^{-4}$, 0.5, and 1 (hence β = 1, 0.5, and 10^{-4}).

In this specific case we will be setting the voltage at bus 1 to 1.00 pu like before, but will be monitoring the voltage at bus 19 as it fluctuates with the proxy limited lines. If the voltage constraint cannot be met, the results of minimizing the LMP 2 norm and dispatch 2 norm do not make sense. Therefore, when the voltage constraint isn't met, the values of the row of the tables below are left blank. The results posed below are calculated by varying the limits on the respective lines to match the voltage constraint while minimizing the respective norms. The minimum of the LMP 2 norm ($\alpha = 1$) the weighted average of the LMP and dispatch 2 norm ($\alpha = .5$), and the dispatch 2 norm ($\alpha = 0$) are calculated and shown below. Note here that the norms will be different than above because we are comparing the ACOPF with the voltage constraint to the proxy limited ACOPF without the voltage constraint.

Table 8. Voltage constraint matching with one-line proxy limit

α	Line #	Voltage Constraint met?	Line MW Limit	Original MW Flow	LMP 2 Norm	Dispatch 2 Norm
1	6	Yes	19.3540	23.2543	0.6863	11.4679
.5	15	No	NA	NA	NA	NA
10^{-4}	15	No	NA	NA	NA	NA

Table 9. Voltage constraint matching with two-line proxy limits

α	Line #	VC met?	Line 1 MW Limit	Original 1 MW Flow	Line 2 MW Limit	Original 2 MW Flow	LMP 2 Norm	Dispatch 2 Norm
1	6, 30	Yes	19.7214	23.2543	11.1071	10.2041	0.6892	10.5248
.5	2, 35	Yes	22.2321	21.4480	5.5429	11.4533	6.0034	11.0369
10^{-4}	1, 15	Yes	10.6967	22.1578	20.1875	11.2697	1.3246	18.8658

Table 10. Voltage constraint matching with three-line proxy limits

α	Line #	VC met?	Line 1 MW Limit	Original 1 MW Flow	Line 2 MW Limit	Original 2 MW Flow	Line 3 MW Limit	Original 3 MW Flow	LMP 2 Norm	Dispatch 2 Norm
1	6, 22, 30	Yes	19.6875	23.2543	7.7500	7.3895	11.1563	10.2041	0.6918	10.6157
.5	7, 14, 35	Yes	21.2188	19.5380	7.6250	7.8747	6.0500	11.4533	5.3602	9.8060
10^{-4}	7, 14, 35	Yes	21.2083	19.5380	7.6667	7.8747	6.0667	11.4533	0.9117	9.7287

Several features of these results to highlight.

- In all cases, when the proxy-limited lines for LMP matching were chosen, the voltage constraint was met.
- After two lines for the α = .5 and 10^{-4} cases the voltage constraint was met, but could not be met with only one line.
- As lines were increased, the LMP 2 norm and dispatch 2 norm didn't necessarily reduce. This is due to the fact that the switch from DC to ACOPF doesn't translate well.

- In the two line case the dispatch doesn’t necessarily get better as alpha decreases. This could also be due to limiting the line more to achieve the voltage constraint.

4. Multiple Constraints

Another result that can be tested is whether or not patterns arise during multiple constraints. Buses with minimum voltage constraints in the same vicinity will be tested, as well as then testing them both together. This is to test whether or not the results are additive.

4.1 Voltage Constraint at bus 17

The first scenario will contain the original 30 bus system with a new voltage constraint of 1.0 pu at bus 17, while the voltage constraint at 19 has been removed. The original binding constraint at bus 1 of 1.00 pu will remain. This results in a binding constraint at bus 17. It is also important to note that bus 17 and bus 19 are not connected (shortest path is through three lines), therefore a specific proxy limited constraint might show up for one and not the other. However, bus 19 and bus 17 are in the same area, so if the results might be additive, similar lines should show up for both cases. The 30 bus case for a minimum voltage constraint at bus 17 is shown below following a similar format of that in section 3. Table 11 shows below the results for the best weighted solution for proxy limits for various values of α and β. The top row corresponds to a strongly LMP weighted solutions and the last row to a strongly weighted dispatch solution. The error described is the weighted 2-norm between the proxy limited solution and that of the original ACOPF. In tables 12, 13, 14, and 15 similar results are discussed for their respective 2, 3, 4, and 5-line proxy limit solutions.

Table 11. Results for one-line proxy limit at bus 17

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.31	7.15	6
0.9	0.1	0.43	1.13	12
0.8	0.2	0.49	0.84	15
0.7	0.3	0.49	0.84	15
0.6	0.4	0.49	0.84	15
0.5	0.5	0.49	0.84	15
0.4	0.6	0.49	0.84	15
0.3	0.7	0.49	0.84	15
0.2	0.8	0.49	0.84	15
0.1	0.9	0.49	0.84	15
10^{-4}	1.0	0.49	0.84	15

Table 12. Results for two-line proxy limit at bus 17

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.25	6.16	6, 30
0.9	0.1	0.33	0.96	2, 35
0.8	0.2	0.34	0.70	2, 35
0.7	0.3	0.35	0.67	2, 35
0.6	0.4	0.35	0.66	2, 35
0.5	0.5	0.35	0.66	2, 35
0.4	0.6	0.35	0.66	2, 35
0.3	0.7	0.47	0.60	1, 15
0.2	0.8	0.47	0.60	1, 15
0.1	0.9	0.47	0.60	1, 15
10^{-4}	1.0	0.47	0.60	1, 15

Table 13. Results for three-line proxy limit at bus 17

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.22	2.08	6, 36, 38
0.9	0.1	0.27	0.79	2, 22, 35
0.8	0.2	0.31	0.43	1, 6, 36
0.7	0.3	0.31	0.39	1, 6, 36
0.6	0.4	0.31	0.38	1, 6, 36
0.5	0.5	0.50	0.11	7, 16, 35
0.4	0.6	0.51	0.10	7, 16, 35
0.3	0.7	0.51	0.10	7, 16, 35
0.2	0.8	0.51	0.10	7, 16, 35
0.1	0.9	0.51	0.10	7, 16, 35
10^{-4}	1.0	0.51	0.10	7, 16, 35

Table 14. Results for four-line proxy limit at bus 17

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.17	2.04	6, 25, 36, 38
0.9	0.1	0.20	0.73	2, 22, 35, 38
0.8	0.2	0.26	0.17	1, 3, 22, 36
0.7	0.3	0.29	0.04	1, 6, 32, 36
0.6	0.4	0.29	0.02	1, 6, 32, 36
0.5	0.5	0.30	0.01	1, 6, 32, 36
0.4	0.6	0.30	0.01	1, 6, 32, 36
0.3	0.7	0.30	0.004	1, 6, 32, 36
0.2	0.8	0.30	0.004	1, 6, 32, 36
0.1	0.9	0.32	0.004	2, 6, 31, 36
10^{-4}	1.0	0.73	0.0002	7, 18, 21, 41

Table 15. Results for five-line proxy limit at bus 17

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.13	1.93	6, 25, 35, 36, 38
0.9	0.1	0.17	0.29	1, 3, 22, 36, 38
0.8	0.2	0.17	0.17	1, 3, 22, 36, 38
0.7	0.3	0.17	0.04	1, 6, 32, 36, 38
0.6	0.4	0.22	0.02	1, 6, 32, 36, 38
0.5	0.5	0.22	0.01	1, 6, 32, 36, 38
0.4	0.6	0.22	0.01	1, 6, 32, 36, 38
0.3	0.7	0.22	0.01	1, 6, 32, 36, 38
0.2	0.8	0.22	0.001	1, 3, 22, 30, 35
0.1	0.9	0.25	0.0002	1, 3, 22, 30, 35
10^{-4}	1.0	0.25	0.0000	1, 3, 22, 30, 35

Several features of these results to highlight.

- The lines that best match the LMPs in the bus 17 constraint are very similar to that when matched to a voltage constraint at bus 19. Only the three-lined proxy limited case had two of the three lines different (Table 13).
- The lines that best match dispatch were also very similar. In the three-lined proxy limited case one of the three lines was different, and in the four-lined case two of the four lines were different. See the last row of Tables 13 and 14.
- Again, by choosing five lines, there are enough degrees of freedom to exactly match the generator dispatch. See last row of Table 15.

4.2 Combining Voltage Constraints at bus 17 and 19

Next, both binding voltage constraints will be inserted together into the same scenario. Both bus 17 and 19, along with bus 1, will be fixed at 1.0 pu while other voltage constraints that may show up will be relaxed. It is important to note here that in sections 3.2 and 4.1 the voltages at bus 17 and 19 were being raised to 1.0 pu respectively. This resulted in a binding constraint for each that was a lower bound. In the situation where both voltages are now fixed at 1.0 pu, the constraint at one bus could effectively raise the voltage at the other bus above 1.0 pu, therefore the binding constraint could be an upper bound. In this particular situation, creating a lower bound of 1.0 pu at bus 19 effectively solves bus 17 voltage to be 1.008 pu. Solving the case where both voltage constraints will be fixed at 1.0 pu will create a lower bound on bus 19, while creating an upper bound on bus 17.

The 30 bus case for a binding voltage constraint at both bus 17 and 19 is shown below. Table 16 shows below the results for the best weighted solution for proxy limits for various values of α and β. The top row corresponds to a strongly LMP weighted solutions and the last row to a strongly weighted dispatch solution. The error described is the weighted 2-norm between the proxy limited solution and that of the original ACOPF. In tables 17, 18, 19, and 20 similar results are discussed for their respective 2, 3, 4, and 5-line proxy limit solutions.

Table 16. Results for one-line proxy limit at bus 17 and 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.50	1.91	25
0.9	0.1	0.50	1.90	25
0.8	0.2	0.50	1.89	25
0.7	0.3	0.85	1.05	15
0.6	0.4	0.85	1.05	15
0.5	0.5	0.85	1.05	15
0.4	0.6	0.85	1.05	15
0.3	0.7	0.85	1.05	15
0.2	0.8	0.85	1.05	15
0.1	0.9	0.85	1.05	15
10^{-4}	1.0	0.85	1.05	15

Table 17. Results for two-line proxy limit at bus 17 and 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.32	6.06	6, 25
0.9	0.1	0.43	0.81	11, 25
0.8	0.2	0.43	0.74	11, 25
0.7	0.3	0.44	0.68	11, 25
0.6	0.4	0.45	0.65	11, 25
0.5	0.5	0.46	0.64	11, 25
0.4	0.6	0.47	0.63	11, 25
0.3	0.7	0.47	0.63	11, 25
0.2	0.8	0.47	0.63	11, 25
0.1	0.9	0.48	0.63	11, 25
10^{-4}	1.0	0.48	0.63	11, 25

Table 18. Results for three-line proxy limit at bus 17 and 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.27	5.57	6, 22, 25
0.9	0.1	0.38	0.82	11, 25, 35
0.8	0.2	0.42	0.46	1, 11, 25
0.7	0.3	0.42	0.39	1, 11, 25
0.6	0.4	0.43	0.35	1, 11, 25
0.5	0.5	0.44	0.33	1, 11, 25
0.4	0.6	0.45	0.33	1, 11, 25
0.3	0.7	0.76	0.15	1, 3, 35
0.2	0.8	0.76	0.15	1, 3, 35
0.1	0.9	0.76	0.15	1, 3, 35
10^{-4}	1.0	0.76	0.15	1, 3, 35

Table 19. Results for four-line proxy limit at bus 17 and 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.26	2.24	6, 25, 35, 38
0.9	0.1	0.29	0.58	2, 22, 25, 35
0.8	0.2	0.30	0.52	2, 22, 25, 35
0.7	0.3	0.30	0.51	2, 22, 25, 35
0.6	0.4	0.45	0.13	1, 12, 25, 31
0.5	0.5	0.46	0.08	1, 12, 25, 31
0.4	0.6	0.46	0.05	1, 12, 25, 31
0.3	0.7	0.46	0.04	1, 12, 25, 31
0.2	0.8	0.46	0.04	1, 12, 25, 31
0.1	0.9	0.60	0.01	7, 16, 25, 35
10^{-4}	1.0	0.81	0.0008	2, 7, 11, 32

Table 20. Results for five-line proxy limit at bus 17 and 19

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.21	1.53	6, 22, 25, 35, 38
0.9	0.1	0.22	0.54	2, 22, 25, 35, 38
0.8	0.2	0.29	0.14	1, 6, 22, 25, 35
0.7	0.3	0.29	0.08	2, 6, 22, 25, 35
0.6	0.4	0.29	0.05	2, 6, 22, 25, 35
0.5	0.5	0.29	0.03	2, 6, 22, 25, 35
0.4	0.6	0.29	0.02	2, 6, 22, 25, 35
0.3	0.7	0.29	0.02	2, 6, 22, 25, 35
0.2	0.8	0.36	0.0007	1, 3, 18, 25, 31
0.1	0.9	0.36	0.0001	1, 3, 18, 25, 31
10^{-4}	1.0	0.36	0.0000	1, 3, 18, 25, 31

Some features of these results to highlight.

- The proxy limited lines that were best matched to LMPs had in each case exactly one line included that was not found in either the bus 17 or bus 19 LMP cases. This may conclude that although the results are not strictly additive for matching LMPs, it follows a similar pattern to match LMPs.

- The proxy limited lines that best matched dispatch resulted differently. In the two and three line cases, there were each two lines that were not found in the bus 17 or 19 dispatch matching cases. Likewise, in the four and five line dispatch matching cases, there were three lines that were not found in the separate bus 17 or 19 voltage constraint cases.

5. Selecting a subset of lines

In the next section ideas are discussed on whether a reasonable proxy limit approximation for a voltage constraint can be found from a subset of lines. Three possible scenarios will be discussed. The first will be choosing lines that are close to the voltage constraint. For this approach, the only lines that can be selected will be lines that are within two buses or three line lengths away from the original constraint. The second scenario will be selecting lines only connected to a generator. Finally, the third approach will be to simulate randomness. Lines that can be selected in this approach will be every third line starting with line number one. In each of these three scenarios, we would like to determine if a method of selecting from a subset of lines stands out from the others, and if so how closely can it match the optimal solution from all lines.

5.1 Selecting lines close to constraint

First, a selection of lines within three line lengths of bus 19 will be calculated. The list of lines that can be selected is shown below.

Lines = 12, 14, 18, 20, 22, 23, 24, 25, 26, 27, 28, 30

The setup and calculation for selecting the best lines out of these 12 will be the same as in chapter 3. Table 21 below shows the best line selected for various α and β. The top row corresponds to the best LMP matching, while the bottom indicates a dispatch matching set. Tables 22 through 25 indicate the same details for their respective two, three, four, and five line combinations.

Table 21. Results for one-line proxy limit close to constraint

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.42	1.12	12
0.9	0.1	0.42	1.12	12
0.8	0.2	0.45	1.12	12
0.7	0.3	0.45	1.12	12
0.6	0.4	0.45	1.12	12
0.5	0.5	0.45	1.12	12
0.4	0.6	0.45	1.12	12
0.3	0.7	0.45	1.12	12
0.2	0.8	0.45	1.12	12
0.1	0.9	0.45	1.12	12
10^{-4}	1.0	0.45	1.12	12

Table 22. Results for two-line proxy limit close to constraint

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.36	2.53	12, 30
0.9	0.1	0.39	0.86	12, 28
0.8	0.2	0.40	0.80	12, 28
0.7	0.3	0.40	0.79	12, 28
0.6	0.4	0.40	0.79	12, 28
0.5	0.5	0.45	0.72	12, 26
0.4	0.6	0.46	0.72	12, 26
0.3	0.7	0.46	0.72	12, 26
0.2	0.8	0.73	0.64	26, 27
0.1	0.9	0.73	0.64	26, 27
10^{-4}	1.0	0.73	0.64	26, 27

Table 23. Results for three-line proxy limit close to constraint

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.34	2.74	12, 22, 30
0.9	0.1	0.37	0.90	12, 25, 30
0.8	0.2	0.39	0.66	12, 28, 30
0.7	0.3	0.39	0.64	12, 28, 30
0.6	0.4	0.39	0.64	12, 28, 30
0.5	0.5	0.39	0.64	12, 28, 30
0.4	0.6	0.39	0.64	12, 28, 30
0.3	0.7	0.39	0.64	12, 28, 30
0.2	0.8	0.39	0.64	12, 28, 30
0.1	0.9	0.39	0.64	12, 28, 30
10^{-4}	1.0	0.39	0.64	12, 28, 30

Table 24. Results for four-line proxy limit close to constraint

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.33	2.54	12, 18, 22, 30
0.9	0.1	0.37	0.85	12, 25, 28, 30
0.8	0.2	0.38	0.66	12, 25, 28, 30
0.7	0.3	0.38	0.64	12, 25, 28, 30
0.6	0.4	0.38	0.64	12, 25, 28, 30
0.5	0.5	0.38	0.64	12, 25, 28, 30
0.4	0.6	0.38	0.64	12, 25, 28, 30
0.3	0.7	0.38	0.64	12, 25, 28, 30
0.2	0.8	0.38	0.64	12, 25, 28, 30
0.1	0.9	0.38	0.64	12, 25, 28, 30
10^{-4}	1.0	0.38	0.64	12, 25, 28, 30

Table 25. Results for five-line proxy limit close to constraint

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.33	2.75	12, 18, 22, 23, 30
0.9	0.1	0.36	0.86	12, 14, 25, 28, 30
0.8	0.2	0.37	0.66	12, 14, 25, 28, 30
0.7	0.3	0.38	0.64	12, 14, 25, 28, 30
0.6	0.4	0.38	0.64	12, 14, 25, 28, 30
0.5	0.5	0.38	0.64	12, 14, 25, 28, 30
0.4	0.6	0.38	0.64	12, 14, 25, 28, 30
0.3	0.7	0.38	0.64	12, 14, 25, 28, 30
0.2	0.8	0.38	0.64	12, 14, 25, 28, 30
0.1	0.9	0.38	0.64	12, 14, 25, 28, 30
10^{-4}	1.0	0.38	0.64	12, 14, 25, 28, 30

Some features to note:

- Even with five lines, dispatch could not match (Last row Table 25). This is due to the fact that the combination of lines chosen were not optimal. In fact, dispatch did not better itself at all after two lines.
- The lines that best chose LMP started fairly decent with line number 12, but after the two lines of 12 and 30, the error hardly decreased. See the first row of Tables 21-25.

5.2 Selecting lines close to generators

The next scenario tested is that of selecting lines connected to generators. The lines for the 30 bus case that could be chosen are shown below.

Lines = 1, 2, 3, 5, 6, 16, 28, 29, 30, 31, 32, 35, 36, 37, 38

Again, the setup and calculation for selecting the best lines out of these 15 will be the same as in chapter 3. Table 26 below shows the best line selected for various α and β. The top row corresponds to the best LMP matching, while the bottom indicates a dispatch matching set.

Tables 27 through 30 indicate the same details for their respective two, three, four, and five line combinations.

Table 26. Results for one-line proxy limit close to generators

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.31	7.14	6
0.9	0.1	0.41	2.24	2
0.8	0.2	0.44	1.90	2
0.7	0.3	0.45	1.88	2
0.6	0.4	0.45	1.88	2
0.5	0.5	0.46	1.88	2
0.4	0.6	0.46	1.88	2
0.3	0.7	0.46	1.88	2
0.2	0.8	0.46	1.88	2
0.1	0.9	0.46	1.88	2
10^{-4}	1.0	0.46	1.88	2

Table 27. Results for two-line proxy limit close to generators

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.25	6.15	6, 30
0.9	0.1	0.33	0.94	2, 35
0.8	0.2	0.34	0.69	2, 35
0.7	0.3	0.35	0.65	2, 35
0.6	0.4	0.35	0.65	2, 35
0.5	0.5	0.35	0.65	2, 35
0.4	0.6	0.35	0.65	2, 35
0.3	0.7	0.35	0.65	2, 35
0.2	0.8	0.35	0.65	2, 35
0.1	0.9	0.35	0.65	2, 35
10^{-4}	1.0	0.35	0.65	2, 35

Table 28. Results for three-line proxy limit close to generators

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.22	2.08	6, 36, 38
0.9	0.1	0.26	0.95	2, 35, 38
0.8	0.2	0.30	0.41	1, 6, 36
0.7	0.3	0.31	0.37	1, 6, 36
0.6	0.4	0.31	0.37	1, 6, 36
0.5	0.5	0.31	0.36	1, 6, 36
0.4	0.6	0.33	0.36	2, 6, 36
0.3	0.7	0.33	0.35	2, 6, 36
0.2	0.8	0.33	0.35	2, 6, 36
0.1	0.9	0.33	0.35	2, 6, 36
10^{-4}	1.0	0.33	0.35	2, 6, 36

Table 29. Results for four-line proxy limit close to generators

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.18	2.03	6, 30, 35, 38
0.9	0.1	0.22	0.64	1, 6, 36, 38
0.8	0.2	0.29	0.10	1, 6, 32, 36
0.7	0.3	0.29	0.04	1, 6, 32, 36
0.6	0.4	0.29	0.02	1, 6, 32, 36
0.5	0.5	0.30	0.01	1, 6, 32, 36
0.4	0.6	0.30	0.01	1, 6, 32, 36
0.3	0.7	0.30	0.004	1, 6, 32, 36
0.2	0.8	0.30	0.004	1, 6, 32, 36
0.1	0.9	0.30	0.004	1, 6, 32, 36
10^{-4}	1.0	0.30	0.004	1, 6, 32, 36

Table 30. Results for five-line proxy limit close to generators

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.16	1.39	6, 30, 35, 36, 38
0.9	0.1	0.22	0.23	1, 6, 32, 36, 38
0.8	0.2	0.22	0.08	1, 6, 32, 36, 38
0.7	0.3	0.22	0.04	1, 6, 32, 36, 38
0.6	0.4	0.22	0.02	1, 6, 32, 36, 38
0.5	0.5	0.22	0.009	1, 6, 32, 36, 38
0.4	0.6	0.22	0.006	1, 6, 32, 36, 38
0.3	0.7	0.22	0.004	1, 6, 32, 36, 38
0.2	0.8	0.22	0.004	1, 6, 32, 36, 38
0.1	0.9	0.22	0.004	1, 6, 32, 36, 38
10^{-4}	1.0	0.27	0.000	1, 3, 16, 30, 35

Some features to note:

- With five lines, the dispatch error could be completely minimized like that of the optimal solution with all lines (Last row Table 30).
- The five line solution for matching LMPs is more than twice as good as that when selecting lines closer to the constraint. Compare first row of Table 25 to first row of Table 30.
- Improvement for both LMPs and dispatch was consistent after adding another line in each case. See first and last rows of Tables 26-30.

5.3 Selecting random subset of lines

The final test is to emulate that of a random reduction of line selection. Every third line in the 30 bus 41 line case will be chosen and are shown below.

Lines = 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40

Again, the setup and calculation for selecting the best lines out of these 14 will be the same as in chapter 3. Table 31 below shows the best line selected for various α and β. The top row corresponds to the best LMP matching, while the bottom indicates a dispatch matching set. Tables 32 through 35 indicate the same details for their respective two, three, four, and five line combinations.

Table 31. Results for random subset of one-line proxy limit

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.45	3.30	22
0.9	0.1	0.49	1.89	4
0.8	0.2	0.49	1.88	4
0.7	0.3	0.49	1.88	4
0.6	0.4	0.49	1.88	4
0.5	0.5	0.49	1.88	4
0.4	0.6	0.49	1.88	4
0.3	0.7	0.58	1.83	25
0.2	0.8	0.58	1.83	25
0.1	0.9	0.59	1.83	25
10^{-4}	1.0	0.59	1.83	25

Table 32. Results for random subset of two-line proxy limit

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.39	5.04	1, 22
0.9	0.1	0.46	0.89	4, 40
0.8	0.2	0.47	0.76	4, 40
0.7	0.3	0.47	0.73	4, 40
0.6	0.4	0.47	0.73	4, 40
0.5	0.5	0.47	0.73	4, 40
0.4	0.6	0.47	0.73	4, 40
0.3	0.7	0.47	0.73	4, 40
0.2	0.8	0.54	0.71	4, 7
0.1	0.9	0.89	0.64	19, 28
10^{-4}	1.0	0.89	0.64	19, 28

Table 33. Results for random subset of three-line proxy limit

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.36	4.54	1, 19, 25
0.9	0.1	0.41	0.89	4, 25, 31
0.8	0.2	0.42	0.68	4, 25, 31
0.7	0.3	0.49	0.41	1, 4, 40
0.6	0.4	0.50	0.37	1, 4, 40
0.5	0.5	0.50	0.37	1, 4, 40
0.4	0.6	0.50	0.36	1, 4, 40
0.3	0.7	0.59	0.31	4, 7, 28
0.2	0.8	0.59	0.31	4, 7, 28
0.1	0.9	0.87	0.27	1, 19, 28
10^{-4}	1.0	0.87	0.27	1, 19, 28

Table 34. Results for random subset of four-line proxy limit

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.34	4.14	1, 22, 37, 40
0.9	0.1	0.37	0.93	4, 25, 31, 37
0.8	0.2	0.39	0.68	4, 25, 31, 37
0.7	0.3	0.49	0.22	1, 4, 31, 40
0.6	0.4	0.50	0.13	1, 4, 31, 40
0.5	0.5	0.50	0.10	1, 4, 31, 40
0.4	0.6	0.50	0.09	1, 4, 31, 40
0.3	0.7	0.60	0.03	4, 7, 16, 28
0.2	0.8	0.60	0.02	4, 7, 16, 28
0.1	0.9	0.61	0.02	4, 7, 16, 28
10^{-4}	1.0	1.53	0.01	7, 19, 25, 31

Table 35. Results for random subset of five-line proxy limit

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.31	3.71	1, 19, 25, 37, 40
0.9	0.1	0.35	0.98	4, 25, 31, 34, 37
0.8	0.2	0.42	0.42	4, 25, 28, 31, 40
0.7	0.3	0.44	0.20	4, 22, 25, 28, 31
0.6	0.4	0.45	0.14	4, 22, 25, 28, 31
0.5	0.5	0.50	0.04	1, 4, 28, 31, 40
0.4	0.6	0.50	0.02	1, 4, 28, 31, 40
0.3	0.7	0.48	0.02	4, 16, 25, 28, 31
0.2	0.8	0.48	0.01	4, 16, 25, 28, 31
0.1	0.9	0.48	0.001	4, 16, 25, 28, 31
10^{-4}	1.0	0.48	0.000	4, 16, 25, 28, 31

Some features to note:

- The lines that best match the dispatch could effectively match after five lines were chosen (Last row Table 35).
- The lines that were chosen to match up best with the LMPs did decrease with each introduced line, but not nearly as fast or effective as chosen from all lines. See first row of Tables 31-35.

5.4 Results of Line Selection

Some very important conclusions can be made regarding the selection of lines. First of all, it must be noted that in the case where all lines could be selected the number of available lines to choose from is 41. In the case of choosing lines near the constraint, the number is 12. The number for lines connected to generators is 15, and the number when choosing every third line is 14. This is important because when deciding on what subset of lines to choose from, the more lines, the more degrees of freedom available. Therefore, the number of lines available in each of the three subsets was tried to keep as close to the others as possible.

Table 36 below highlights how well a single line matched the LMPs. Tables 37 and 38 follow the same pattern, but with three and five proxy limited lines total. The error shown remains the 2 norm error. Each of the cases are trying to match the LMPs related to the voltage constraint at bus 19.

Table 36. Comparing single-line proxy limits – LMP matching

Subset Taken	LMP Error	Proxy Lines
All	0.31	6
Close to Constraint	0.42	12
Connect to Generators	0.31	6
Every Third Line	0.45	22

Table 37. Comparing three-line proxy limits – LMP matching

Subset Taken	LMP Error	Proxy Lines
All	0.22	6, 22, 30
Close to Constraint	0.34	12, 22, 30
Connect to Generators	0.22	6, 36, 38
Every Third Line	0.36	1, 19, 25

Table 38. Comparing five-line proxy limits – LMP matching

Subset Taken	LMP Error	Proxy Lines
All	0.13	6, 25, 35, 36, 38
Close to Constraint	0.33	12, 18, 22, 23, 30
Connect to Generators	0.16	6, 30, 35, 36, 38
Every Third Line	0.31	1, 19, 25, 37, 40

It can be seen from these results that picking lines close to the actual constraint to match LMPs is no better than reducing the entire set in a uniform fashion (in this specific case it's actually worse). When the lines that could be selected are chosen near generators, the results are nearly that of choosing the best overall lines from the entire set. In the three line proxy limited case, choosing lines near generators actually produced a result equal (when rounded) to that in the optimal solution. In the five line proxy limited case, the LMP error was within 24% of the optimal solution, while bettering itself than the every third line approach by over 51%.

Next, the results for dispatch matching are shown. Table 39 below highlights how well a single line matched the dispatch. Tables 40 and 41 follow the same pattern, but with three and five proxy limited lines total. The error shown remains the 2 norm error. Each of the cases are still trying to match the dispatch related to the voltage constraint at bus 19.

Table 39. Comparing single-line proxy limits – dispatch matching

Subset Taken	Dispatch Error	Proxy Lines
All	0.85	15
Close to Constraint	1.12	12
Connect to Generators	1.88	2
Every Third Line	1.83	25

Table 40. Comparing three-line proxy limits – dispatch matching

Subset Taken	Dispatch Error	Proxy Lines
All	0.10	7, 14, 35
Close to Constraint	0.64	12, 28, 30
Connect to Generators	0.35	2, 6, 36
Every Third Line	0.27	1, 19, 28

Table 41. Comparing five-line proxy limits – dispatch matching

Subset Taken	Dispatch Error	Proxy Lines
All	0.000	1, 3, 22, 30, 35
Close to Constraint	0.64	12, 14, 25, 28, 30
Connect to Generators	0.000	1, 3, 16, 30, 35
Every Third Line	0.000	4, 16, 25, 28, 31

There are several interesting results from these findings. Initially choosing the line closest to the constraint resulted in the lowest dispatch error for a single line (1.12). While it would normally take four lines to completely eliminate dispatch error, when choosing lines close to the constraint, the dispatch error cannot be minimized. However, choosing lines randomly (every third line) or choosing lines close to the generators results in error that can be minimized. In this particular situation, choosing the random subset of lines as opposed to choosing lines close to the generators actually results in a lower dispatch error for both one (1.83 to 1.88) and three (0.27 to 0.35) chosen lines.

Overall, it seems that choosing lines closest to the generators would be the best overall fit for both LMP and dispatch matching. After five lines, the dispatch is completely matched, while in the LMP case the error minimization is nearly that of choosing from all lines.

6. Selecting a subset of lines

In this section, other sized bus systems will be tested. A modified 14 bus case and a modified 118 bus case from the Matpower distribution will be tested for how well four lines will match the LMPs, dispatch, and a combination of both [2]. Also noted in this section will be the time it takes to run the various simulations, so that a better handle on how the systems could scale up or down would be achieved.

6.1 118 Bus Case

The first case tested will be the modified 118 bus, 186 line case from Matpower [2]. The changes made to the case includes setting a maximum voltage of 1.00 pu at bus 17 while all other bus voltages will solve at however the optimal solution wishes. It is important to note that some buses will solve at their maximum limit to reduce losses. All line flows and generator limits are not binding. Also, only generators at buses 10, 12, 25, 26, 31, 46, 49, 54, 59, 61, 65, 66, 69, 80, 87, 89, 100, 103, and 111 will be turned on. A more detailed description of the model is included in the appendix.

The testing will be conducted like that of section three to find the best one, two, three, and four lines to match LMPs, dispatch, and different weighted combinations of both. Results only show up to four line combinations due to the computational time required to solve for the optimal lines. These results will prove to be useful for a system that is nearly four times the size of the 30 bus system in buses and 4.5 times in lines. Table 42 below shows the best line selected for various α and β. The top row corresponds to the best LMP matching, while the bottom indicates a dispatch matching set. Tables 43 through 45 indicate the same details for their respective two, three, and four line combinations.

Table 42. Results for one-line proxy limit for 118 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	18.90	351.59	1
0.9	0.1	20.97	81.10	141
0.8	0.2	21.44	75.60	141
0.7	0.3	25.24	61.04	8
0.6	0.4	25.28	60.98	8
0.5	0.5	26.68	59.14	4
0.4	0.6	26.69	59.13	4
0.3	0.7	26.70	59.13	4
0.2	0.8	26.71	59.13	4
0.1	0.9	26.71	59.13	4
10^{-4}	1.0	26.71	59.13	4

Table 43. Results for two-line proxy limit for 118 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	16.03	311.33	1, 154
0.9	0.1	19.19	53.33	36, 154
0.8	0.2	22.38	29.28	8, 154
0.7	0.3	22.64	27.13	8, 154
0.6	0.4	22.74	26.83	8, 154
0.5	0.5	22.78	26.77	8, 154
0.4	0.6	22.80	26.76	8, 154
0.3	0.7	24.54	25.90	12, 154
0.2	0.8	24.56	25.89	12, 154
0.1	0.9	24.56	25.89	12, 154
10^{-4}	1.0	24.56	25.89	12, 154

Table 44. Results for three-line proxy limit for 118 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	13.62	251.69	1, 98, 154
0.9	0.1	16.06	53.40	36, 51, 154
0.8	0.2	18.47	27.03	8, 51, 154
0.7	0.3	18.64	25.58	8, 51, 154
0.6	0.4	22.00	19.77	8, 105, 154
0.5	0.5	22.05	19.67	8, 105, 154
0.4	0.6	22.08	19.65	8, 105, 154
0.3	0.7	22.09	19.65	8, 105, 154
0.2	0.8	22.10	19.65	8, 105, 154
0.1	0.9	22.10	19.65	8, 105, 154
10^{-4}	1.0	24.08	19.47	12, 94, 154

Table 45. Results for four-line proxy limit for 118 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	12.10	207.14	1, 66, 98, 164
0.9	0.1	16.10	32.80	8, 51, 141, 164
0.8	0.2	16.42	25.05	8, 51, 141, 164
0.7	0.3	16.52	24.40	8, 51, 141, 164
0.6	0.4	20.87	16.12	8, 98, 106, 154
0.5	0.5	22.26	14.15	8, 91, 105, 154
0.4	0.6	22.31	14.09	8, 91, 105, 154
0.3	0.7	22.49	14.00	8, 91, 106, 154
0.2	0.8	22.50	13.99	8, 91, 106, 154
0.1	0.9	22.51	13.99	8, 91, 106, 154
10^{-4}	1.0	22.51	13.99	8, 91, 106, 154

Results to note:

- Dispatch did not even come close with four lines. It did reduce however, from 59.99 in the single line case to 13.99 in the four line case, which is a reduction of 4.23 times. (See last row of Table 42 and Table 45). This is due to the fact that there are much more than five generators in this large system.
- Although the LMP error for one line was higher (18.90) than the 30 bus case (First row Table 42), it was able to reduce to 12.10 using four lines (First row Table 46). This results in a reduction of 1.56 times. In the 30 bus case, the LMP error reduction from one to five lines was only 2.38.

6.2 14 Bus Case

The final case tested will be the 14 bus, 20 line case from Matpower [2]. In this system, bus 1 will be set to 1.00 pu while bus 11 will have an upper limit of 1.00 pu at which it will be binding. All other line flows and generator limits are not be binding. This effectively acts like an upper limit voltage constraint at bus 11.

The testing will be conducted like that of section three to find the best one, two, three, four, and five lines to match LMPs, dispatch, and different weighted combinations of both. The results will prove to be useful for a system that is nearly half the size of the 30 bus system in both buses and lines. Table 46 below shows the best line selected for various α and β. The top row corresponds to the best LMP matching, while the bottom indicates a dispatch matching set. Tables 47 through 49 indicate the same details for their respective two, three, four, and five line combinations.

Table 46. Results for one-line proxy limit for 14 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	2.27	64.70	2
0.9	0.1	3.26	8.04	1
0.8	0.2	3.30	7.28	1
0.7	0.3	3.31	7.23	1
0.6	0.4	3.31	7.23	1
0.5	0.5	3.31	7.22	1
0.4	0.6	3.32	7.22	1
0.3	0.7	3.32	7.22	1
0.2	0.8	3.32	7.22	1
0.1	0.9	3.32	7.22	1
10^{-4}	1.0	3.32	7.22	1

Table 47. Results for two-line proxy limit for 14 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	1.59	21.15	2, 3
0.9	0.1	1.67	12.77	2, 3
0.8	0.2	3.04	6.68	1, 4
0.7	0.3	3.11	6.31	1, 4
0.6	0.4	3.14	6.27	1, 4
0.5	0.5	3.15	6.26	1, 4
0.4	0.6	3.15	6.26	1, 4
0.3	0.7	3.15	6.26	1, 4
0.2	0.8	3.15	6.26	1, 4
0.1	0.9	3.15	6.26	1, 4
10^{-4}	1.0	3.15	6.26	1, 4

Table 48. Results for three-line proxy limit for 14 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	1.19	5.46	2, 3, 13
0.9	0.1	1.22	3.00	2, 3, 13
0.8	0.2	1.50	0.39	2, 6, 7
0.7	0.3	1.51	0.21	2, 6, 7
0.6	0.4	1.51	0.17	2, 6, 7
0.5	0.5	1.51	0.16	2, 6, 7
0.4	0.6	1.51	0.16	2, 6, 7
0.3	0.7	1.51	0.16	2, 6, 7
0.2	0.8	1.51	0.16	2, 6, 7
0.1	0.9	2.22	0.03	2, 6, 14
10^{-4}	1.0	2.22	0.03	2, 6, 14

Table 49. Results for four-line proxy limit for 14 bus case

α	β	LMP Error	Dispatch Error	Proxy Lines
1.0	10^{-4}	0.66	12.18	2, 6, 7, 17
0.9	0.1	0.80	1.68	2, 6, 7, 17
0.8	0.2	0.83	0.40	2, 6, 7, 17
0.7	0.3	0.84	0.15	2, 6, 7, 17
0.6	0.4	0.84	0.09	2, 6, 7, 17
0.5	0.5	0.84	0.07	2, 6, 7, 17
0.4	0.6	0.84	0.07	2, 6, 7, 17
0.3	0.7	0.84	0.07	2, 6, 7, 17
0.2	0.8	1.02	0.006	1, 3, 6, 17
0.1	0.9	1.02	0.001	1, 4, 6, 17
10^{-4}	1.0	1.02	0.0000	1, 3, 4, 17

Some features to note:

- The lines that best match the dispatch could effectively match after four lines were chosen (Last row Table 49).
- Although the LMP error for one line was higher (2.27) than the 30 bus case (First row Table 46), it was able to reduce to .66 using four lines (First row Table 49). This results in a reduction of 3.44 times. In the 30 bus case, the LMP error reduction from one to four lines was only 1.82.

6.3 Computational Time

In this section the computation times for each bus model will be compared. Solving times for finding the optimal four lines of the 14, 30, and 118 bus systems are included. It is important to note that the numbers here are from an Intel Core 2 Quad Processor at 2.4GHz. Table 50 below lists the time to compute in seconds.

Table 50. Computation time for four-line proxy limit cases

Bus System	Number of Total Lines	Time (s)
14	20	1.25
30	41	26.04
118	186	15784

It is interesting here to note that the number of computations required here go as follows:

$$\frac{n!}{4!(n-4)!} = \frac{n(n-1)(n-2)(n-3)}{4!} \tag{19}$$

where n is the number of lines. If a fifth line is added it would change to:

$$\frac{n!}{5!(n-5)!} = \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} \tag{20}$$

And so on. This is important to note because with large systems, the computation time required to find the best lines would be very lengthy. One way around this is to use methods from section 5 to limit the lines checked to those near generators or a random subsection. Either way, this is a costly disadvantage of selecting optimal lines.

7. Conclusion

Using the MIP method described above to solve for optimal proxy limits has great advantages. Depending on the size of the model, the number of lines to accurately depict dispatch is equal to the number of generators - 1. Minimizing the dispatch error is not unique, however, and may actually cause more discrepancies to the LMP solution if an incorrect set of lines is chosen. Matching the LMPs is relatively harder, but with a few lines (14 and 30 bus model) the error is greatly minimized.

Other results show that when trying to minimize LMP error, the voltage constraint is usually met. There were several cases in which the voltage constraint was met for a single line through five lines where α = 1. This was concluded by taking the chosen set of lines and constraining them in the ACOPF case and testing the voltage constraint at the given bus. However, since minimizing the dispatch error is not unique, the voltage constraint was not met in the ACOPF case where α = .5 or 10^{-4}. In the single line case with the voltage constraint at bus 19, the voltage actually decreased as the line limit increased (line 15). Only after adding an additional line, could the voltage constraint could be met in the ACOPF.

The next results discussed how adding an additional voltage constraint affected the previous result. The lines of the first voltage constraint (bus 19) were compared to the lines of the second voltage constraint (bus 17), and finally were compared to the results when both voltage constraints were simultaneously binding. Here it seemed that similar lines were chosen for both voltage constraints separately. In the one line to five line cases only one total line was different in the LMP error minimization, while only three total lines were different in the dispatch error minimization. However, when both constraints were made binding at the same time, the lines chosen seemed to follow a different path. Five lines were completely different when minimizing LMP error, and ten were different when minimizing dispatch error.

Third, different subsets of lines were chosen, and the results were compared. The three different scenarios that were chosen were choosing lines close the constraint, lines close to generators, and a random reduction in lines that could be chosen. The obvious winner in minimizing LMPs was choosing lines close to the generators. It may seem like choosing lines near the constraint may be a smart choice, but the results show that this was the worst of the three scenarios for minimizing LMP error. In minimizing dispatch error, the results were not as clear, although choosing lines close to the generators or choosing from a random subset of lines should allow complete minimization of error with enough lines.

Finally, results were shown as to how well the model scales up or down with the size of the system. This would be ideal for small systems (50 buses or smaller), but could be adapted to larger systems if more lines were chosen to help reduce the error of the LMPs or dispatch. This comes with a cost of computational time, but could be further minimized. Lines chosen could be closer to the generator or randomly chosen from a subset.

With that said, there are some disadvantages to this method. The first being that one must first solve the ACOPF with the voltage constraint included in order to correctly map the error to which the DCOPF proxy-limited lines must minimize. While this may be possible for studies in forecasting applications, it may not be possible most large scale real time applications.

Even with the disadvantages stated above, the methods described in this paper provide an accurate description of how proxy-limited lines can be better chosen in the future. This could provide companies and industry tools to better select lines not only to ensure a more effective and accurate electricity market, but also to have a greater impact on the efficient use of the grid itself.

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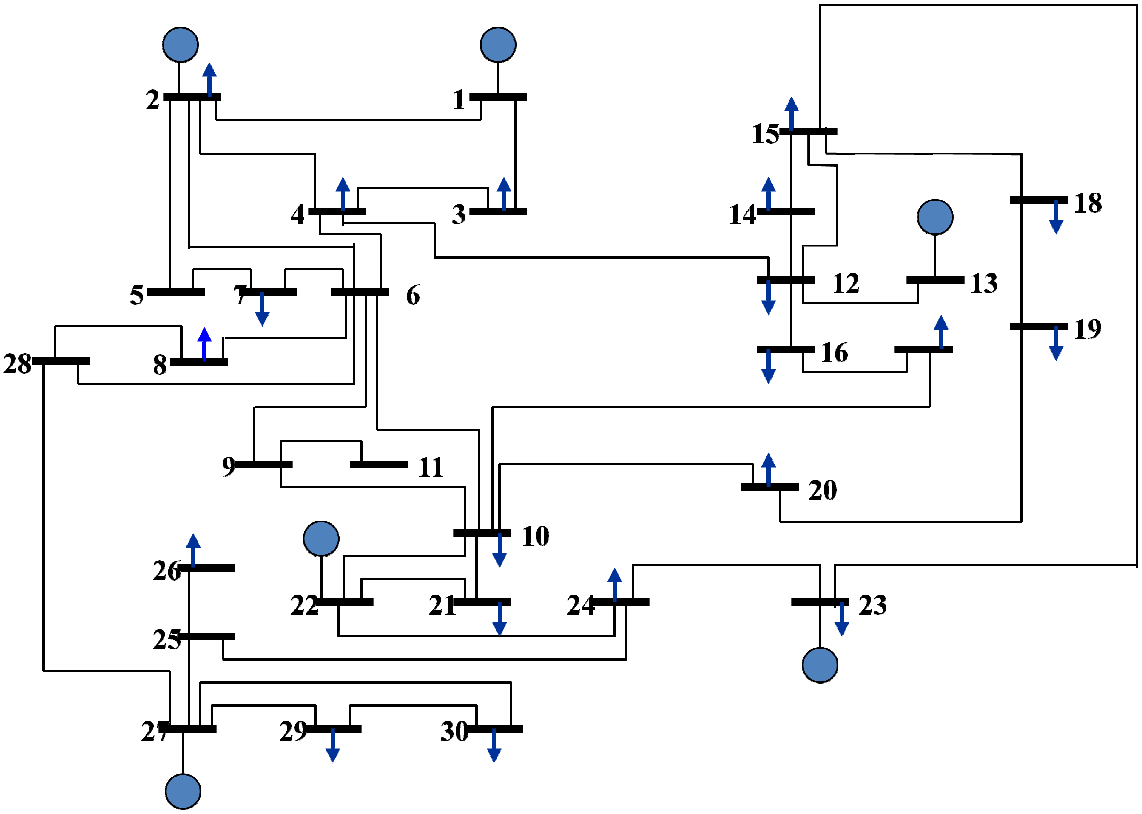
Appendix A – 30 Bus Model

A diagram of the 30 bus model including line numbers to show which buses are connected is shown below.

line	buses	line	buses	line	buses	line	buses	line	buses
1	1-2	10	6-8	19	6-11	28	10-22	37	27-29
2	1-3	11	6-9	20	6-12	29	21-22	38	27-30
3	2-4	12	6-10	21	6-13	30	15-23	39	29-30
4	3-4	13	9-11	22	7-8	31	22-24	40	8-28
5	2-5	14	9-10	23	7-9	32	23-24	41	6-28
6	2-6	15	4-12	24	19-20	33	24-25		
7	4-6	16	12-13	25	10-20	34	25-26		
8	5-7	17	12-14	26	10-17	35	25-27		
9	6-7	18	12-15	27	10-21	36	28-27		

Detailed 30 Bus Model:

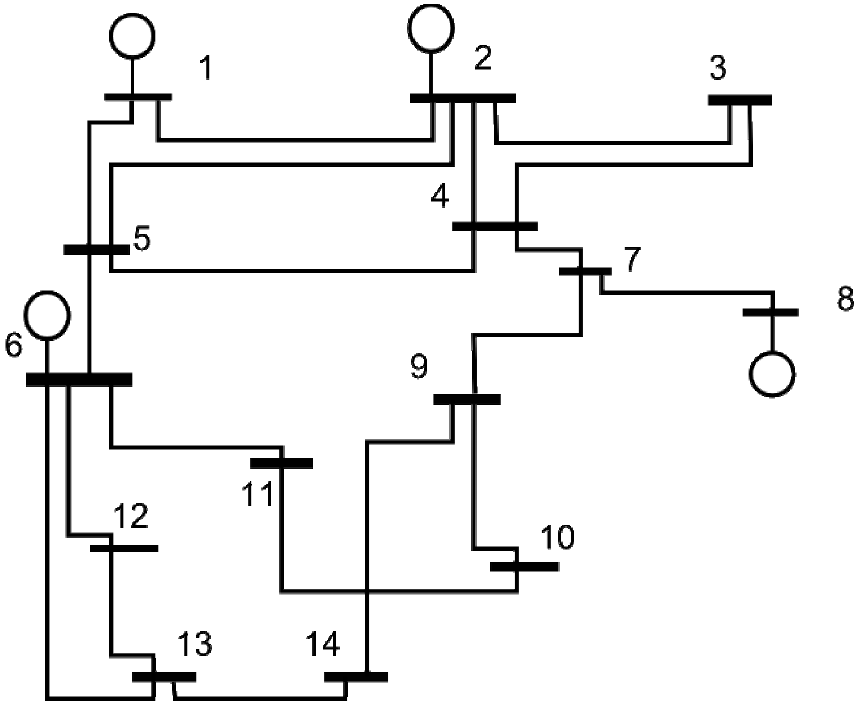


Appendix B – 14 Bus Model

A diagram of the 14 bus model including line numbers to show which buses are connected is shown below.

line	buses	line	buses	line	buses	line	buses
1	1-2	6	3-4	11	6-11	16	9-10
2	1-5	7	4-5	12	6-12	17	9-14
3	2-3	8	4-7	13	6-13	18	10-11
4	2-4	9	4-9	14	7-8	19	12-13
5	2-5	10	5-6	15	7-9	20	13-14

Detailed 14 Bus Model:



Appendix C – 118 Bus Model

A list of line numbers and generator locations for the modified 118 bus is shown below.

line	buses	line	buses	line	buses	line	buses	line	buses	line	buses
1	1-2	32	26-25	63	46-47	94	63-64	125	79-80	156	98-100
2	1-3	33	25-27	64	46-48	95	64-61	126	68-81	157	99-100
3	4-5	34	27-28	65	47-49	96	38-65	127	81-80	158	100-101
4	3-5	35	28-29	66	42-49	97	64-65	128	77-82	159	92-102
5	5-6	36	30-17	67	42-49	98	49-66	129	82-83	160	101-102
6	6-7	37	8-30	68	45-49	99	49-66	130	83-84	161	100-103
7	8-9	38	26-30	69	48-49	100	62-66	131	83-85	162	100-104
8	8-5	39	17-31	70	49-50	101	62-67	132	84-85	163	103-104
9	9-10	40	29-31	71	49-51	102	65-66	133	85-86	164	103-105
10	4-11	41	23-32	72	51-52	103	66-67	134	86-87	165	100-106
11	5-11	42	31-32	73	52-53	104	65-68	135	85-88	166	104-105
12	11-12	43	27-32	74	53-54	105	47-69	136	85-89	167	100-106
13	2-12	44	15-33	75	49-54	106	49-69	137	88-89	168	104-105
14	3-12	45	19-34	76	49-54	107	68-69	138	89-90	169	105-106
15	7-12	46	35-36	77	54-55	108	69-70	139	89-90	170	105-107
16	11-13	47	35-37	78	54-56	109	24-70	140	90-91	171	105-108
17	12-14	48	33-37	79	55-56	110	70-71	141	89-92	172	106-107
18	13-15	49	34-36	80	56-57	111	24-72	142	89-92	173	108-109
19	14-15	50	34-37	81	50-57	112	71-72	143	91-92	174	103-110
20	12-16	51	38-37	82	56-58	113	71-73	144	92-93	175	109-110
21	15-17	52	37-39	83	51-58	114	70-74	145	92-94	176	110-111
22	16-17	53	37-40	84	54-59	115	70-75	146	93-94	177	110-112
23	17-18	54	30-38	85	56-59	116	69-75	147	94-95	178	17-113
24	18-19	55	39-40	86	56-59	117	74-75	148	80-96	179	32-113
25	19-20	56	40-41	87	55-59	118	76-77	149	80-97	180	32-114
26	15-19	57	40-42	88	59-60	119	69-77	150	80-98	181	27-115
27	20-21	58	41-42	89	59-61	120	75-77	151	80-99	182	114-115
28	21-22	59	43-44	90	60-61	121	77-78	152	92-100	183	68-116
29	22-23	60	34-43	91	60-62	122	78-79	153	94-100	184	12-117
30	23-24	61	44-45	92	61-62	123	77-80	154	95-96	185	75-118
31	23-25	62	45-46	93	63-59	124	77-80	155	96-97	186	76-118

Generator Locations at buses:

10, 12, 25, 26, 31, 46, 49, 54, 59, 61, 65, 66, 69, 80, 87, 89, 100, 103, and 111

Detailed 118 Bus Model: