STRUCTURAL CHANGE AND INEQUALITY:
SKILL PREMIA, FIRM SELECTION AND POLITICAL CONSEQUENCES

By

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Much of what we have comes from those who built before us.
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Part 1. Is Skill Dispersion a Source of Productivity and Exporting in Developing Countries?

ABSTRACT. Recent literature claims that skill mix within firms, in contrast to average human capital, influences the entire economy. This paper provides theoretical and empirical evidence of the linkage from skill mix to output, inequality, productivity and exports. I develop a multisector model of firms who employ teams of workers in production. First, I consider what impact changes in the skill distribution from migration, education or outsourcing have on output. I find an increase in industry specific workers boosts output, but in contrast to classical models, worker spillovers to other industries may attenuate output. Increases in diversity increase productivity and decrease inequality. Second, I consider the impact of price changes as caused by tariff reductions or subsidies. I show a rise in output prices raises the total wages of a worker team, but changes relative wages within the team. This is because wages depend on the relative supply of high and low skill team members. Inequality will increase if the supply of high skilled workers is tight. This possibility of a sector boom coincident with higher inequality provides a new explanation of inequality trends beyond skill biased technical change. Empirically, my model motivates a novel specification that characterizes industries as “intensive in skill diversity” or “intensive in skill similarity.” Productivity differences explained by skill mix intensity are comparable to the magnitude of training and imported inputs combined. I also find skill mix differences explain intrasector export variation better than physical or human capital.

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1. INTRODUCTION

This paper revisits the theory of the firm to incorporate the role of worker teams in production. The mix of skills employed within a firm may range from one extreme of skill similarity to another extreme of skill diversity. By skill similarity I refer to production processes which benefit when worker teams are composed of similarly skilled workers. An example is a production line where jobs have been broken down into equally difficult tasks, as typified by the O-Ring production process of Kremer (1993). By skill diversity I refer to superstar production processes which depend on the most skilled member of a team. An example is hierarchical production structures where success is heavily tied to the quality of individuals at the top, an idea which goes back to Rosen’s Economics of Superstars (Rosen 1981). Such a fundamental distinction between methods of production has important implications for growth, labor markets with imperfect information, theories of the firm, wage inequality and trade.

This paper contributes new theoretical implications by considering the role of worker scarcity when firms form teams. In addition, this paper supports the theoretical literature by providing an empirical linkage from skill mix to productivity within and across industries.

Theoretically I build on Grossman and Maggi (2000) who consider worker teams employed in either skill similar or skill diverse production. Each firm hires from a population of skill levels to form worker teams and produce goods. I extend their model to multiple sectors and arbitrary skill distributions to exhibit new, rich forms of the Rybczynski and Stolper-Samuelson theorems. The Rybczynski Theorem under skill diversity predicts an increase in the mass of industry specific workers increases industry output. However, spillovers of workers into other sectors may attenuate this increase in output. The Stolper-Samuelson theorem under skill diversity shows that a rise in output prices raises the total wages of worker teams but changes relative wages within the team. Furthermore, an increase in industry output price may decrease the wages of some workers in the industry due to changes in the division of surplus. This possibility of growth concurrent with decreasing wages provides a new channel for inequality beyond skill biased technical change.

The primary empirical focus of this paper is establishing that skill mix, in contrast to the average level of skill, is an important determinant of productivity. There is a paucity of systematic evidence for the role of skill similarity and skill diversity at the firm level, although some evidence is suggestive. The theoretical model motivates an empirical specification that characterizes industries as “intensive in skill diversity” or “intensive in skill similarity.” I use the production structure of the model to arrive at estimable primitives of a skill diverse, multisector economy. The approach is to specify production in a neoclassical form where skill mix enters as labor augmenting technology. The labor augmenting technology may be “similar skill loving”, “diverse skill loving” or neutral for each industry. The specification is estimated for a cross section of firms in over thirty developing countries using the Enterprise Surveys collected by the World Bank. In developing countries productivity differences due to skill mix should be pronounced due to both greater heterogeneity in educational attainment and labor abundant production.

I find that over two-thirds of developing country firms belong to sectors which are significantly characterized as either “similar skill loving” or “diverse skill loving.” The estimates provide a ranking of intensity across industries from skill similar to skill diverse. This parallels the concept of factor intensity in classical trade theory. As the model predicts linkages from such intensities to inequality, the magnitudes of the estimates are particularly relevant in a developing country context. Within industries, I rank firms by the level of productivity explained by skill mix. Using this ranking, I find the difference between the 75th and 25th percentiles are 9-13%, comparable to the effects of training and imported inputs combined.

The second empirical focus of this paper is testing the linkage from skill mix to comparative advantage at the firm level. Recent theoretical work has shown that skill mix may predict patterns

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1. Andersson et al. (2009) find that firms with high potential payoffs for selecting the right products (a task assisted by worker talent) pay higher starting salaries and select superstars who have a history of success. Martins (2008) also uses matched employer-employee data to tackle competing theories of the relationship between wage dispersion on firm performance, finding a positive relationship which becomes negative once firm and worker fixed effects are considered.

2. The relationship between trade and inequality has generated a vast literature. For surveys, see Kremer and Maskin (2003), Winters et al. (2004) and Goldberg and Pavcnik (2007).

3. Again evidence is sparse or suggestive. Mamoon and Murshed (2008) find developing countries with a high level of schooling experience smaller increases in wage inequality following trade. This is consistent with the idea that high skill countries have a comparative advantage in complementary production relative to low skill countries which best utilize scarce high skill workers in diverse production.
of trade and expected wage shifts resulting from trade\footnote{Ohnsorge and Trefler\citeyear{OhnsorgeTrefler2007} construct a model in which a high correlation between worker attributes amounts to a relative abundance of one of the factors, resulting in a pattern of trade based on a second moment of the skill distribution. In\cite{BougheasRiezman2007} both first order stochastic dominance and mean preserving spreads which alter the skill distribution predict a pattern of trade.}. My approach to testing trade implications joins two strains of the trade literature. The first literature emphasizes the role of heterogeneous firm level productivity as a selection mechanism for exports in the presence of trade frictions (e.g.\cite{Melitz2003a}). The second literature explains productivity differences as outcomes of heterogeneous worker matches to jobs\footnote{From a comparative advantage perspective, both\cite{ManasseTurrini2001} and\cite{Yeaple2005} result in the selection of high skill inputs into export activities. Both papers are competing explanations for the stylized facts of a growing skill premium and productivity differences between exporters and non-exporters.\cite{CostinotVogel2009} consider a continuum of workers and goods produced with different skill intensities and considers the implications of skill and technology shifts in autarky, North-South and North-North trade. In contrast, I investigate a different channel based on the\textit{skill mix of worker teams} rather than the skill of individual workers.}. If productivity differences arise from a relatively high endowment of diverse or similar labor, the predicted pattern of trade depends on moments of the skill distribution beyond the mean. I use a non-linear function of skill mix to explain firm productivity and account for firm exports, revealing a new connection from skill similarity and skill diversity to trade. Productivity differences explained by skill mix predict intrasector exports better than physical or human capital.

The rest of this paper is organized in six sections. Section 2 lays out the model setting while Section 3 develops implications in the form of new Rybczynski and Stolper-Samuelson theorems. Section 4 puts forth the production specification and estimates the relative importance of skill mix within industries. Section 5 estimates the relationship between propensity to export and skill mix. Section 6 concludes.

## 2. A Skill Diverse Multi-Sector Economy

This section begins with the model setting where firms form production teams from a heterogeneous pool of workers. After defining a perfectly competitive equilibrium, I consider the allocation of workers within and across firms. Finally, I construct the wage schedule which supports the efficient equilibrium. General equilibrium implications are pursued in the following section.

### 2.1. Model Setting.

Consider an economy populated with a mass $L$ of workers of varying skill levels $q$. Denote the distribution of skills within the population by $\Phi(q)$, and assume that $\Phi$ has...
a continuous pdf with full support on $[0, \infty)$ and finite mean. Equilibrium wages received by a worker of skill level $q$ are written as $w(q)$. Workers supply labor inelastically, and choose employment at the highest wage available in one of $N + 1$ sectors $S_i$, $i \in \{0, \ldots, N\}$. This results in an optimal sorting problem of workers within firms and across sectors.

Each sector $i$ produces a single type of good, whereas goods within a sector $i$ are indistinguishable. Workers incomes consist of their wages $w(q)$ and have identical homothetic preferences $U(y_0, \ldots, y_N)$ over bundles of outputs $y \equiv (y_0, \ldots, y_N)$. Output prices are denoted $\{p_i\}_{i=0}^N$. For the moment I take prices $\{p_i\}_{i=0}^N$ as exogeneous, consistent with the assumption of a small open economy, in order to focus on the production side.

A firm within sector $i$ has a production technology $F^i$ which produces goods by pairing workers with skill levels $q$ and $q'$, producing a quantity $F^i(q, q')$. Each production technology $F^i$ is symmetric, homogeneous of degree one and twice continuously differentiable. All firms maximize profits. Firms face perfectly competitive output prices $p_i$ and input prices $w(q)$. Thus each firm takes $p_i$ and $w(q)$ as set by the market and chooses to pair workers of skills $q, q'$ in order to maximize profits $\pi^i(q, q')$

\[\pi^i(q, q') \equiv p_iF^i(q, q') - w(q) - w(q') \tag{2.1}\]

I now formalize the idea that different industries might better use different mixes of skill within their workforce. This is done by adding structure to the production technologies, which will result in a ranking of the skill diversity content across sectors. $S_0$ is a complementary sector where the production technology $F^0$ supermodular in skill inputs. In this case, supermodularity of $F^0$ is equivalent to $F^0_{12}(q, q') \geq 0$ so worker skills are complementary. Grossman and Maggi (2000) show that the revenue maximizing skill pairing within such a supermodular sector is to pair workers of identical skills and this pairing also occurs in the equilibrium of this model. It is only necessary to consider a single supermodular sector $S_0$ as investigation shows one such complementary sector dominates the others and crowds them out in equilibrium.

For each sector $S_i$, with $i > 0$ the production technology $F^i(q, q')$ is submodular. Submodularity of $F^i$ is equivalent to $F^i_{12}(q, q') \leq 0$ so workers skills are imperfect substitutes for each other. I show in the appendix that the value of production in the economy is bounded if and only if $\Phi$ has a finite mean so this assumption entails no loss of generality.

Note the often used Cobb-Douglas form for skill inputs cannot distinguish skill inputs as being supermodular or submodular since the cross partials are always positive (if output is homogeneous and increasing in inputs), implying
implies production becomes mostly dependent on the highest skill worker of the team. Holding total skills \( q + q' \) constant, firm revenues in these sectors increase as the skill levels of employed workers diverge. However, the allocation of labor across many sectors creates a conflict: which sectors will get the most diverse workers and how will they be paired? I show that the most important difference between sectors is their input intensities, akin to other models based on factor intensity. Intuitively, the “most submodular” sector can best utilize a diverse workforce and should therefore be allocated the most diverse workers. This intuition bears out in equilibrium, provided the production technologies satisfy the following diversity ranking \( \succeq \) between sectors.

**Definition (Diversity Ranking).** \( S_i \) is more diverse than \( S_j \), written \( S_i \succeq S_j \), iff \( p_iF^i(1,x)/p_jF^j(1,x) \) is strictly increasing for \( x \geq 1 \).

If for each pair of sectors \( S_i \) and \( S_j \) either \( S_i \succeq S_j \) or \( S_i \preceq S_j \) then \( \succeq \) is a complete diversity ranking. Intuitively, \( S_i \succeq S_j \) says that the relative output of \( S_i \) over \( S_j \) increases as more diverse workers are employed. In order to make this ranking concrete, consider the ranking in the context of CES production functions. In this case it is straightforward to show that the elasticity of substitution measures the capacity of a technology to efficiently use diverse labor. The diversity ranking induced is given in the following Lemma.

**Lemma 1 (CES Ranking).** Suppose each \( F^i \) is CES, specifically \( F^i(q,q') \equiv A_i(q'^{\rho_i} + q^{\rho_i})^{1/\rho_i} \). Then \( S_i \succeq S_j \) if and only if \( \rho_i > \rho_j \) so CES production technologies imply a complete diversity ranking.

**Proof.** See Appendix. \( \square \)

The supermodularity of \( F^0 \) guarantees that \( S_i \succeq S_0 \) for all \( i > 0 \). This makes sense as the productivity of a submodular technology \( F^i \) increases in diversity, while the productivity of \( F^0 \) is maximized when identical workers are employed. It is also clear that \( \succeq \) is transitive, i.e. \( S_i \succeq S_j \)

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9Of course, if some sectors strictly dominate others, for example through a superior Hicks neutral technology or high price, such dominant sectors will soak up all the workers. Consequently the most interesting sectors are those which employ workers in equilibrium and therefore consideration is restricted to producing sectors.
and $S_j \succeq S_k$ implies $S_i \succeq S_k$, so $\succeq$ provides a natural way to rank all of the sectors $S_i$. Consequently, I hereafter maintain Assumption 1.

Assumption 1. The diversity ranking $\succeq$ is complete and $S_N \succeq S_{N-1} \succeq \ldots \succeq S_1 \succeq S_0$.

I consider competitive equilibria in which all firms maximize profits and economy wide revenue is maximized. To account for the sorting of workers, define two assignment maps $M(q) = q'$ and $\iota(q) = i$. $M(q)$ pairs a worker of skill $q$ to a worker of skill $q'$ while $\iota(q)$ assigns the paired workers $(q, M(q))$ to sector $i$. A competitive equilibrium therefore consists of assignments $M(q), \iota(q)$ and wages $w(q)$ which satisfy the following conditions:

Definition. An efficient, competitive equilibrium is a wage schedule $w(q)$, team compositions $(q, M(q))$, and sector assignments $\iota(q)$ which satisfy:

1. (Profit Maximization) Each triple $(q, M(q), \iota(q))$ is consistent with profit maximization:
   \[ \pi^{i(q)}(q, M(q)) \geq \pi^{i'}(q, q') \quad \forall i, q' \]

2. (Perfect Competition) Each triple $(q, M(q), \iota(q))$ yields zero profits, $\pi^{i(q)}(q, M(q)) = 0$.

3. (Efficiency) Economy wide revenue is maximized.

Conditions (1) and (2) together guarantee a "no arbitrage" condition for workers. They guarantee that breaking up an equilibrium pair of workers $(q, M(q))$ and reassigning them to different sectors or teams can only yield weakly negative profits. Under conditions (1) and (2) workers cannot be lured away from equilibrium through arbitrage. Condition (3) is an equilibrium selection constraint that restricts attention to first best equilibria.

I now provide implications for the allocation of workers to each other through teams $(q, M(q))$ and across sectors through $\iota(q)$. Shortly these allocations will be supported by wages so that the allocative results discussed here are the equilibrium skill allocation, fixing production and endogenous sorting in the economy.

2.2. Allocation of Skill Diversity by Sector. I now derive the allocation of workers to teams and sectors. This allocation hinges heavily on the concept of skill dispersion. While there are

10A more empirically motivated criterion which implies completeness (and is almost equivalent) is to define the “elasticity of diversity” for each sector $i$ by $\xi_i \equiv \partial \ln F(1, x) / \partial x$. Then there is a complete ranking of sectors $S_N \succeq S_{N-1} \succeq \ldots \succeq S_1 \succeq S_0$ so long as $\xi_N \geq \xi_{N-1} \geq \ldots \geq \xi_0$. 
potentially many ways to measure skill dispersion within a firm, this paper uses the following idea. For an equilibrium matching function \( M(q) \), suppose firm \( i \) employs workers with skill levels \((q_i, M(q_i))\) and firm \( j \) employs the team \((q_j, M(q_j))\), where by convention \( q_i \leq M(q_i) \) and \( q_j \leq M(q_j) \). Then a natural definition is that firm \( i \) employs more diverse labor than firm \( j \) if
\[
\frac{M(q_i)}{q_i} \geq \frac{M(q_j)}{q_j}.
\]
If this holds when comparing every firm \( i \) in sector \( S_i \) with every firm \( j \) in sector \( S_j \), then sector \( S_i \) employs more diverse labor than sector \( S_j \). Assumption 1 guarantees that entire sectors can be ranked by skill diversity using the ranking \( \succeq \).

**Proposition 1.** Under Assumption 1, any equilibrium allocation exhibits skill ratios \( \{\hat{t}_i\} \), increasing in \( i \), where if \((q, M(q))\) are assigned to \( S_i \) then \( M(q)/q \in [\hat{t}_i, \hat{t}_{i+1}) \). The ratios are fixed for \( i > 1 \) by
\[
p_i F^i(1, \hat{t}_i) = p_{i-1} F^{i-1}(1, \hat{t}_i).
\]

**Proof.** See Appendix. \( \square \)

Figure 2.1(a) illustrates Proposition 1 for a hypothetical \( M(q) \). For a team \((q, M(q))\), the x-axis tracks the skill level of the low skill worker \( q \) and the y-axis plots the skill level of his partner \( M(q) \). As the Figure shows, teams where the skill ratio \( M(q)/q \) is the highest (close to \( q = 0 \)) must work in the most diverse sector, \( S_5 \). If the skill ratio within a team decreases slightly, profit maximizing firms will employ the team in the next most diverse sector, \( S_4 \) and so on. In conclusion, skill intensity ratios play a similar role as factor intensities in the multi-sector Heckscher-Ohlin model, except that in Heckscher-Ohlin there is a unique intensity in each sector whereas here there is a range of skill ratios.

So far, Proposition 1 has pinned down \( \iota(q) \) (the equilibrium sector of a worker of skill \( q \)), given \( M(q) \). This follows because when \( M(q) \) is known, \( M(q)/q \) is known and consequently the skill ratio cutoffs \( \{\hat{t}_i\} \) imply the sectoral assignment (2.2).

\[
\iota(q) = \begin{cases} 
N, & M(q)/q \in [\hat{t}_N, \infty) \\
\vdots & \vdots \\
1, & M(q)/q \in [\hat{t}_1, \hat{t}_2) \\
0, & M(q)/q \in [1, \hat{t}_1) 
\end{cases}
\]

(2.2)

I now discuss how \( M(q) \) is determined in order to fix the equilibrium allocation \((q, M(q), \iota(q))\).

Getting from the assignment (2.2) to the matching function \( M(q) \) proceeds in three steps:
(1) Aggregate the sectors \( \{S_i\}_{i \geq 1} \) into a single submodular sector using Assumption 1.

(2) Show in the \( S_0 \) sector workers pair assortatively, i.e. \( M(q) = q \). In the sectors \( \{S_i\}_{i \geq 1} \) workers pair according to maximal cross matching.

(3) Show efficiency requires the skill set of workers assigned to the \( S_0 \) sector is of the form \([t_0, \bar{t}_0]\).

With (2), this fixes \((q, M(q))\) for the sectors \( \{S_i\}_{i \geq 1} \). Proposition 1 then assigns workers among the \( N \) sectors since \( M(q) \) is known.

As these steps are somewhat involved, they are left to the appendix. Here I focus on the resulting form of the matching function \( M(q) \). Within the \( S_0 \) sector, workers are paired with identical skills. As the diversity loving sectors \( \{S_i\}_{i > 0} \) best utilize pairs of low and high skill workers, it is sensible that the skill range of workers employed in \( S_0 \) is the range \([t_0, \bar{t}_0]\) with

\[
\frac{1}{2} \leq \text{median skill level of population} \leq \bar{t}_0
\]

So the “middle of the road” workers are employed in \( S_0 \). I now detail an intuitive argument for fixing the cutoffs \((t_0, \bar{t}_0)\). The revenue generated per unit of skill in the \( S_0 \) sector must be at least as high as in any other sector. This results in skill cutoffs \((t_0, \bar{t}_0)\) as fixed in Equation (2.3):

\[
(2.3) \quad \frac{p_0 F^0(1, 1)}{2} = \frac{p_1 F^1(t_0, \bar{t}_0)}{(t_0 + \bar{t}_0)}
\]

Revenue per unit of skill in \( S_0 \) Revenue per unit of skill in least submodular sector

The remaining workers are employed in pairs \((q, m^S(q))\), where \( m^S(q) \) is the maximal cross matching of workers unassigned to \( S_0 \). The function \( m^S(q) \) pairs the highest and lowest skilled
workers first, the second highest skilled with the second lowest skilled, and so forth. Formally, letting $\Phi^S(t) \equiv \int_0^t 1_{[0,t] \cup [t,\infty)} d\Phi$ be the distribution of workers employed in the sectors $\{S_i\}_{i>0}$, the maximal cross matching $m^S(q)$ must satisfy $\Phi^S(q) + \Phi^S(m^S(q)) = 1$. Therefore $m^S(q)$ can be written $m^S(q) = (\Phi^S)^{-1}(1 - \Phi^S(q))$. The final allocation is therefore given by Equation (2.4), where the skill cutoffs $t_i, \bar{t}_i$ are fixed by $(t_0, \bar{t}_0)$ and Equation (2.2).

\[
M(q) \equiv \begin{cases} 
  m^S(q), & q \leq t_0 \\
  q, & q \in (t_0, \bar{t}_0) \\
  m^S(q), & q \geq \bar{t}_0
\end{cases} \quad \iota(q) = \begin{cases} 
  N, & q \in (0, t_{N-1}] \\
  \vdots & \vdots \\
  0, & q \in (t_0, \bar{t}_0) \\
  \vdots & \vdots \\
  N, & q \in [\bar{t}_N, \infty)
\end{cases}
\] (2.4)

Although the geometry of Equation (2.4) is simple, as illustrated in Figure 2.1(b). Once the worker match function $M(q)$ is fixed by Equation (2.4), Proposition 1 then pins down $\iota(q)$ as in Figure 2.1(a). This induces the skill cutoff ranges for each sector $S_i$ of low skill workers $q \in (t_i, \bar{t}_i-1]$ paired with high skill workers $M(q) \in [M(t_i-1), M(t_i)]$. These allocations are next supported by wages.

2.3. Wages Under Multisector Diversity. Up until now, I have only derived necessary conditions for a competitive equilibrium, which fix a unique assignment of workers to firms. I now construct wages across sectors which support this assignment so that the allocation of workers described above is a competitive equilibrium. The assumption of multiple sectors will yield differing skill premiums across sectors for both low and high skill workers. This highlights a principle of worker symbiosis and parasitism: the division of surplus between high and low skill workers depends on the joint skills of the workers, as well as aggregate labor market conditions.

For brevity I only consider wages $w(q)$ which are continuous and differentiable in the interior of each sector. Since there is a continuum of firms, competitive behavior must drive profits to zero. In the complementary $S_0$ sector where $q \in (t_0, \bar{t}_0)$, workers are paired with identical skills so the zero profit condition implies that

\[ w(q) + w(q) = p_0 F^0(q, q) = p_0 F^0(1, 1) \cdot q \]

So for $q \in (t_0, \bar{t}_0)$, $w(q) = w_0 q$ where $w_0 \equiv (p_0/2) \cdot F^0(1, 1)$. However, wages in the sectors $\{S_i\}_{i\geq1}$ are more interesting. This is because a firm in $\{S_i\}_{i\geq1}$ must lure both high and low skill workers...
away from $S_0$ by offering wages above $w_0 q$, and I term the additional wages required a diversity premium. Notice that the diversity premium is different from a skill premium: both low and high skill workers receive a diversity premium in addition to $w_0 q$ which increases in skill.

Next I construct wages recursively starting with sector $S_1$ and proceeding to $S_2, S_3$, and so on. Workers in $S_1$ have skills $q \in (t_1, t_0] \cup [M(t_0), M(t_1))$ and once wages are found for $q \in (t_1, t_0]$, the zero profit condition fixes wages for $q \in [M(t_0), M(t_1))$. First, a worker with skill $t_0$ employed at the cusp between $S_0$ and $S_1$ must be employable in either sector, so $w(t_0) = w_0 t_0$ since $w_0$ is the wage rate paid per unit of skill in the $S_0$ sector. Second, for any wages $w$ the first order necessary conditions for profit maximization by firms must hold. The profit maximization conditions are Equations (2.5) for $i \geq 1$:

$$p_i F_i(q, M(q)) = w'(q) \quad \text{and} \quad p_i F_2(q, M(q)) = w'(M(q))$$

Thus, for $q \in (t_1, t_0]$, equilibrium requires that $w'(q) = p_1 F_1(q, M(q))$ and integrating from $t_0$ down to $q < t_0$ gives

$$w(q) = w(t_0) - \int_q^{t_0} p_1 F_1(s, M(s)) ds, \quad q \in (t_1, t_0]$$

Proceeding inductively, the recursive relationship for wages across sectors at skill levels below the median are given by

$$w(q) = \begin{cases} w_0 q, & q \in (t_0, t_0^1) \\ w(t_0) - \int_q^{t_0} p_1 F_1(s, M(s)) ds, & q \in (t_1, t_0] \\ w(t_1) - \int_q^{t_1} p_2 F_2(s, M(s)) ds, & q \in (t_2, t_1] \\ \vdots & \vdots \\ w(t_{N-1}) - \int_q^{t_{N-1}} p_N F_N(s, M(s)) ds, & q \in (0, t_{N-1}] \end{cases}$$

Equation (2.6) highlights the fact that the wage an individual receives depends not only on his raw skill but also the wages of those in less diverse sectors. Consider a worker who is the lower skill member of a team in sector $S_i$, $i \geq 0$. His wage is a sector specific wage $w(t_i)$ plus an integral related to the curvature of $F_i$. Since $w(t_{i-1})$ is tied to wages in sector $S_{i-1}$, wages in sector $S_i$ are recursively “pegged” to wages in $S_{i-1}$ and so on up to $S_0$.

\(^{11}\)Other approaches to obtaining wages for heterogeneous workers in the absence of firm competition include bargaining between workers (e.g. Delacroix, 2003) and the $N$-worker coalitional model of Sherstyuk (1998).
Equilibrium wage pegs are better understood by decomposing wages into variable and sector level components. First, define the shadow wage a worker would receive in an economy which has access to a single technology \( F_i \), which I label \( w_i^{\text{Shadow}}(q) \). From Equation (2.6), shadow wages for the diversity loving sectors \( S_i \) are fixed by

\[
 w_i^{\text{Shadow}}(q) \equiv p_i F_i(q_{\text{med}}, q_{\text{med}}) / 2 - \int_q^{q_{\text{med}}} p_i F_i(s, M(s)) \, ds
\]

where \( q_{\text{med}} \) is the median of the skill distribution. Then wages (Equation 2.6) can be written recursively as

\[
 w(q) = \begin{cases} 
 w_0 q, & q \in (t_0, \bar{t}_0) \\
 w_i(t_{i-1}) - w_i^{\text{Shadow}}(t_{i-1}), & q \in (t_i, t_{i-1}) \\
 \text{Variable Component} & \text{Sector-wide Component}
\end{cases}
\]  

(2.7)

Equation (2.7) shows that wages are composed of a variable shadow wage and a sector-wide component. The shadow wage is fixed by the technology of \( S_i \) in isolation of other technologies. The sector-wide component is fixed by the allocation of skill across all sectors which determines the cutoffs \( t_{i-1} \).

Although “pegged”, wages given by Equation (2.7) have the usual properties. For instance, they are strictly increasing in skill. Wages are also bounded below by the \( S_0 \) shadow wages of \( w_0 q \) since the sectors \( \{S_i\}_{i \geq 1} \) must lure away workers from \( S_0 \). Wages are also convex in skill for the following economic reason: wages are fixed by the profit maximization Equations (2.5). Through duality wages are also fixed by cost minimization in skill choice, implying some variety of convexity. The specific property is that \( w'(q) \) is (essentially) increasing. These properties of the wage structure are summarized in Proposition 2.

**Proposition 2.** The wage schedule \( w(q) \) has the following properties:

1. \( w(q) \) is strictly increasing and convex.
2. \( w(q) \) is bounded below by the \( S_0 \) shadow wages \( w_0 q \).
3. \( w'(q) \) is increasing where defined, and elsewhere \( \lim_{q \to x_-} w'(q) \leq \lim_{q \to x_+} w'(q) \).

\[\text{At the cusp levels of skill \{t_i\} and \{M(t_i)\}, w'(q) \text{ is not defined but essentially increases discontinuously as the cusp from lower to higher skill is crossed. Technically, the one sided derivatives of } w(q) \text{ are defined at the cusps \{t_i\} and \{M(t_i)\} and the left hand limit is less than the right hand limit.}\]
Proof. See Appendix.

The wage schedule of Equation (2.6) supports a competitive equilibrium. In order to show this, it is sufficient to guarantee that if a firm deviates from a skill pairing \((q, M(q))\), then the firm obtains non-positive profits. This can be done by exploiting the convexity of the wage structure as given by Proposition 2. The basic argument relies on checking the first order conditions of any firm who deviates from equilibrium. Leaving details to the appendix, the fact that wages support the proposed equilibrium is stated as Proposition 3.

**Proposition 3.** Wages given by Equation (2.6) support an efficient competitive equilibrium.

Proof. See Appendix.

This section has detailed the allocation of workers to both teams and sectors in any efficient, competitive equilibrium. The next section proceeds to consider the effects of price and endowment changes in general equilibrium, deriving variations of the Rybczynski and Stolper-Samuelson theorems.

### 3. General Equilibrium Implications

This section extends canonical results of the trade literature, the Rybczynski and Stolper Samuelson theorems. The Rybczynski theorem predicts how production changes in response to a change in endowments. In this model setting, the most interesting changes in the skill distribution are those which in some way change the diversity of skills in the workforce. Here I provide a definition of such “changes in diversity” and show that increases in availability of a sector’s inputs increase the mass of workers employed in the sector. Associated with the Rybczynski theorem is the “magnification effect”, which means that an increase in sector endowments results in a more than proportional increase in output. In this section, I show changes in the skill distribution cause spillovers of workers across sectors. Such spillovers may amplify or eliminate magnification effects, conditional on economy wide endowments.

I then present two variations on the Stolper-Samuelson theorem. The first shows an increase in output price increases total factor returns in a sector, namely total wages per worker pair. This increase in wages is independent of endowments, in the spirit of the Stolper-Samuelson theorem (e.g. the review in Lloyd 2000). Unlike the Stolper-Samuelson theorem, an increase in output price
also reallocates workers to the more profitable sector. In contrast, the second variation of the Stolper-Samuelson theorem shows that surplus sharing within worker teams generally changes in response to price changes. In contrast to Grossman and Maggi (2000), the wages of either member of a worker team may decrease once a sector booms, providing a new channel from growth to inequality.

3.1. Rybczynski Under Skill Diversity. Characterizing precisely what diversity means for a given distribution of skills $\Phi$ is a difficult task. This is because without additional structure on $\Phi$ it is hard to predict equilibrium changes, or come to terms about the meaning of a particular change. I settle on a “results based” definition which allows for a Rybczynski type prediction, although there could easily be others. To simplify the exposition I first consider only two sectors $S_0$ (which pairs identical workers) and $S_1$ (which pairs diverse workers). Having seen the mechanics of the two sector case, the multisector Rybczynski theorem is presented.

Define a change in a skill endowment $\Phi$ to a new skill endowment $\tilde{\Phi}$ a skill shock and normalize the mass of labor to one for both distributions. I define three types of skill shifts from $\Phi$ to $\tilde{\Phi}$: Low Skill Shocks, High Skill Shocks and Diverse Skill Shocks.

**Definition (Skill Shocks).** Let $\Phi$ and $\tilde{\Phi}$ be two skill distributions and $(t_0, \tilde{t}_0)$ the range of worker skills employed in $S_0$ under $\Phi$.

1. (Low Skill Shock) $\Phi(t_0) \leq \tilde{\Phi}(t_0)$ and $\tilde{\Phi}(M(t_0)) = \Phi(M(t_0))$.
2. (High Skill Shock) $\Phi(t_0) = \tilde{\Phi}(t_0)$ and $\tilde{\Phi}(M(t_0)) \leq \Phi(M(t_0))$.
3. (Diverse Skill Shock) $\Phi(t_0) \leq \tilde{\Phi}(t_0)$ and $\tilde{\Phi}(M(t_0)) \leq \Phi(M(t_0))$.

In the case of a Low Skill Shock, the pool of workers in the low skill range of $S_1$, $q \in (0, t_0]$, expands at the expense of $S_0$. An High Skill Shock is similar. In both cases, the addition of workers to $S_1$ will increase the mass of workers employed in $S_1$ in the post-Shock equilibrium. A Diverse Skill Shock is the simultaneous combination of a Low Skill Shock and High Skill Shock. A special case of a Diverse Skill Shock is a “median preserving spread.” The fact that all of these skill shocks increase the mass of workers in $S_1$ is recorded as Lemma. The Lemma also shows that changes in the skill distribution induces spillovers of the new workers across sectors.

---

13Upward sloping supply of labor in each sector is an important feature missing from classical trade theory, as discussed by Ohnsorge and Trefler (2007).
Lemma 2. Low, High and Diverse Skill Shocks increase the mass of workers in the diverse sector. Low Skill Shocks downskill the similarity loving sector, while High Skill Shocks upskill the similarity loving sector.

Proof. See Appendix.

The effect of new workers with more than two sectors is shown in Figure 3.1. Figure 3.1(a) displays the allocation of workers across sectors before a skill shock, while Figure 3.1(b) displays the reallocation after the endowment of high skill workers in \( S_4 \) increases. Figure 3.1(c) shows that adding high skill workers to the economy shifts up the matching function \( M(q) \). This is because low skill workers can be matched with new, higher skill team members, increasing the ratio of skills per team. Once \( M(q) \) shifts up, it determines new skill ranges for each sector. As shown, the range of low skill workers \([t_4, t_5]\) employed in \( S_4 \) increases by shifting \( t_3 \). The range of high skill workers \([M(t_3), M(t_4)]\) expands as well, maintaining skill diversity in \( S_4 \). While new low skill workers join \( S_4 \), some high skill \( S_4 \) workers will spill over into \( S_3 \).

By keeping track of how workers are employed as in Figure 3.1, I arrive at a new version of the Rybczynski Theorem. Consider an economy with an initial skill endowment \( \Phi \) and a mass of workers \( L \), to which is added a distribution of workers \( \Psi \) with mass \( P \). This added endowment of workers might be migrants, newly trained workers, or the result of outsourcing. To mirror the concept of a Diverse Skill Shock, further assume \( \Psi \) is composed of workers with the right skill levels for one sector \( S_i \). Formally, \( \Psi \) has support on some set \([t_i, t_{i-1}] \cup [M(t_{i-1}), M(t_i)]\). Once \( \Psi \) is pooled with the initial endowment \( \Phi \), the supply of workers to \( S_i \) has increased. The new equilibrium allocation will result in more workers employed in \( S_i \), but more precisely, the total output in the sector will increase. This result is summarized in Proposition 4.

A stronger form of the Rybczynski theorem is the “magnification effect,” meaning that an increase in sector specific endowments results in a more than proportional increase in output. As seen above, the addition of workers to the economy causes spillovers of workers across sectors. Such spillovers preclude any unconditional magnification effect because added endowments can contain a high percentage of workers who will spillover into other sectors.\(^{14}\) Thus worker

\(^{14}\)For instance, suppose the skill distribution is \( \Phi \) and a distribution of workers \( \Psi \) migrate to join the \( \Phi \) workers. To be specific, say the new \( \Psi \) workers have low skills in the range \([t_i, t_{i-1}]\), those skills used by the \( S_i \) sector. A Heckscher-Ohlin setting would predict a boom in the \( S_i \) sector through magnification. However, in this model, the outcome depends on the supply of high skill workers available to work with the new migrants in \( S_i \). If the host country’s distribution \( \Phi \) is thin on workers in the range \([M(t_{i-1}), M(t_i)]\), then new migrants will force spillovers into other sectors successively; first the \( S_{i-1} \) sector then \( S_{i-2} \), etc. Conversely, if the host country is abundant in the \([M(t_{i-1}), M(t_i)]\) skill range, then magnification effects could be large since the migrants might be very efficient additions to the sector.
spillovers may attenuate or strengthen output increases in the sector, altering magnification effects.

However, if the worker spillover effect is purged, new and more efficient resorting of workers will lead to magnification. To see this, suppose the skill endowment increases by adding a skill distribution $\Psi$. To purge the worker spillover effect outside of $S_i$, assume again that $\Psi$ has support on $[L_i, L_{i-1}] \cup [M_\Phi(L_i), M_\Phi(L_{i-1})]$, and additionally assume equal masses of low and high skill workers. Then no worker spillovers occur, but the new allocation of workers in $S_i$ is optimal and therefore output is higher than if $\Phi$ and $\Psi$ are allocated to $S_i$ separately. I summarize these results regarding magnification in Proposition 4.
Proposition 4. Increases in sector specific skills increase sector output. Worker spillovers may prevent magnification effects. Absent spillovers, output is magnified due to more efficient worker sorting.

Proof. See Appendix.

These results are in sharp contrast to model settings which ignore the team aspect of production. From the team based perspective, migration and education policies designed to support particular industries or growth should consider the relative scarcity of differently skilled workers. Policies targeting groups of fairly homogeneous skills (refugees, the uneducated, specialist workers) should keep in mind the availability of their potential co-workers as this may influence their final industry of employment.

Finally, consider the effect of adding more diverse workers to $S_i$. Again suppose $\Psi$ has support on $[\ell_i, \ell_{i-1}] \cup [M\Phi(\ell_{i-1}), M\Phi(\ell_i)]$ with equal masses of low and high skill workers. Denote the respective worker matching functions as $M\Phi$ and $M\Psi$, making the dependence of matches on the skill distribution explicit. In the case that $M\Psi(q) \geq M\Phi(q)$ for all $q$, $\Psi$ generates more diverse teams than $\Phi$ (i.e. $M\Psi(q) / q \geq M\Phi(q) / q$ for all $q$). Therefore productivity (output per unit of skill) under $\Psi$ is uniformly greater than under $\Phi$ because of submodularity. Formally for all $q$:

$$F^i(q, M\Psi(q)) / (q + M\Psi(q)) \geq F^i(q, M\Phi(q)) / (q + M\Phi(q))$$

It can be shown that once the more diverse $\Psi$ is added to $\Phi$, a uniform increase in productivity still holds.

The second effect of adding more diverse workers is to increase shadow wages $w^\text{Shadow}_i(q)$ for low skill workers, while decreasing $w^\text{Shadow}_i(M(q))$. To see this, recall

$$w^\text{Shadow}_i(q) = p_i F^i(q_{\text{med}}, q_{\text{med}}) / 2 - \int q_{\text{med}} p_i F^i(s, M(s)) ds$$

and $F^i_{12} \leq 0$ so if the match function increases, low skill shadow wages increase. Since there are no worker spillovers, the sector wide component of wages is unaffected in $S_i$. However, the lowest skill worker $t_{i-1}$ has a higher wage which propagates to low skill workers in $S_j$ for $j > i$ through $w_i(t_{i-1})$. In summary, adding more diverse workers decreases inequality in more diverse sectors. These effects of adding more diverse workers are summarized in Proposition 5.

Proposition 5. Suppose sectoral skill diversity increases by adding equal masses of low and high skill workers. Then
(1) Productivity increases across all teams in the sector.
(2) Low skill wages rise in the receiving sector.
(3) Wage inequality decreases in sectors more diverse than the receiving sector.

In addition, the distribution of skills influences relative wages within teams when output prices change, which I now turn to.

3.2. **Stolper-Samuelson under Skill Diversity.** The Stolper-Samuelson Theorem, along with modifications and extensions, has a long history (e.g. Neary 2004). I now develop two Theorems in the spirit of Stolper-Samuelson with surprising implications. The first is in line with the canonical result that a rise in the world price $p_i$ of a good increases returns to the intensive factor of a sector. In this case I consider the aggregate wages of the two workers $(q, M(q))$ paired in a sector relative to all other sectors. The second version I consider concerns the wage distribution between low and high skill workers. A rise in a sector’s good price will raise aggregate wages in the sector, but will also change relative wages within worker teams due to the endogenous reallocation of workers across sectors. I show that the indirect reallocation effect of worker sorting may dominate the direct “Stolper-Samuelson” effect of a rise in good price. In particular, the welfare implications of Grossman and Maggi (2000) depend on the assumption of a symmetric skill distribution and are overturned once empirically motivated distributions are considered.

For the first Stolper-Samuelson result, consider a rise in output prices $p_i$ in a diverse sector $S_i$ with $i \geq 1$. Increases in $p_i$ make production in $S_i$ more profitable, thereby expanding the range of worker teams employable in the sector. This effect is depicted in Figure 3.2(a). As more worker teams become profitable to employ the sector expands, stealing away worker teams from adjacent sectors.\footnote{Note there is only one instance in which a change in output prices can affect the composition a worker team. This is when the team is either created by pulling $(q, M(q))$ from the $S_0$ where workers are paired with identical skills, or a team is destroyed by moving from $S_1$ into $S_0$. Outside of this case, the effect of a change in $p_i$ on the range of workers $[L_i, L_i-1]$ employed in $S_i$ can be quantified by defining the following skill and revenue elasticities:
\[
\varepsilon_{L, p_i} \equiv \frac{d \ln L_i}{d \ln p_i}, \quad \varepsilon_{L, p_{i-1}} \equiv \frac{d \ln L_{i-1}}{d \ln p_i}, \quad \varepsilon_{R, L_i} \equiv \frac{d p_j F_j(t_i, M(t_i))}{d \ln L_i},
\]
in which case it can be shown that
\[
\varepsilon_{L, p_i} = \left[\varepsilon_{R, L_i} - \varepsilon_{R, L_{i-1}}\right]^{-1} < 0 \quad \text{and} \quad \varepsilon_{L, p_{i-1}} = \left[\varepsilon_{R, L_{i-1}} - \varepsilon_{R, L_{i-2}}\right]^{-1} > 0.
\]}


Now consider the effect of output prices on wages. If the output price $p_i$ rises to $\tilde{p}_i$, then for any team $(q, M(q))$ employed in $S_i$, revenues $p_iE^i(q, M(q))$ rise. Perfect competition implies that firm revenue equals $w(q) + w(M(q))$ and therefore total wages increase with output price. At the same time, "new" teams $(q, M(q))$ who join $S_i$ after the price increase also receive $w(q) + w(M(q))$ equal to firm revenue. As revenue is maximized in equilibrium, it must be that the "new" team receives total wages at least as high as the revenue generated in their "old" employment. Finally, revenues remain unchanged outside of $S_i$, so total wages paid by any firm outside of $S_i$ do not change. Therefore total wages $w(q) + w(M(q))$ for incumbent and new employees in $S_i$ increase. Investigation shows total real wages $[w(q) + w(M(q))]/p_i = E^i(q, M(q))$ stay constant. These effects of a rise in output price are recorded as Proposition 6.

**Proposition 6.** If output price $p_i$ increases then output and employment in $S_i$ expand. In addition:

1. Total nominal wages for each team in $S_i$ increase, but are constant outside of $S_i$.
2. Total real wages $[w(q) + w(M(q))]/p_i$ for each team in $S_i$ stay constant.

In contrast to models where inputs are mobile but homogeneous, Proposition 6 shows that output price effects on total factor returns are quarantined to the affected sector. What is surprising is that despite this quarantine on total wages for a team, individual wages outside the affected sector change. This occurs because worker reallocation changes the division of total wages within a team. Thus, there is a second Stolper-Samuelson effect on input prices which I now consider.

3.3. **Stolper-Samuelson within Sectors.** Although total wages increase following an increase in good price, it is not necessarily true that all wages rise. In fact, the wages of a high or low skill
worker employed in a “booming” sector may fall. A rise in good price causes the sector to expand, drawing in more workers and changing the relative scarcity of low and high skill workers in the sector. This changes the division of wages within teams. The crucial determinant of how sector expansion changes wages is the elasticity of matching $\varepsilon_{M,q}$ where

$$\varepsilon_{M,q} \equiv d \ln M(q)/d \ln q = \frac{\Phi'(q)/\Phi'(M(q))}{(-q/M(q))}.$$  

**Relative Scarcity of Skills**

$\varepsilon_{M,q}$ captures the elasticity of high skill matches $M(q)$ in response to a small change in $q$. It measures how quickly high skill matches $M(q)$ decrease as $q$ increases. As a sector expands, it draws in worker teams at the margins of the sector, i.e. the teams $(t_{i-1}, M(t_{i-1}))$ and $(t_i, M(t_i))$. Wage changes within teams are characterized by the elasticity of matching $\varepsilon_{M,t_{i-1}}$, which I now detail.

Increases in sector output price $p_i$ will have two effects on wages: direct effects through $p_i$ within the sector and indirect effects through $t_{i-1}$ by drawing new workers into the sector. As seen in Figure 3.2.a, the indirect effect will expand the skill range $(t_i, t_{i-1}] \cup [M(t_{i-1}), M(t_i))$ of workers employed in $S_i$. The two effects are decomposed in Equation (3.2).

$$
\frac{dw_i(q)}{dp_i} = \frac{\partial w_i}{\partial p_i} + \frac{\partial w_i}{\partial t_{i-1}} \cdot \frac{dt_{i-1}}{dp_i} = - \int_q^{t_{i-1}} F_i(s, M(s))ds + \frac{w(t_{i-1}) + w(M(t_{i-1}))}{1 - \varepsilon_{M,t_{i-1}}} \tag{3.2}
$$

The decomposition of Equation (3.2) is striking because the direct effect of an increase in output price is to decrease wages for low skill workers\textsuperscript{16}. Paradoxically, when revenues rise, low skill wages fall in the absence of entry by new workers. These lost wages “trickle up” to each low skill worker’s team member. From the perspective of the firm, the importance of high skill workers increases with revenues since they are the essential ingredient to a diversity loving technology. For high skill workers, the direct effect of an increase in output prices is to capture all new revenues, and also some wages formerly paid to low skill workers.

This direct “trickle up” effect is counteracted by the indirect effect from new workers entering the sector. This indirect effect can be understood in terms of the relative supply of low and high

\textsuperscript{16}Equation (3.2) holds for workers in $S_1$ by interpreting $p_0 F_1^0(t_0, M(t_0)) = p_0 F_1^0(t_0, t_0)$. 

skill workers to the sector, as captured by the elasticity of matching in Equation (3.1). If low skill workers are abundant, the indirect effect will be small since even a small increase in wages will draw many low skill workers to the sector. Potential low skill entrants have skills in the range \( (t_{i-1}, t_{i-1} - \epsilon) \) for small \( \epsilon \) and the mass of such entrants is approximately \( \Phi'(t_{i-1}) \). Large masses of low skill entrants \( \Phi'(M(t_{i-1})) \) will increase \( |\mathcal{E}_{M,t_{i-1}}| \) unless a large mass of high skill workers \( \Phi'(M(t_{i-1})) \) are available to work with. Therefore high values of \( \Phi'(t_{i-1}) \) attenuate the indirect effect on wages. In contrast, if high skill entrants are abundant, as indicated by large values of \( \Phi'(M(t_{i-1})) \), then the indirect effect will be strengthened since \( \mathcal{E}_{M,t_{i-1}} \) is close to zero. These direct and indirect effects on wages emphasize the joint role of prices, technology and labor supply in determining wages. This second version of the Stolper-Samuelson theorem is stated in Proposition 7.

**Proposition 7 (Nominal Wages).** Suppose output prices \( p_i \) rise in a diversity loving sector. Nominal wage effects can be decomposed into a direct price effect and an indirect entry effect where:

1. The direct effect increases high skill and decreases low skill wages.
2. The indirect effect increases low skill wages and decreases high skill wages.
3. The indirect effect is magnified when high skill entrants are relatively abundant.

Finally, the relationship of the match elasticity to real wages is especially clear. Solving for the effect of a price change for low skill workers \( d \left( \frac{w_i(q)}{p_i} \right) / dp_i \) yields

\[
\frac{dw_i(q)}{dp_i} = \left[ \frac{w(t_{i-1}) + w(M(t_{i-1}))}{1 - \mathcal{E}_{M,t_{i-1}}} - w_i(t_{i-1}) \right] / p_i^2
\]

which implies real wages increase for low skill workers whenever \( |\mathcal{E}_{M,t_{i-1}}| < w_i(M(t_{i-1}))/w_i(t_{i-1}) \). When the supply of high skill workers high, \( \Phi'(M(t_{i-1})) \) is large and \( \mathcal{E}_{M,t_{i-1}} \) is small so real wages increase for low skill workers. In contrast, when the supply of low skill workers is high, \( \mathcal{E}_{M,t_{i-1}} \) is large and real wages decrease for low skill workers.

**Proposition 8 (Real Wages).** Suppose output prices \( p_i \) in a diversity loving sector. Real wages for low skill workers increase if and only if high skill entrants are sufficiently abundant.

In contrast to the rich wage dynamics just discussed, the \( S_0 \) sector operates identically to a “piece-rate” sector with constant wages \( w_0 = p_0 F_0(1,1)/2 \) per unit of skill. Here the effects of a rise in output price are more conventional. The direct effect of an increase in \( p_0 \) is simply
\[ \frac{\partial w_0 q}{\partial p_0} = \frac{w_0 q}{p_0}, \] so wages in \( S_0 \) increase. The range of skills employed \( [t_0, T_0] \) have no influence on wages in the \( S_0 \) sector, so the indirect effect of a price rise is zero. Therefore nominal wages in \( S_0 \) rise exactly in proportion to the increase in \( p_0 \), and gains are equally captured by all workers. This implies real wages \( \frac{w(q)}{p_0} \) in \( S_0 \) do not change with prices.

### 3.4. The role of skill symmetry.

I now compare the Stolper-Samuelson theorem in this model to Grossman and Maggi’s model, who consider symmetric skill distributions. In their survey of organizational economics and trade, Antras and Rossi-Hansberg (2009) point out the assumption of symmetric skills limits analysis of the role of heterogeneous workers. In the context of Stolper-Samuelson, symmetric skill distributions cause the indirect effect to dominate the direct effect. Under symmetry the relative scarcity component of the indirect effect is neutralized. To see this, note that if \( \Phi \) is symmetric, then \( \Phi'(q) = \Phi'(M(q)) \) for all \( q \). Referring back to the elasticity of matching in Equation (3.1), this forces the relative scarcity of skills to unity. In contrast, general skill distributions allow the scarcity term \( \Phi'(t_{i-1})/\Phi'(M(t_{i-1})) \) to range from zero (where the indirect effect dominates) to infinity (where the direct effect dominates). Symmetry restricts the scarcity term \( \Phi'(t_{i-1})/\Phi'(M(t_{i-1})) \) to unity, causing the indirect effect to dominate. This surprising implication of symmetric skills is recorded as Proposition 9.

**Proposition 9.** If the skill distribution is symmetric then the indirect Stolper-Samuelson effect dominates the direct effect. An increase in output price raises nominal low skill wages.

**Proof.** See Appendix. □

Contrasting Propositions 7 and 9 shows that considering general asymmetric skill distributions can have surprising implications. Introducing general skill distributions into worker sorting models is important to capture empirically motivated skill distributions, such as those from wage regressions. Also of empirical interest is the relationship between worker scarcity and predicted wage changes. This relationship depends on both the immediate sector and adjacent sectors which employ similarly skilled workers. In particular, this model shows that heterogeneous worker teams may generate unintended consequences with regard to inequality. To highlight one example, the Stolper-Samuelson results show that subsidization of an industry will cause the sector to

\[ \text{Analytical wage effects across two sectors can be worked out for the Log-Normal and Pareto distributions.} \]
expand, but low skill wages may fall as high skill workers in the sector are disproportionately rewarded.

Having laid out the model and described several testable implications, I now step back and look for structural evidence of the production side. An essential assumption of this setting is the presence of sectors which benefit from skill diversity in varying degrees. Does productivity vary by team composition holding average human capital constant? If so, does skill diversity affect productivity in different ways across sectors? I address these questions in the next section.

4. Estimation: Production and Skill Mix

This section tests for evidence of production technologies which benefit from skill diversity or skill similarity. The model above (Lemma 1) shows that the CES form allows a clear interpretation of productivity differences arising from skill complementarity or skill dependence. In this section I first develop a simple, but novel, specification to estimate productivity differences explained by skill mix, then incorporate controls for differences in foreign and domestic markups. This provides direct evidence for the microeconomic foundations of the model, while leaving tests of the theoretical implications for further work. I then discuss the econometric approach for productivity estimation and describe the data used. The section concludes with the production estimation results which show that most sectors vary in productivity with skill diversity.

4.1. Productivity Specification. I begin with a general form \( Y_i = G(K_i, L_i, \psi_i) \) which relates value added output \( Y_i \) for a firm \( i \) to capital \( K_i \), labor \( L_i \) and a firm specific skill mix measure \( \psi_i \). While the model above considers worker teams with two types of workers, more generally firms employ several different types of workers. Interpreting the skill mix measure \( \psi_i \) more broadly as the distribution of workers in the firm, this paper uses data which differentiates between four different educational levels of workers. Consequently \( \psi_i \) is defined as a \( k \)-dimensional distribution of skill levels, \( \psi_i = (\psi_{1,i}, \ldots, \psi_{k,i}) \) measured as a percentage of total employment by the firm. Although \( \psi_i \) denotes the distribution of worker skills rather than skill levels of discrete workers, a diversity and similarity interpretation of skill mix holds as developed shortly.

\[18\] The closest work I am aware of is Iranzo et al. (2008) who examine skill dispersion and firm productivity using Italian employer-employee panel match data. They find that productivity is associated with a higher overall dispersion of skills and evidence of complementarity between production and non-production workers.
Modeling the effect of $\psi_i$ as a labor augmenting factor $\phi(\psi_i)$, I rewrite the production function using a neo-classical production function $F$

$$Y_i = G(K_i, L_i, \psi_i) = F(K_i, \phi(\psi_i) \cdot L_i)$$

(4.1)

Letting $F$ be the Cobb-Douglas form $F(K, L) = AK^\alpha L^\beta$ and allowing both $F$ and $\phi$ to be specific to each sector $S$, denoted $F_S$ and $\phi_S$, value added output $Y_i$ for a firm $i$ in sector $S$ is given by Equation (4.2).

$$Y_i = F_S(K_i, \phi_S(\psi_i) \cdot L_i) = A_S K_i^\alpha L_i^\beta \phi_S(\psi_i)^{\beta_S}$$

(4.2)

In Equation (4.2), the contributions of capital and labor are assumed to be sector specific, with both sector specific and firm idiosyncratic productivity terms $A_S$ and $\phi_S(\psi_i)^{\beta_S}$.

I now connect the productivity term $\phi_S(\psi_i)^{\beta_S}$ of Equation (4.2) to the composition of skills employed in the firm via supermodularity and submodularity in skill inputs. This is done by assuming $\phi_S$ be the CES form, with a sector specific substitution parameter $\rho_S$. Specifically, I assume $\phi_S$ takes the form

$$\phi_S(\psi_i) = \left( \frac{1}{k} \psi_{1,i}^{\rho_S} + \frac{1}{k} \psi_{2,i}^{\rho_S} + \ldots + \frac{1}{k} \psi_{k,i}^{\rho_S} \right)^{1/\rho_S}$$

(4.3)

The CES specification (4.3) parallels that of the model above, but with a twist that $\psi_i$ is the distribution of workers across skill groups instead of the skill levels of individual workers. To help fix ideas about the meaning of $\rho_S$, consider the limiting cases as $\rho_S \rightarrow \infty$ and $\rho_S \rightarrow -\infty$. As is well known, these limiting cases of the CES are

$$\lim_{\rho_S \rightarrow \infty} \phi_S(\psi_i) = \max\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\} \quad \text{Similarity Loving}$$

(4.4)

$$\lim_{\rho_S \rightarrow -\infty} \phi_S(\psi_i) = \min\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\} \quad \text{Diversity Loving}$$

(4.5)

Therefore a firm in a sector with $\rho_S > 1$ will approximately choose $\psi_i$ to solves $\max\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\}$ subject to prevailing wage rates. Such a choice of $\psi_i$ is typified by vectors of the form $\left( \begin{array}{c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right)$, etc. which is to say a mix of workers with similar skill levels. Thus firms in a sector with $\rho_S > 1$ benefit from skill similarity and correspond to sector $S_0$ of the model.

Conversely, in a sector with $\rho_S < 1$, firms will pick $\psi_i$ to roughly maximize $\min\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\}$. This implies a mix of workers with diverse skills, and a representative choice of $\psi_i$ would be
Therefore sectors with $\rho_S < 1$ correspond to the skill diverse sectors $S_i$ for $i \geq 1$ of the model. Finally, at $\rho_S = 1$, $\phi_S(\psi_i)$ collapses to $\sum \psi_i = 1$, implying that differences in skill mix have no influence on productivity. Thus $\phi_S$ nests the null hypothesis that skill mix is irrelevant for productivity at $\rho_S = 1$. To summarize, for sectors where $\rho_S < 1$, skill diversity increases productivity. For $\rho_S > 1$, skill diversity decreases productivity.

Combining the specification (4.2) with the basic production Equation (4.2) and adding an idiosyncratic productivity term $\epsilon_i$ for estimation yields

\[
Y_i = F_S(K_i, \phi_S(\psi_i) \cdot L_i)\epsilon_i = \frac{A_S K_i^\alpha S L_i^\beta S}{\text{Cobb-Douglas}} \cdot \left(\sum \psi_i^{\rho_S} \right)^{\beta_S / \rho_S} \cdot e^{\epsilon_i}
\]

The specification (4.6) allows identification of “diversity loving” and “similarity loving” technologies through estimation of $\rho_S$. The relative magnitudes of $\rho_S$ rank sectors as $S_0, \ldots, S_N$. In addition, the specification allows for testing against the null hypothesis $\rho_S = 1$, which implies $\left(\sum \psi_i^{\rho_S} \right)^{1/\rho_S} = 1$. Equation (4.6) shows that the null hypothesis $\rho_S = 1$ corresponds to a standard neo-classical form with no labor augmentation from skill mix. This paper uses the term $\phi_S(\psi_i)^\beta_S = \left(\sum \psi_i^{\rho_S} \right)^{\beta_S / \rho_S}$ to explain intrasector productivity and later, export propensities.

4.2. Econometric considerations and Data Description. In the data, only the value of sales generated by a firm are observed instead of the direct quantities of different goods produced. Fernandes and Pakes (2008) have also used a similar data set and emphasize that the data allows for estimation of the “sales generating function” rather than the production function. For brevity I stick to the label “production function.” Since sales are observed, it is important to control for the effect of producing for both the domestic and foreign market. For this reason, as further developed in the appendix, I include sector level controls ($M_S$) for markups on firm level exports ($X_i$).

\[\text{Explained productivity can also be interpreted in terms of weighted Theil index } T(\rho) = \sum (x_i^s / \bar{x}_i) \ln(x_i^s / \bar{x}_i). \text{ } T(\rho) \text{ increases as the percentage of workers are concentrated in fewer skill bins and therefore measures how similar workers in a firm are. The productivity term } \ln \phi_S(\psi_i) \text{ can be rewritten in terms of } T(\rho) \text{ as }\]

\[
\ln \phi_S(\psi_i) = \ln \min \{x_i\} + \int_{-\infty}^{0} T(p) / p^2 dp
\]

This shows at the diversity loving extreme of $\rho = -\infty$, $\ln \phi_S(\psi_i) = \ln \min \{x_i\}$. As $\rho$ increases, the $\ln \phi_S(\psi_i)$ measure includes a higher weight on Theil indices and will associate similar workers with higher productivity.

The controls used here are of an accounting nature, although more involved derivations are possible. Depending on the focus and data available, various methods can be used. For instance, Melitz (2000) is primarily concerned with washing out biases in measuring firm productivity for differentiated product firms. In a similar vein, De Loecker (2009) estimates productivity gains from liberalization using an explicit demand system which implies constant markups, the latter being an assumption I impose below.
the firms considered produce in developing countries, generally one should expect $M_S \geq 0$. This is because of the high value exported to the economic North suggesting $M_S^X \geq M_S^D$ (see OECD (2006)).

After a transformation of Equation (4.6) by logs, letting lower case letters represent log-values and incorporating the control for the effect of exports on sales, $M_S X_i$, the specification derived is

$$y_i = M_S X_i + \frac{\beta_S}{\rho_S} \ln \sum_{e=1}^{4} \psi_e^{\rho_S} + a_S + \alpha_S k_i + \beta_S l_i + \epsilon_i \tag{4.7}$$

Here $a_S$ is a sector level effect while $k_i$ and $l_i$ are measured by the value of capital and labor inputs. The skill diverse or skill similar term $\left( \sum_{e=1}^{4} \psi_e^{\rho_S} \right) \frac{\beta_S}{\rho_S}$ is specialized to reflect the data which has the percentage of workers within four educational bins by firm. Equation (4.7) is non-linear in the CES coefficients $\rho_S$ which leads to some choices about the estimation method.\(^{21}\) Equation (4.7) is estimated via non-linear least squares using feasible generalized least squares to control for heteroskedasticity across all country-sector pairs.\(^{22}\)

The most closely related paper in approach is Iranzo et al. (2006) who estimate a linearized CES specification for Italian firms which allows techniques to control for firm level fixed effects and other endogeneity issues (for a brief overview of such techniques see Arnold (2005)). In comparison to my approach, Iranzo et al. have a larger sample across time and so suffer from fewer endogeneity issues, although questions regarding theoretical implications of their linearized CES system remain. In this respect, the scarcity of cross-country firm level panels is a limitation to controlling for endogeneity issues in this paper.\(^{23}\)

While endogeneity may be a problem for estimates of $a_S$ and $\beta_S$, it is less important for the main parameters of interest, namely $\rho_S$. Even if unexpected shocks in productivity $\epsilon_i$ influence labor choices $L_i$ and capital is fixed before the shock, the term $\left( \sum_{e=1}^{4} \psi_e^{\rho_S} \right) \frac{\beta_S}{\rho_S}$ is Hicks neutral and

\(^{21}\)Estimation of CES production technologies goes back to Kmenta (1967) who surmounts computational issues using a second order approximation to the production function. This became a popular technique, e.g. Klump et al. (2007). However, simulation work indicates that Kmenta’s approximation suffers from efficiency problems, resulting in “unacceptable standard errors” (Hansen and Knowles, 1998; Tsang and Persky, 1975; White, 1980).

\(^{22}\)Identification issues for nonlinear least squares are outlined in Cameron and Trivedi (2005) and Amemiya (1983). There are a variety of “assumption bundles” to choose from in order to establish consistency of the NLS estimator. See for instance Jennrich (1969). I have settled on those presented in Malinvaud (1970).

\(^{23}\)Although there is a large literature on production function estimation, the techniques beginning with Olley and Pakes (1996) have been developed using panel data and assuming Cobb-Douglas forms, continuing on through Levinsohn and Petrin (2003) and more recently Ackerberg et al. (2006).
should not be affected by $\epsilon_i$ if the firm faces competitive input markets. For example, productivity shocks to the firm might alter the number of man hours employed, but are less likely to alter the optimal composition of the workforce. This argument is additionally supported by specification tests of the model below.

As far as I am aware, this paper is unique in the breadth of developing country firms examined ($\approx 6700$ firms across 36 countries). The main data set consists of firm level data generally called the Enterprise Surveys, conducted by the World Bank. To the best of my knowledge, this is the largest set of cross country data with firm level distributions of employed skill. The countries included in my sample were selected by income and data availability; see the appendix for details. Firms in the surveys were randomly sampled, in some cases with stratification. The country/year pairs in the survey span from 2002-2005 as tallied in Table 10 of the Appendix. The break down of firms and sales by sector are presented in Table 1. Monetary values have been converted to 2004 US Dollars (CPI adjusted) based on the 2008 International Financial Statistics published by the IMF.

<table>
<thead>
<tr>
<th>Table 1. Observations and Sales Percentages in Sample by Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Agro-industry</strong></td>
</tr>
<tr>
<td>Observation %</td>
</tr>
<tr>
<td>Sales %</td>
</tr>
<tr>
<td>Value Added %</td>
</tr>
<tr>
<td><strong>Metals/ Non-metal/ Other manufact.</strong></td>
</tr>
<tr>
<td>Sales %</td>
</tr>
<tr>
<td>Value Added %</td>
</tr>
</tbody>
</table>

The factor endowments of firms vary considerably across sectors, as shown in Table 2. Capital-labor ratios, measured as the dollar value of capital per dollar of wage, range from labor intensive (Garments and Leather) industries up to capital intensive (Agroindustry). Skill, as measured by mean years of education, range from low skill intensive in Garments and Leather to high skill intensive in Chemicals, Electronics and (surprisingly) Paper production. Skill dispersion, as measured by the Gini of education, ranges from skill similar in Agroindustry to skill diverse in Paper.

For discussion of issues and findings regarding manufacturing firms in developing countries see Tybout (2000).

For some countries, this publication simultaneously provides market and official exchange rates. The market rate was preferred when available.
production. Finally, Table 2 shows that the typical developing country firm is a marginal exporter at best, underscoring the well known fact that most exports accrue to a disproportionately small share of firms.

<table>
<thead>
<tr>
<th>TABLE 2. Endowment Intensities and Exports by Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agro-industry</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Median K/L</td>
</tr>
<tr>
<td>Median Educ Gini</td>
</tr>
<tr>
<td>Median Export %</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Metals/ Non-metal/ Other manufact.</th>
<th>Paper</th>
<th>Textiles</th>
<th>Wood/ Furniture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median K/L</td>
<td>2.822</td>
<td>4.869</td>
<td>6.210</td>
</tr>
<tr>
<td>Median Yrs Educ</td>
<td>7.872</td>
<td>9.700</td>
<td>10.150</td>
</tr>
<tr>
<td>Median Educ Gini</td>
<td>0.502</td>
<td>0.492</td>
<td>0.487</td>
</tr>
<tr>
<td>Median Export %</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

I now briefly discuss variable construction. Value added sales were constructed as total sales reported, less raw material costs and energy costs, with the caveat that energy costs are only known to be non-zero for about 70% of observations. Sector-year and country controls were included to pick up a variety of effects. Five controls, indicators for product line and technology upgrading, ISO certification, internal worker training and firm size (>50 permanent workers) are used to capture productivity differences associated with these characteristics. ISO certification in particular has been found to be associated with increases in exports, quality upgrading and higher productivity firms; for evidence and a theoretical mechanism see Verhoogen (2008). Three continuous controls are also used. First, the imported fraction of inputs control for quality and technology differences of imported processes and inputs. Second, the fraction of foreign ownership helps control for imported management, organization and general know-how. Third, foreign versus domestic markup differences are controlled for using the fraction of exports, as discussed above.

4.3. Production Estimates. The production estimates are provided in Table 3. Parameter estimates have been segmented into three groups: Controls, Production and Markups, and the CES skill parameter by sector. The association of value added sales with ISO certification and training have the expected positive and significant signs. ISO certification explains an additional 6.6% of increased productivity while training accounts for 2.5%. I also find a productivity increase of roughly 4.8% if a firm has upgraded their product line in the last three years. The effect of new production technologies is surprisingly insignificant. Also as expected, the percentage of inputs
which are imported has a significant positive sign but is modest: roughly a 14% increase in imported inputs is associated with a 1% increase in value added sales.\footnote{This estimate is lower than found by \cite{Halpern2009} who use the universe of data on imported goods in Hungary and model foreign versus domestic input substitution explicitly. The authors find imported inputs can explain about 11% of productivity differences but only about 60% of this difference is due to imperfect substitution, making the estimate here based on coarser data comparable.} The controls for firm size and foreign ownership are also positive.

The estimates for production and markups show highly significant capital and labor production parameters which have a sum slightly less than one for each sector, except Leather which has a sum slightly greater than one. Wald tests of the restriction $\alpha_S + \beta_S = 1$ fail to reject constant returns in eight of the sectors.\footnote{The constant returns sectors are Beverages, Chemicals/Pharma, Food, Garments, Metals/Machinery, Other manufacturing, Textiles and Wood/Furniture.} This suggests constant or slightly decreasing returns to scale in capital and labor in almost all sectors. The controls for the effects of markups ($\tilde{M}_S$) are generally insignificant, with the notable exceptions of Garments, Leather, Paper and Textiles where they are positive as expected, and Non-Metals/Plastics which has a negative sign. This surprising finding for Non-Metals/Plastics would be consistent if this industry produces a large quantity of intermediate inputs for strong domestic markets, but this is purely conjecture.

The final group of estimates characterizes sectors as “diverse skill loving” ($\rho_S < 1$) or “similar skill loving” ($\rho_S > 1$). Here tests of significance that $\rho_S \neq 1$ are often highly significant and are jointly significant at the 1% level. The $\rho_S$ estimates actually have even more coverage than Table 3 suggests: skill mix explains productivity differences in 11 of 14 sectors which comprise over 90% of firms and 90% of sales in the sample. This high proportion of sales is largely driven by Food, which comprises the lions share of both sales and value added sales.

The significance of each $\rho_S$ and whether each $\rho_S$ is greater or less than 1 is fairly robust across controls. Similar estimates are obtained using optimal GMM. As an additional robustness check, the estimates of Table 3 are repeated using the translog form for $F(K, L)$ in Table 11 of the Appendix. For the translog, ten of fourteen sectors are found to have $\rho_S$ significantly different from one.
TABLE 3. Non-linear FGLS production function estimates

<table>
<thead>
<tr>
<th>Controls</th>
<th>Value Added Sales</th>
<th>Other Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td></td>
</tr>
<tr>
<td>Upgraded Products</td>
<td>0.0476***</td>
<td>(0.0132)</td>
</tr>
<tr>
<td>New Production Tech</td>
<td>0.0059</td>
<td>(0.0127)</td>
</tr>
<tr>
<td>ISO Certification</td>
<td>0.0656***</td>
<td>(0.0165)</td>
</tr>
<tr>
<td>Worker Training</td>
<td>0.0254**</td>
<td>(0.0122)</td>
</tr>
<tr>
<td>% Imported Inputs</td>
<td>0.0697***</td>
<td>(0.0175)</td>
</tr>
<tr>
<td>Large Firm</td>
<td>0.0547***</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Foreign Own</td>
<td>0.0938***</td>
<td>(0.0211)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obs: 6687</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pseudo $R^2$: 0.8951</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production and Markups</th>
<th>Capital ($\alpha_S$)</th>
<th>Labor ($\beta_S$)</th>
<th>Markup ($M_S$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agroindustry</td>
<td>0.3810***</td>
<td>0.5690***</td>
<td>−0.1622</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autos &amp; components</td>
<td>0.2461***</td>
<td>0.7354***</td>
<td>0.3060</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beverages</td>
<td>0.8068***</td>
<td>0.1486***</td>
<td>−0.0596</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals/Pharma</td>
<td>0.4491***</td>
<td>0.5126***</td>
<td>0.1260</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electronics</td>
<td>0.3011***</td>
<td>0.6748***</td>
<td>−0.0256</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>0.4478***</td>
<td>0.5230***</td>
<td>−0.0345</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Garments</td>
<td>0.3650***</td>
<td>0.5966***</td>
<td>0.0894**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leather</td>
<td>0.2606***</td>
<td>0.7687***</td>
<td>0.1502*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metals and machinery</td>
<td>0.5228***</td>
<td>0.4533***</td>
<td>−0.0073</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-metal/Plastics</td>
<td>0.3896***</td>
<td>0.5923***</td>
<td>−0.2473***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other manufacturing</td>
<td>0.2912***</td>
<td>0.6447***</td>
<td>−0.0349</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paper</td>
<td>0.5768***</td>
<td>0.4195***</td>
<td>0.1486***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Textiles</td>
<td>0.3263***</td>
<td>0.6339***</td>
<td>0.1018**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wood and furniture</td>
<td>0.2921***</td>
<td>0.6764***</td>
<td>0.0700</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*/**/*** denote .1/.05/.01 Significance levels

Finally, if skill mix can help explain productivity through a labor augmenting effect, are the labor coefficients biased when skill mix is not accounted for? I approach this question by comparing the labor augmented specification (4.2) with the restricted specification that $\phi(\psi_i) \equiv 1$, which ignores the labor augmenting role of skill mix. The labor coefficients for both models are extremely close, and applying a Hausman specification test fails to reject the null hypothesis that labor coefficients are affected by skill mix. This suggests that skill mix explains productivity rather than unaccounted for elements of a firm’s wage bill. The evidence also suggests the presence of endogeneity bias in production coefficients is unlikely to affect $\rho_S$.

4.4. Interpretation of Explained Productivity. I now examine how the labor augmenting productivity term $\phi_S(\psi_i)^{\beta_S}$ explains within sector productivity differences. The terms $\phi_S(\psi_i)^{\beta_S}$ can explain within sector variation, but are not directly comparable in levels across sectors because $\phi_S(\psi_i)^{\beta_S}$ is decreasing in $\rho_S$ (see Hardy et al. (1988)). This implies the average level value of labor
augmenting productivity in a sector is indistinguishable from the sector fixed effect, so only variation within $\phi_S(\psi_i)^{\beta_S}$ is identified. While $\rho_S$ can pick up the submodularity and supermodularity of skill mix within a sector, this is inherently a sector specific measure.

Of practical importance is the magnitude of productivity differences explained by skill mix. For example, consider the inter-quartile range: is the productivity difference between the most productive 25% and least productive 25% of firms (as accounted for by skill mix) the same magnitude as other productivity controls? To answer this question for each sector, I first introduce the shorthand $P^S_i \equiv \phi_S(\psi_i)^{\beta_S}$. We can examine the ratio $P^S_{(75)} / P^S_{(25)}$, where $P^S_{(x)}$ denotes the $x^{th}$ percentile of explained productivity. This forms a measure of productivity differences. If $P^S_{(75)} / P^S_{(25)}$ is equal to 1.17 then any firm $H$ picked from the top 25% of explained productivity and any firm $L$ picked from the bottom 25% must have a ratio of productivity $P^S_H / P^S_L$ of at least 1.17. This translates into at least a 17% productivity difference between the firms $H$ and $L$. Accordingly, the productivity ratios $P^S_{(x)} / P^S_{(1-x)}$ for $x \geq 50\%$ are graphed in Figure 4.1.

In interpreting Figure 4.1 consider the expected patterns of productivity ratios. First, sectors with $\rho_S$ terms close to unity, especially if they are insignificant, imply a production technology which is skill mix neutral. Thus if $\rho_S \approx 1$ then the productivity ratios accounted for by skill mix should be close to 1. These are the Agroindustry, Garments, Leather, Metals/Machinery, Paper and Textiles sectors. It is also appropriate to include the Beverages sector which has a significant $\rho_S$ but stands out as being exceedingly capital intensive so labor augmentation does not amount to large differences. Second, with regard to significant sectors (excluding Beverages), differences in productivity of at least 5-9% at the inter-quartile range would imply that skill mix is as important as any of the controls considered individually. In fact, Table 4 shows inter-quartile productivity differences of 9-13% under the inter-quartile measure $P^S_{(75)} / P^S_{(25)}$ for significant sectors. Thus productivity differences explained by skill mix are comparable to the magnitude of training and imported inputs combined. These results are emphasized in Table 4. By comparison, the inter-quartile measure in the capital intensive Beverage sector is roughly comparable to the effect of training. Of course, all of these differences become more pronounced if we consider the 90%/10% firm measure in Table 4. These results support Iranzo et al. (2006) who find that firms in the last productivity decile have dispersion almost 35% higher than those in the first decile.

Table 4 shows that the majority of sectors best utilize diverse skills. However, since the largest sector by sales is Food which best utilizes similarly skilled workers, it would be inaccurate to
conclude that the bulk of developing country sectors are “diversity intensive.” Rather, the amount of diversity intensive production depends on the stage of development or export orientation into sectors beyond Food and basic manufactures. In the long run, it is likely countries transition into manufacturing which allows for specialized jobs that encourage employment of a diverse workforce. If the theory, which implies a convex structure of wages in diverse sectors, is correct then the expansion of the manufacturing sector caused by growth and trade implies a widening wage gap. This suggests further work looking at the link between inequality and the growth of diverse sectors as enumerated in Table 4.

This section has provided structural evidence at the firm level for the model of this paper as well as the literature on the role of skill mix in production. The fact that sectors have been ranked by $\rho_S$ opens the door to testing other implications of the model, in particular the new versions of the Rybczynski and Stolper-Samuelson Theorems. Taking a step in this direction, I look for evidence...
Table 4. Productivity Ratios Explained by Skill Mix

<table>
<thead>
<tr>
<th>Sector</th>
<th>Skill Mix Estimate</th>
<th>Diverse or Similar Skill Intensive</th>
<th>75%/25% Explained Productivity Ratio</th>
<th>90%/10% Explained Productivity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agroindustry</td>
<td>1.2710</td>
<td>-</td>
<td>1.0538</td>
<td>1.0934</td>
</tr>
<tr>
<td>Autos/Components</td>
<td>0.6799**</td>
<td>Diverse</td>
<td>1.1384</td>
<td>1.2542</td>
</tr>
<tr>
<td>Beverages</td>
<td>0.6332***</td>
<td>Diverse</td>
<td>1.0289</td>
<td>1.0593</td>
</tr>
<tr>
<td>Chemicals/Pharma</td>
<td>0.6243***</td>
<td>Diverse</td>
<td>1.1299</td>
<td>1.2502</td>
</tr>
<tr>
<td>Electronics</td>
<td>0.7120**</td>
<td>Diverse</td>
<td>1.1226</td>
<td>1.2611</td>
</tr>
<tr>
<td>Food</td>
<td>1.5072***</td>
<td>Similar</td>
<td>1.0911</td>
<td>1.1720</td>
</tr>
<tr>
<td>Garments</td>
<td>1.1321*</td>
<td>Similar</td>
<td>1.0330</td>
<td>1.0610</td>
</tr>
<tr>
<td>Leather</td>
<td>0.9856</td>
<td>-</td>
<td>1.0049</td>
<td>1.0100</td>
</tr>
<tr>
<td>Metals/Machinery</td>
<td>0.8501**</td>
<td>Diverse</td>
<td>1.0306</td>
<td>1.0679</td>
</tr>
<tr>
<td>Non-metal/Plastics</td>
<td>0.6394***</td>
<td>Diverse</td>
<td>1.1492</td>
<td>1.3143</td>
</tr>
<tr>
<td>Other manufacturing</td>
<td>0.6716***</td>
<td>Diverse</td>
<td>1.1236</td>
<td>1.2858</td>
</tr>
<tr>
<td>Paper</td>
<td>1.3376</td>
<td>-</td>
<td>1.0551</td>
<td>1.1054</td>
</tr>
<tr>
<td>Textiles</td>
<td>0.8896*</td>
<td>Diverse</td>
<td>1.0354</td>
<td>1.0682</td>
</tr>
<tr>
<td>Wood/Furniture</td>
<td>0.7663***</td>
<td>Diverse</td>
<td>1.0887</td>
<td>1.1874</td>
</tr>
</tbody>
</table>

*/**/*** denote .1/.05/.01 Significance levels that $\rho_S \neq 1$.

of comparative advantage in the next section by testing if differences in productivity explained by skill mix result in greater exports for firms.

5. Estimation: Exports and Skill Mix

The last section showed that a skill diverse workforce is positively related to productivity in some sectors and vice versa for others. Is there evidence of increased exports for firms which capture such productivity gains? This section tests whether skill dispersion is a basis for exports. I quickly detail a two-tailed Tobit specification explaining exports through skill mix, controlling for overall skill and capital intensity. I then estimate the specification and assess the relative magnitude of skill dispersion as a determinant of exports.

5.1. Productivity-Export Linkage. While there are many possible relationships between skill mix and propensity to export, I frame the question by considering the selection mechanism caused by export costs. As discussed by Roberts and Tybout (1997), export costs are one of the most important determinants of exports in developing countries. In the presence of trade frictions, only the most productive firms export. Therefore higher productivity should assist in amortizing trade frictions, as should higher physical and human capital intensity. These relationships are depicted with solid lines in Figure 5.1. Productivity differences arising from skill mix will increase

[28] For instance Bernard and Jensen (1999a) find that US exporters are “larger, more productive, more capital-intensive, more technology-intensive, and pay higher wages” (Pg. 2)
exports when the skill endowment available is comparatively advantageous. These relationships are depicted with dashed lines in Figure 5.1.

**Figure 5.1. Effects of Endowments and Diversity on Firm Level Exporting**

I now use explained productivities $\phi_S(\psi_i)^{\beta_S}$ to capture the effect of productivity differences through skill diversity. Since the percentage of exports is between zero and one, any linear model examining exports is necessarily truncated so the specification is a two-tailed tobit:

$$\text{Export } \% \text{of Sales}_i = \text{Sector Effects} + \text{Country Effects} + \alpha \cdot K_i / L_i + \beta \cdot \text{Skill}_i$$

$$+ \gamma \cdot \text{Productivity(Skill Mix)}_i + \delta \cdot \text{Unexplained Productivity}_i$$

This specification tests whether productivity explained by skill mix predicts exports after controlling for average skill and capital intensity. In order to operationalize Equation (5.1) using the explained productivities $\phi_S(\psi_i)^{\beta_S}$, I address the fact that $\phi_S(\psi_i)^{\beta_S}$ only identifies within sector variation as mentioned above. Accordingly, define the Z-score for each $\phi_S(\psi_i)^{\beta_S}$ within a sector by $Z^S_i \equiv (\phi_S(\psi_i)^{\beta_S} - \mu^S) / \sigma^S$, where $\mu^S$ and $\sigma^S$ are the estimated mean and standard deviation of $\phi_S(\psi_i)^{\beta_S}$. A one unit increase in $Z^S_i$ is precisely an increase of one standard deviation in productivity. Since $Z^S_i$ has an approximately normal distribution, the inter-quartile difference between the 75th and 25th percentiles, $Z^S_{(75)} - Z^S_{(25)}$, is approximately 1.35. Finally, $Z^S_i$ increases in productivity so positive estimates of $\gamma$ imply a positive relationship between exports and productivity explained by skill mix.\footnote{This two stage estimation process is suboptimal in the sense that standard errors of the second stage are not derived from joint estimation. This issue could be addressed in the future using the strategy of Newey (1984).}
5.2. Export Propensity and Productivity From Diversity. The estimates of Equation (5.1) are reported in Table 5, with and without controls such as ISO certification which have been linked to exports.

To assess the magnitude of skill mix effects, I appeal to the “rule of thumb” inter-quartile difference $Z_{75}^S - Z_{25}^S \approx 1.35$, suggesting the difference in exports explained by inter-quartile skill mix differences is 6-7%. In contrast, the remaining unexplained productivity from the production estimates do not explain exports as well. This supports the claim that skill mix is an especially important determinant of trade. Furthermore, skill mix is robust in predicting exports under the inclusion of controls, unlike physical and human capital. In order to compare the predictive power of skill mix to physical and human capital, I provide the inter-quartile firm differences by sector in Table 6. Considering the significant effect of physical capital on exporting, I find an implied export propensity of .7-2.8%, very small compared to the magnitude explained by skill mix. Similarly, mean skill shows inter-quartile differences of 1.7-3.5%. Therefore inter-quartile differences of exporting due to skill mix are more than those from physical and human capital combined. I conclude that skill diversity is a relatively strong determinant of exports through its effects on firm productivity.


### Table 6. Inter-quartile Endowment Differences by Sector

<table>
<thead>
<tr>
<th>Sector</th>
<th>Interquartile K/L</th>
<th>Interquartile Mean Skill</th>
<th>Interquartile Skill Mix</th>
<th>Sector</th>
<th>Interquartile K/L</th>
<th>Interquartile Mean Skill</th>
<th>Interquartile Skill Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agroindustry</td>
<td>14.233</td>
<td>3.775</td>
<td>1.333</td>
<td>Leather</td>
<td>4.9195</td>
<td>2.850</td>
<td>1.309</td>
</tr>
<tr>
<td>Autos/Components</td>
<td>7.012</td>
<td>2.516</td>
<td>1.521</td>
<td>Metals/Machinery</td>
<td>5.9239</td>
<td>2.255</td>
<td>1.241</td>
</tr>
<tr>
<td>Beverages</td>
<td>8.198</td>
<td>2.105</td>
<td>1.237</td>
<td>Non-metal/Plastics</td>
<td>9.5572</td>
<td>2.530</td>
<td>1.412</td>
</tr>
<tr>
<td>Chemicals/Pharma</td>
<td>10.612</td>
<td>3.005</td>
<td>1.407</td>
<td>Other manufacturing</td>
<td>7.2056</td>
<td>1.800</td>
<td>1.295</td>
</tr>
<tr>
<td>Electronics</td>
<td>6.376</td>
<td>2.300</td>
<td>1.455</td>
<td>Paper</td>
<td>8.6811</td>
<td>1.983</td>
<td>1.429</td>
</tr>
<tr>
<td>Food</td>
<td>10.563</td>
<td>2.788</td>
<td>1.433</td>
<td>Textiles</td>
<td>7.8246</td>
<td>3.153</td>
<td>1.414</td>
</tr>
<tr>
<td>Garments</td>
<td>3.616</td>
<td>2.858</td>
<td>1.434</td>
<td>Wood/Furniture</td>
<td>5.3937</td>
<td>2.330</td>
<td>1.369</td>
</tr>
</tbody>
</table>

6. **Conclusion and Directions for Further Work**

This paper has developed a model in which the technical capacity to use diverse workers varies by sector. The model serves the joint role of extending the theoretical literature and motivating a structural form for estimation. Theoretically, this paper generates new predictions about how worker teams are employed and what team members are paid. I provide a new version of the Rybczynski Theorem: a one percent increase in workers with skills specific to a sector increases output, but potentially by less than one percent due to worker spillovers. I also provide new versions of the Stolper-Samuelson theorem. An increase in sector output price will cause a sector to boom, but superstar effects and entrant workers change the pay structure of the sector. The new pay structure will increase team wages but may decrease individual wages.

These results suggest growth and trade can exacerbate inequality. In particular, this paper places emphasis on the role of worker teams formed from asymmetric skill distributions. Since low and high workers cannot efficiently produce without the other, growth depends on the available supply of both groups. Wages also hinge on the relative scarcity of low and high skill workers. Therefore this paper establishes ideas about the structure of production which have fundamental implications for inequality, among other themes. This suggests further work looking at the link between inequality and changes in the size of “skill diversity loving” sectors through growth and trade. Further work might also consider a political economy framework considering protection as pro/anti-tariff coalitions can form which cut along both sector and skill lines.
Empirically, I characterize manufacturing sectors by skill mix intensity (diversity or similarity) helps explain productivity. I find that greater than two thirds of firms in a large cross country sample belong to sectors where skill mix is an important determinant of productivity. Inter-quartile productivity differences explained by skill mix are comparable to the magnitude of training and imported inputs combined, and the magnitude in four sectors is comparable to training, imported inputs and ISO combined. Furthermore, the majority of sectors best utilize diverse skills, and theory suggests this explains higher wage inequality through “superstar” wage effects.

Having established a linkage from skill mix to productivity, this paper also evaluates the effect of productivity differences on exports. I find that differences in skill mix explain intrasector export variation better than the physical and human capital combined. I conclude that skill diversity is a relatively good determinant of exports, through its effects on firm productivity. This result clearly has implications for the human capital content of traded goods. Put together, the results of this paper show that a more detailed view of human capital, beyond that of a simple average, yields insights into both productivity and export patterns.
Part 2. Is Selection on Firm Productivity a Third Gain from Trade?

ABSTRACT. Empirical studies provide strong evidence that trade liberalization reallocates resources towards high productivity firms. Theoretical models of international trade have incorporated firm heterogeneity to explain such selection. This paper addresses two questions: Do Selection Effects yield new Gains from Trade distinct from Comparative Advantage and Scale Effects? How does Selection caused by trade compare with domestic policy options? Examining heterogeneous firm models, we find the answers depend not on the production structure rather the demand structure. Linear demand as in Melitz and Ottaviano (2008) generates returns to scale which favor the most productive firms, making Selection in this model a Scale Effect. In contrast, the original model of Melitz (2003) exhibits Selection Effects as a genuinely new Gain from Trade. Selection in a Melitz economy reflects the optimal internalization of trade frictions. We show the Melitz model is efficient, independent of the productivity distribution of firms. The CES demand of Melitz (2003) is necessary for efficiency as is the taste for variety imposed in the model. The results highlight the role of demand and provide empirical implications for determining when Selection Effects are distinct and optimal.

Acknowledgement. We thank Bob Staiger for continued guidance. This paper has benefited from helpful comments of Costas Arkolakis, Thomas Chaney, Steven Durlauf, Charles Engel, Rob Feenstra and John Kennan. The authors are responsible for all remaining errors.
Empirical studies provide strong evidence that trade liberalization induces exit of low productivity firms and increases sales of high productivity firms. To explain these observations, theoretical models have incorporated firm heterogeneity to explain selection of higher productivity firms through international trade. The key insight of this literature is that opening the economy to trade increases competition among firms, leading to a rise in average productivity through reallocation of resources towards more productive firms. Such Selection Effects increase average productivity and result in lower average prices, yielding welfare gains for consumers. This is in contrast to standard monopolistic competition models where trade does not give rise to selection since all firms have the same productivity. Thus, recent models with heterogeneous firms suggest Selection Effects are a new channel for Gains from Trade (GFT). In this paper, we examine the nature of Selection Effects as a source of GFT.

Do Selection Effects provide new welfare gains that cannot be attained without trade? For example, if selection is solely due to tougher competition from larger markets, then selection is actually a scale effect which cannot be attained without trade. Concretely, in the model of Melitz and Ottaviano (2008b), increases in domestic scale replicate selection induced by trade. Selection effects are a consequence of larger market size through trade rather than a new GFT.

On the other hand, domestic scale is distinct from Selection Effects in Melitz (2003b), implying selection gives potentially new GFT. This raises the question whether welfare gains from selection can be achieved without trade. A thought experiment makes this point clear. Consider a closed economy divided into two identical parts that can trade with each other. In standard comparative advantage and scale effects models, this division does not provide any welfare gains since relative technologies, endowments and scale of the economy stay the same. Therefore comparative advantage and scale effects induced by international trade cannot be replicated in autarky. However, Selection Effects can be replicated in autarky. Suppose firms must pay taxes matching the variable and fixed costs of international trade when exporting to the other part of the country.

\[\text{\textsuperscript{31}}\text{Considering 48 countries exporting to the US in 1980-2000,} \text{Feenstra and Kee (2007) estimate that rise in export variety accounts for an average 3.3 per cent rise in productivity and GDP for the exporting country.}\]
Then Selection Effects arise in autarky. However, we show that the closed market equilibrium of Melitz (2003b) is efficient so a planner will never choose to emulate selection caused by trade.

Why then does Selection caused by trade generate welfare gains? We find that efficiency in autarky carries over to the open economy with export costs. In the presence of trade costs, the firms a planner would weed out in the open economy are exactly those that would not survive in the open market. Thus market selection is the optimal internalization of trade frictions.

To sum up, Selection is distinct from Scale in Melitz and arises as an optimal internalization of trade frictions. In Melitz and Ottaviano, Selection is a form of Scale implying trade expands production possibilities of the economy. What drives these different outcomes? By ruling out differences on the production side, we find the difference arises from the demand structure of the economy. We show Selection Effects are distinct from Scale Effects under separable preferences. Separable preferences imply that relative demand for varieties does not respond to exogenous increases in market size. Therefore, increasing the scale of the economy does not alter profitability of firms, leaving selection unchanged. This result is consistent with work in the context of consumption-leisure choices (e.g. Deaton, 1981). In contrast, the non-separable preferences of Melitz and Ottaviano (2008b) imply selection is a form of Scale Effects. In this case, selection occurs because of tougher competition in larger markets. In Melitz, selection arises due to trade frictions and not scale.

As the nature of selection depends so heavily on the demand structure, we then focus on consumer preferences. We show the CES form is unique in ensuring that selection is optimal. Selection Effects of trade may reduce welfare under separable preferences other than those of Melitz (e.g. Benassy-CES of Bilbiie et al., 2005a).

This paper contributes to the long tradition of examining welfare implications of trade and to recent work focusing on welfare in new trade models. Comparing Selection to Comparative Advantage and Scale Effects, we answer when welfare gains from Selection Effects constitute a third GFT. Recent work has drawn attention to the question of whether Selection Effects would increase empirical estimates for welfare gains. Focusing on standard methods used to measure GFT, Arkolakis et al. (2009) show that selection would not change existing empirical measures. Our paper contributes to this literature from a theoretical standpoint. Selection Effects provide welfare gains by optimally internalizing trade frictions in Melitz. In the presence of trade frictions, Selection

\[^{32}\text{Atkeson and Burstein (2005) consider a first-order approximation and numerical exercises for market outcomes in a Melitz economy to show that productivity increases are offset by reductions in variety. We provide an analytical}\]
solves the informational problem of who gets additional resources from trade. The market selects the same firms as a planner with complete information. Therefore, Selection in Melitz differs from Comparative Advantage and Scale Effects which provide welfare gains by expanding consumption or production possibilities.

The optimality results follow the line of query in Dixit and Stiglitz (1977) and are closely related to Bilbiie et al. (2005b) for symmetric firms. Within the heterogeneous firm literature, Baldwin and Nicoud (2005) and Feenstra and Kee (2006) discuss certain efficiency properties of the Melitz economy. Taking their analysis further, we find market outcomes are socially optimal in a Melitz economy and highlight the special role played by CES demand. These results show the demand side is crucial in determining the nature of welfare gains from selection. In fact, departing from CES preferences, we find Selection Effects are a form of scale and provide real gains through an expansion of production possibilities in Melitz and Ottaviano. Thus our findings echo the observation of Feenstra (2010) who suggests moving beyond the CES case to capture social gains from lower markups. In fact, we find that a variable markup setting increases real welfare through lower markups as well as selection.

The paper is organized as follows. Section 2 recaps the theoretical structure of trade models with firm heterogeneity. In Section 3, we present the relationship between Scale and Selection discussed above and explain how selection on firm productivity generates welfare gains by inducing optimality. Section 4 characterizes the deeper features of demand that relate Selection Effects to Scale and determine the optimality of Selection Effects. Section 5 discusses the results by placing them into the context of the literature and presents conclusions.

8. TRADE MODELS WITH HETEROGENEOUS FIRMS

Trade models with heterogeneous firms differ from earlier trade models with product differentiation in two significant ways. First, costs of production are unknown to firms before sunk costs of entry are incurred. Second, firms are asymmetric in their costs of production. This asymmetry treatment for the market equilibrium and further examine the social optimum to show that selection cannot increase production and consumption possibilities.

Arkolakis et al. (2009) and Atkeson and Burstein (2005) make CES assumptions which we show are unique in their welfare implications. Compared to these papers, we only focus on GFT from Selection Effects as our purpose is to examine whether selection constitutes a third source of gain from trade.
of costs induces Selection Effects within an industry. In this section we briefly recap the implications of asymmetric costs for consumers, firms and equilibrium outcomes. Readers familiar with the models may wish simply to refer to a summary of the equilibria in these models provided in Tables [12 and 13] in the Appendix.

8.1. **Consumers.** A mass \( L (L^*) \) of identical consumers in the home (foreign) economy are each endowed with one unit of labor and face a wage rate \( w \) normalized to one. Preferences are identical in the home and foreign countries. Each consumer has preferences over differentiated goods \( U(M_e, q) \) which induce an inverse demand \( D(q(c)) \) for each good indexed by \( c \). Preferences are CES in the Melitz economy and quasilinear in Melitz and Ottaviano (MO hereafter). The forms are given respectively in definitions (CES) and (QL).

\[
U(M_e, q) \equiv M_e^{1/\rho} \left( \int (q(c))^\rho dG \right)^{1/\rho} \quad \text{(CES)}
\]

\[
U(M_e, q) \equiv q_0 + \alpha M_e \int q(c) dG - \frac{\gamma}{2} M_e \int q(c)^2 dG - \frac{\eta}{2} \left( M_e \int q(c)dG \right)^2 \quad \text{(QL)}
\]

where \( q_0 \) denotes the numeraire good and \( \alpha, \gamma, \eta \) are positive parameters. The significance of the parameters \( \alpha, \gamma, \eta \) in MO deserves some mention. An increase in \( \alpha \) or a decrease in \( \eta \) shifts demand up for the differentiated good relative to the numeraire good. On the other hand, \( \gamma \) indexes substitution possibilities within the differentiated goods sector with \( \gamma = 0 \) implying that varieties are perfect substitutes.\(^{34}\) Given \( \gamma \), lower average prices and higher number of competing firms increase the price elasticity of demand. In MO’s terminology, “toughness” of competition has an impact on pricing decisions.

These preferences induce demand curves \( D(q(c)) \). CES preferences over differentiated goods yield the familiar demand curve \( D(q(c)) = IQ^{-\rho} q(c)^{\rho - 1} \) which depends on the elasticity of substitution \( 1/(1 - \rho) \) and the aggregate bundle of goods \( Q \equiv U \). Quasilinear preferences induce a linear demand \( D(q(c)) = \alpha - \gamma q(c) - \eta Q \) which depends on the aggregate bundle \( Q \equiv M_e \int q(c)dG \). Following MO we assume throughout that an interiority condition is met in the market equilibrium so that consumers demand positive amounts of the numeraire good.

\(^{34}\)These preferences differ significantly from the CES case as marginal utilities are bounded so that consumers need not demand positive amounts of any particular variety. MO note that compared to the CES case, the price elasticity of demand is not solely a function of the elasticity of substitution between varieties (\( \gamma \) in this case).
8.2. Firms. There is a continuum of firms which may enter the market for differentiated goods, by paying a sunk entry cost of $f_e$. We denote the mass of all entering firms by $M_e$. Upon entry, each firm receives a marginal cost draw of $c$ drawn from a distribution $G$ with continuously differentiable pdf $g$. In MO, $G$ is assumed to be Pareto$(k, c_M)$ with 
\[ g(c) = \frac{k c^{k-1}}{c_M^k}, \quad c \in [0, c_M]. \]

Each firm acts as a monopolist of a distinct variety. Accordingly, we index each differentiated good by $c$ and the quantity and price respectively by $q(c)$ and $p(c)$. After entry, should a firm produce for the domestic market it faces a cost function $TC(q(c)) = cq(c) + f$ where $f$ denotes the fixed cost of production. Each firm faces an inverse demand of $p(c) = D(q(c))$ and acts as a monopolist, charging a markup over cost denoted by $\mu(c) = p(c) - c$. Post-entry profit of the firm from domestic sales is $\pi(c)$ where $\pi(c) = \max_{q(c)}[p(c) - c]q(c) - f$. When the economy opens to trade, firms incur an iceberg transport cost $\tau^* > 1$ and a fixed cost $f_x \geq 0$ in order to export to other countries. As a result, firms face a cost function $TC(q_x(c)) = \tau^* c q_x(c) + f_x$ and a demand function $D(q_x(c))$ for sales to the export market ($x$).

Profit maximization implies that firms produce for the domestic and export markets if they can earn non-negative profits from sales in the domestic and export markets respectively. We denote the cutoff cost level of firms that are indifferent between producing and exiting from the domestic market as $c_a$ in the closed economy and $c_d$ in the open economy. The cutoff cost level for firms indifferent between exporting and not producing for the export market is denoted by $c_x$. Formally, let $i = a, d, x$ denote autarky and the domestic and export markets in the open home economy respectively. Each $c_i$ is fixed by the Zero Profit Condition (ZPC).

\[ \pi_i(c_i) = 0 \quad \text{for } i = a, d, x \quad \text{(ZPC)} \]

Since firms with cost draws higher than the cutoff level do not produce, the mass of domestic producers ($M_i$) supplying to market $i$ is $M_i = M_e G(c_i)$.

In summary, each firm faces a two-stage problem: in the second stage it maximizes profits from domestic and export sales given a known cost draw, and in the first stage it decides whether to enter given the expected profits in the second stage. Finally, we maintain the standard free entry condition imposed in monopolistic competition models. Specifically, let $\Pi(c)$ denote the total expected profit from sales in all markets for a firm with cost draw $c$, then ex ante average $\Pi$ net of

---

\[ ^{35} \text{Some additional regularity conditions on } G \text{ are required for existence of a market equilibrium in Melitz.} \]
sunk entry costs must be zero.

\[ \int \Pi(c) dG = f_c \quad \text{(FE)} \]

The equilibrium outcomes are summarized in Tables 12 and 13 in the Appendix.

9. SELECTION EFFECTS AS A GAIN FROM TRADE

In this Section we disentangle Selection from Scale Effects, showing when Selection is a form of previously known Scale effects or a new GFT. Concretely, we show selection in Melitz and Ottaviano (2008b) is replicated by scale implying Selection Effects arise due to an increase in market size. In Melitz, we show Selection is a new source of GFT and characterize Selection as an optimal response to trade frictions. Finally, we show that the different roles of Selection in Melitz and MO are due to differences in demand rather than firm costs. This sets the stage for the following section which shows how demand structures influence selection.

9.1. When Selection is a Scale Effect. In order to highlight the nature of selection on firm productivity, we consider trade between countries with a single factor of production and ex ante identical cost distributions. This rules out Heckscher-Ohlin and Ricardian comparative advantage, allowing us to focus on the relationship between Scale Effects and Selection Effects. In MO, Selection is a new form of previously known Scale Effects. In Melitz, Selection is a consequence of trade frictions and not Scale. In this sense, the MO and Melitz frameworks are orthogonal.

**Proposition 1.** Scale Effects of Trade in Heterogeneous Firm Models

(1) Average productivity and markup of the open Melitz and Ottaviano (2008b) economy can be replicated in autarky by an appropriate change of scale \((L)\).

(2) Average productivity and markup of the closed and open Melitz (2003b) economies are independent of scale \((L)\).

Proof. We detail the relationship between scale \((L)\) and average productivity in autarky \((\bar{c}_a)\) and in the open economy \((\bar{c}_t)\). Recall \(c_a\) denotes the autarkic domestic cost cutoff and \(c_d\) denotes the open economy cost cutoff for domestic sales. In both models, average productivity is fixed by cutoff cost levels of the lowest productivity firms. For Melitz, this is shown algebraically in the Appendix. For MO, the relationships are particularly simple, namely \(\bar{c}_a = \frac{k}{k+1} c_a\) and \(\bar{c}_t = \frac{k}{k+1} c_d\). In both models, the cost cutoffs are fixed by the conditions determining the zero profit level and free entry \((ZPC = FE)\). The ZPC = FE conditions are provided in Table 7.
Table 7. Cutoff Cost Levels and Scale Effects

<table>
<thead>
<tr>
<th></th>
<th>Autarky</th>
<th>Open Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melitz</td>
<td>$f_e / G(c_a) = f k(c_a)$</td>
<td>$f_e / G(c_d) = f k(c_d) + n G(c_x(c_d)) f_{k(c_x(c_d))} / G(c_d)$</td>
</tr>
<tr>
<td>MO</td>
<td>$c_d^{k+2} = 2(k + 1)(k + 2) \gamma c_M f_e / L$</td>
<td>$c_d^{k+2} = \frac{1 - (\tau^<em>)^{-k}}{1 - (\tau^</em>)^{-k} - \tau} \cdot 2(k + 1)(k + 2) \gamma c_M f_e / L$</td>
</tr>
</tbody>
</table>

Table 7 shows that $ZPC = FE$ is independent of scale in the Melitz economy so that $c_a$ does not change with $L$. Therefore the open economy average productivity level cannot be attained by scaling market size in Melitz. In contrast, Table 7 shows that in the MO economy scale does affect cost cutoffs and thus average markup and productivity.

We now show that by appropriate scaling of a closed MO economy of size $L$ to size $sL$, the autarkic productivity of the scaled economy, $\tilde{c}_s$ can replicate the open economy productivity $\tilde{c}_t$. Referring to Table 7, $\tilde{c}_s = \tilde{c}_t$ when

$$s = \frac{1 - (\tau^*)^{-k}(\tau)^{-k}}{1 - (\tau^*)^{-k}}$$

(MO Scale)

Thus scaling up resources in a closed MO economy from $L$ to $sL$ yields the open economy average productivity level in autarky. As expected when $\tau = \tau^* = 1$, the scaling factor $s = 2$.

The first consequence of Proposition 1 is Selection Effects cannot be a new source of GFT in the MO economy since they are really Scale Effects. The second consequence of Proposition 1 is that in the Melitz economy, Selection Effects are unrelated to scale. These results are summarized below.

**Corollary.** Interdependence of Scale and Selection

1. Selection Effects are a form of Scale Effects in MO. In particular they can be replicated by scaling autarkic resources.
2. Selection Effects are distinct from Scale Effects in Melitz (2003).

9.2. How Selection Generates GFT. We now examine how international trade generates welfare gains through Selection. As shown above, in a MO economy Selection Effects are a form of Scale

\[\text{This Corollary explains the observation of Baldwin and Forslid, 2006 that reduction in trade costs can have an "anti-variety effect" in a Melitz economy. Indeed, Arkolakis et al., 2008 provide evidence of only small gains from variety in Costa Rica. These findings are a consequence of the relationship between Selection and Scale. In a Melitz economy, Selection Effects are distinct from Scale Effects. Thus, there is no reason for them to move together. In fact, higher fixed exporting costs intensify selection but reduce entry due to lower expected profits. On the other hand, in a MO economy, Selection Effects are a form of Scale Effects so trade frictions affect variety and selection in the same direction.}\]
Effects. Therefore, international trade provides welfare gains from selection by expanding production possibilities of a MO economy. In a Melitz economy, Selection Effects are independent of scale and the mechanism for welfare gains differs from traditional GFT. Consequently, we provide a detailed exposition of how Selection generates welfare gains in the Melitz model.

We first show there is nothing special about international trade in inducing selection. In fact, Selection Effects can be replicated in autarky by using taxes similar to trade costs. To see this, consider a closed Melitz economy divided into two identical parts that can trade with each other. Suppose firms must pay taxes \((\tau, f_x)\) when exporting to the other part of the country. Then the firm problem is exactly as in an open Melitz economy, though the scale of this economy is lower than the open economy. As shown in Proposition 1, scale does not matter for cost cutoffs \(c_d\) and consequently resource allocation across firms in the economy is exactly as in the open Melitz economy. Selection Effects of the open economy are replicated.

In many cases, the optimal choice of domestic taxes such as \(\tau\) can improve welfare by inducing firms to internalize social benefits (Ramsey 1927). We have just argued that tax policy can replicate firm selection found in the open economy. It is also known that Selection is welfare improving in the open economy. One might expect Selection can improve welfare in autarky. In fact, this is not the case; we find that the autarkic market equilibrium is efficient. This implies that Selection associated with trade is undesirable in autarky. It reduces entry and generates too little variety. However, the open economy market equilibrium is efficient as well. How can Selection in the open economy be inefficient in autarky? As discussed below, the difference occurs because GFT from Selection is an optimal internalization of trade frictions. Conditional on trade costs, the market induces optimal Selection Effects in a Melitz economy. We state the efficiency results as Propositions 2 and 3.

**Proposition 2.** Every market equilibrium of a closed Melitz economy is socially optimal.

**Proposition 3.** Every market equilibrium of an open Melitz economy is socially optimal.

**Proofs.** See Appendix.

Although the proofs of Propositions 2 and 3 are involved, the reasoning is similar to that in homogeneous firm monopolistic competition models. From that literature we know that CES preferences yield "synchronization of markups" across firms (Bilbiie et al. 2005a). The synchronized markups and free entry imply that changes in consumer surplus and profits exactly balance each other, leading to an efficient market equilibrium (Grossman and Helpman 1993; see also Baldwin...
Propositions 2 and 3 show these properties are preserved in a heterogeneous firm setup with sunk costs.

Efficiency of the market equilibrium in a Melitz economy is tied to the CES assumption. CES preferences and free entry imply market allocations are socially optimal. The market makes an “optimal departure from marginal cost pricing” and replicates the social optimum. Firms charge markups but free entry implies the proceeds from markups exactly finance the fixed and sunk costs of production. Isoelasticity of preferences ensures constant markups across firms. Thus, prices are proportionate to costs and the market quantities are proportionate to the quantities produced under marginal cost pricing. This result is consistent with the insight of [Baumol and Bradford (1970)] that in the presence of fixed costs, Pareto optimal utilization of resources requires proportionate reductions in all quantities.

In conclusion, both closed and open market outcomes are efficient. Differences in Selection occur because variable profits in the open economy reflect additional trade frictions. The market internalizes these frictions and eliminates firms with low profitability. We detail this process next.

9.3. Selection as an Optimal Response to Trade Frictions. Why does Selection yield welfare gains in an open economy? We address this question in the original setting of Melitz (2003). Here Selection arises as an optimal response to trade frictions, yielding positive welfare gains.

To highlight this clearly, consider a Melitz economy which stays closed but has twice its original scale $2L$. This controls for Scale Effects from two identically sized economies trading with each other. Having controlled for Scale Effects, divide the scaled closed economy into two parts of size $L$. Suppose a planner implements a tax policy where firms must pay variable and fixed costs $(\tau, f_x)$ to trade with each other. In such a scaled closed economy, the firm problem is identical to an open Melitz economy, and open economy firm outcomes are replicated. Consequently, controlling for scale, Selection can be generated in autarky by imposing frictions such as taxes $(\tau, f_x)$. Put differently, international trade by itself does not induce selection. Selection is a consequence of frictions associated with trade and can be generated even in autarky.

From Proposition 2, a no tax policy is optimal in a scaled closed economy since the market is efficient. This implies welfare in a scaled closed economy (with no selection) is higher than welfare in an open Melitz economy with selection. However, Proposition 3 implies that if an increase in scale can only be attained at a cost of exogenous frictions $(\tau, f_x)$, Selection Effects are optimal. We summarize this result as Proposition 4.
Proposition 4. In a Melitz economy, Selection Effects induced by international trade can be generated in autarky but are not socially optimal. If new markets can only be accessed by paying \((\tau, f_x)\) then these Selection Effects are optimal.

Proposition 4 shows Selection Effects in a Melitz economy are an internalization of trade frictions and do not reflect “real” gains on the production or consumption sides. However, Selection Effects are an optimal internalization of trade frictions and hence yield positive GFT in the presence of trade frictions. Thus, Selection Effects in Melitz perform the function of allocating additional resources optimally without any informational requirements. We now contrast these results with selection in a MO economy.

9.4. Why Selection Differs in Melitz and MO. There are three key differences between the Melitz and MO economies:

1. Fixed costs of exporting \((f_x)\) in Melitz,
2. The presence of a homogeneous good in MO,
3. Demand for differentiated goods.

We harmonize MO to Melitz along the lines of fixed costs and homogeneous goods. As detailed below, we find that Selection is still a Scale Effect. Therefore the independence of Selection from Scale in Melitz and not in MO can only be due to demand for differentiated goods.\(^{37}\)

Adding Fixed Costs of Exporting to MO. In the Melitz economy firms incur a fixed cost to enter export markets while in the MO economy these fixed costs of exporting are assumed to be zero. Selection Effects vanish in the Melitz model when \(f_x = 0\) so we might expect that once \(f_x\) is incorporated into the MO model, Selection Effects may be independent of scale. Accordingly, we consider a case where all assumptions of the MO model are retained but now firms have to incur a fixed cost for exporting \((f_x > 0)\). It turns out that after introducing such fixed costs in the MO model, Proposition 1 still holds so Selection Effects are still a form of Scale Effects. In MO, the presence of homogeneous goods implies the supply of labor to the differentiated goods sector is elastic. So it is not surprising that fixed exporting costs \(f_x\) play a role similar to iceberg transport costs \(\tau\).

\(^{37}\)The assumption of a single sector in Melitz is not driving the novelty.\(^{[2007]}\) use a two-sector Cobb-Douglas Melitz framework and derive cost cutoff levels for each sector. Examination shows these cutoffs are independent of scale implying that Melitz (2003) may be extended to a multisector framework which preserves Proposition 1.
Removing Homogeneous Goods from MO. Unlike Melitz, MO consider preferences defined over a homogeneous good in addition to the differentiated good. We relax this assumption to examine whether the presence of a homogeneous good prevents independence of Selection Effects from Scale Effects in MO. For illustrative purposes, we consider only symmetric resource endowments and trade costs. After removing homogeneous goods from MO, we find preferences continue to be non-separable so Selection Effects remain Scale Effects.

In conclusion, the independence of Selection as a GFT is not driven by the supply side of the model. Rather, different demand schedules induce monopolistically competitive firms to respond differently following a change in market size. In the next Section, we further explore the role of preferences in generating independence and efficiency.

10. Demand: Selection, Scale and Efficiency

In a Melitz economy, Selection Effects are distinct from scale and are optimal in the presence of trade frictions. As pointed out in the last Section, these properties are a consequence of demand schedules. In this Section, we detail the role of demand in distinguishing selection from scale and inducing efficient selection.

First, we show firm selection is tied to how scale shifts demand for varieties. We then show that separable preferences imply demand for varieties is independent of scale. Therefore separable preferences make selection independent of scale, as in Melitz (2003). In contrast, scale shifts demand in MO, inducing selection.

Second, we show that isoelasticity is necessary to ensure market efficiency. Trade leads to optimal Selection only when preferences have the CES form. However, isoelasticity alone does not produce optimal Selection. Incorporating a “taste for variety” along the lines of Benassy (1996) will cause the market to over or under select firms.

The results in this section emphasize how properties of consumer demand impact Selection. We summarize the results in the context of the wider literature in the next section.

38 In particular, independence from scale is not due to the assumption of isoelasticity.

39 Alternative generalizations could include an additive numeraire (Bilbiie et al. 2005b), terms of trade effects (Demirova and Rodriguez-Clare 2007), asymmetric countries or trade costs etc.
10.1. **Demand Shifts and Selection.** Firm selection is clearly tied to the demand for varieties \( p(q(c)) \). For instance, if a change in scale \( L \) increases the price the least productive firm \( c_a \) receives \( \left( \frac{\partial p(q(c_a))}{\partial L} > 0 \right) \) then \( c_a \) will make positive variable profits and less productive firms will enter \( \left( \frac{\partial c_a}{\partial L} > 0 \right) \). Similarly, shifts in the entire demand curve for varieties \( \left( \frac{\partial p(q(c))}{\partial L} \right) \) change \( c_a \) through the effect on firm revenues. To see this explicitly, recall the free entry condition

\[
\int_{0}^{c_a} \{ [p(q(c)) - c] q(c) - f \} \, dG = f_e
\]

Differentiating with respect to scale \( L \) and appealing to the envelope theorem (since \( q(c) \) is the optimal quantity of a firm) we know

\[
\int_{0}^{c_a} \left[ \frac{\partial p(q(c))}{\partial L} \cdot q(c) \right] dG + \left[ \frac{\partial c_a}{\partial L} \right] \cdot \left\{ [p(q(c_a)) - c_a] q(c_a) - f \right\} \cdot G'(c_a) = 0
\]

By definition, a firm with cost draw \( c_a \) makes zero profits \([p(q(c_a)) - c_a] q(c_a) - f = 0\) implying

\[
\int \frac{\partial p(q(c))}{\partial L} \cdot q(c) \, dG = -\frac{\partial c_a}{\partial L} \cdot \int \frac{\partial p(q(c))}{\partial L} \cdot q(c) \, dG
\]

As monopolistic competition implies additional competitors lower prices \( \left( \frac{\partial p(q(c))}{\partial c_a} < 0 \right) \), it follows that \( \frac{\partial c_a}{\partial L} \) and \( \int \frac{\partial p(q(c))}{\partial L} \cdot q(c) \, dG \) have the same signs. The term \( \int \frac{\partial p(q(c))}{\partial L} \cdot q(c) \, dG \) is the effect of scale on revenue at current quantities. For example, if scale shifts demand up \( \left( \frac{\partial p(q(c))}{\partial L} > 0 \right) \) then higher revenues allow higher cost firms to produce. This result relates the effect of scale on selection to demand shifts, summarized in Proposition 5.

**Proposition 5.** Selection is independent of scale if and only if demand shifts from scale are revenue neutral at current quantities. Formally,

\[
\frac{\partial c_a}{\partial L} = 0 \quad \text{iff} \quad \int_{0}^{c_a} \frac{\partial p(q(c))}{\partial L} \cdot q(c) \, dG = 0
\]

This proposition helps explain why selection is distinct from scale in Melitz (2003) but not in MO. Consider the general class of “Melitz-type” preferences of the form given by Equation (10.1):

\[
(10.1) \quad U(M_e, c_a, q) = v(M_e, c_a) \int_{0}^{c_a} u(q(c)) \, dG
\]
Here $u$ denotes utility from the bundle of differentiated goods while the function $v$ denotes utility from mass of variety. Melitz-type preferences of Equation (10.1) are weakly separable in the bundle of differentiated goods $q(c)$ and mass of entrants $M_e$. As in two-stage budgeting problems, a change in market size only changes the shares allocated to mass of variety and the unit bundle, but not substitution within the bundle. Formally, $\partial p(q(c))/\partial L = 0$, so Proposition 5 shows that selection is unresponsive to $L$. Of course, Equation (10.1) includes Melitz as a special case. We state this formally in Proposition 6.

**Proposition 6.** Let preferences be separable as in Equation (10.1). Then scale does not matter for the cost cutoff level so Selection Effects are distinct from Scale Effects.

*Proof.* See Appendix.

In contrast, separability breaks down in MO; an increase in market size changes both quantities and variety. Relative demand for varieties shifts with scale. To see that these scale-induced demand shifts must affect selection, we appeal again to Proposition 5. Suppose $\partial c_a/\partial L = 0$ so prices for the $c_a$ firm must also be constant ($\partial p(q(c_a))/\partial L = 0$). At the same time, we know demand is linear and pivoting to a new line through the point $(q(c_a), p(q(c_a)))$. Since $\partial p(q(c))/\partial L$ represents a line pivoting through a point, either demand is shifting up ($\partial p(q(c))/\partial L > 0$) or down ($\partial p(q(c))/\partial L < 0$), which violates revenue neutrality. We must conclude $\partial c_a/\partial L \neq 0$, so scale effects selection in MO.

### 10.2. Optimal Selection: Isoelasticity and Taste for Variety.

The welfare-enhancing effects of international trade and their efficiency properties have figured prominently in the literature. Within heterogeneous firm models, Feenstra and Kee (2006) show that quantities and cost cutoffs are optimal in a Melitz economy when industry prices are held constant. We have taken their analysis further by showing that the market equilibrium of an autarkic and open Melitz economy is efficient and that Selection Effects are an optimal internalization of trade frictions. The significance of CES preferences of Melitz cannot be understated. Departing from CES preferences, the market equilibrium and the adjustment to trade frictions are no longer socially optimal. This holds even for general Melitz-type preferences of Equation (10.1) as stated in Proposition 7.

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40We assume $v$ is positive and continuously differentiable and $u$ satisfies usual regularity conditions which guarantee that each monopolist will have a unique optimal quantity in a market equilibrium. Note that the functions $u$ and $v$ can be defined to yield variable own-price elasticities that need not be equal across firms or to the elasticity of substitution.
**Proposition 7.** Consider an economy with preferences of Equation (10.1). A necessary condition for the market equilibrium to be socially optimal is that $u$ is CES.

*Proof.* See Appendix. □

Proposition 6 implies if utility from the unit bundle is not CES, i.e. $u(q(c)) \neq q(c)^\rho$, then Selection Effects cannot be optimal. Except in the CES case, trade does not lead to an optimal internalization of trade frictions. When the economy opens to trade, Selection Effects arise in the market equilibrium but there is no guarantee that they are desirable. Depending on the utility specification of $u$ and $\nu$, Selection Effects may not even be welfare-improving. In such cases, the market selects too much or too little and variety gets too low or too high.

We now detail a specific example of inefficient Selection arising from “taste for variety”. Following [Bilbiie et al., 2005a], we specialize the separable preferences of Equation (10.1) to CES-Benassy preferences which disentangle “taste for variety” from market power (Benassy, 1996). The specialization assumes isoelasticity and defines $\nu(M_e, c_d) \equiv \left[M_e G(c_d)\right]^{\rho(\nu_B + 1) - 1} M_e$ which results in preferences similar to that of Melitz with the addition of a Benassy parameter $\nu_B$. The Benassy parameter $\nu_B$ captures taste for variety, while market power is given by the markup to cost ratio $(1 - \rho)/\rho$. In the special case of Melitz preferences, taste for variety exactly equals the market power of producers ($\nu_B = (1 - \rho)/\rho$), in which case $\nu(M_e, c_d) = M_e$.

Once taste for variety is incorporated, isoelasticity is no longer sufficient to produce optimal selection. Proposition 8 shows Selection is optimal iff taste for variety equals market power.

**Proposition 8.** Consider an economy with preferences of Equation (10.1) and Benassy $\nu$. The market equilibrium is socially optimal only if $u$ is CES and taste for variety equals market power.

*Proof.* See Appendix. □

Proposition 8 is the classic Chamberlinian “efficiency versus diversity” problem re-interpreted in the presence of selection (Chamberlin, 1950). It highlights whether the market provides the optimal levels of quantity versus variety. With the exception of $\nu_B = (1 - \rho)/\rho$, the market fails to optimally select the cost cutoff for firms and induces suboptimal levels of quantity versus variety. With the results for Selection in hand, the next section places Selection in the context of the wider literature on gains from trade.
11. DISCUSSION AND CONCLUSION

In this Section we summarize our results on Selection and place them in the context of other Gains from Trade, in particular Comparative Advantage and Scale Effects (see Table 8). Before concluding, we discuss empirical implications of our results.

11.1. GFT: Comparative Advantage and Scale Effects. As shown in Section 3, Selection Effects in Melitz and Ottaviano (2008b) are a form of Scale Effects while Selection is distinct from Scale in Melitz (2003). These results are presented in the bottom of Column (a) of Table 8. In what follows, we relate key results for Selection in each model with other sources of GFT.

<table>
<thead>
<tr>
<th>GFT Source</th>
<th>(a) Distinct From Other GFT</th>
<th>(b) GFT Mechanism</th>
<th>(c) Pareto Efficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Comparative Advantage</strong></td>
<td>Yes</td>
<td>Expands Consumption Set</td>
<td>Yes First Welfare Thm</td>
</tr>
<tr>
<td>Ricardo/HO</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. <strong>Scale Effects</strong></td>
<td>Yes</td>
<td>Expands Consumption Set</td>
<td>Yes CES Preferences</td>
</tr>
<tr>
<td>Krugman (1980)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. <strong>Selection Effects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Melitz (2003)</td>
<td>Yes Separable Prefs</td>
<td>Optimally Internalize Trade Frictions</td>
<td>Yes† CES Preferences</td>
</tr>
<tr>
<td>b. MO (2008)</td>
<td>No Scale Effect</td>
<td>Expands Production Set</td>
<td>No Variable Markups</td>
</tr>
</tbody>
</table>

† Efficiency is unique to both CES preferences and the specific “taste for variety” considered by Melitz.

**Comparative Advantage.** In standard comparative advantage models, aggregate gains from trade on the production and consumption sides result in a Pareto improvement in the economy once distributional issues are addressed. In the absence of trade, resources can be reallocated within countries but there is no change in consumption possibilities. Thus trade based on Comparative Advantage expands the consumption set, yielding welfare gains that are unattainable without trade. Comparative Advantage results in a trading equilibrium that is Pareto superior to

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41 See Facchini and Willmann (1999); Helpman and Krugman (1985, Chap. 9) for a survey of Gains from Trade in constant returns to scale and increasing returns to scale models of trade, respectively. Also see Feenstra (2006) for a survey of the empirical evidence on gains from trade.

42 Kemp and Wan (1972); Dixit and Norman (1988). For a detailed survey regarding distributional policies see Facchini and Willmann (1999).
the autarky equilibrium. In fact, the trading equilibrium is efficient implying no role for trade and industrial policy to improve world welfare. Other sources of GFT are absent in standard Ricardian and Heckscher-Ohlin models implying Comparative Advantage is a new GFT. We summarize these facts in Row (1) of Table 8.

**Scale Effects.** In standard models with Scale Effects, international trade yields welfare gains that are unattainable without trade (e.g. Helpman and Krugman [1985]). In the absence of a domestic growth mechanism, an economy cannot expand its scale and hence its production or consumption set without engaging in trade. Efficiency of market equilibria have been studied at length in symmetric firm models with Scale Effects. Recently, Bilbiie et al. (2005a) show the market equilibrium is socially optimal if and only if preferences are CES. We generalize their result to models with heterogeneous firms.

**Selection Effects.** Like Comparative Advantage and Scale Effects, Selection Effects in Melitz (2003) are distinct from other sources of GFT and yield a Pareto efficient equilibrium. In autarky, Selection Effects in Melitz can be emulated through domestic policy, but only at a welfare cost. Thus, they do not provide “real” gains in production or consumption possibilities. However, Selection Effects in an open Melitz economy provide welfare gains to the extent that they are an optimal internalization of trade frictions. This implies a laissez faire trade and industrial policy is jointly optimal for the world economy.

However, efficiency holds if and only if preferences are CES. Surprisingly, efficiency is unrelated to the productivity distribution of firms. Allowing for the Benassy-CES preferences of Bilbiie et al. (2005a), we show the market may select too many or too few firms. We summarize these findings in Row (3a) of Table 8.

In contrast, Selection Effects in a Melitz-Ottaviano economy are a form of Scale Effects which yield real gains on the production side as summarized in Row (3b) of Table 8. In MO, the GFT from Selection is not fully realized by the market. The reader may verify that linear demand provides

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43 Dixit and Stiglitz (1977); Spence (1976); Bilbiie et al. (2005b); Behrens and Murata (2006).

44 However, terms of trade externalities may exist and lead to a breakdown of laissez faire policies. Demidova and Rodriguez-Clare (2007) incorporate terms of trade considerations and provide domestic policies to obtain the first-best allocation in an open Melitz economy with Pareto cost draws.
firms with excess market power which drives up profits and eliminates too few firms. This opens the door to industrial policy both in autarky and under trade.\footnote{In similar work on optimal policy, Chor (2006) shows domestic tax and subsidy schemes increase welfare in the open economy of Helpman et al. (2004b).}

11.2. **Implications for Empirical Work.** We briefly outline key empirical implications for the role of trade and demand in Selection and for the role of demand in Gains From Trade. Our results shed light on answering:

1. How much selection is induced by trade?
2. How does consumer demand impact selection?
3. How do productivity gains translate into welfare gains?

Suppose we are interested in the extent to which trade induces firm selection. Propositions 1 and 6 show that when consumers have separable preferences, observed selection effects are due to changes in trade frictions. Since these Selection Effects are the optimal response to frictions in Melitz (Proposition 4), welfare depends heavily on trade barriers. Therefore additional controls for changes in other frictions (e.g. domestic frictions before export) are justified as they too are important determinants of selection. On the other hand, if the researcher has good reason to suspect that scale economies are at play then any separable preferences must be rejected (Proposition 6). In this case, growth in domestic scale will influence selection independent of trade. Therefore isolating Selection Effects from trade vs growth of the domestic market using available observables is an important concern.

As discussed in Section 3.4, the measurement of selection effects depends heavily on the demand side, rather than production side. In general, shifts in demand which change available market revenues (at current quantities) determine selection effects (Proposition 5). Empirically, such demand shifts may occur for factors not considered in basic models, such as exogeneous shocks to consumer income which induce selection through the demand effect. Therefore including controls for such shocks may be important. Thinking about demand as a primitive also opens the empirical avenue of recovering shifts in demand curves across periods (for instance, non-parametrically) and using the estimates to explain firm selection or other firm choices tied to selection.

Finally, the fragility of efficiency and optimal Selection shown in Section 4.2 points to difficulties in translating productivity gains into welfare gains. To draw the stronger conclusion of welfare
gains requires a better understanding of real world demand patterns, especially substitution patterns across varieties and between varieties and quantities of goods.

11.3. **Conclusion.** We have examined the nature of Gains from Trade through Selection Effects in benchmark models with heterogeneous firms. We find that the nature of Selection Effects depends heavily on the structure of consumer demand. With linear demand of Melitz and Ottaviano (2008b), Selection Effects are a form of Scale Effects. In the CES model of Melitz, Selection Effects are a new source of GFT, independent of Scale Effects. CES preferences are not unique in their ability to induce this independence of selection and scale. For a general class of separable preferences, demand and entry changes offset each other leading to independence of selection and scaling. So for a broad class of preferences, Selection Effects are a new source of GFT. Unlike traditional sources of GFT, Selection Effects in the Melitz economy are an internalization of trade frictions. The market acts as an information aggregator by allocating additional resources optimally. However, this optimality property is unique to CES preferences, and breaks down even if “taste for variety” is considered. So in principle, Selection Effects need not be optimal or even welfare-improving.

We conclude Selection Effects differ from traditional sources of Gains from Trade. The demand structure and hence, the channel through which Selection operates is crucial in determining the nature of Selection Effects as a third Gain from Trade.
Part 3. Left, Right, Left: Income Dynamics and the Evolving Political Preferences of Forward-looking Bayesian Voters

ABSTRACT. The political left turn, which lagged Latin America’s transition to liberalized market economies by a decade, challenges conventional economic explanations of voting behavior. This paper provides a theoretical framework to help understand these complex political-economic dynamics. We build on forward-looking voter models and analyze political preferences under a general family of income transition functions. We show that non-concave functions, which offer no prospect of upward mobility for some voters, result in stronger support for redistributive policies than otherwise anticipated. Numerical analysis of estimated transition functions suggests much stronger support for redistribution than materialized over the first decade of economic liberalization. We thus eschew the assumption that voters had full information on their new economic reality, and model voters as Bayesian learners. We show that starting from a prior that was consistent with the so-called Washington Consensus vision of liberalization, voters would be expected to exhibit the sort of political dynamics observed in most of Latin America over the last two decades.

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12. INTRODUCTION

Most Latin American countries had transitioned to market economies by the early 1990’s. The largely center right political leadership that instituted these transitions continued to win national elections and persisted in power throughout the 1990s and into the early 2000’s. Since that time, electoral politics have turned sharply left. The recent suite of presidential elections have seen left-leaning candidates defeat more conservative opponents in Brazil, Bolivia, Chile, Ecuador, El Salvador, Nicaragua, Paraguay, Peru and Venezuela. Not only have these elections ushered in a political shift, they have in many instances been hotly contested by candidates offering fundamentally different economic visions. The goal of this paper is to provide a theoretical framework to help us understand the economic forces that underlie these complex political dynamics.

The influential body of political economy literature that focuses on economic inequality as a force that determines both political institutions and voting patterns would seem to offer a window into these political patterns (Acemoglu and Robinson, 2006; Boix, 2003). However, the fact that inequality measures tend to be remarkably stable over time makes it unlikely that inequality can explain the right-left voting dynamics of Latin America. A recent paper by Robert Kaufman confirms the inconvenient empirical fact that existing measures of economic inequality do a very poor job of explaining both political institutions and voting patterns in Latin America (Kaufman, 2009).

Although we could abandon the search for economic explanations of contemporary voting patterns and appeal to other factors to explain changing voting patterns, we instead take our cue from (Benabou and Ok, 2001) who model voters as forward-looking agents who (1) formulate their political preferences based on how redistributive schemes will influence their future stream of well-being; and (2) have full information about their economy’s income distribution dynamics. From this perspective, voters should be driven by income dynamics, not by the current level of income inequality or other features of the contemporaneous income distribution.

While the contemporary Latin American left cannot be defined by a shared economic model, this new left does share a largely populist impulse and desire to shift resources and opportunity to those at the bottom of the income distribution.

For a type of hybrid approach, see for instance Chapter 10 of (Roemer, 2001).

In a recent paper, Gary Fields makes this point even more strongly as he shows how economic inequality can actually increase during the early stages of a period of upward mobility that would surely dampen political preferences for redistribution (Fields, 2007).
At first glance, this analytical direction might seem odd given that Benabou and Ok provide theoretical foundations for surprisingly conservative voting behavior in the face of high inequality, not for the surprising shift towards redistribution in Latin America. Benabou and Ok specifically show that concave income distribution dynamics that offer the prospect of upward mobility (or, POUM) can account for anti-redistribution conservatism. Under POUM, forward-looking voters who would benefit from redistribution in the short run do not benefit in the long run and therefore vote against long term redistributive policies. A first contribution of this paper is to generalize the class of income transition functions considered by these authors. We show that non-concave income transition functions of the sort suggested by poverty trap theory, which offer no prospect of upward mobility (or, NoPOUM), can result in a surprisingly and increasingly redistributive electorate. Our result shows that redistributive dynamics are determined by smoothed envelopes drawn around income transition functions. This generalizes the connection between redistribution and income beyond the usual concepts of concavity and convexity.

In an effort to corroborate this theoretical intuition, we estimate income dynamics in Latin American countries. These estimates indeed reveal the sort of NoPOUM dynamics that would be expected to generate an increasingly pro-redistribution electorate. Surprisingly, applying these dynamics to our full information, forward-looking voting model indicates that the demand for redistribution should have been stronger and should have occurred well in advance of the recent suite of Latin American presidential elections. This result presents a puzzle that questions fundamental assumptions about how economic voters process and react to economic prospects. Survey research which assesses voter’s subjective expectations about prospects has found “POUM captures hopes and expectations as well as realistic socioeconomic assessments” (Graham and Pettinato, 2002). A second contribution of this paper is to explore this puzzle within a rational voter framework.

We argue it is the assumption that voters have full information about their economy’s income distribution dynamics that is most problematic, especially in transition economies where the electorate had little prior experience of a liberalized market economy (e.g. Przeworski, 1991). In such circumstances, voters have little choice but to fall back on ideological priors about how such an

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49Complementary endogenous explanations for anti-redistributive positions include disincentives for labor supply (Meltzer and Richards, 1981), asset formation (Persson and Tabellini, 1994), and inefficient levels of public goods (Alesina and Rodrik, 1994). As in Benabou and Ok, we ignore the incentive effects of taxation to highlight the roles of income dynamics and learning.
Indeed, (Tucker, 2006) shows that voting in the post-Soviet bloc reflects economic experiences: areas with poor economic outcomes tend to support “Old Regime” parties while good outcomes provide support for liberal “New Regime” parties. In Latin America, the shift to the liberal economic model was sold on the grounds that it would boost incomes and well-being in the lower half of the income distribution. In an effort to better capture political dynamics, we model voters as Bayesian learners who begin with this “POUM prior,” and then experientially update their priors based on their own stochastic income realizations. This model first shows how beliefs about income dynamics convert into redistributive preferences. We then provide sharp conditions which characterize when income transitions are to the Right or Left of one another. We show that this model of forward-looking, Bayesian voters offers an empirically tenable explanation of the recent right to left political evolution in Latin America. A key ingredient in this explanation is the volatility that dead weight loss induces in uncertain environments, a new effect that the voter learning approach reveals.

The remainder of this paper is organized as follows. Section 2 develops a basic framework for individual and aggregate income dynamics in the presence of transient shocks, and models political support for redistributive policies by both myopic and forward looking voters who enjoy full information about the income dynamic process. Section 3 then introduces both concave (POUM) and poverty trap (NoPOUM) dynamics, and derives results on the political preferences of fully-informed forward looking voters. The analysis of Section 3 is applied to Latin American income dynamics in Section 4. Section 5 then relaxes the full information assumption, and explores political dynamics as voters learn about the true income distribution dynamics that characterize their economy. Section 6 shows this model of forward-looking Bayesian voters who confront a NoPOUM world can give rise to the political polarization and sudden political shifts that have been observed in 21st century Latin America.

13. Forward-looking Voters and the Demand for Redistribution

This section lays out machinery needed to discuss changing patterns in majoritarian voting when the electorate can choose among income redistribution schemes. Our emphasis is the role

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Our approach to modeling ideology as an idiosyncratic evolving process echoes Bates et al. (1998) as a means to complement cultural and ideological political theories with rational choice. The Latin American case is also compounded by a recent history of autocracy which likely bounded the range of publicly admissible priors, a possibility our framework can incorporate.
of income dynamics, rather than static measures of inequality (e.g. [Moene and Wallerstein 2001]). The setting is a continuum of voters whose income evolves over time and fluctuates with idiosyncratic shocks each period. We consider the fraction of the voter population who rationally prefer income redistribution, which we term the demand for redistribution.

Each individual voter considers their individual income path, aggregate income of the economy and the longevity of a proposed redistributive scheme. Changes in the economy over time thereby induce changes in voting patterns, and support for a given policy is dependent on expected economic conditions while in effect. To help unpack these relationships, this section defines income transitions, redistributive schemes and forward looking demand for redistribution. We analyze these concepts in the next section.

13.1. **Stochastic Income Transitions.** Individuals are indexed by \( i \in [0, 1] \) and have initial incomes \( y_{i0} \). The initial distribution of income is given by a absolutely continuous cdf \( F_0 \) with mean \( \mu_0 \). Individual \( i \)'s income at time \( t \) is denoted by \( y_{it} \) and evolves according to the following relationship:

\[
y_{it} = I(d_t(y_{i0}), \epsilon_{it})
\]  

(13.1)

where \( I \) is a continuous, increasing function relating current income to last period’s income, and \( d_t \) is a deterministic function around which income fluctuates each period due to the current period shock \( \epsilon_{it} \). Note that under this specification, the impacts of \( \epsilon_{it} \) are transitory, affecting contemporaneous income, but not future income. This simplifying assumption makes the evolution of an individuals income independent of his or her past history of shocks.

We further assume that the deterministic function is simply the period 0 expectation of time \( t \) income \( (d_t(y_{i0}) \equiv E_0[y_{it}]) \) and that the shocks \( \epsilon_{it} \) are identically distributed. Equation (13.1) then reduces to:

\[
y_{it} = I(E_0[y_{it-1}], \epsilon_{it})
\]  

(13.2)

Defining \( f(y) \equiv E[I(y, \epsilon)] \) as the expected income next period given \( y \), and taking expectations of each side of Equation (13.2) yields

\[
E_0[y_{it}] = E_0[I(E_0[y_{it-1}], \epsilon_{it})] = f(E_0[y_{it-1}])
\]  

(13.3)
Equation (13.3) shows $f(y)$ maps expected income this period to expected income next period, and recursion shows $E_0[y_{it}] = f^{(t)}(E_0[y_{i0}]) = f^{(t)}(y_{i0})$. For this reason we refer to $f$ as the income transition function. Substituting into (13.2) yields

$$y_{it+1} = I(f^{(t)}(y_{i0}), \epsilon_{it+1})$$

Equation (13.4) shows that conditional on initial income, current income fluctuates around expected income $f^{(t)}(y_{i0}) = E_0[y_{it}]$ with the same distribution.

13.2. “One-shot”/Myopic Demand for Redistribution. We now consider the political preferences of pocketbook voters whose incomes evolve according to a known income transition function $f$ of the type just outlined. Pocketbook voters choose policies which maximize their income, and for simplicity we assume voters are risk neutral. Following Benabou and Ok (2001), we define redistribution schemes composed of a flat tax $\tau$ and a lump sum transfer to all voters. Such redistribution schemes are denoted $r_{\tau}$ where if $r_{\tau}$ is enacted in period $t$, each voter $i$ receives income $r_{\tau}(y_{it})$ given by Equation (13.5).

$$r_{\tau}(y_{it}) \equiv (1 - \tau) \cdot y_{it} + \tau \cdot (1 - D)\mu_t$$

Here $\mu_t$ denotes the mean income of the population at time $t$ and $D$ denotes any dead weight loss under the redistributive scheme. Of primary interest are the “laissez faire” scheme $r_0$ and the “complete equalization” scheme $r_1$. In any case, it is clear the poor will tend to prefer $r_1$ to $r_0$.

Election timing plays an important role in the demand for redistribution. Consider a majoritarian vote taken between $r_1$ and $r_0$ in period $t$. Also suppose voting takes place ex ante before period $t$, so voter $i$ prefers $r_1$ to $r_0$ exactly when $E[y_{it}] = f^{(t)}(y_{i0}) \leq (1 - D)\mu_t$. Therefore all voters $i$ of the same initial income level $y_{i0}$ vote identically. We assume that voting takes place ex ante over redistribution, with the unrealistic consequence that if $f$ is known perfectly, voters with the same initial income $y_{i0}$ vote identically.\footnote{In contrast, the vote could be modeled as ex post after the realization of the shocks $\epsilon_{it}$. Then voter $i$ prefers $r_1$ to $r_0$ exactly when $y_{it} \leq (1 - D)\mu_t$ which from Equation (13.4) is when $I(f^{(t-1)}(y_{i0}), \epsilon_{it}) \leq (1 - D)\mu_t$. Therefore in the ex post case a voter’s redistributive preferences depend on the outcome of idiosyncratic shocks.}

The demand for redistribution will clearly depend on the distribution of income each period. The most important aspect of the income distribution is the distribution of expected time-$t$ income,
\[ F_t(y) = \Pr(E[y_{it}] \leq y) \]

Given a continuum of individuals \( i \) with incomes \( y_{i0} \) that follow a distribution \( F_0(y) = \Pr(y_{i0} \leq y) \) at time 0, we have

\[ F_t(y) = \Pr(E[y_{it}] \leq y) = F_0(f^{(-t)}(y)) \]

“One-shot” redistributive preferences are the proportion of the population who, sitting at time \( t-1 \) and taking an ex ante vote, would prefer \( r_1 \) to \( r_0 \) at period \( t \). Thus one-shot preferences would reflect repeated annual polling for policies which last one year. Voter \( i \) will prefer \( r_1 \) to \( r_0 \) at time \( t \) if and only if their expected income at time \( t \), \( E[y_{it}] = f^{(t)}(y_{i0}) \) is less than the redistributed income at time \( t \), \((1 - D)\mu_t\). Therefore the income of an individual who is indifferent between \( r_1 \) and \( r_0 \) at time \( t \) is \((1 - D)\mu_t\) and his income at time 0 is \( f^{(-t)}((1 - D)\mu_t) \).

Finally, within our linear taxation and redistribution scheme, if a voter prefers \( r_1 \) to \( r_0 \) he or she prefers \( r_\tau \) to \( r_{\tau'} \) for any \( \tau > \tau' \). Thus the conditions which summarize redistributive preferences at period \( t-1 \) are given by Equation (13.6).

(13.6)

\[
\begin{align*}
\text{Policy Preferences} & \quad \text{Income Cutoff:} \quad \text{% of Population:} \\
\text{at time } t-1 & \quad f(y_{it-1}) \leq (1 - D)\mu_t \quad F_0(f^{(-t)}((1 - D)\mu_t)) \\
& \quad f(y_{it-1}) \geq (1 - D)\mu_t \quad 1 - F_0(f^{(-t)}((1 - D)\mu_t))
\end{align*}
\]

Equation (13.6) reveals that \( f^{(-t)}(\mu_t) \) dictates the proportion of the population who wants redistribution, \( F_0(f^{(-t)}(\mu_t)) \). Clearly \( F_0(f^{(-t)}(\mu_t)) \) depends on the initial distribution of income \( F_0 \) as well as the sequence \( \{\mu_t\} \) which is fixed by \( \mu_t = \int f^{(t)}(y)dF_0(y) \). Since voters’ preferences evolve with \( f^{(-t)}(\mu_t) \), the outcome of an election depend not only on the current period of the vote, but the longevity of the policies presented to voters. For instance, if the demand for one period redistribution will follow an upward trend for the next decade, a ten year redistributive policy should garner more immediate support than a one or five year policy. This is because longer policies force voters to commit to what they want “most of the time” through aggregation of their income dynamics.

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\[ \text{Note that in principal the distribution of expected time-t income could vary depending on the information available in different periods. Our assumed structure relieves us of these complications since the expected income of an individual at time } t \text{ becomes deterministic.} \]
13.3. **Forward Looking Demand for Redistribution.** We now consider redistributive preferences for policies that last from period 0 through period $T$. We assume voters have additively separable utility with discount rate $\delta$. The discounted stream of income a voter $i$ receives from period 0 to $T$ depends on both initial income $y_{i0}$ and a history of idiosyncratic shocks:

\[
\text{Discounted Income Stream} = \sum_{t=0}^{T} \delta^t y_{it} = \sum_{t=0}^{T} \delta^t f^{(t-1)}(y_{i0}, \epsilon_{it}) 
\]

For convenience we introduce a function $g^T(y)$ which is the expected discounted income of a voter with initial income $y$. Following from Equation (13.7) and $E_0[y_{it}] = f^{(t)}(y_{i0})$, $g^T$ is given by Equation (13.8)

\[
\text{Expected Income Stream} = g^T(y_{i0}) = E_0 \left[ \sum_{t=0}^{T} \delta^t y_{it} \mid y_{i0} \right] = \sum_{t=0}^{T} \delta^t f^{(t)}(y_{i0}) 
\]

For notational convenience we also denote $(g^T)^{-1}$ by $g^{-T}$. Similarly, define $\mu^T$ to be the discounted mean of population income over periods 0 to $T$ given by

\[
\text{Population Expected Income Stream} = \mu^T = \int E_0[\sum_{t=0}^{T} \delta^t y_{it}] dF_0(i) = \sum_{t=0}^{T} \delta^t \mu_t 
\]

Redistributive preferences for a policy which takes effect in period 0 and continues through period $T$ take into account discounting and the evolution of income during the policy. An individual with income $y_{i0}$ in period 0 receives a discounted income stream of $g^T(y_{i0})$ and if complete redistribution is enacted would receive a discounted income stream of $(1 - D)\mu^T$. Consequently, this individual prefers redistribution over periods 0 to $T$ if and only if $g^T(y_{i0}) \leq (1 - D)\mu^T$. This implies redistributive preferences when considering periods 0 to $T$ as given by Equation (13.9) which is analogous to Equation (13.6) for “one-shot” redistribution.

\[
\text{Policy Preferences at time } t - 1 = \begin{cases} 
\text{Income Cutoff:} & \text{\% of Population:} \\
\text{Redistribute,} & g^T(y_{i0}) \leq (1 - D)\mu^T \quad F_0(g^{-T}((1 - D)\mu^T)) \\
\text{Laissez Faire,} & g^T(y_{i0}) \geq (1 - D)\mu^T \quad 1 - F_0(g^{-T}((1 - D)\mu^T)) 
\end{cases}
\]

Equation (13.9) shows that the fraction of the population who wants redistribution, $F_0(g^{-T}(\mu^T))$ changes as longer periods of policy commitment are considered. $g^{-T}(\mu^T)$ is a generalized average of the terms $f^{(-t)}(\mu_t)$ which determine the demand for redistribution in the “one-shot” case.
Therefore the role of the initial income distribution $F_0$ is much the same: it affects the demand for redistribution though $\mu^T$ and small perturbations in $F_0$ can give rise to large changes in the demand for redistribution in the presence of poverty traps, especially for large $T$. In the next section we consider how the shape of $f$ determines changes in the demand for redistribution.

14. Political Evolution with Forward-looking Voters under Full Information

The Solow model of neoclassical economic growth relies on an assumption of diminishing capital returns to hypothesize that poorer nations will tend to catch up over time, or converge, with the incomes of richer nations. When transported to the individual or microeconomic level, the Solow assumptions imply a process of convergence among the population of a single country.

![Figure 14.1. POUM and NoPOUM Income Transitions](image)

Figure 14.1(a) illustrates a typical income dynamic implied by accumulation under decreasing returns. Note that this concave transition process implies a unique long-term or steady state income level, $y^*$, at the point where $f_c(y)$ crosses the 45-degree line. Under this transition process, individuals who begin with incomes below the steady state level will converge towards it, while those who begin above the steady state level will drop back towards it. Note that this sort of concave income distribution process offers prospects of upward mobility (POUM) to voters whose initial income levels are less than the steady state income level.

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[^5]: For an early review of both the theoretical and empirical controversies, see [Romer, 1994](#). A more recent review with a theoretical emphasis is [Azariadis and Stachurski, 2005](#).
This Prospect of Upward Mobility for the poor to achieve convergence with the population at large can serve to lessen preferences for redistribution. This mechanism allows Benabou and Ok (2001) to connect POUM income dynamics and aversion to redistribution. Subsequent evidence for the income based approach to voting has been mixed, and its reach can be extended by modeling more general income dynamics.\footnote{Fong (2001) finds that variables reflecting a personal benefit from redistribution are insignificant in predicting redistributive preferences in the US. On the other hand, Checchi and Filippini (2004) conduct experiments, finding some support that the POUM reduces chosen taxation rates and that longer time horizons tend to decrease chosen rates under POUM. Beckman and Zheng (2003) also find tentative support for the POUM hypothesis using surveys administered to undergraduates (primarily business and economics majors). At the international level, Wong (2002) examines the GSS and World Values Survey for redistributive preferences and finds the expected signs across incomes, but find no evidence of the “tipping behavior” implied by median voter or POUM models and that current. Wong also finds that expected income indicators are small in magnitude in explaining redistributive preferences.}

In contrast to Figure 14.1(a), individuals need not face uniformly decreasing returns in asset accumulation. The increasingly well developed theory of poverty traps suggests a number of mechanisms that can trap households at low living standards (see the review in Carter and Barrett (2006)). Central to all of these theories of poverty traps is exclusion from financial markets.\footnote{There is now a plethora of theory about why financial markets are often thin, missing and, or biased against low wealth agents. Banerjee and Newman (1994) provide an example of a debt based poverty trap which generates income dynamics. For a recent contribution, see Boucher et al. (2007).} Put differently, if households have access to loan markets and insurance instruments, then even when confronted by locally increasing returns to scale and risk, they can successfully engineer a strategy to obtain the assets needed to jump to a high level equilibrium. But absent access to those financial markets, households below a critical initial asset level will remain stuck in a low level, poverty trap equilibrium.

The result of such poverty trap models is Figure 14.1(b) which illustrates income transition dynamics with multiple steady states.\footnote{For empirical examples see Lybbert et al. (2004) and Adato et al. (2006). Banerjee and Newman (2000) construct a macroeconomic model of “dynamic institutional change” which implies non-concave income dynamics and exhibits path dependence on the distribution of wealth.} The non-concave income transition function, \( f_n(y) \), maps incomes in period \( t \) into incomes in period \( t + 1 \). As can be seen, this non-concave transition function has multiple crossings of the 45-degree line and thus admits multiple equilibria: \( y^*_H \) is the high income steady state; \( y^*_L \) is the low level steady state. Bifurcation occurs around the unstable point \( y_b \). Households that being with incomes in excess of \( y_b \) will tend toward the high level equilibrium while those that begin below this critical threshold will head towards the low level,
poverty trap equilibrium, \( y^*_L \). This implies No Prospect of Upward Mobility (NoPOUM) for voters below the threshold \( y_b \). In contrast to an economy with a concave income transition function, economic polarization will occur and inequality can deepen if income transitions are governed by a non-concave function like \( f_n(y) \).

This section considers any continuous income transition function, allowing for both \( f_c \) and \( f_n \) types of income transitions and then derives a general set of results with political implications. We show that relaxing Benabou and Ok’s assumption of concavity can generate rich patterns in the demand for redistribution. We then provide a theorem showing how these new income transitions create both increases and decreases in the demand for redistribution, even when not concave or convex. This result generalizes the political implications of income transition dynamics by analyzing their upper and lower envelopes.

14.1. Demand for Redistribution. From Section 2, the fraction of the population who demands redistribution at time \( t \) is given by \( F_0(f^{(-t)}(\mu_t)) \). It follows that the path of redistributive preferences, namely if they are increasing or decreasing is determined by \( f^{(-t)}(\mu_t) \). In a POUM world, the concavity of \( f \) implies (through Jensen’s inequality) the relationship \( f^{(-(t+1))}(\mu_{t+1}) \leq f^{(-t)}(\mu_t) \) in each period so the demand for redistribution is always decreasing. Similarly if voters are forward looking, the fraction of the population who wants redistribution \( F_0(g^{-T}(\mu^T)) \) monotonically decreases as the duration of a policy increases. Therefore in a POUM world, the demand for redistribution decreases with time in two senses: as “one-shot” evaluations each period and as policy longevity increases. This is the type of behavior that Benabou and Ok set out to explain. We summarize these two salient aspects of redistributive dynamics in a POUM world in a Corollary.

**Corollary.** [POUM Dynamics] Let \( F_0 \) be absolutely continuous and suppose \( f \) is concave. Then:

1. The demand for “one-shot” redistribution decreases over time.
2. The demand for redistribution over a \( T \) period horizon decreases in \( T \).

However, non-convexities under general income transitions can break the POUM relationship leading to more complex redistributive dynamics. In contrast to a POUM world in which the

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57 Strictly speaking, this non-concave income transition function implies increasing polarization, not necessarily increasing inequality, as Esteban and Ray (1994) discuss.

58 These statements follow respectively from Proposition 2 and Theorem 3 of Benabou and Ok (2001).
path of $F_t(\mu_t)$ is decreasing, in a NoPOUM world this path can be increasing in the presence of non-convexities, and need not even be monotone. Thus, the twist in a NoPOUM world versus a POUM world is more complex movement in the demand for redistribution. In the appendix we show that for a large class of income dynamics, the demand for redistribution is increasing or decreasing depending on the initial distribution of income. We illustrate the point here with a concrete example.

Consider an income distribution $F_0$ composed of three equally sized groups with incomes $Y_1, Y_2, Y_3$ where $0 < Y_1 < Y_2 < Y_3 < \bar{Y}$. Then $\mu_t = \sum(1/3) \cdot f^{(t)}(Y_i)$ and so clearly depends on the initial distribution of income $F$ interacting with the evolution of group incomes $f^{(t)}(Y_i)$. Small changes in $F$ can also cause large qualitative changes in the demand for redistribution over time. Now also suppose $f$ has three fixed points $Y_{\text{Trap}} < Y_{\text{Escape}} < \bar{Y}$ where for $y < Y_{\text{Escape}}$, $f^{(t)}(y) \rightarrow Y_{\text{Trap}}$ and for $y > Y_{\text{Escape}}$, $f^{(t)}(y) \rightarrow \bar{Y}$. In this case all groups are going to either $Y_{\text{Trap}}$ or $\bar{Y}$. To fix ideas, assume $Y_1 < Y_{\text{Escape}} < Y_3$ so the $Y_1$ group converges to $Y_{\text{Trap}}$ while $Y_3$ converges to $\bar{Y}$. Clearly $Y_1$ prefers the complete redistribution scheme $r_1$ while $Y_3$ prefers $r_0$. This leaves open the middle class of “swing voters” $Y_2$. If $Y_2 > Y_{\text{Escape}}$ then the middle class eventually climbs the income ladder to $\bar{Y}$ and joins the (now majoritarian) voting block of $Y_3$. Otherwise, if $Y_2 < Y_{\text{Escape}}$ there is a thinning of the middle class and swing voters eventually join with $Y_1$, implying the median voter prefers redistribution. Note the fragility of the eventual voting outcomes: a small income difference $\delta$ in $Y_2$ can push $Y_2 + \delta$ to be greater or less than $Y_{\text{Escape}}$. This eventually results in a large fraction $p_2$ of swing voters to switch their vote as income evolves. Similar consequences can arise if voters lack perfect information about $F_0$ or $f$ so that small changes in beliefs can give rise to large changes in redistributive preferences.

To better describe this complex process, we connect the changing demand for redistribution to the upper and lower envelopes of an income dynamic. Our characterization explains POUM as a special case. However the characterization shows that when income dynamics exhibit convexity around the average income then NoPOUM occurs and the demand for redistribution increases. The take away message is that unlike the POUM world, the demand for redistribution is not a foregone conclusion and may manifest in volatile political patterns. This points to evaluating the relationship between income dynamics and political choices in light of the conditions voters face on a country specific basis.
Our characterization relies on the current location of mean income $\mu_t$ relative to the income transition $f$. We first define the upper envelope of $f$, $U$ as the envelope created by tracing all lines which are above, but do not cross $f$. The lower envelope $\underbar{U}$ is defined similarly. Both types of envelopers are illustrated in Figure 14.2(b). Formally, these envelopes are defined in Equations (14.1).

\[
U \equiv \inf \{h(x) : h \text{ is a line and } h \geq f\} \quad \underbar{U} \equiv \sup \{h(x) : h \text{ is a line and } f \geq h\}
\]

Clearly for each $y$ we have $\underbar{U}(y) \geq f(y) \geq U(y)$ and necessarily $\overline{U}$ is concave and $\underline{U}$ is convex. Two special cases stand out. When $f$ is concave, $\overline{U}$ and $f$ coincide. When $f$ is convex, $\underline{U}$ and $f$ coincide. Therefore in a POUM world, $\overline{U} = f$. We define the sets of incomes where $f$ and $\overline{U}$ exactly coincide as $\pi$. Similarly define $\underline{u}$ as incomes where $f$ and $\underline{U}$ coincide. The relationship of $\pi$ and $\underline{u}$ to the path of redistributive preferences is given in Proposition 10.

**Proposition 10.** If $\mu_t \in \pi$ then the demand for redistribution decreases in period $t$ relative to period $t - 1$. Conversely, if $\mu_t \in \underline{u}$ then the demand for redistribution increases.

**Proof.** We will consider $\mu_t \in \pi$ as the other case is similar. We want to show that $f^{(-t+1)}(\mu_{t+1}) \leq f^{(-t)}(\mu_t)$. This holds iff $\mu_{t+1} \leq f(\mu_t)$ and by assumption $\overline{U}(\mu_t) = f(\mu_t)$ so we want that $\mu_{t+1} = f^{(-t+1)}dF_0 \leq \overline{U}(\mu_t)$. Since $f \leq \overline{U}$ we know that $\int f^{(-t+1)}dF_0 \leq \int \overline{U} \circ f^{(-t)}dF_0$ and so it is enough that $\int \overline{U} \circ f^{(-t)}dF_0 \leq \overline{U}(\mu_t) = \overline{U}(\int f^{(-t)}dF_0)$ which follows from Jensen’s inequality. \qed
Proposition 10 says that if the mean income next period $\mu_t$ lies in the region $\pi$ corresponding to the upper envelope, the demand for redistribution decreases. Conversely, if $\mu_t$ lies in the lower envelope region, the demand for redistribution increases. In this sense, the upper and lower envelopes of $f$ are natural definitions of Right and Left income transitions based on $f$. This also highlights the differences between POUM and NoPOUM income dynamics. In a POUM world, $f$ is concave and therefore corresponds to $\overline{U}$. This implies that any $\mu_t$ always lies in the upper envelope region so the demand for redistribution is always decreasing (See Figure [14.2a]). In contrast a NoPOUM income dynamic has regions which coincide with both the upper and lower envelopes. Depending on where $\mu_t$ lies the next period, the demand for redistribution can either increase or decrease (See Figure [14.2b]). In the next section we apply this insight about curvature to actual income dynamics in Latin America.

15. Demand for Redistribution in Latin America

We have just seen from the prior section that political dynamics for forward looking voters will depend on both the income transition and the initial distribution of income $F_0$. This section asks if these two considerations can help us understand recent electoral dynamics in Latin America.59

Based on income decile data from Latin American national surveys we estimate simple income dynamics and evaluate the implications in light of the above results.60 We emphasize here that the estimates are very rough, leaving details to the Appendix. The results for Chile and Peru are presented graphically in Figure [15.1]. The dashed diagonal line is the 45 degree line representing the break-even points on the income transition function. A benchmark of five years was chosen for graphical purposes as this roughly corresponds to the presidential election cycle in the countries we consider. Vertical lines fixed by the mid-points between first year deciles divide the ten income bins used for the estimation. Finally, the solid line represents the estimated income dynamic ($\hat{f}$) over five years (in other words, $\hat{f}(5)$) for each country. As can be seen, the estimated dynamics for Peru show convexity for much of the income distribution and therefore exhibit NoPOUM

59See (Lora and Olivera, 2005) for a review of the empirical literature and estimates of the political impact of specific economic policies. More recently, (Greene and Baker, 2009) construct vote-revealed leftism (VRL) from ideological ratings of presidents and parliamentary parties in Latin America from 1996-2008, showing that the left has a clear economic policy mandate to halt or partially reverse recent neoliberal economic policies.

60The data comes from SEDLAC (CEDLAS and The World Bank) and includes both monetary and non-monetary income as well as private/public Transfers and Imputed rent.
dynamics. In contrast, the estimated dynamics for Chile show at least some prospects for absolute if not relative, mobility for all deciles of the income distribution.

**Figure 15.1. Estimated Income Transition Functions**

As shown above, the nature of political preferences by forward-looking voters will depend fundamentally on the shape of the income distribution dynamic. Figure 15.2 graphs the demand for redistribution in Chile and Peru for policies which last for various lengths of time. In order to calculate these outcomes, we begin with the estimated income dynamic \( \hat{f} \) for each country and an estimate of the distribution of income \( \hat{F}_0 \) which is interpolated from the data. Computing the demand for redistribution across time and for varying policy lengths then follows the development above. Explicitly, we calculate the estimated analogue of Equation (13.9) by setting \( \hat{g}^T(y_{i0}) \equiv \sum_{t=0}^{T} \delta^t \hat{f}^{(t)}(y_{i0}) \) for \( \delta = .95 \) and solving Equation (15.1) for each period and a range of values for \( T \):

\[
\text{(15.1) Fraction of Population by Preference} = \begin{cases} 
\text{Redistribute,} & \hat{F}_0(\hat{g}^{-T}((1-D)\mu^T)) \\
\text{Laissez Faire,} & 1 - \hat{F}_0(\hat{g}^{-T}((1-D)\mu^T))
\end{cases}
\]

Figure 15.2 therefore illustrates the relationship of estimated income dynamics to redistributive preferences. At present we assume that redistribution incurs no dead weight loss so that \( D = 0. \)
We now connect the income dynamics of Figure 15.1 to the demand for redistribution in both countries at the national level, associating the demand for redistribution with the politically Left.

First, consider the “one-shot” demand for redistribution in each country, or rather the fraction of voters who would prefer redistribution for one year. Chile shows a fairly linear pattern, which implies fairly flat redistributive preferences over the period as in Figure 15.2(a). In contrast, Figure 15.1 shows fairly convex income dynamics in Peru which imply an increasing demand for redistribution as we see.

Second, consider the demand for redistribution as the longevity of the redistributive period increases. Again we see barely any change in Chile, as implied by a fairly linear income dynamic. The convex dynamics for Peru imply redistributive demand increases with policy longevity: future redistributive pressures are averaged in with immediate demand for redistribution.

Consistent with the theory, the leftward political trend in each country facing income dynamics of Figure 15.2 is supported by survey evidence of ideological polarization and shifting perceptions of household economic status. However, Figure 15.2 also shows that existing levels of inequality at the national level should easily have created majoritarian support for redistribution in all time

\[ \text{Time trends for these survey questions across the countries are depicted in the Supplemental Appendix.} \]
periods and countries. Thus the leftward shift observed in countries was late in coming if voters were aware of the income dynamics they faced. To put this in perspective, we consider the level of dead weight taxation loss that would provide majoritarian support for laissez faire policies. The levels of dead weight loss corresponding to Figure 15.2 are 46-51% in Chile and 45-51% in Peru which are all exceedingly high in comparison to existing estimates of dead weight loss (See for instance Olken (2006)). This rules out dead weight loss as the sole explanation of support for laissez faire.

We address this issue in the next two sections by modeling voters as Bayesian learners who started with a POUM prior as advocated by the Washington Consensus in the early 1990’s but who have updated their beliefs to reflect their experience, creating a rapid shift in favor of redistribution.

16. Learning and Voting Under Imperfect Information

While the analysis here helps make sense of the recent course of politics in Latin American countries under general income dynamics, our model predicts extremely high levels of preference for redistribution, even by the mid-1990s. This finding thus throws into sharp relief the question as to why so many voters voted for largely laissez faire policies prior to the early part of this century. One explanation of course is that the political space was tightly constricted during that initial time period, or perhaps that voters were simply fooled and voted against their economic self-interest

Another, perhaps complementary, explanation is to recognize the intrinsic complexity of income distribution dynamics. Income dynamics and prospects of (no) upward mobility are complex and hard to understand, especially in economies that have altered their economic model. Our analysis so far has followed Benabou and Ok and assumed that voters know the true income transition function and use this knowledge to construct their forward looking income forecast and vote accordingly. We now generalize this approach and model the evolving political preferences of voters who optimally learn from their own experience and continually update their understanding or mental model of the income distribution process in their economy.

To keep this problem manageable, we assume voters face a known family of possible income transition functions $I_\lambda(y, \epsilon)$ indexed by the parameter $\lambda$. Analogous to our earlier analysis, for

---

62For consideration of a host of additional possibilities, see Puterman (1996). A careful study of how economic information is mediated by larger sets of social relations has be conducted by Herrera (2005) in the context of post-USSR Russia.
any value of $\lambda$, we can define the expected income transition as

$$f_\lambda(y) \equiv \int I_\lambda(y, \epsilon) dP(\epsilon)$$

where as before the individual idiosyncratic income shock $\epsilon_{it}$ has distribution $P$. The family of expected income transitions is assumed to be bracketed by two extreme specifications, one representing a right perspective or vision of how the economy operates ($\lambda = 1$) and the other a left perspective ($\lambda = 0$). We refer to these specifications as “ideologies,” using this word to denote a model or understanding of how the world works. We assume that the any income transition function can be expressed as a linear combination of the left ($f_L$) and the right ($f_R$) ideologies:

$$f_\lambda(y) = (1 - \lambda)f_L(y) + \lambda f_R(y) \quad (16.1)$$

At any point in time $t$, the individual’s understanding of the economy can be represented by a probability distribution $\pi_{it}(\lambda)$ over possible values of $\lambda$ while the true value of $\lambda$, labeled $\lambda_0$, is unknown. The individual’s expected income in period $t + 1$ thus becomes:

$$E[y_{it+1}|y_{it}, \pi_{it}] = \int_0^1 E[y_{it+1}|y_{it}, \lambda]\pi_{it}(\lambda) d\lambda = \int_0^1 f_\lambda(y_{it})\pi_{it}(\lambda) d\lambda$$

Note that this specification naturally describes someone with a left view of the world as putting a lot of probability weight on low or left values of $\lambda$, whereas a right view of the world would have probability massed near the right side of the spectrum or 1. This specification of how voters predict their future income under incomplete information will be incorporated into our model of forward-looking voters. However, we first consider how the critical new element, the voter’s probability distribution over $\lambda$, is formed and evolves over time.

16.1. A Bayesian Model of Voter Learning. We assume each voter $i$ begins with a prior distribution $\pi_{i0}(\lambda)$ over possible values of $\lambda$ while $\lambda_0$, the true value of $\lambda$, is the same for all voters but is unknown. We also assume that voters keep track of their idiosyncratic income histories $H_{it} = \{y_{i0}, \ldots, y_{it}\}$. The history $H_{it}$ is used to update beliefs each period to a posterior belief $\pi_{it}(\lambda|H_{it})$ according to Bayes rule. In our context, we can think of $\pi_{i0}(\lambda)$ as an initial ideology a voter has about the income transitions they face, while $\pi_{it}(\lambda|H_{it})$ is the voter’s new ideology after $t$ periods of learning the true income dynamic.
In order to make this learning process concrete, we now analyze it assuming an explicit structure of the transient income shocks and their relationship to income each period in Assumption 2.

**Assumption 2 (Personal Dynamics).** The income dynamic each voter faces is characterized by:

1. **At the true value of \( \lambda \), incomes are**
   \[ y_{it+1} = f^{(t)}(y_{i0})\epsilon_{it+1}. \]
2. **The shock \( \epsilon_{it+1} \) is distributed Uniform(1 - \( \sigma \), 1 + \( \sigma \)) for some \( \sigma \in (0, 1) \).**
3. **Voters know the value of \( \sigma \).**

Under Assumption 2 if the true value of \( \lambda \), namely \( \lambda_0 \), were known then
\[ E[y_{it+1}] = f^{(t)}(y_{i0}) \]
since \( E[\epsilon_{it+1}] = 1 \). This coincides exactly with the perfect information case. Therefore the income dynamic specified in Assumption 2 can be interpreted as receiving some random percentage of the “true” income one should receive given \( \lambda \). Depending on the magnitude of \( \sigma \), the amount of fluctuation is large or small.

Now consider how voters update their beliefs under Assumption 2. If \( \lambda_0 \) is the true value of \( \lambda \) then since
\[ y_{it+1}/f^{(t)}_\lambda(y_{i0}) = \epsilon_{it+1} \]
each voter knows that
\[
|y_{it+1} - f^{(t)}_\lambda(y_{i0})| / f^{(t)}_\lambda(y_{i0}) \leq \sigma
\]
Equation (16.2) encapsulates the fact that a voter knows his observed income \( y_{it+1} \) must be within \( \sigma \)% of expected income \( f^{(t)}_\lambda(y_{i0}) \) under \( \lambda_0 \). Therefore any \( \lambda \) for which
\[ |y_{it+1} - f^{(t)}_\lambda(y_{i0})| / f^{(t)}_\lambda(y_{i0}) > \sigma \]
\( \sigma \) cannot be the true value of \( \lambda \). Eliminating these impossible values of \( \lambda \) is exactly what Bayes rule dictates as the updating rule.

**16.2. Demand for Redistribution under Imperfect Information.** In general, neither \( f_L \) nor \( f_R \) need reflect reality but rather idealized versions of what Left and Right ideologues might represent in a manifesto. Clearly the relative strength of a voter’s belief in these world views influences their voting behavior. These beliefs are measured by \( \pi_{it}(\lambda|H_{it}) \) which connect ideology to voting. In order to emphasize the role of beliefs in deriving a voter’s expected income, we now illustrate the decisions of a pocketbook voter who is also a Bayesian learner.

If a voter knows the true value of \( \lambda \), namely \( \lambda_0 \), then expected income in period 1 is given by \( E[y_{i1}|y_{i0}, \lambda] = f_{\lambda_0}(y_{i0}) \). However, each voter does not know \( \lambda_0 \) with certainty but has a prior distribution \( \pi_{i0}(\lambda) \) over possible values of \( \lambda \). The expected income in period 1 of a voter with income \( y_{i0} \) is a weighted average of income over plausible values of \( \lambda \), namely \( E[y_{i1}|y_{i0}, \lambda] = \)
\[
f_{\lambda}(y_{i0}) \text{ weighted by } \pi_{i0}(\lambda). \text{ Specifically,}
\]
\[
E[y_{i1} | y_{i0}, \pi_{i0}] = \int_0^1 E[y_{i1} | y_{i0}, \lambda] \pi_{i0}(\lambda)d\lambda = \int_0^1 f_{\lambda}(y_{i0}) \pi_{i0}(\lambda)d\lambda
\]

At the end of periods 1 to \( t \), a voter updates his prior \( \pi_{i0}(\lambda) \) to a posterior \( \pi_{it}(\lambda) \) using his new history \( H_{it} = \{y_{i0}, \ldots , y_{it}\} \). Therefore expected income in period \( t+1 \) is given by

\[
(16.3) \quad E[y_{it+1} | H_{it}, \pi_{i0}] = \int \underbrace{f_{\lambda}^{(t+1)}(y_{i0})}_{\text{Expected Income} | \lambda} \cdot \underbrace{\pi_{it}(\lambda|H_{it})}_{\text{History Dependent Beliefs}} d\lambda
\]

Equation \( (16.3) \) highlights the two dynamic factors which influence a voter’s beliefs about expected income. The first element, expected income given \( \lambda \) is the true state of the world, is deterministic as in the first part of this paper. The second element, a voter’s ideological beliefs, evolve as information is collected in the form of the idiosyncratic income history \( H_{it} \).

As in Section 3, each voter has a future expected income of \( E[y_{it+1} | H_{it}, \pi_{i0}] \) next period while he believes the mean income next period will be \( E[\mu_{t+1} | H_{it}, \pi_{i0}] = \int \int E[y_{it+1} | y_{i0}, \lambda] \pi_{it}(\lambda)d\lambda dF(y_{i0}) \). Therefore after accounting for any dead weight loss \( D \), a voter will prefer \( r_1 \) to \( r_0 \) if and only if

\[
(16.4) \quad \int E[(1 - D)\mu_{t+1} - y_{it+1} | y_{i0}, \lambda] \pi_{it}(\lambda)d\lambda \geq 0
\]

Since \( E[(1 - D)\mu_{t+1} - y_{it+1} | y_{i0}, \lambda] \) is the expected transfer under \( r_1 \) given \( \lambda \) is the true state of nature, \( r_1 \) is preferred to \( r_0 \) whenever the expected transfer is positive, given a voter’s income and beliefs. Note that two voters with the same initial incomes need not have the same redistributive preferences: whether Equation \( (16.4) \) holds depends on each voter’s income history through their ideology \( \pi_{it}(\lambda) \). This implies the popularity of redistributive policies varies in a non-trivial way across initial incomes. Voter preferences conditional on their history \( H_{it} \) are summarized in Equation \( (16.5) \).

\[
(16.5) \quad \text{Redistributive Preferences} \mid y_{i0}, H_{it} = \begin{cases} \text{Redistribute}, & \int E[(1 - D)\mu_{t+1} - y_{it+1} | y_{i0}, \lambda] \pi_{it}(\lambda)d\lambda \geq 0 \\ \text{Laissez Faire}, & \int E[(1 - D)\mu_{t+1} - y_{it+1} | y_{i0}, \lambda] \pi_{it}(\lambda)d\lambda \leq 0 \end{cases}
\]
Equation (16.5) allows us to make a clear connection from ideological beliefs to demand for redistribution through the following assumption:

**Assumption 3.** Increases in $\lambda$ imply relative income position improves ($\frac{d}{d\lambda} f^{(t)}(y_{i\lambda}) \geq \frac{d}{d\lambda} E[\mu_{t+1}|\lambda]$) for all swing voters defined as $y_{i\lambda} \in \left[ f^{(-t)}(E[\mu_{t+1}|\lambda = 1]), f^{(-t)}(E[\mu_{t+1}|\lambda = 0]) \right]$. This Assumption says that as $\lambda$ increases (moves to the Right), each voter believes his expected income $f^{(t)}(y_{i\lambda})$ increases relatively more than mean income $E[\mu_{t+1}|\lambda]$. Furthermore, we only require this to hold for voters who might potentially change their vote: the votes of both destitute ($f^{(t)}(y_{i\lambda}) < E[\mu_{t+1}|\lambda]$ for all $\lambda$) and well-to-do ($f^{(t)}(y_{i\lambda}) > E[\mu_{t+1}|\lambda]$ for all $\lambda$) are unaffected by belief.

Assumption 3 implies the expected transfer $E[\mu_{t+1} - y_{it+1}|y_{i\lambda}, \lambda]$ is decreasing in $\lambda$. It follows that for a voter $j$ with beliefs $\pi_j(\lambda)$ “to the Right” of a voter $i$ with beliefs $\pi_i(\lambda)$ that voter $j$ tends to prefer less redistribution than voter $i$. To make this precise, assume that $\pi_j$ stochastically dominates $\pi_i$ and $y_{j\lambda} = y_{i\lambda} = y_0$. Since $E[\mu_{t+1} - y_{it+1}|y_{j\lambda}, \lambda] = E[\mu_{t+1} - y_{it+1}|y_{i\lambda}, \lambda]$ is decreasing in $\lambda$, the stochastic dominance of $\pi_j$ over $\pi_i$ implies

$$\int E[\mu_{t+1} - y_{it+1}|y_0, \lambda] \pi_j(\lambda) d\lambda \leq \int E[\mu_{t+1} - y_{it+1}|y_0, \lambda] \pi_i(\lambda) d\lambda$$

Therefore in the absence of dead weight loss $D$, voter $j$ prefers less redistribution than voter $i$. This result which connects ideological belief to redistributive demand continues to hold for any level of dead weight loss provided $f_R \geq f_L$ and is summarized as Proposition 11.

**Proposition 11.** Suppose $f_R \geq f_L$ and Assumption 3 holds. If voters $i$ and $j$ are identical except voter $j$’s belief $\pi_j$ stochastically dominate voter $i$’s belief $\pi_i$ then $j$ prefers less redistribution than $i$.

Proposition 11 shows that that further to the ideological Right a voter is, the less redistribution they prefer. A natural extension, of particular relevance to Latin America, is the possibility that voters’ true ideological positions are not expressed in the political process. For instance, voters may fear possible retribution for revealing their ideological “type” because of potential political policing (e.g. a potential return to dictatorship in early 1990’s Chile). In this case, one could model the ideological space in period $t$ as being constrained in an ideological spectrum of $[\lambda_{it}, \bar{\lambda}_t]$ which expands with “political thawing” and voter faith in democratic institutions. In the Latin American case, the gradual expansion of publically admissible “Left” views may also help explain the Leftward shift.
In this framework, one would expect that the speed of learning would be related to both the variability of income signals and the gap between left and right predictions for an individual’s future income position. These expectations imply a rich set of testable implications about the evolution of political preferences and voting that we hope to explore in future work.

17. THE RIGHT LEFT POLITICAL SHIFT IN LATIN AMERICA

This section employs the model of forward-looking, Bayesian voters to analyze the striking right to left political shift observed across contemporary Latin America. To do this, we first provide an empirically-grounded approach for representing left and right political ideologies. Second, we argue that economic crises of the 1980s put the left in disarray, and at the time of the market transitions voters adopted a POUUM prior as the economic crises of the 1980s left no credible alternative to the emergent pro-market model. Applying these assumptions to Peru, we show that voter learning over the course of a dozen years would be expected to generate up to a 30 percentage point shift in the fraction of the electorate preferring redistributive to free market policies.

17.1. Empirical Approximation of Left and Right Ideologies. In order to arrive at plausible left and right ideological models of income dynamics, we construct two functions \( f_R \) and \( f_L \) that literally surround the true (estimated) income transition function that we denote as \( \hat{f}(y) \). We begin by characterizing the right income transition model as one that offers greater prospects for upward mobility and implies less demand for redistribution than does \( \hat{f} \). For a given \( f_R \), we then residually construct \( f_L \) so that the true function can be expressed as a linear combination of the left and right ideologies as specified in Equation (16.1) above.

While this approach implies an element of arbitrariness, we keep our modeling options fairly open by defining \( f_R \) using a continuum of transition functions \( g_{\rho}(y) \) indexed by the parameter \( \rho \). Successively higher values of \( \rho \) correspond to more exaggerated right ideologies that promise greater upward mobility and imply less demand for redistribution. Given our method for residually calculating the left ideology, higher values of \( \rho \) also imply greater ideological polarization in the sense that the left and right positions become more sharply differentiated.

The conditions which characterize any such \( g_{\rho}(y) \) are surprisingly sharp as provided here:

**Proposition 12.** For any class of income transitions \( g_{\rho}(y) \) indexed by \( \rho \), demand for redistribution decreases in \( \rho \) for all income distributions if and only if
\[
\frac{\partial}{\partial \rho} \ln \frac{\partial}{\partial y} g_{\rho}(y) \leq 0 \] provided:
(1) $\frac{\partial}{\partial y} g_\rho(y)$ is positive and bounded.
(2) $\frac{\partial^2}{\partial y^2} g_\rho(y)$ is non-zero and continuous.

Proof. See Appendix. □

We now describe the construction of $f_R$ and $f_L$ from the empirical income transition $\hat{f}(y)$. First derive $\bar{f}(y)$, the upper envelope of $\hat{f}(y)$ (which is necessarily concave) and subtract a line $sy$ to arrive at the component $\bar{f}(y) - sy$. The component $\bar{f}(y) - sy$ is concave and therefore POUM, and is added to the upper envelope $\bar{f}$ to arrive at $f_R$. Weighting the component $\bar{f}(y) - sy$ by a factor $\rho$ and adding it to $\bar{f}$ allows us to define $f_R$ as in Proposition 12 as

\begin{equation}
(17.1) 
 f_R(y) \equiv \bar{f}(y) + \rho[\bar{f}(y) - sy]
\end{equation}

Applying Proposition 12 to Equation (17.1) we have

\begin{equation}
\frac{\partial}{\partial \rho} \frac{\partial}{\partial y} \ln \frac{\partial}{\partial y} f_R(y) = s[\bar{f}'(y) - s\bar{f}(y)](\bar{f}'(y) + \rho[\bar{f}'(y) - s])^2 \leq 0
\end{equation}

Therefore our definition of $f_R$ implies lower demand for redistribution as $\rho$ increases. $f_R$ is therefore a POUM dynamic, which grows increasingly POUM as $\rho$ increases. As this suggests, $f_R$ defined in Equation (17.1) implies that voters who find $f_R$ credible demand less redistribution than under $\hat{f}$.

In order to find $f_L$, we fix the true value of $\lambda$ to $1/2$, implying $\hat{f}(y) = \frac{1}{2}f_R(y) + \frac{1}{2}f_L(y)$. Therefore $f_L$ is fixed by the Right transition $f_R$ and the empirical transition $\hat{f}$ as

\begin{equation}
(17.2) 
 f_L(y) = 2\hat{f}(y) - \bar{f}(y) - \rho[\bar{f}(y) - sy]
\end{equation}

$f_L(y)$ is the empirical transition $\hat{f}(y)$ plus a component $[\hat{f}(y) - \bar{f}(y)] - \rho[\bar{f}(y) - sy]$, so that if $\hat{f}$ is POUM then $f_L(y) = \bar{f}(y) - \rho[\bar{f}(y) - sy]$, the mirror of $f_R$. Voters finding $f_L$ credible demand more redistribution than under $\hat{f}$. Furthermore, this redistributive gap between $f_R$ and $f_L$ increases with $\rho$, making $\rho$ an index of the ideological polarization between Right and Left positions.

\footnote{The line $sy$ ensures the positivity of $f_L$, and the second derivative of $sy$ vanishes so curvature is unaffected.}
Figure 17.1 illustrates the application of this approach to Chile and Peru. Using the estimated income transition functions described in the appendix, we derive the right ideology assuming that $\rho_{\text{Chile}} = 80$ and $\rho_{\text{Peru}} = 5$ which are the largest values that imply $f_L$ is positive.\(^{64}\)

**Figure 17.1. Stylized Right and Left Income Ideologies**

(A) Chile Transitions

(B) Peru Transitions

17.2. **Learning and Political Dynamics under a ‘TINA’ Prior.** The final element needed to permit analysis of Latin American political dynamics is a specification of voters’ initial beliefs about the prospects, or lack thereof, for upward mobility at the beginning of the 1990s. To illustrate the implications of our model, we take seriously the then common observation that there was an exhaustion of credible political alternatives to a liberal economic regime. As Margaret Thatcher famously intoned: “TINA—there is no alternative” to free markets (Margaret Thatcher Foundation, 1984-85). While perhaps an exaggeration, Thatcher’s statements motivates what we call the TINA prior, meaning an initial set of beliefs, $\pi_{10}(\lambda)$, that heavily weight the right perspective on the income process and its promise of upward mobility. In the numerical analysis that follows, we assume that all voters begin with the prior probability distribution illustrated in Figure 17.2.\(^{65}\)

With this TINA prior in hand, and the empirically grounded representations of left and right ideologies in Figure 17.1, we are now in a position to numerically simulate political dynamics in

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\(^{64}\)The $s$ parameters are established by setting $s = \frac{f(y)}{y}$ where $y \equiv 2 \times 10$th Income Decile of Base Year. We have imposed an income cap of 250% of the top decile to avoid discussing income dynamics far out of the data range.

\(^{65}\)This figure illustrates an exponential prior, but the results are fairly robust to any prior highly weighted to the Right.
Figure 17.2. The TINA Prior

Chile and Peru. Under the assumption that income process is noisy with an idiosyncratic income shock parameter $\sigma = 1/2$, Figure 17.3 shows the simulated evolution of political preferences for Peru in the initial period (circa 1990), six years later and twelve years later\footnote{The longevity of the policy assumed is 10 years with a discount rate of $\delta = .95$.} The vertical axis in each figure represents the fraction of the electorate that would prefer redistributive economic policies using the forward-looking perspective developed earlier. The solid line in each figure represents what political preferences would look like under the assumption that all voters know the true income transition function; this solid line thus represents the same information discussed in section 4 above.

The dashed line in the Figures show the political preferences for voters who begin with the TINA prior and then experientially update the distribution they hold over the critical income distribution parameter, $\lambda$. Interestingly, under the assumptions made, the median, forward-looking voter would have initially voted against redistribution given TINA and substantial prospects for upward mobility. However, after six years of living and learning from the actual income distribution process, the median voter, and most voters in the lower 60\% of the income would have
favored redistributive policies. After a dozen years, the preferences of most voters approach those that would hold under full information.

**Figure 17.3. Demand for a 10 Year Redistributive Policy by Initial Income**

(A) Peru: Year 0  
(B) Peru: Year 6  
(C) Peru: Year 12

Figure 17.4 provides another look at the political dynamics implied by our model of forward-looking, Bayesian voters. The vertical axis now displays the fraction of the electorate at each point in time that is expected to vote for redistribution. As can be seen, over the 1998 to 2010 simulation period in Peru, the fraction voting for redistribution rises by some 16% points, again approaching the levels that would be expected under full information by 2010.

These sharp swings in policy preferences are of course driven by swing voters’ radical reevaluation of their prospects for upward mobility as they learn from the actual operation of the Peruvian economy. An interesting contrast to these results is provided by undertaking a similar exercise for the Chilean economy. The estimated Chilean income transition function is one that shows absolute upward income mobility for all classes, though not much relative improvement for the initially lower income deciles. While simulated preferences for redistribution in the Chilean case are strong, they remain quite stable over time, offering a vision of much more stable politics in Chile than in a country with a polarizing income distribution process.
17.3. **Dead Weight Loss and Political Volatility.** Our final simulation exercise explores the impact of dead weight losses attributable to redistribution policies on political dynamics. For purposes of the numerical analysis, we assume a modest 10% dead weight loss is known to accompany redistributive programs. The dotted lines in Figures 17.3 and 17.4 illustrate the simulated political dynamics for this dead weight loss case.

Not surprisingly, the presence of a dead weight loss dampens support for redistribution, initially reducing political support by almost 20 percentage points. More surprising is that dead weight losses increase political volatility in the case of Peru. A dozen years of learning by voters (again assumed to begin with the TINA prior) returns support to redistribution almost to the levels expected when dead weight losses are zero. In the Peruvian simulation, this learning effect in the presence of dead weight losses creates an almost 30 percentage point swing in the fraction of the forward-looking electorate that prefers redistributive policies.

That dead weight loss \( (D) \) increases support for laissez faire is clear, but the large increase in volatility is perhaps surprising. As analyzed in the appendix below, this volatility effect is explained by the asymmetric effect that \( D \) would have on a Right partisan with a strong belief...
in \( f_R \) in comparison to a Left partisan with a strong belief in \( f_L \). Increases in \( D \) attrit support for redistribution much faster for a Right partisan than for a Left partisan, creating a wider gulf to cross as voters learn. As individuals learn and their beliefs move toward \( f_L \), their sensitivity to dead weight losses evaporates, further powering a large shift in support to redistributive policies.

18. Conclusion

Adopting the perspective that voters are forward-looking and pay attention to income dynamics, not just their static place in the income distribution, this paper has explored the left-right-left shift in the politics of Latin American countries over the last three or four decades. Two analytical innovations are key to this exploration. The first is the generalization of earlier work on forward-looking voters to model political preferences under general families of income distribution dynamics, not just under concave dynamics that offer prospects of upward mobility. This generalization, motivated by empirical evidence of polarizing, non-concave dynamics that offer no prospects of upward mobility for segments of the population, shows that preferences for redistributive policies may increase, not decrease over time when voters are forward looking. However, detailed analysis of the case of Peru suggests that there would have been initially strong support for redistribution had voters been fully informed about the nature of the income distribution dynamics, making it extremely hard to account for the elections in Peru and elsewhere in Latin America in the 1990s that brought more conservative parties and candidates to power.

This observation motivates this paper’s second innovation, namely its modeling of voters as Bayesian learners who update their understanding of income distribution dynamics based on their own lived experience. Given that most voters in Peru (and elsewhere in Latin America where the late 1980s and early 1990s saw a transition to a market economy) had little prior experience with the new economic model, we assume that they initially adopted a prior probability distribution that put substantial weight on a right wing ideological position that attached strong prospects for upward mobility to the region’s new economic model. Numerical simulation of political preferences as voters received noisy draws from the true (estimated) income distribution process shows that a numerically substantial shift from strong right political majority to a strong left political majority over the course of about a dozen years. Somewhat surprisingly, simulated political volatility for Peru is actually increased when the electorate believes that redistributive policies carry dead
weight losses. While there can certainly be no claim that these patterns are to be expected everywhere, the modeling approach does offer new ways to think about political economy, especially in transition or other economies where voters’ prospects for upward mobility are largely initially unknown to them.
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Part 4. Appendices

APPENDIX A. SUPPLEMENTAL EMPIRICAL DETAILS

A.1. Firm distribution by Sector and Country. This selection of country-year pairs were subject to the constraints of data availability, and in a few cases when more than one year of suitable data was available from the same country the most recent set with the largest number of observations was chosen. Detailed reasons for exclusion are tabulated in the following table.

<table>
<thead>
<tr>
<th>Country/Year:</th>
<th>Code:</th>
<th>Country/Year:</th>
<th>Code:</th>
<th>Country/Year:</th>
<th>Code:</th>
<th>Exclusion Code List:</th>
</tr>
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<td>Albania2002</td>
<td>2</td>
<td>Hungary2002</td>
<td>1</td>
<td>Russia2002</td>
<td>8</td>
<td>2 Incomplete, missing, or incompatible education data.</td>
</tr>
<tr>
<td>Armenia2002</td>
<td>1</td>
<td>India2002</td>
<td>6</td>
<td>Russia2005</td>
<td>8</td>
<td>3 Incomplete or missing labor costs.</td>
</tr>
<tr>
<td>Azerbaijan2002</td>
<td>3</td>
<td>Indonesia2003</td>
<td>2</td>
<td>Senegal2003</td>
<td>6</td>
<td>4 Incomplete or missing asset data.</td>
</tr>
<tr>
<td>Bangladesh2002</td>
<td>1</td>
<td>Kazakhstan2002</td>
<td>1</td>
<td>Serbia2003</td>
<td>1</td>
<td>6 Missing ISO or production upgrading data.</td>
</tr>
<tr>
<td>Belarus2002</td>
<td>1</td>
<td>Kazakhstan2005</td>
<td>4</td>
<td>Slovenia2002</td>
<td>4</td>
<td>7 Included separate survey from same year.</td>
</tr>
<tr>
<td>BiH2002</td>
<td>2</td>
<td>Kyrgyzstan2002</td>
<td>4</td>
<td>Slovenia2005</td>
<td>4</td>
<td>8 Excluded as Russian economy reorganizing from USSR, results robust in inclusion.</td>
</tr>
<tr>
<td>BiH2005</td>
<td>4</td>
<td>Kyrgyzstan2005</td>
<td>4</td>
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The distribution of firms by sector and country is presented in Table 10. In order to facilitate heteroskedasticity by sector-country pairs, sector-country pairs containing exactly one observation were dropped from the sample, making the minimum number of observations two for each pair.
### Table 10. Distribution of Firms by Country and Sector

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<th>Beverages</th>
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**Sector Totals**: 126 126 343 414 107 861 1170 280 1169 462 215 157 567 690 6687

A.2. **Controlling for the sales effects of exporting.** Ideally, the data set would include production information that differentiates the use of inputs for domestic and foreign use. Of course, many
inputs are used in both domestic and foreign production activities, so the true counterfactuals of exporting are not directly observable. I therefore impose structure on the domestic and foreign decomposition of revenues to control for market segmentation.

Suppose firm $i$ produces a quality adjusted quantity $q_i$, of which a fraction $X_i$ is sold to foreign markets at price $p_i^X$, and the remainder is sold domestically at price $p_i^D$. Then value added sales $Y_i$ are given by Equation (A.1)

$$Y_i = X_i \cdot [p_i^X - c_i^X]q_i + (1 - X_i) \cdot [p_i^D - c_i^D]q_i$$

where $c_i^X$ and $c_i^D$ are foreign and domestic material input costs. Defining the foreign ($M_i^X$) and domestic ($M_i^D$) markups in the usual way by $M_i^X \equiv (p_i^X - c_i^X) / c_i^X$ and $M_i^D \equiv (p_i^D - c_i^D) / c_i^D$, rewrite Equation (A.1) as

$$Y_i = (p_i^D - c_i^D)q_i \cdot \left(1 + \frac{M_i^X(c_i^X / c_i^D) - M_i^D}{M_i^D}X_i \right)$$

Value added sales at domestic prices and input costs

Export weight to account for foreign markups and input costs

Equation (A.2) shows value added sales decomposed into domestic value added sales times an export weight that depends on markups and input costs. Introducing the shorthand

$$M_i \equiv \left[M_i^X(c_i^X / c_i^D) - M_i^D\right] / M_i^D$$

and taking logs, Equation (A.2) becomes

$$\ln Y_i \approx \ln(p_i^D - c_i^D)q_i + M_iX_i$$

where the approximation is the commonly used fact that $\ln(1 + x) \approx x$ for small values of $x$. This approximation halves the number of non-linear parameters to recover. Although the individual elements of $M_iX_i$ are not identified, $X_i$ is observed. Assuming the term $M_i$ does not vary across some grouping, (as would be implied by constant markups within the grouping and $c_i^X / c_i^D$ fixed), $M_i$ can be used to control for the sales effects of exporting. In what follows, I assume $M_i \equiv M_S$ for
all firms $i$ in sector $S$ in order to focus on within-sector differences. Since the firms considered produce in developing countries, generally one should expect $M^X_S \geq M^D_S$ (see OECD (2006)) and because of potentially higher costs for "export quality" goods ($c^X_S \geq c^D_S$).

A.3. **Translog production estimates.** After adding the labor augmenting effect of skill mix, the translog specification implies

$$\ln F(K, \phi(\psi_i) \cdot L) \equiv \alpha_S \ln K_i + \beta_S \ln (\phi(\psi_i)L_i) + \sum_{i,j} \gamma_{ij} \ln K_i \ln (\phi(\psi_i)L_j)$$ (A.3)

Estimates are reported in Table 11.

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<th>Markup ($M_S$)</th>
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<th>Labor$^2$</th>
<th>Capital/Labor</th>
<th>CES Param ($\rho_S$)</th>
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<td>0.2956</td>
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<td>.0351</td>
<td>-0.0031</td>
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<td>Autos &amp; comp</td>
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<td>.0153</td>
<td>-1.163***</td>
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<tr>
<td>Beverages</td>
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<td>.0780***</td>
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*/**/*** denote .1/.05/.01 Significance levels

---

68Constant markups for both exporting and domestic production are, for example, consistent with [Melitz (2003a)](https://doi.org/10.1162/000287703322538245), but no longer hold once that model is modified to allow for scale effects as in [Melitz and Ottaviano (2008a)](https://doi.org/10.1086/526414). For a concise summary and comparison of these models see [Dhingra and Morrow (2008)](https://doi.org/10.1162/rte.a.2008.10.1.3).
This specification coincides with that of the main text, with the exception that firm size is not included as a control since the specification (A.3) already captures much variation in possible scale effects. Note that the estimates are somewhat inefficient as the share equations associated with this specification were not used in estimation, see Kim (1992).
This section of the appendix includes most proofs from the main text. The construction of wages as those which support the efficient competitive equilibrium is considerably involved and available from the author upon request.

Lemma. If $\Phi$ has finite expectation then the value of all production in the economy is bounded.

Proof. For the result it is necessary to show that for all admissible allocations $A \equiv (q, M(q), \iota(q))$ of workers to firms that

\[(B.1) \quad \sup_A \int p_{i(q)} F^{(q)}(q, M(q)) d\Phi < \infty\]

Since the integrand in equation (B.1) is always positive, for the result it is sufficient to show it holds for $N + 1$ different economies with technologies $F_i$ and the entire distribution of skill to allocate. Formally, it is sufficient to show for the allocations $A_i, i = 0, \ldots, N$ which employ all workers in each sector $i$ that

\[(B.2) \quad \sup_{A_i} \int p_i F_i^i(q, M(q)) d\Phi < \infty \quad \forall i = 0, \ldots, N\]

Since each $F_i$ is increasing in each argument and is homogeneous of degree one, $p_i F_i^i(q, M(q)) \leq \max\{q, M(q)\} \cdot p_i F_i^i(1, 1) \leq (q + M(q)) \cdot p_i F_i^i(1, 1)$. Thus for equation (B.2) to hold, since the terms $p_i F_i^i(1, 1)$ are irrelevant for boundedness, it is enough that

\[(B.3) \quad \sup_{A_i} \int (q + M(q)) d\Phi < \infty \quad \forall i = 0, \ldots, N\]

Examining equation (B.3), $\int q d\Phi$ is the mean of $\Phi$ which is assumed finite so showing

\[\sup_M \int M(q) d\Phi < \infty\]

will prove the result. I claim in fact that $\int M(q) d\Phi = \int q d\Phi$ for any admissible $M$. This follows from the allocative nature of $M$ which requires $\int_{\{q: M(q) \in [a, b]\}} 1 d\Phi = \Phi(b) - \Phi(a)$ so in particular for any integers $j$ and $K$,

\[
\left| \int_{j/K}^{(j+1)/K} q d\Phi - \int_{\{q: M(q) \in [j/K, j/K+1]\}} M(q) d\Phi \right| \leq \left| \frac{j + 1}{K} - \frac{j}{K} \right| \int_{j/K}^{(j+1)/K} 1 d\Phi - \int_{\{q: M(q) \in [j/K, j/K+1]\}} 1 d\Phi \\
\leq \left| \Phi((j + 1)/K) - \Phi(j/K) \right| / K
\]
Summing both sides over \( j \) implies
\[
\left| \int_0^\infty q d\Phi - \int_{\{q; M(q) < \infty\}} M(q) d\Phi \right| \leq \lim_{j \to \infty} \Phi((j+1)/K)/K = 1/K
\]
and letting \( K \to \infty \) shows \( \int M(q) d\Phi = \int q d\Phi \) which is finite, proving the result. \( \square \)

**Lemma (CES Ranking).** Suppose each \( F^i \) is CES, specifically \( F^i(q, q') \equiv A_i(q^\rho_i + q'^\rho_i)^{1/\rho_i} \). Then \( S_i \geq S_j \) if and only if \( \rho_i > \rho_j \) so CES production technologies imply a complete diversity ranking.

**Proof.** It is clear that a necessary condition for \( S_i \geq S_j \) is \( \rho_i \neq \rho_j \). Some algebra along with the differentiability and homogeneity of each \( F^i \) shows that another necessary condition is that \( F^i_2(1, z)/F^i_1(1, z) \geq F^i(1, z)/F^i(1, z) \) for all \( z > 1 \) or rather
\[
\frac{A_i z^{\rho_i-1}(1 + z^{\rho_i})^{1/\rho_i} - 1}{A_i z^{\rho_i-1}(1 + z^{\rho_i})^{1/\rho_i} - 1} \geq \frac{A_i(1 + z^{\rho_i})^{1/\rho_i} / A_i(1 + z^{\rho_i})^{1/\rho_i}}{A_i(1 + z^{\rho_i})^{1/\rho_i}}
\]
which holds iff \( z^{\rho_i}/(1 + z^{\rho_i}) \geq z^{\rho_i}/(1 + z^{\rho_i}) \). Since \( x/(1 + x) \) is strictly increasing for \( x > 0 \), the inequality holds iff \( z^{\rho_i} \geq z^{\rho_i} \). Since \( \rho_i \neq \rho_j \) equality cannot hold and since \( z > 1 \) this holds iff \( \rho_i > \rho_j \). Working backwards, \( \rho_i > \rho_j \) is sufficient for \( S_i \geq S_j \), giving the result. \( \square \)

**Proposition.** Under Assumption [\( \square \)] any equilibrium allocation exhibits skill ratios \( \{\hat{t}_i\} \), increasing in \( i \), where if \( (q, M(q)) \) are assigned to \( S_i \) then \( M(q)/q \in [\hat{t}_i, \hat{t}_{i+1}] \). The ratios are fixed for \( i > 1 \) by
\[
p_i F^i(1, \hat{t}_i) = p_{i-1} F^{i-1}(1, \hat{t}_i)
\]

**Proof.** First consider any worker team \( (q, M(q)) \) and consider assigning the team to either \( S_i \) or \( S_j \) with \( j > i \). Note that for any fixed wage structure, \( \pi^i(q, M(q)) \geq \pi^i(q, M(q)) \) iff \( p_j F^j(q, M(q)) \geq p_i F^i(q, M(q)) \) iff \( p_j F^j(1, x) \geq p_i F^i(1, x) \) where \( x \equiv M(q)/q \). Therefore, conditional on wages \( w(q) \), the most profitable assignment depends on the skill ratio \( x \). Assumption [\( \square \)] implies \( F^i(1, x) \) and \( F^j(1, x) \) cross at most once, and they cross at least once \( ^{[\square]} \) so let \( \chi(i, j) \) denote the skill ratio at which the crossing occurs. Since \( p_j F^j(1, x)/p_i F^i(1, x) \) is increasing by Assumption [\( \square \)] it follows that \( p_j F^j(1, x) \geq p_i F^i(1, x) \) iff \( x \geq \chi(i, j) \). Working backwards to the profit conditions for fixed wages, we conclude
\[
\pi^i(q, M(q)) \geq \pi^i(q, M(q)) \text{ iff } M(q)/q \geq \chi(i, j)
\]

\(^{[\square]}\)Otherwise one technology is always dominated at current prices, so it may be omitted without loss of generality.
Now consider which skill ratios may be assigned to \( S_N \). Since \( S_N \supseteq S_j \) for any \( j \), it follows that if \( M(q)/q \geq \max_i \chi(i, N) \), \( S_N \) is the most profitable sector to employ \((q, M(q))\). Conversely, if \( M(q)/q < \max_i \chi(i, N) \) there is some \( S_j \) which is more profitable to employ \((q, M(q))\). Therefore \( S_N \) consists of the most diverse teams where \( M(q)/q \geq \max_i \chi(i, N) \). Proceeding recursively for \( S_{N-1} \) (which satisfies \( S_{N-1} \supseteq S_j \) for any \( j < N \)), \((q, M(q))\) is most profitably employed in \( S_{N-1} \) iff 

\[
M(q)/q \in [\max_{i<N-1} \chi(i, N-1), \max_i \chi(i, N)].
\]

Continuing in this fashion produces the skill ratio cutoffs \( \{\hat{t}_i\} \) as promised where by convention \( \hat{t}_{N+1} \equiv \infty \).

Finally, the recursive nature of assignment fixes each \( \hat{t}_i \) to where “\( S_{i-1} \) crosses \( S_i \)”, namely 

\[
p_i F_i(1, \hat{t}_i) = p_{i-1} F_{i-1}(1, \hat{t}_i).
\]

This is exactly equivalent to \( \hat{t}_i = \chi(i - 1, i) \). To see the result, suppose \( \hat{t}_i = \chi(k, i) > \chi(i - 1, i) \) so 

\[
p_i F_i(1, \hat{t}_i) > p_{i-1} F_{i-1}(1, \hat{t}_i)
\]

where the inequality follows from the fact that \( \hat{t}_i > \hat{t}_j \) for \( j < i \). Equation (B.4) yields a contradiction and it holds that 

\[
p_i F_i(1, \hat{t}_i) = p_{i-1} F_{i-1}(1, \hat{t}_i) \text{ for each } i.
\]

\[
1 = \frac{p_i F_i(1, \hat{t}_i)}{p_k F_k(1, \hat{t}_i)} = \prod_{j=k+1}^{i} \frac{p_j F_j(1, \hat{t}_i)}{p_{j-1} F_{j-1}(1, \hat{t}_i)} > \prod_{j=k+1}^{i-1} \frac{p_j F_j(1, \hat{t}_i)}{p_{j-1} F_{j-1}(1, \hat{t}_i)} = 1
\]

\[
\text{ where the inequality follows from the fact that } \hat{t}_i > \hat{t}_j \text{ for } j < i. \quad \Box
\]

**Proposition.** The wage schedule \( w(q) \) has the following properties:

1. \( w(q) \) is strictly increasing and convex.
2. \( w(q) \) is bounded below by the \( S_0 \) shadow wages \( w_0q \).
3. \( w'(q) \) is increasing where defined, and elsewhere \( \lim_{q \rightarrow x^-} w'(q) \leq \lim_{q \rightarrow x^+} w'(q) \).

**Proof.** The first claim: That \( w(q) \) is strictly increasing is obvious from the form. Convexity follows from the third claim combined with Theorem 24.2 of [Rockafellar 1970] by pasting across sectors.

The second claim: The result clearly holds for workers in \( S_0 \) so consider workers in the sectors \( \{S_i\}_{i \geq 1} \). First note that for \( i \geq 1 \), \( F_{i2} \leq 0 \) and given that \( F_i \) is homogeneous of degree one implies \( F_{i1} \geq 0 \). Second, since at least some firms produce in the \( S_0 \) sector, efficiency implies that for all \( i \), \( p_i F_i(1, 1) \leq p_0 F_0(1, 1) = 2w_0 \). Using these facts, for any worker of skill \( q \) which is below the median of \( \Phi \) it holds that

\[
F_i(q, M(q)) = F_i(q/M(q), 1) \leq F_i(1, 1) = F_i(1, 1)/2 \leq w_0/p_i
\]
We conclude that \( p_i F_i^i(q, M(q)) \leq w_0 \). This implies for any \( q \) below the median employed in \( \{ S_i \}_{i \geq 1} \) wages are bounded below by

(B.6) \[
  w(q) = w(t_0) - \int_q^{t_{j-1}} p_i F_i^i(s, M(s))ds \geq w(t_{j-1}) - \int_q^{t_{j-1}} w_0ds = w(t_{j-1}) - w_0[t_{j-1} - q]
\]

Consequently, if \( q \) is employed in \( S_1 \), using the fact that \( w(t_0) = w_0 t_0 \), Equation (B.6) implies \( w(q) \geq w_0 q \). Proceeding inductively for \( q \) below the median, suppose \( w(q) \geq w_0 q \) for sector \( S_{i-1} \) so for \( S_i \), Equation (B.5) implies

(B.7) \[
  w(q) \geq w(t_{j-1}) - w_0[t_{j-1} - q] = w_0 q
\]

Therefore by induction, Equation (B.7) implies \( w(q) \geq w_0 q \) for all \( q \) below the median of \( \Phi \). A similar method applies to \( q \) above the median of \( \Phi \) by exploiting the analogous inequality of Equation (B.5) using \( F_i^2 \).

**Third Claim:** For \( q \) not at a cusp \( \{ t_j \} \) or \( \{ M(t_j) \} \) this follows from \( F_{11}^1, F_{22}^2 \geq 0 \) for \( q \) in \( \{ S_i \}_{i \geq 1} \) and from \( w'(q) = w_0 \) for \( q \) in \( S_0 \). Now suppose \( q = t_i \) for some \( i \). For \( \delta > 0 \),

(B.8) \[
  w(t_i - \delta) = p_{i+1} F_i^{i+1}(t_i - \delta, M(t_i - \delta)) \quad \text{and} \quad w(t_i + \delta) = p_i F_i^i(t_i + \delta, M(t_i + \delta))
\]

Therefore it is sufficient to show that

\[
\lim_{\delta \to 0} p_{i+1} F_i^{i+1}(t_i - \delta, M(t_i - \delta)) \leq \lim_{\delta \to 0} p_i F_i^i(t_i + \delta, M(t_i + \delta))
\]

and since each \( F_i^i \) is continuous this inequality holds if \( p_{i+1} F_i^{i+1}(t_i, M(t_i)) \leq p_i F_i^i(t_i, M(t_i)) \). Since \( S_{i+1} \subset S_i \), by definition \( \frac{\partial}{\partial x} \frac{p_{i+1} F_i^{i+1}(1, x)}{p_i F_i^i(1, x)} \geq 0 \). This implies through homogeneity that

(B.9) \[
  \frac{p_{i+1} F_i^{i+1}(y, z)}{p_i F_i^i(y, z)} \geq \frac{p_{i+1} F_i^{i+1}(y, z)}{p_i F_i^i(y, z)} \quad \forall z / y \geq t_{i+1}
\]

Also by definition, \( M(t_i)/t_i = t_{i+1} \) and \( p_{i+1} F_i^{i+1}(t_i, M(t_i)) = p_i F_i^i(t_i, M(t_i)) \) so letting \( y = t_i \) and \( z = M(t_i) \) in Equation (B.9) the denominators cancel and it holds that \( p_{i+1} F_i^{i+1}(t_i, M(t_i)) \geq p_i F_i^i(t_i, M(t_i)) \). Homogeneity then implies \( p_{i+1} F_i^{i+1}(t_i, M(t_i)) \leq p_i F_i^i(t_i, M(t_i)) \) as desired. The cusps \( \{ M(t_i) \} \) can be shown similarly.

**Lemma.** Let \( \Phi \) and \( \Phi \) be two skill distributions and \( t_0, \hat{t}_0 \) be the respective skill cutoffs in equilibrium. Consider a Skill Shock from \( \Phi \) to \( \Phi \). Then:

(1) Following a DSS the mass of workers in \( S_1 \) increases.
(2) Following a HSS, $\bar{I}_0 \geq t_0$ and $M(\bar{I}_0) \geq M(t_0)$ (and $S_1$ workers increase).
(3) Following a LSS, $\bar{I}_0 \leq t_0$ and $M(\bar{I}_0) \leq M(t_0)$ (and $S_1$ workers increase).

Proof. I first show Claim 2. Let $\tilde{M}$ denote the equilibrium skill pairings under $\Phi$. Under a HSS, by assumption $\Phi(t_0) = \Phi(\bar{I}_0)$ and $\Phi(M(t_0)) \leq \Phi(M(\bar{I}_0))$ so that $\Phi(t_0) + \Phi(M(t_0)) \leq 1$. In equilibrium necessarily $M(t_0)/L_0 = \tilde{M}(\bar{I}_0)/L_0 = \hat{t}_1$ since the $\{t_i\}$ are fixed by prices and production technologies that are independent of the skill distribution. Therefore $\Phi(t_0) + \Phi(\hat{t}_1t_0) = \Phi(t_0) + \Phi(M(t_0)) \leq 1$ which implies

$$\Phi(t_0) + \Phi(\hat{t}_1t_0) \leq 1 = \Phi(\bar{I}_0) + \Phi(\hat{t}_1\bar{I}_0)$$

and since $\Phi$ is increasing, conclude $\bar{I}_0 \geq t_0$. Therefore $\Phi(\bar{I}_0) \geq \Phi(t_0)$ so the mass of workers employed in $S_1$ under $\Phi$ is larger than under $\Phi$ as claimed. Claim 3 then follows by a symmetric argument. Claim 1 follows by decomposing any DSS into a HSS for changes above the median and a LSS for changes below the median and applying Claims 2 and 3 in succession. □

**Proposition** (Rybczynski under Diversity). If the endowment of skills specific to a sector increases, then output of the sector increases. However, worker spillovers can prevent or amplify magnification effects.

Proof. (Sketch) This is clear for the $S_0$ sector. For other sectors, it can be shown that the mass of workers employed increases in a similar fashion as the skill shock Lemma, but with careful accounting of set of workers. Now fix a sector $S_i$ for $i \geq 1$, let $\Phi$ be given and first assume $\Psi$ has support on $[M(\underline{t}_{i-1}), M(\underline{t}_i)]$. Normalize $\Phi$ and $\Psi$ to have mass one, forcing the normalized combined distribution to be $\hat{\Phi} = \frac{L}{L+P} \Phi + \frac{P}{L+P} \Psi$ and note $\hat{\Phi}(q) = \frac{L}{L+P} \Phi(q)$ for $q \in [\underline{t}_i, \underline{t}_{i-1}]$. Let $\{\hat{t}_i\}$ be the new skill cutoffs between sectors and it is clear as in the text that $\hat{t}_i = \hat{t}_i$ and $\hat{t}_{i-1} \geq \underline{t}_{i-1}$. Also let $\tilde{M}$ denote the new matching function under $\Phi$ and the support of $\Psi$ implies $\tilde{M}(q) \geq M(q)$ for $q \in [\underline{t}_i, \underline{t}_{i-1}]$. Using these facts we have

$$\int_{\underline{t}_i}^{\underline{t}_{i-1}} \int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, M(q))d\Phi \geq \int_{\underline{t}_i}^{\underline{t}_{i-1}} \int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, M(q))d\Phi \geq \int_{\underline{t}_i}^{\underline{t}_{i-1}} \int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, M(q))d\Phi = \frac{L}{L+P} \int_{\underline{t}_i}^{\underline{t}_{i-1}} \int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, M(q))d\Phi$$

The un-normalized value of the last line coincides with sector output before the skill shift. Thus for $\Psi$ with support on $[M(\underline{t}_{i-1}), M(\underline{t}_i)]$ the result follows, and for $\Psi$ with support on $[\underline{t}_i, \underline{t}_{i-1}]$ the result follows by symmetry. Combining the two cases gives the result. □
Proposition. If the endowment of skills specific to a sector increases with equal masses of low and high skill workers then no worker spillovers occur. Output is magnified beyond what the initial and added endowments produce separately due to more efficient worker sorting.

Proof. Let \( Y \) denote the combined skill distribution of \( \Phi \) and \( \Psi \), namely the skill distribution with mass \( L + P \) and (proper) distribution \( \lambda \Phi + (1 - \lambda)\Psi \) where \( \lambda \equiv \frac{L}{L + P} \). Also let \( M_{\Phi} \) denote the match function \( \Phi \) generates in isolation and similarly \( M_{\Psi} \) and \( M_{Y} \). I first claim

\[
\min\{M_{\Phi}, M_{\Psi}\} \leq M_{Y} \leq \max\{M_{\Phi}, M_{\Psi}\} \quad (B.10)
\]

To see this, note that by definition

\[
\Phi + \Phi \circ \min\{M_{\Phi}, M_{\Psi}\} \leq 1 \quad \text{and} \quad \Psi + \Psi \circ \min\{M_{\Phi}, M_{\Psi}\} \leq 1
\]

and therefore for any \( \lambda \in [0, 1] \)

\[
\lambda \Phi + (1 - \lambda)\Psi + [\lambda \Phi + (1 - \lambda)\Psi] \circ \min\{M_{\Phi}, M_{\Psi}\} \leq 1
\]

which with \( Y + Y \circ M_{Y} = 1 \) shows \( \min\{M_{\Phi}, M_{\Psi}\} \leq M_{Y} \) and similarly \( M_{Y} \leq \max\{M_{\Phi}, M_{\Psi}\} \) so Equation (B.10) holds. Investigation shows there are no worker spillovers iff

\[
\frac{M_{\Phi}(t_{i-1})}{t_{i-1}} \leq \frac{M_{Y}(t_{i-1})}{t_{i-1}} \quad \text{and} \quad \frac{M_{Y}(t_{i})}{t_{i}} \leq \frac{M_{\Phi}(t_{i})}{t_{i}} \quad (B.11)
\]

Equation (B.10) implies that Equation (B.11) holds so long as

\[
M_{\Phi}(t_{i-1}) \leq M_{Y}(t_{i-1}) \quad \text{and} \quad M_{Y}(t_{i}) \leq M_{\Phi}(t_{i}) \quad (B.12)
\]

Since the support of \( \Psi \) is on \( [t_{i}, t_{i-1}] \cup [M_{\Phi}(t_{i-1}), M_{\Phi}(t_{i})] \), clearly \( M_{\Psi}(t_{i}) \leq M_{\Phi}(t_{i}) \). Since there are equal masses of low and high skill workers in \( \Psi \), \( \lim_{x \to 5^+} \Psi^{-1}(x) \geq M_{\Phi}(t_{i-1}) \) so Equation (B.12) holds. Therefore there are no worker spillovers. Magnification follows from the optimality of \( M_{Y} \). \( \square \)

Proposition. If the skill distribution is symmetric then the indirect Stolper-Samuelson effect dominates the direct effect.
Proof. The indirect effect dominates when for any \( i \geq 1 \) and \( q \in (t_{i-1}, t_i] \):

\[
\int_q^{t_{i-1}} F_i^1(s, M(s)) \, ds \leq \frac{F_i^1(t_{i-1}, M(t_{i-1})) \cdot M(t_{i-1}) / t_{i-1}}{M(t_{i-1}) / t_{i-1} + 1}
\]  

(B.13)

Noting that for any \( s, F_i^1(s, M(s)) = F_i^1(s / M(s), 1) \) and \( F_{11} \geq 0 \), it follows \( F_i^1(s, M(s)) \leq F_i^1(t_{i-1}, M(t_{i-1})) \). Therefore

\[
\int_q^{t_{i-1}} F_i^1(s, M(s)) \, ds \leq \int_q^{t_{i-1}} F_i^1(t_{i-1}, M(t_{i-1})) \, ds \leq F_i^1(t_{i-1}, M(t_{i-1})) t_{i-1}
\]  

(B.14)

Using the fact that \( F_2^1(x, y) = F_1^1(y, x) \) it is also clear that

\[
F_i^1(t_{i-1}, M(t_{i-1})) = F_i^1(t_{i-1}, M(t_{i-1})) t_{i-1} + F_2^1(t_{i-1}, M(t_{i-1})) M(t_{i-1})
\]

(B.15)

Applying the inequalities of Equation (B.14) to the LHS of Equation (B.13) and Equation (B.15) to the RHS of Equation (B.13) it is sufficient for the result that

\[
F_i^1(t_{i-1}, M(t_{i-1})) t_{i-1} \leq F_i^1(t_{i-1}, M(t_{i-1}) [t_{i-1} + M(t_{i-1})] M(t_{i-1}) / M(t_{i-1}) + t_{i-1})
\]

which clearly holds. \( \square \)
APPENDIX C. APPENDIX: PROOFS

C.1. A Folk Theorem. In this context we need to define the Social Planner’s policy space. Provided $M_e$ and $q(c)$, and assuming without loss of generality that all of $q(c)$ is consumed, all allocations are determined. The only question remaining is what class of $q(c)$ the SP is allowed to choose from. A sufficiently rich class for our purposes are $q(c)$ which are positive and continuously differentiable on some closed interval and zero otherwise. This follows from the basic principle that a SP will utilize low cost firms before higher cost firms. Formally, we restrict $q \in Q \equiv \bigcup_{c_d \in \mathbb{R}} Q_{[0,c_d]}$ where

$$Q_{[0,c_d]} \equiv \{q \in C^1, > 0 \text{ on } [0,c_d] \text{ and } 0 \text{ otherwise}\}$$

so $Q_{[0,c_d]}$ denotes all smooth, strictly positive quantity allocations on $[0,c_d]$. For brevity we will use the shorthand $G(x) \equiv \int_0^x g(c)dc$ and $R(x) \equiv \int_0^x c^{\rho-1} g(c)dc$ throughout the proofs. Finally, we maintain Melitz’s assumptions which imply a unique market equilibrium in each economy.

Proposition. Every market equilibrium of a closed Melitz economy is socially optimal.

Proof. Assume a market equilibrium exists, which guarantees that $R(c)$ is finite for all $c$. We first remark that in both the market equilibrium and SP problem, $L/M_e - f_e - fG(c_d) = 0$ implies utility of zero so in both cases we must have $L/M_e - f_e - fG(c_d) > 0$. The SP problem is equivalent to

$$(SP) \max_{M_e,c_d \in Q_{[0,c_d]}} M_e \int_0^{c_d} q(c)\rho g(c)dc \text{ subject to } f_e + fG(c_d) + \int_0^{c_d} cq(c)g(c)dc = L/M_e$$

We will exhibit a globally optimal $q^*(c)$ for each fixed $(M_e,c_d)$ pair, reducing the SP problem to a choice of $M_e$ and $c_d$. We then solve for $M_e$ as a function of $c_d$ and finally solve for $c_d$.

Finding $q^*(c)$ for $M_e, c_d$ fixed. For convenience, define the functionals $V(q), H(q)$ by

$$V(q) \equiv \int_0^{c_d} v(c,q(c))dc \quad H(q) \equiv \int_0^{c_d} h(c,q(c))dc$$
where \( h(c, x) \equiv xc g(c) \) and \( v(c, x) \equiv x^\rho g(c) \). One may show that \( V(q) - \lambda H(q) \) is strictly concave \( \forall \lambda \). Now for each fixed pair \((M_e, c_d)\), consider the problem of finding \( q^* \) in \( \mathcal{Q}_{[0,c_d]} \) given by

\[
\max_{M_e, c_d, q \in \mathcal{Q}_{[0,c_d]}} V(q) \text{ subject to } H(q) = L/M_e - f_e - fG(c_d)
\]

Following Troutman\cite{Troutman}, if some \( q^* \) maximizes \( V(q) - \lambda H(q) \) on \( \mathcal{Q}_{[0,c_d]} \) for some \( \lambda \) then it is a solution to the above problem provided the constraint is met by \( q^* \). Again following Troutman, for any \( \lambda \) a sufficient condition for some \( q^* \in \mathcal{Q}_{[0,c_d]} \) to be a global maximum on \( \mathcal{Q}_{[0,c_d]} \) is

\[
(C.1) \quad D_2v(c, q^*(c)) = \lambda D_2h(c, q^*(c))
\]

This follows because \([C.1]\) implies for any such \( q^* \), \( \forall \xi \) s.t. \( q^* + \xi \in \mathcal{Q}_{[0,c_d]} \) we have \( \delta V(q^*; \xi) = \lambda \delta H(q^*; \xi) \) (where \( \delta \) denotes the Gateaux derivative in the direction of \( \xi \)) and \( q^* \) is a global max since \( V(q) - \lambda H(q) \) is strictly concave. The condition \([C.1]\) is nothing but \( \rho q^*(c)^{p-1} g(c) = \lambda cg(c) \) which is equivalent\footnote{By abuse of notation we allow \( q^* \) to be \( \infty \) at \( c = 0 \) since reformulation of the problem omitting this single point makes no difference to allocations or utility which are all eventually integrated.} to \( q^*(c) = \left( \frac{\lambda c}{\rho} \right)^{\frac{1}{p-1}} \). From above, this \( q^* \) serves as a solution to \( \max V(q) \) provided that \( H(q^*) = L/M_e - f_e - fG(c_d) \). This will be satisfied by appropriate choice of \( \lambda \) since for fixed \( \lambda \) we have

\[
H(q^*) = \int_0^{c_d} \left( \frac{\lambda c}{\rho} \right)^{\frac{1}{p-1}} cg(c) dc = \left( \frac{\lambda}{\rho} \right)^{\frac{1}{p-1}} R(c_d)
\]

so that choosing \( \lambda^* \) as \( \lambda^* \equiv \rho \left[ \frac{L/M_e - f_e - fG(c_d)}{R(c_d)} \right]^{p-1} \) will make \( q^* \) a solution provided \( L/M_e - f_e - fG(c_d) > 0 \). In summary, for each fixed \((M_e, c_d)\) a globally optimal \( q^* \) satisfying the resource constraint given by

\[
q^*(c) = \frac{L/M_e - f_e - fG(c_d)}{R(c_d)} c^{\frac{1}{p-1}}
\]

which must be \( > 0 \) since \( L/M_e - f_e - fG(c_d) \) must be \( > 0 \) as discussed at the beginning of the proof.

\footnote{Since \( h \) is linear in \( x \), \( H \) is linear and since \( v \) is strictly concave in \( x \) (using \( \rho < 1 \) so is \( V \), and therefore \( V(q) - \lambda H(q) \) is strictly concave \( \forall \lambda \).}
Finding $M_e$ for $c_d$ fixed. We may therefore considering maximizing $W(M_e, c_d)$ where, we have

$$W(M_e, c_d) \equiv M_e \int_0^{c_d} q^*(c)^p g(c) dc = M_e [L/M_e - f_e - fG(c_d)]^p R(c_d)^{1-p}$$

Direct investigation yields

$$D_1 W(M_e, c_d) = [L/M_e - f_e - fG(c_d)]^{p-1} \{(1 - \rho)L/M_e - f_e - fG(c_d)\} R(c_d)^{1-p}$$

$$D_{11} W(M_e, c_d) = -\rho(1 - \rho) \frac{L^2}{M_e^3} [L/M_e - f_e - fG(c_d)]^{p-2} R(c_d)^{1-p}$$

So we conclude that $W$ is strictly concave in $M_e$ and that for each $c_d$, the globally optimal $M_e$ is given by $M_e(c_d) = (1 - \rho)L/(f_e + fG(c_d))$.

Finding $c_d$. Finally, we have maximal welfare for each fixed $c_d$, explicitly $Z(c_d)$ where

$$Z(c_d) \equiv \rho^p (1 - \rho)^1 R(c_d)^{1-p} \{f_e + fG(c_d)\}^{p-1}$$

We may easily rule out $c_d = 0$ as an optimum since this yields zero utility. Since $Z(c_d) > 0$ for $c_d > 0$ we may find a maximum for $Z(c_d)$ by maximizing $\ln Z(c_d)$. Some algebra shows that any maximum of $\ln Z(c_d)$ is a maximum of $B(c_d)$ where $B(c_d) \equiv \ln R(c_d) - \ln [f_e + fG(c_d)]$ so we need maximize only $B(c_d)$ which is easily seen to be continuously differentiable on $(0, \infty)$. Now direct inspection shows that

$$B'(c_d) = \{R(c_d)[f_e/f + G(c_d)]\}^{-1} \{R'(c_d)[f_e/f + G(c_d)] - R(c_d)g(c_d)\}$$

Our strategy to find an optimal $c_d$ will be as follows: We will show that $\lim_{c_d \to 0} B'(c_d) > 0$ and $\lim_{c_d \to \infty} B'(c_d) < 0$ so that for some interval $[a, b]$ with $a > 0$ that $[a, b]$ contains a critical point and we have $\forall c > 0$ that

$$\sup_{x \in [c, a]} B(x), \sup_{x \in [b, \infty)} B(x) < \sup_{x \in [a, b]} B(x)$$

We may therefore conclude that $B$ attains a maximum on $[a, b]$ compact since it is continuous. Then since $B$ is continuously differentiable, its maximum must occur at a critical point in $[a, b]$ or at $a$ or $b$. Since maxima at $a$ or $b$ are ruled out by the above inequalities, we conclude that at least one critical point of $B$ in $[a, b]$ is a global maximum. Finally, by showing that $B$ has a unique critical point, we conclude that $B$ takes on a unique global maximum on $(0, \infty)$. 
We now show \( \lim_{c_d \to 0} B'(c_d) > 0 \). Inspection of the expression for \( B' \) shows for the first expression it is sufficient to show that for sufficiently small \( c_d \) we have
\[
(C.2) \quad \frac{R'(c_d)}{g(c_d)} [f_e/f + G(c_d)] > R(c_d)
\]
Since \( R(c_d) \) is bounded, it is sufficient that to show that \( \lim_{c_d \to 0} \frac{R'(c_d)}{g(c_d)} = c_d^{\rho} \) so we conclude \( C.2 \).

We now show \( \lim_{c_d \to \infty} B'(c_d) < 0 \). As above it is sufficient to show that
\[
\lim_{c_d \to \infty} \frac{f_e/f + G(c_d)}{\int_0^{c_d} \frac{G(c)}{c} dc} < 1
\]
Considering the denominator, since \( \frac{G(c)}{c} \) has support on \((0, \infty)\) we conclude that \( \lim_{c_d \to \infty} \int_0^{c_d} \frac{G(c)}{c} dc = \infty \) and we conclude \( \lim_{c_d \to \infty} B'(c_d) < 0 \).

All that remains is to show that \( B \) has at most one critical point. As above \( B'(c_d) = 0 \) iff
\[
(C.3) \quad f_e/f = \int_0^{c_d} \frac{G(c)}{c} dc - G(c)
\]
and direct inspection shows that the RHS of Equation \( C.3 \) has a strictly positive derivative in \( c_d \) so \( B \) has as most one critical point.

Finally, we leave it to the reader to verify that the implicit equation determining \( c_d \) is the same as in the market equilibrium, which also determines \( M_e \) and \( q^* \) to exactly coincide with the market equilibrium.

\section*{C.2. Converse of the Folk Theorem.} We now consider general consumer preferences of the form given by Equation \( C.4 \).
\[
(C.4) \quad U(M_e, c_d, q) \equiv v(M_e, c_d) \int_0^{c_d} u(q(c))g(c)dc
\]
We assume the following regularity conditions on \( u \) which guarantee that each monopolist will have a unique optimal quantity in a market equilibrium.

\textbf{Definition.} (Regular Preferences) \( u \) satisfies the following conditions:

1. \( u \) is twice continuously differentiable with \( u' > 0 \) and \( u'' < 0 \) and satisfies the inada conditions.
(2) \( u \) is s.t. each monopolist’s FOC uniquely determines his optimal quantity supplied.\(^{72}\)

(3) \( \nu \) is positive and \( C^1 \).

**Proposition 5.** Consider a Melitz economy with preferences as in (10.1). The market equilibrium is socially optimal only if \( u \) is CES.

*Proof.* Assume an equilibrium exists which is socially optimal with \( M_e \) and \( c_d \) fixed by that equilibrium. Also let \( q^*(c) \) denote equilibrium quantities. If the equilibrium is efficient for these fixed \( M_e \) and \( c_d \), the quantities \( q_p(c) \) a planner would choose must be optimal. For convenience, define the functional \( H(q) \) as in the above proof and let \( U^*(q) \equiv U(M_e, c_d, q) \) be as in Equation [C.4].

By Theorems 5.11 and 5.15 of Troutman, a necessary condition for \( q_p \) to be optimal is that either

\[
\delta H(q_p; \xi) = 0 \quad \forall \xi \in C^1[0, c_d]
\]

or

\[
\exists \lambda \text{ s.t. } \delta U^*(q_p) = \lambda \delta H(q_p; \xi) = 0 \quad \forall \xi \in C^1[0, c_d].
\]

We will rule out the first and exploit an implication of the second.

**Case 1:** \( \delta H(q_p; \xi) = 0 \quad \forall \xi \in C^1[0, c_d] \). \( \forall \xi \) we have that

\[
\delta H(q_p; \xi) = \int_0^{c_d} \xi(c)cg(c)dc = 0
\]

which implies \( cg(c) \) is identically zero on \([0, c_d]\) by Corollary 4.3 of Troutman which is clearly not optimal.

**Case 2:** \( \delta U^*(q_p) = \lambda \delta H(q_p; \xi) \forall \xi \in C^1[0, c_d] \). For any fixed \( M_e \) and \( c_d \) and \( \forall \xi \) we have that

\[
v(M_e, c_d) \int_0^{c_d} \xi(c)u'(q_p(c))g(c)dc = \lambda M_e \int_0^{c_d} \xi(c)cg(c)dc
\]

so for \( \lambda' \equiv \frac{\lambda M_e}{v(M_e, c_d)} \) we have \( \int_0^{c_d} [u'(q_p(c)) - \lambda' c]g(c)\xi(c)dc = 0 \) and since \( g \) is \( C^1 \) and strictly positive, again by Corollary 4.3 of Troutman we conclude

(C.5) \[
\begin{equation}
    u'(q_p(c)) = \lambda' c
\end{equation}
\]

Using similar reasoning, the solution to the consumer’s problem in the market equilibrium must necessarily satisfy \( u'(q^*(c)) = \mu p(c) \) for equilibrium prices \( p(c) \) and some \( \mu \). Inverse demand \( D(q(c)) \) for a monopolist with cost \( c \) is therefore \( D(q(c)) = \frac{u'(q(c))}{\mu} \). In equilibrium a monopolist

\(^{72}\)Sufficient conditions for this are \( 2u'' + u'''q < 0 \) or that \( u \) is the integral of a strictly decreasing and concave function.
with costs $c$ picks $q_m(c)$ according to

\[(\text{MP}) \quad \max_{q_m(c)} [D(q_m(c)) - c]q_m(c) = \max_{q_m(c)} \frac{u'(q_m(c))}{\mu} - c]q_m(c)\]

so long as the resulting profit covers $f$. By assumption, the FOC \( [u'(q_m(c)) - c] + \frac{u''(q_m(c))}{\mu} q_m(c) = 0 \) uniquely determines each monopolist’s optimal quantity which must be $q^*(c)$ in equilibrium. We conclude that $q^*(c)$ is implicitly determined by the monopolist FOC as given in Equation (C.6).

\[(\text{C.6}) \quad u'(q^*(c)) + u''(q^*(c))q^*(c) = \mu c\]

We now show $q^* = q_p$. Since $H(q_p) = H(q^*)$ and $H(q)$ is linear in $q$, any convex combination $q_a \equiv aq^* + (1-a)q_p$ has $H(q_a) = H(q_p) = H(q^*)$ and so is attainable. Since $u$ is strictly concave, a standard concavity argument shows that the optimality of $q_p$ and $q^*$ implies $q_p = q_a = q^* \quad \forall a \in [0,1]$. Now comparing Equations (C.5) and (C.6) with the knowledge that $q^* = q_p$ and dividing the second by the first we see Equation (C.7) holds on $[0, c_d]$.

\[(\text{C.7}) \quad 1 + \frac{u''(q_p(c))q_p(c)}{u'(q_p(c))} = \frac{\mu}{\lambda'}\]

Equation (C.7) implies for some constant $k_0$ that for each $c \in [0, c_d]$ that

\[u''(q_p(c))q_p(c) = k_0u'(q_p(c))\]

Equation (C.6) paired with $u'' < 0$ shows that $q(c)$ is strictly decreasing so we have that $q([0, c_d]) = [q(c_d), q(0)]$. Consequently, $\forall x \in [q(c_d), q(0)]$ we have that $u''(x)x = k_0u'(x)$. Standard solution techniques imply that the unique continuously differentiable solution for $u$ on $[0, c_d]$ is $u(x) = \alpha + \beta x^\gamma$ for constants $\alpha, \beta, \gamma$, which is precisely the CES form up to an affine transformation.


Proof. We will show that for any two Melitz economies of size $L$ and $L'$, the market equilibrium outcomes are the same excepting $M$ and $M_e$. First consider a market equilibrium of an economy of size $L$ which has characterizing conditions (1), (2) and (3):
(1) Consumers maximize utility given prices and income. Thus \( u'(q(c)) = \lambda p(c) \) where \( \lambda \) is a positive constant which solves \( \frac{1}{M_i} = \int_0^{e_i} p(c)(u')^{-1}(\lambda p(c))dG \) and \( q(c) = (u')^{-1}(\lambda p(c)) \).

(2) Subsequent to entry, firms maximize profits which implies for \( h(x) \equiv (u')^{-1}(x), h(\lambda p(c)) + \lambda (p(c) - c)h'(\lambda p(c)) = 0 \).

(3) Prior to entry, expected industry profits are zero. \( \int_0^{c_D} (p(c) - c)h(\lambda p(c))dG = fG(c_D) + f_e \).

In order to construct the equilibrium for an economy of size \( L' \), let denote \( ' \) denote the equilibrium values under \( L' \) and begin with let \( M'_e = \frac{L'}{L}M_e \) and \( p'(c) \equiv p(c) \). Then \( \frac{L'}{M'_e} = \frac{L}{M_e} = \int_0^{c_D} p(c)h(\lambda p(c))dG \) so from Condition (1), \( \lambda' = \lambda \) and \( q'(c) = q(c) \). This implies at \( \lambda', q'(c) \) both Conditions (2) and (3) hold so \( \{M'_e = \frac{L'}{L}M_e, p'(c) = p(c), q'(c) = q(c)\} \) constitute and equilibrium for an economy of size \( L' \).

**C.4. Melitz Open Economy. Proposition.** Assuming \( \int f / f_x \frac{1}{\tau} < \tau \) (as in Melitz). Then the market equilibrium of an open Melitz economy is socially optimal.

**Proof.** Fix a country \( i \) and that country’s contribution to aggregate welfare is given by \( W^i \) where

\[
W^i(q^i, M^i, c^i_d) \equiv M^i_c \int_0^{c^i_d} q^i(c)e^{\rho}g(c)dc + \sum_{j \neq i} \tau^{-\rho}M^i_c \int_0^{c^j_d} q^j(c)e^{\rho}g(c)dc
\]

Since labor is not mobile, maximizing \( W \equiv \sum W^i(q^i, M^i, c^i_d) \) is equivalent to maximizing each \( W^i \) separately. In particular, if we can exhibit a maximum for \( W^i \) which corresponds to the market equilibrium then by symmetry we are done. Defining \( U^i(q^i, M^i, c^j_d) \equiv M^i_c \int_0^{c^j_d} q^j(c)e^{\rho}g(c)dc \), we have \( W^i = U^i + \tau^{-\rho} \sum_{j \neq i} U^j \). Explicitly, suppressing some arguments the problem of maximizing \( W^i \) is given by Equation (C.8) where \( H^i \) is given by Equation (C.9).

\[
\text{(C.8)} \quad \max_{q^i, M^i, c^i_d} U^i + \tau^{-\rho} \sum_{j \neq i} U^j \quad \text{subject to} \quad H^i(q^i, M^i, c^i_d) = 0
\]

\[
\text{(C.9)} \quad H^i(q^i, M^i, c^i_d) \equiv L - \sum_j M^i_c \int_0^{c^j_d} cq^j(c)g(c)dc - \max_j \{M^i_c\} [f_e + G(\max_j c^j_d)f] - f_x \sum_{j \neq i} G(c^j_d)M^i_c
\]

\[\text{\footnote{Our strategy is to reduce optimal welfare} W^i \text{ to a function of} M \text{ and the} L_j. \text{ This reduces Equation} \text{ (C.8) to a simpler maximization problem whose solution is that of the market equilibrium.}}]
Again suppressing the arguments $q^i, M_e, c_d^i$ and for brevity defining the two “max” terms as

\[
\bar{M} \equiv \max_j \{M_e^j\} \quad \bar{c} \equiv \max_j \{c_d^j\}
\]

we can decompose the allocation of labor by export destination and a fixed cost component $L_f$ as

\[
H^i = L - [L_f + \sum L_j]
\]

where

\[
L_f(\bar{M}, \bar{c}) \equiv \bar{M}[f_e + G(\bar{c})] \quad L_i \equiv \int_0^{c_d^j} cq^i(c)g(c)dc \quad L_j \equiv \int_0^{c_d^j} c q^i(c)g(c)dc + f_x G(c_d^j)
\]

For the present fix $\bar{M}$ and the $L_j$ and consider the problem of maximizing each $U^i$ individually whenever $L_j > 0$. Maximizing each $U^i$ for $j \neq i$ are equivalent problems, with maximizing $U^i$ being slightly different since fixed costs $f_x$ are not incurred. The two maximization problems for $U^i$ and $U^j$ are given respectively by Equations (C.10) and (C.10) with the additional constraints $M_e^j, M_e^i \leq \bar{M}$ and $c_d^j, c_d^i \leq \bar{c}$.

(C.10) \[
\max_{q^i, M_e, c_d^j} U^i(q^i, M_e^j, c_d^j) \text{ subject to } L_j = M_e^j[\int_0^{c_d^j} cq^i(c)g(c)dc + f_x G(c_d^j)]
\]

(C.11) \[
\max_{q^j, M_e, c_d^j} U^j(q^j, M_e^j, c_d^j) \text{ subject to } L_i = M_e^j[\int_0^{c_d^j} cq^j(c)g(c)dc]
\]

Following the same strategy to solve Equations (C.10) and (C.11) as in the autarky case, by fixing $M_e^j, M_e^i \leq \bar{M}$ and $c_d^j, c_d^i \leq \bar{c}$ we have that the unique solution for $q^i(c)$ and $q^j(c)$ which are sufficient for an optimum for $M_e^j, M_e^i$ and $c_d^j, c_d^i$ fixed (provided $L_j > M_e^j f_x G(c_d^j)$ which must hold as above so long as $L_j > 0$) are given by Equation (C.12) (where as above $R(x) \equiv \int_0^x c^{\frac{1}{1-\rho}} g(c)dc$).

(C.12) \[
q^i(c) = \frac{1}{1-\rho} \left[ \frac{L_j}{M_e^j} - f_x G(c_d^j) / R(c_d^j) \right] \quad q^j(c) = \frac{1}{1-\rho} \left[ \frac{L_i}{M_e^i} \right] / R(c_d^j)
\]

Equation (C.12) respectively gives welfare $U^i$ and $U^j$ of

(C.13) \[
U^i = (M_e^j)^{1-\rho} (L_j - M_e^j f_x G(c_d^j))^{\rho} R(c_d^j)^{1-\rho} \quad U^j = (M_e^i)^{1-\rho} L_i^{\rho} R(c_d^j)^{1-\rho}
\]

so long as $L_j > 0 \ \forall j$. In fact, since $\rho < 1$, so long as $L_j > 0$ for at least one $j \neq i$, it is easy to show that a necessary condition is that $L_j > 0 \ \forall j \neq i$ by reallocating labor from the positive $L_j$ to some $L_k = 0$ so either all $L_j \neq i$ are strictly positive or zero. Clearly $f_x > 0$ also implies a necessary condition is that $L_i \geq \max_{j \neq i} \{L_j\}$ so the only cases we need to consider are $L_j > 0 \ \forall j$ and $L_j = 0 \ \forall j \neq i$. 
We now summarize what we have shown so far. We have reduced the problem (C.8) to two finite dimensional problems (C.14, C.15) given below corresponding to the cases above regarding the $L_j$, respectively.\footnote{For Problem (C.15) it is clear that $M_c^i = \overline{M}$ and $c_d^j = \overline{c}$.}

\[(C.14) \max_{\{L_k, M_i^c, c_d^j, \overline{M}, \overline{c}\}} (M_i^c)^{1-\rho} L_i^f R(c_d^j)^{1-\rho} + \tau^{-\rho} \sum_{j \neq i} (M_i^c)^{1-\rho} (L_j - M_c^j f_x G(c_d^j))^{\rho} R(c_d^j)^{1-\rho} \]

subject to $L_f(M, \overline{c}) + \sum L_j = L$ and $M_c^j \leq \overline{M}$, $c_d^j \leq \overline{c}$ \forall j

\[(C.15) \max_{L_i, \overline{M}, \overline{c}} (\overline{M})^{1-\rho} L_i^f R(\overline{c})^{1-\rho} \quad \text{subject to } L_f(\overline{M}, \overline{c}) + L_i = L \]

Now for either problem (C.14, C.15) and any fixed pair $(\overline{M}, \overline{c})$ the remaining choice variables are restricted to a compact set $K(\overline{M}, \overline{c})$ so that continuity of the objective function (by defining $U_j = 0$ when $L_j = 0$) guarantees a solution to the problem exists for each pair $(\overline{M}, \overline{c})$ and we denote the value of the objective function at the maximum by $S(\overline{M}, \overline{c})$. In fact, $K(\overline{M}, \overline{c})$ can be shown to be a continuous correspondence so by the Theorem of the Maximum $S(\overline{M}, \overline{c})$ is continuous on $L_f^{-1}([0, L])$ which is compact since $L_f$ is continuous and therefore a global max of $S(\overline{M}, \overline{c})$ exists which is the Social Optimum by the above arguments (see for instance, Berge and Karreman (1963)).

Problem (C.15) corresponds exactly to a Planner in autarky and we have detailed the solution above, namely an allocation corresponding to the market equilibrium in autarky. As shown in Melitz, under our parameter assumption the open economy market equilibrium (clearly attainable by a social planner) yields higher welfare than the closed market equilibrium and therefore the solution to Problem (C.14) yields higher welfare than Problem (C.15).

As for Problem (C.14), having shown existence, we now return to maximizing Equations (C.13) for fixed $L_i, L_j$. The solution for the $j = i$ problem is clearly $M_i^c = \overline{M}$, $c_d^i = \overline{c}$. Consider any $j$ where $L_j > 0$ so clearly we must have $M_c^j, c_d^j > 0$ at any optimum. One may show using standard techniques that for fixed $c_d^j$, $U_j$ is strictly concave in $M_i^c$ with critical point \( \frac{(1-\rho)L_j}{f_x G(c_d^j)} \), so for any fixed $c_d^j$, $M_c^j = \min \{ \frac{(1-\rho)L_j}{f_x G(c_d^j)}, \overline{M} \}$. Now if the optimal $c_d^j$ is s.t. $c_d^j \in (0, \overline{c})$ we must have for fixed $M_c^j$ that
\[ \frac{\partial U}{\partial c_d} = 0. \] Some algebra shows that this FOC is equivalent to Equation (C.16).

\[ [L_j - M^{i*}_e f_x G(c_d^j)] = \frac{\rho}{1 - \rho} \frac{M^{i*}_e f_x R(c_d^j)}{(c_d^j)^\frac{r}{\rho - 1}} \]

Similarly if for fixed \( c_d^j \) we have \( M^{i*}_e = \frac{(1 - \rho)L_j}{f_x G(c_d^j)} < \overline{M} \) we must have

\[ [L_j - M^{i*}_e f_x G(c_d^j)] = \frac{\rho}{1 - \rho} M^{i*}_e f_x G(c_d^j) \]

Equations (C.16) and (C.17) together would imply \( \frac{R(c_d^j)}{(c_d^j)^\frac{r}{\rho - 1} G(c_d^j)} = 1 \) but \( \frac{R(c_d^j)}{(c_d^j)^\frac{r}{\rho - 1} G(c_d^j)} > 1 \) for \( c_d^j > 0 \), so we conclude that \( c_d^j < \bar{c} \) implies \( M^{i*}_e < \overline{M} \) and \( \frac{(1 - \rho)L_j}{f_x G(c_d^j)} < \overline{M} \) implies \( c_d^j = \bar{c} \). Now considering \( M^{i*}_e = \frac{(1 - \rho)L_j}{f_x G(c_d^j)} < \overline{M} \) as a function of \( c_d^j \), from above we have

\[ \text{sgn}\{ \frac{\partial U}{\partial c_d^j} \} = \text{sgn}\{ (1 - \rho) \frac{(c_d^j)^\frac{r}{\rho - 1} G(c_d^j)}{L_j - M^{i*}_e f_x G(c_d^j)} \} \]

\[ = \text{sgn}\{ (1 - \rho) \frac{(c_d^j)^\frac{r}{\rho - 1} G(c_d^j)}{R(c_d^j)} \} < 0 \]

since again \( \frac{R(c_d^j)}{(c_d^j)^\frac{r}{\rho - 1} G(c_d^j)} > 1 \) for \( c_d^j > 0 \). This implies that for \( M^{i*}_e = \frac{(1 - \rho)L_j}{f_x G(c_d^j)} < \overline{M} \), \( c_d^j = \bar{c} \) is not an optimum and therefore at any optimum, \( M^{i*}_e \leq \overline{M} \leq \frac{(1 - \rho)L_j}{f_x G(c_d^j)} \). Since \( c_d^j = 0 \) is clearly not optimal, this in turn implies that either Equation (C.16) holds or \( c_d^j = \bar{c} \). With these reductions we revisit Problem (C.14) which has been reduced to Problem (C.18) with the added constraints of either \( c_d^j = \bar{c} \) or Equation (C.16) holds \( \forall j \neq i \).

\[ \max_{\{l_i\},c_d^j,\overline{M},\overline{x}} \overline{M}^{1 - \rho} \{ L_i^\rho R(\bar{c})^{1 - \rho} + \tau^{-\rho} \sum_{j \neq i} (L_j - \overline{M} f_x G(c_d^j))\rho R(c_d^j)\}^{1 - \rho} \text{ sub to } L_f(\overline{M},\bar{c}) + \sum L_j = L \]

Now consider solving Problem (C.18) with \( c_d^j \) unconstrained. Using a standard Lagrangian approach, we find a candidate solution from necessary conditions in which \( c_d^{j*} = \left( \frac{f_j}{f_x} \right)^{\frac{1 - \rho}{\rho}} \bar{c} \) and since it is assumed \( f_j \geq f_x > 0 \), \( c_d^{j*} < \bar{c} \). The candidate solution with \( c_d^j \) unconstrained also yields Equation (C.16) so the unconstrained candidate solution satisfies the omitted constraints. We conclude the necessary conditions embodied in the candidate solution are also necessary for any solution.
to Problem (C.18). It turns out these necessary conditions are exactly those which fix market equilibria so if the market equilibrium exists and is unique, there is a unique solution to the necessary conditions and since existence of a solution to Problem (C.18) was shown above, the planner and market allocations coincide.

C.5. Converse of Optimality of Selection Effects with CES-Benassy. Proposition 6. Consider a Melitz economy with preferences as in (10.1) and Benassy $v$. The market equilibrium is socially optimal only if $u$ is CES and $v_B = (1 - \rho) / \rho$ as in Melitz (2003).

Proof. Under Benassy preferences, specifically $U(M_e, c_d, q)$ where

$$U(M_e, c_d, q) \equiv \{[M_e G(c_d)]^{\rho^{(v+1)}-1} M_e \int_{c_d}^{c_d} q(c)^{\rho} g(c) dc\}^{\rho}$$

one may show that the market equilibrium $M_e, c_d$ and $q(c)$ correspond to the market equilibrium of Melitz (2003). One may also show that for fixed $M_e$ and $c_d$ a social planner will choose $q^*$ under Benassy as under Melitz as in the proof that a Melitz economy is first best. Having fixed $q^*$ as in that proof, we consider the reduced SP problem which involves finding an optimal $M_e$ and $c_d$.

Finding $M_e$ for $c_d$ fixed. We may therefore consider maximizing $W(M_e, c_d)$ where we have

$$W(M_e, c_d) \equiv [M_e G(c_d)]^{\rho^{(v+1)}-1} M_e \int_{0}^{c_d} q^*(c)^{\rho} g(c) dc$$

and for $c_d$ fixed we can maximize $\tilde{W}(M_e) \equiv M_e^{\nu} \{\gamma - M_e\}$ where $\gamma \equiv L / (f_e + f G(c_d))$ since the arg max of $\tilde{W}(M_e)$ maximizes $W(M_e, c_d)$ for $c_d$ fixed. Looking at the FOC we have

$$\tilde{W}'(M_e) = M_e^{\nu-1} \{\nu \gamma - (v + 1) M_e\}$$

so the unique maximum of $\tilde{W}$ is

$$M_e^\nu(c_d) = \frac{v}{v + 1} L / (f_e + f G(c_d))$$
**Inefficiency of the market equilibrium.** Under Melitz preferences we have shown the SP chooses \( c_d \) to maximize \( Z(c_d) \) where

\[
Z(c_d) \equiv R(c_d)^{1-\rho} [f_e + f G(c_d)]^{\rho - 1}
\]

and the arg max of \( Z \) corresponds to the \( c_d \) fixed by the market, say \( c_d^M \). Now we can compute an explicit measure of welfare using \( M_e(c_d) \) which is

\[
W(M_e(c_d), c_d) = \left[ \frac{G(c_d)}{f_e + f G(c_d)} \right]^{\rho (v+1) - 1} L^{\rho (v+1)} \left[ \frac{v}{v+1} \right]^{\rho v} \left[ \frac{1}{v+1} \right]^{\rho (f_e + f G(c_d))^{\rho - 1}}
\]

\[
\propto \left[ \frac{G(c_d)}{f_e + f G(c_d)} \right]^{\rho (v+1) - 1} Z(c_d)
\]

Since \( W(M_e(c_d), c_d) \) and \( Z(c_d) \) are continuously differentiable in \( c_d \), by the above remarks we must have \( Z'(c_{d}^{ME}) = 0 \) and if the market is efficient under Benassy that \( \frac{\partial}{\partial c_d} W(M_e(c_d^M), c_d^M) = 0 \). But then

\[
\frac{\partial}{\partial c_d} W(M_e(c_d), c_d) \propto \frac{\partial}{\partial c_d} \left[ \left[ \frac{G(c_d)}{f_e + f G(c_d)} \right]^{\rho (v+1) - 1} Z(c_d) \right]
\]

\[
= \left[ \frac{G(c_{d}^{M})}{f_e + f G(c_{d}^{M})} \right]^{\rho (v+1) - 2} Z'(c_d^M)
\]

\[
= (\rho (v+1) - 1) \left[ \frac{G(c_{d}^{M})}{f_e + f G(c_{d}^{M})} \right]^{\rho (v+1) - 2} Z(c_d^M)
\]

This last expression is zero iff \( v = \frac{1-\rho}{\rho} \) so we conclude that even if preferences are CES, the market is efficient under Benassy preferences iff \( v = \frac{1-\rho}{\rho} \).

**C.6. Market Equilibrium in Melitz and MO.** The equilibrium outcomes are summarized in Tables 12 and 13. We denote the domestic autarky outcomes by subscript \( a \) and the open economy outcomes by \( t \). The domestic and export markets of the open economy are denoted by \( d \) and \( x \) respectively. For brevity, we define \( \xi_i^{\rho/(\rho-1)} = \int_0^{c_i} \xi_j^{\rho/(\rho-1)} dG/G(c_i) \) and \( k(c_i) = (\xi_i/c_i)^{\rho/(\rho-1)} - 1 \) for \( i = a, d, x \). Note that \( f = 0 \) in MO.
Table 12. Market Outcomes in Autarky (a)

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_a$</td>
<td>$f_c / G(c_a) = f_k(c_a)$</td>
</tr>
<tr>
<td>$p(c)$</td>
<td>$c / \rho$</td>
</tr>
<tr>
<td>$\tilde{p}_a$</td>
<td>$\tilde{c}_a / \rho$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>$f_{G(c_a)}(1-\rho)L$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\gamma_a(c_a)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\gamma_a(c_a)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\gamma_a(c_a)$</td>
</tr>
</tbody>
</table>

Table 13. Market Outcomes in the Open Economy (t)

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_d$</td>
<td>$f_c / G(c_d) = f_k(c_d) + nG(c_x)f_k(c_x)/G(c_d)$</td>
</tr>
<tr>
<td>$c_x$</td>
<td>$c_d(f_x)^{1-\rho}$</td>
</tr>
<tr>
<td>$\tilde{c}_t(c_d)$</td>
<td>$\left{ \frac{M_d}{M_t}\rho^{\eta-1} + n\frac{M_d}{M_t}\beta \frac{\rho^{\eta-1}}{1-\rho} \right}^{\eta^{1-\eta}}$</td>
</tr>
<tr>
<td>$p_x(c)$</td>
<td>$\tau c / \rho$</td>
</tr>
<tr>
<td>$\tilde{p}_t$</td>
<td>$\tilde{c}_t / \rho$</td>
</tr>
<tr>
<td>$M_t$</td>
<td>$f_{G(c_t)}(1-\rho)L$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\gamma_a(c_a)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\gamma_a(c_a)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\gamma_a(c_a)$</td>
</tr>
</tbody>
</table>

APPENDIX D. EMPIRICAL DETAILS

D.1. Estimation of Simple Income Dynamics.

D.1.1. A Simple Class of Income Dynamics. We assume income evolves as $y_{it+1} = f_{\beta}(y_{it})$ where $\beta$ are parameters of an increasing income transition $f_{\beta}$. For expositional purposes, we have chosen a particularly simple form for $f_{\beta}$ to capture the rates of income change within each income decile. We assume $f$ is continuous and piecewise linear on segments $[\gamma_{i-1}, \gamma_i]$ which correspond to the $i^{th}$ income decile, and for convenience we define $\gamma_0 \equiv 0$ and $\gamma_{10} \equiv \infty$. The slope of $f_{\beta}$ on each segment $[\gamma_{i-1}, \gamma_i]$ must be positive and so is defined as $\beta_i$ where $\beta_i$ is the $i^{th}$ coordinate of $\beta$. Therefore $\beta$ is simply a vector containing the rate of income growth for each decile. The equation for $f_{\beta}$ is given in Equation (D.1).

(D.1) \[ f_{\beta}(x) \equiv \sum_{i=1}^{10} \left[ \sum_{j=1}^{i-1} \beta_i \right] \left[ \gamma_{i-1} - \gamma_j \right] \mathbf{1}_{[\gamma_{i-1}, \gamma_i]}(x) \]

D.1.2. Econometric Structure. We are provided income deciles $I_{dtk}$ for periods $\{t_k\}$ and deciles $d$, where we assume for convenience that $I_{dtk}$ are the median of each decile observed with log-normal(0, $\sigma$) errors $\xi_{dt}$. Letting $F_0$ denote the cumulative distribution of income in period 0, our
assumptions imply that the observed \( I_{dt_k} \) are given by Equation (D.2).

\[
I_{dt_k} = f^{(h)}(x_d) \xi_{dt_k}
\]

where \( x_d \equiv F_0^{-1}(0.1d - .05) \) is the median income of the \( d^{th} \) decile starting in period 0. Given Equation (D.1) we recover \( \beta \) through maximum likelihood.

D.1.3. Estimates. Before getting to the estimates, we wish to point out some caveats about our results. Since we are working with income deciles rather than micro-data, the estimates should not be taken too literally. In particular, due to the small number of observations and ten parameters, confidence bounds for the estimates would be essentially meaningless. On the positive side, our estimates do most likely capture far more relevant information about income dynamics than point estimates such as GINI or Polarization measures. We report our country wide estimates of \( \beta \) for each country in Table 14.

<table>
<thead>
<tr>
<th>Country</th>
<th>Slope of Income Transition in each Decile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Chile</td>
<td>1.0369</td>
</tr>
<tr>
<td>Peru</td>
<td>1.0399</td>
</tr>
<tr>
<td>Venezuela</td>
<td>0.9031</td>
</tr>
</tbody>
</table>

APPENDIX E. PROOFS

Lemma. Suppose \( f \) and \( g \) have bounded derivatives. Then \( g \) is concave iff \( g(\int f dF) \geq \int g \circ f dF \) for all distributions \( F \).

Proof. If \( g \) is concave the result holds by Jensen’s inequality. For the converse, assume \( g(\int f dF) \geq \int g \circ f dF \) for all distributions \( F \) and fix \( a, b \in \mathbb{R} \) and \( \lambda \in [0, 1] \). Define the distribution \( H_\delta(x) \equiv \int_{-\infty}^{x} h_\delta(t)dt \) where \( h_\delta(t) = \frac{\lambda}{2\delta} 1_{[f^{-1}(a) - \delta, f^{-1}(a) + \delta]}(t) + \frac{1-\lambda}{2\delta} 1_{[f^{-1}(b) - \delta, f^{-1}(b) + \delta]}(t) \) where \( 1_A(t) \) denotes the indicator for \( t \) contained in the set \( A \). We will show that

\[
g(\lambda a + (1 - \lambda)b) = \lim_{\delta \to 0} g \left( \int f dH_\delta \right) \geq \lim_{\delta \to 0} \int g \circ f dH_\delta = \lambda g(a) + (1 - \lambda)g(b)
\]
In order to evaluate the limits above, we will make use of the fact that for \( \|f'\|_{\infty} \equiv \sup |f'| \), \( \|g'\|_{\infty} \equiv \sup |g'| \) and \( c > 0 \) we have

\[
\text{sup}_{|y| \leq c} |f(x) - f(x + y)| \leq c \|f'\|_{\infty} \quad \text{sup}_{|y| \leq c} |g \circ f(x) - g \circ f(x + y)| \leq c \|g'\|_{\infty} \|f'\|_{\infty}
\] (E.2)

First we evaluate \( g \left( \int f dH_{\delta} \right) \) and consider that

\[
\int f(t) \frac{\lambda}{2\delta} \mathbf{1}_{[f^{-1}(a) - \delta, f^{-1}(a) + \delta]}(t) dt - \lambda a = \int \frac{\lambda}{2\delta} \int_{-\delta}^{\delta} f(f^{-1}(a) + t) - f(f^{-1}(a)) dt
\]

\[
\leq \frac{\lambda}{2\delta} \int_{-\delta}^{\delta} \|f'\|_{\infty} dt = \lambda \delta \|f'\|_{\infty}
\]

where the last line follows from Equations \( \text{E.2} \). It follows that for \( L(\delta) \equiv \lambda a + (1 - \lambda) b - \int f dH_{\delta}, \)

\[
|L(\delta)| \leq \delta \|f'\|_{\infty}
\]

and

\[
\lim_{\delta \to 0} g \left( \int f dH_{\delta} \right) = \lim_{\delta \to 0} g \left( \lambda a + (1 - \lambda) b - L(\delta) \right) = g \left( \lambda a + (1 - \lambda) b \right)
\]

Similarly from Equations \( \text{E.2} \), for \( R(\delta) \equiv \lambda g(a) + (1 - \lambda) g(b) - \int g \circ f dH_{\delta}, \|R(\delta)\| \leq \delta \|g'\|_{\infty} \|f'\|_{\infty} \)

and

\[
\lim_{\delta \to 0} g \circ f dH_{\delta} = \lim_{\delta \to 0} \lambda g(a) + (1 - \lambda) g(b) - R(\delta) = \lambda g(a) + (1 - \lambda) g(b)
\]

Furthermore, for each \( \delta \) we have \( g \left( \lambda a + (1 - \lambda) b - L(\delta) \right) \geq \lambda g(a) + (1 - \lambda) g(b) - R(\delta) \) giving Equation \( \text{E.1} \).

\[\square\]

**Proposition.** For any class of income transitions \( g(y, \rho) \) indexed by \( \rho \), demand for redistribution decreases in \( \rho \) for all income distributions if and only if \( \frac{\partial}{\partial \rho} \frac{\partial}{\partial y} \ln \frac{\partial}{\partial y} g(y, \rho) \leq 0 \) provided:

1. \( g \) is strictly increasing in \( y \) and twice continuously differentiable
2. \( \frac{\partial}{\partial y} g(y, \rho) \) is bounded and \( \frac{\partial^2}{\partial y^2} g(y, \rho) \) non-zero

**Proof.** Let \( f_{\rho}(y) \equiv g(y, \rho) \) and we want to show for all \( \Delta > 0 \) that

\[
f_{\rho}^{-1} \left( \int f_{\rho} dF \right) \geq f_{\rho+\Delta}^{-1} \left( \int f_{\rho+\Delta} dF \right)
\] (E.3)

for all distributions \( F \) iff \( \frac{\partial}{\partial \rho} \frac{\partial}{\partial x} \ln \frac{\partial}{\partial y} f_{\rho}(x) \leq 0 \). Letting \( h \) be defined by \( f_{\rho+\Delta} = h \circ f_{\rho} \) since \( f_{\rho} \) is strictly increasing Equation \( \text{E.3} \) is equivalent to \( \int f_{\rho} dF \geq h^{-1} \left( \int h \circ f_{\rho} dF \right) \) (that \( h \) has an inverse follows from \( g \) strictly increasing in \( y \)). It therefore follows from Lemma \( ?? \) that Equation \( \text{E.3} \)
holds iff \( h = f_{p+\Delta} \circ f_p^{-1} \) is concave. Noting that \( h' \circ f_p = f'_{p+\Delta} / f'_p \) we have

\[
h'' \circ f_p \cdot f'_p = \left[ f''_{p+\Delta} f'_p - f'_{p+\Delta} f''_p \right] / \left( f''_p \right)^2
\]

where \( h'' \) exists by inspection. We conclude \( h \) is concave iff \( h'' \leq 0 \) iff \( f''_{p+\Delta} / f'_{p+\Delta} \leq f''_p / f'_p \) iff

\[
\frac{\partial}{\partial \rho} \frac{\partial}{\partial y} \ln \frac{\partial}{\partial y} g(y, \rho) \leq 0.
\]

E.1. **Non-monotonicity of redistributive demand.** In a NoPoUM world, the determination of whether the demand for redistribution is increasing or decreasing depends on all of \( f, t \) and the initial distribution of income. Although the demand for redistribution may be directly computed, in general it is hard to derive a particular path analytically due to its dependence on the range of possible income distributions. In order to highlight this relationship, we provide an “Impossibility Result.” Our result shows for a fixed NoPOUM income dynamic that the demand for redistribution can be either increasing or decreasing depending on the income distribution. We state our result as

**Proposition 13.** Suppose \( f \) finitely many fixed points \( \{p_i\}_{i=1}^I \) and let \( F \) denote a continuous distribution of income on \([p_1, p_I]\).

1. *(POUM Forever)* If \( \bar{u} \cap [p_2, p_I] \) contains an open set there is an \( F \) where the demand for redistribution always decreases.

2. *(NoPOUM Forever)* If \( u \cap [p_1, p_{I-1}] \) contains an open set there is an \( F \) where the demand for redistribution always increases.

**Proof.** Available upon request. □

This proposition shows that a broad class of dynamics can exhibit either increasing or decreasing demand for redistribution. The deciding factor for redistributive dynamics, even for a fixed dynamic, is the initial distribution of income. This emphasizes the interrelationship between “Upward/No Mobility” in the dynamic role of income transitions and the “existing order” in the role of the income distribution: political implications cannot be drawn without considering both.

**APPENDIX F. DEAD WEIGHT LOSS**

In order to explain this effect, we depict idealized transitions \( f_R \) and \( f_L \) in Figure F.1. This figure supposes \( f_R \) is concave as above while \( f_L \) is convex (which is approximately true in applications). Fix any future mean income \( \mu \) which is a fair margin above median income, and consider the level
of support for redistribution next period as dead weight loss $D$ increases. Under $f_R$, the fraction of the population supporting redistribution falls from $F_0(f_R^{-1}(\mu))$ to $F_0(f_R^{-1}([1 - D]\mu))$ which in Figure F.1 is larger than the drop in support under $f_L$, namely $F_0(f_L^{-1}(\mu))$ to $F_0(f_L^{-1}([1 - D]\mu))$. This asymmetric effect of dead weight loss holds because in the illustrated range, the concavity of $f_R$ implies $f_R$ is much flatter than $f_L$ which is convex. The precise conditions under which this argument apply are stated in Proposition F. Proposition F shows that modeling Right and Left ideologies reveals a second, new insight that dead weight loss can increase political volatility for forward looking voters.

**Figure F.1. Polarization from Dead weight Loss**

![Graph showing polarization from dead weight loss](image)

**Proposition.** Assume $f'_R, f'_L > 0, f''_R < 0, f''_L > 0$ and $f_R(0) = f_L(0) = 0$ with $E[f_R(y)] \geq E[f_L(y)]$. If $f'_R(y_{equal}) = f'_L(y_{equal})$ and $E[f_R(y)] \geq E[f_L(y)]$ for $1 - D \geq f_R(z) / E[f_R(y_{equal})]$.

**Proof.** The difference in redistribution demanded between Left and Right is $f_L^{-1}([1 - D]E[f_L(y)]) - f_R^{-1}([1 - D]E[f_R(y)])$. We need to show that for all suitable values of $D$, $\frac{\partial}{\partial D} f_L^{-1}([1 - D]E[f_L(y)]) \geq \frac{\partial}{\partial D} f_R^{-1}([1 - D]E[f_R(y)])$. Evaluating both sides of the inequality yields

$$\frac{\partial}{\partial D} f_L^{-1}([1 - D]E[f_L(y)]) = -E[f_L(y)]/f'_L \circ f_L^{-1}([1 - D]E[f_L(y)])$$

$$\frac{\partial}{\partial D} f_R^{-1}([1 - D]E[f_R(y)]) = -E[f_R(y)]/f'_R \circ f_R^{-1}([1 - D]E[f_R(y)])$$

By assumption $E[f_R(y)] \geq E[f_L(y)]$ and $f'_R, f'_L > 0$ so it is sufficient to show

(E.1) $f'_R \circ f_R^{-1}([1 - D]E[f_R(y)]) \leq f'_L \circ f_L^{-1}([1 - D]E[f_L(y)])$
Constructing a distribution $G(y) \equiv [1 - D]F(y) + D1_{\{y \leq 0\}}$, Jensen’s inequality with $f_R(0) = f_L(0) = 0$ implies

$$f_R^{-1}([1 - D]E[f_R(y)]) \leq \int xdG = [1 - D]E[y] \leq f_L^{-1}([1 - D]E[f_L(y)])$$

so with $f''_L > 0$ to show (F.1) it is sufficient that

$$f'_R \circ f_R^{-1}([1 - D]E[f_R(y)]) \leq f'_L \circ f_R^{-1}([1 - D]E[f_R(y)])$$

which holds for all $D$ with $f_R^{-1}([1 - D]E[f_R(y)]) \geq y_{equal}$ giving the result. \hfill \Box