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Strategies for Optical Measurements in Rocket Plumes

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Abstract

An ideal rocket engine sensing system might probe several key elements of the rocket engine including: the fuel and oxidizer supply for mass flow rate and impurities, the combustion chamber for pressure and mole fraction of reactants, or the plume for temperature and mole fraction of products. This thesis focuses on the rocket plume component of a rocket engine sensing system. The rocket plume sensor consists of a wavelength scanning laser that spatially sweeps through the rocket plume in order to determine the temperature, water mole fraction, and pressure at different positions in an axially symmetric plume. Because of the low absorbance that will occur in a small rocket plume (<1% for a 2 inch diameter plume), all noise must be minimized. The three largest sources of noise are mode noise, beamsteering, and laser intensity noise. Throughout this research, these three sources of noise are studied in depth. Final recommendations are given for designing a system that will successfully measure these qualities in a rocket plume.
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1. Introduction

The application of absorption spectroscopy has been widely reported in the literature[1, 3-5]. Many measurements have been made in combustion engines where the temperatures are relatively low and the pressures relatively high compared with rocket plumes. Typical HCCI combustion temperatures and pressures might be around 1200K and 40 bar, while rocket plumes are generally around 3000K and 1 bar. Recently, there has been an increased interest in making more spectroscopy measurements in rocket plumes, particularly measurements of water vapor[6, 7]. Rocket engine measurements are more difficult than combustion engine measurements because the lower (atmospheric) pressure makes the water lines very narrow. Not only are the spectral lines narrower, but in the cases investigated for this thesis, the path lengths are always much shorter than paths in the triptane engine. Thus the spectral lines were smaller and harder to measure.

The purpose of this research is to develop the technologies for a system that can measure the absorption spectra of water in rocket plumes from engines like the one shown in Figure 1. The spectra will then be analyzed to determine the water mole fraction, temperature, and pressure of the exhaust plume. Once this sensor is realized, it will be combined with another sensor that measures the purity of the liquid oxygen as it is fed to the rocket engine[8] to complete a sensor system for rocket engine research and development. The information gathered from this sensor system will enable developers like Orbitec and NASA to better
understand the physics of the rocket itself. This understanding will then be used to further develop rocket engine technologies.

Figure 1 – The thrust chamber of one of Orbitec’s vortex rocket engines[1]. This particular setup was specially designed for spectroscopic analysis. The quartz tube allows optical access to the chamber, and it is mounted in such a way as to allow optical access to the plume. All dimensions are given in inches.

The best sensing system to begin with would include a laser that scans quickly through 50-100 nm, on the order of once every 10µs. This would allow measurements to be made on an effectively “frozen” rocket plume. FFT analysis on the signal from a monochromatic beam probing the plume shows that the highest frequency noise from the turbulence is less than 1 MHz. Since the features of interest (i.e. the water lines) are on the order of 1 GHz, the turbulent noise will not be mistaken as a signal when the laser scans over the previously
stated range. Averaging can be used to clean up many other types of noise, such as detector noise.

A robust sensing system should also be immune to beamsteering. Beamsteering is what happens to a beam in an optically turbulent region. Optical disturbances occur when the index of refraction is not uniform and the gradients in the index of refraction are not parallel to the beam path. This causes the light to bend and diverge. If the beamsteering is strong enough, signals can become noisy or lost completely. Although the speed of the beamsteering noise may not be a factor with the proper system, loss of signal would be. Having a good model of beamsteering is critical to designing a beamsteering resistant system.

Finally, the resolution, or bandwidth, of the laser is important. If the bandwidth of the laser is broader than the line width of the water absorption line, than the measured water line will be wider than expected. The laser selected to fit these and above requirements is called a Fourier Domain Mode Locking laser. It is a type of ring laser that has a fiber optic loop in which the light circulates. Each time the light circulates it passes through a gain medium which enables the system to laze. Two necessities for the narrow bandwidth of this type of laser are low dispersion and managed polarization in the fiber loop. Dispersion causes the different wavelengths of light to move at different speeds. As the different wavelengths propagate through the fiber, they will either arrive at the filter at the wrong time and be rejected or pass through the filter and compete with each
other for amplification. Maintaining the polarization is important because the gain medium is highly polarized. As the polarization wanders, the gain in the system can vary intensely. Suggestions are made regarding how to proceed with these problems.
2. Rocket Sensing Background

When light of a certain wavelength is directed a distance $L$ through a medium, some of the light may be absorbed by the medium. The ratio of remaining light (at a given wavelength) to initial incident light is given by Beer’s Law in Equation 1.

\[
\frac{I}{I_0} = \exp\left(-\int_0^L k(\lambda)d\xi\right) \quad (1)
\]

where $k$ is the spectral absorption coefficient, which is a function of wavelength. This form of Beer’s Law accounts for the possibility of varying $k$ throughout the medium along the beam’s path. These variations may arise from different temperatures, mole fraction, or densities, especially in harsh environments like those inside combustion engines or rocket plumes.

When a multi-wavelength laser beam is directed through a test region, different wavelengths will be absorbed different amounts based on the properties of the gases in the test region. For absorption spectroscopy, the general setup is as follows. First there is a light source. In this discussion the source will always be a laser produced by laser diodes or silicon optical amplifiers (SOA). Next there is a pitching element. Although the source itself could be the pitching element, it is often not practical to have the source directly next to a test section. Space constraints around the test section, ease of setup, and sensitivity to the
environment are all possible reasons why fiber coupling to the source might be preferred. When the light is in a fiber, the end of the fiber is the pitching element. There are often optics after the pitching element which condition the light before letting it enter the test section. In most cases, it is a single lens or collimation package. After the pitched light traverses the test section, it comes to a collector or catching element. The collector could be a detector or another fiber which leads to a detector. Just as the pitching element often has optics in front of it, so does the catching element. For almost all discussions in this paper, there will be one lens in front of the pitching element and one lens in front of the catching element.

By assuming the test region to be axially symmetric as in Figure 2, we can use tomography to reconstruct a two dimensional picture of the properties across a plane in the test section. This is done by sweeping the probing beam across the test section from outside to inside. Each bin, or line-of-sight path measured, can be used to find out information about the properties of the medium at a radius $r$. The distance $r$ refers to the closest distance that the bin comes to the center of the test section. The first scan begins on the outside of the plume or test section where the beam is assumed to be unaffected. (Background water in the air is subtracted out.) The second scan, or next bin, is measured slightly closer to the center of the rocket plume than the first. In order to process the data from this second bin, the known properties of the plume at the first bin can be subtracted
out from the measurement so that only the new properties at the inner ring, r are measured.

![Figure 2 - Rays are directed through the beamsteering region at several angles so that different "rings" of the plume can be measured. Each ray represents a measured bin, and the shading emphasizes the axial-symmetry of the plume.](image)

By using tomography and sweeping the beam spatially through the test section, plots like those in Figure 3 can be made. The top plot gives the water mole fraction as a function of radial distance from the center of the plume, and the bottom plot gives the temperature at the same locations. These are the type of measurements that NASA and Orbitec want to make with a rocket plume sensor.
Figure 3 - This data comes from Caswell's initial rocket engine tests[1]. The top plot shows water mole fraction, and the bottom temperature as a function of radial position in the plume. Each number corresponds to a bin with bin 1 being the center of the plume. The purpose of the MSE is simply to show how that the innermost measurements are not as trustworthy due to the small distances measured.

In a typical rocket plume measurement, there is not a lot of absorbance. The rocket plume that has previously been tested at Orbitec has a 2 cm diameter. The absorption coefficient for most water lines is less than 0.01 cm$^{-1}$. This means that only 1% of the light is absorbed after passing through 1 cm of water vapor. On top of this, the spectral width of the water lines narrows as pressure decreases. Since these pressures are atmospheric, the water lines are much
more narrow than in combustion engine measurements, where much higher pressures are observed. The temperatures in the rocket plume are also higher than in combustion. Typically, high temperature water lines are much shorter than the low temperature water lines as is evident in Figure 4.

Figure 4 - Plot showing hot and cold water lines. The numbers in the upper left hand corner correspond to the bin number in Figure 3. Thus, hot water lines can be seen in the top spectrum, and cold water lines can be seen in the bottom spectra. These simulated spectra are taken from Caswell[1].
In order for accurate measurements to be made, measurement noise levels must be kept to a minimum. The main sources of noise in the experiments are from mode noise, beamsteering, and laser behavior. Mode noise comes from multimode fiber (MMF) anywhere in the system when all of the multimode light is not captured by the detector. It will be explained further in section 3.

Beamsteering occurs in the combustion region and in the plume where large optical gradients exist. The gradients bend and scatter the light on a microscopic scale which will misalign and cause additional divergence in the probing beam. Beamsteering is the topic of section 4.

Laser intensity noise comes from the laser itself. Sources of laser intensity noise can be polarization wander and chromatic dispersion in the fiber. These noise sources will be described in section 5.
3. Dual-Clad Fiber

Optical fibers are of great interest in rocket sensing applications because of the ease with which they transfer light from one place to another. Being able to connect a fiber from a laser to a test cell not only simplifies alignment and aids in directing light around obstacles, but it is also safer for those working with the laser (i.e. less chance of directing light into one’s eyes). Dual-clad fiber (DCF) is a special fiber that combines the pitching and the catching fibers into one. Absorption spectroscopy measurements have already been made using DCF[9].

3.1 DCF anatomy

Fibers come in various types, but they are all described by at least two important measurements: core diameter, and numerical aperture (NA). The diameter of the core determines, in part, the wavelength of light that can be guided through the fiber. It also determines for a given wavelength of light whether the fiber is single mode or multimode. Single-mode fiber (SMF) allows the light only one mode of propagation along the fiber. Multimode fiber (MMF) allows light to take multiple paths through the fiber.

Numerical aperture (NA) refers to the sine of the half-angle of the cone of light that can exit or enter the fiber. Any light that enters the fiber at an angle greater than that determined by the NA will not be guided through the core. The NA also plays a role in the wavelength of light that can be guided and the number of
modes supported by the fiber due to its dependence on the index of refractions for the core ($n_0$) and the cladding ($n_1$).

Optical fibers generally consist of an inner core of one index of refraction, surrounded by a cladding with a lower index of refraction. There are many other types which include gradients of index of refraction as well, but for simplicity, only step index fiber will be discussed. If a wavelength is too large for a given core, the light will not be guided through it. In order to guide light through the fiber, either the core must increase in size or the wavelength must decrease. At the point when the light wave can be guided through the fiber, there is only one manner in which the electromagnetic field can travel. This type of fiber is called single mode fiber (SMF) but is only single mode for the given set of wavelengths. As the wavelength decreases for a given core, the cutoff wavelength is approached. Once this point is passed, there begin to be multiple modes of propagation supported by the fiber. For a given core diameter the cutoff wavelength is given by Equation 2. Similarly, for a given wavelength, there is a maximum allowable core defined by Equation 3. When the core is larger than what Equation 3 permits, there become multiple ways for the light to propagate through the fiber. Fibers that support several ways for the light to propagate are called multimode fibers (MMF)[10].

$$\lambda_c = \frac{\pi D}{2.4} \sqrt{n_0^2 - n_i^2}$$  \hspace{1cm} (2)
The DCF, sometimes called a double clad fiber, is an optical fiber that has an inner core, a cladding around the core, and an additional cladding around the first. A close up of the dual clad fiber is shown in Figure 5. It is often used for laser pumping, but it can be used for other purposes as well[11]. For this experiment, the DCF is a compact way of overlapping a pitch signal and catch signal along the same path. This setup bestows the benefits of a single mode fiber for pitching the light (Gaussian light distribution in the beam), and the benefits of a MMF for catching the light (larger diameter and NA).

\[
D < \frac{2.4\lambda}{\pi \sqrt{n_0^2 - n_1^2}} \quad (3)
\]

Figure 5 - Cut-away schematic of dual-clad fiber (DCF) showing diameter and numerical aperture (NA) of inner core and inner cladding for the PTAP FUD-3381 9-105-125-20A-3ETFE fiber provided by Nufern.
Ideally, all of the light that is guided through the fiber to the test section is in the inner core of the DCF. The inner core is single mode for the light used, so the light entering the test section will be free from mode noise. Mode noise is a noise associated with the fact that the light does not uniformly propagate though the fiber. It will be further addressed in section 3.3. When the light returns from the test section, it has a larger diameter with a greater angle of incidence, even in the absence of beamsteering. This is because of both aberrations in the optics and divergence of the beam as it travels through space. It is possible for some of the light to reenter the inner core, but because of the divergence in the test section, the returning light overfills the area and NA of the inner core. This overfill light then enters the cladding. With the use of a typical fiber, light would still enter the cladding; however, the light would leak out of the cladding and would be lost. For small divergence, and no beamsteering, this may not be a problem, but for any practical setup, the amount of signal entering the cladding is magnitudes greater than the light that reenters the core. For this reason, another cladding with an even lower index of refraction is added around the inner cladding, which allows the inner cladding to efficiently transport the light it received through the fiber. The inner cladding is much larger and has a larger NA than the core, allowing for relatively strong disturbances due to divergence and beamsteering without loss of return light.

3.2 DCF in a rocket sensor

It was originally thought that the DCF would be an advantageous way to measure rocket plume absorption spectra because it is both the catching and the pitching
fiber. Not only is one fiber an easier setup than the two fiber system often implemented in combustion engine measurements[2], but the light has to make a double pass, so the signal is twice as strong. The aforementioned design is attached in Appendix A as design 4 and its schematic is shown below in Figure 6.

![Figure 6 - Rocket engine test setup using the DCF. The test section is a cross section of the rocket plume. The red cone is not a sheet, rather it is the area over which a collimated beam sweeps as the lens vibrates back and forth.](image)

The setup shown in Figure 6 works well because it contains only two optical elements. The first is the lens, and the second is the spherical mirror. The use of each optical element, introduces additional possibilities for losses of light and signal due to misalignment, reflection, surface imperfections and dust. Compared with the setup in Figure 6, others have as many as four optics[1]. Also, this setup is more beamsteering resistant because the beam path is kept as short as possible. When the beam path is kept short, small deviations in the
beam’s path have less of an effect on the beam’s destination. This helps maintain coupling efficiency.

Figure 7 shows how the light was coupled into and out of the DCF. Initially, light is coupled into the single mode input, off the first surface of the 50/50 beam splitter, and into the core of the DCF. It is important to reflect the light off of the first surface and into the DCF because then coupling is not wavelength dependent. Single-mode core to single-mode core coupling is very sensitive and even chromatic differences in refraction can misalign the setup, which will result in substantial signal loss. When light returns from the DCF, it passes through the beam splitter to a lens. The lens was placed in the middle of the path from DCF to MMF so that it provided a one-to-one imaging of the DCF onto the MMF. This allowed the lens to compensate for the divergence that the beam undergoes in this long coupling path. After passing through the lens, a mirror is used to help with the alignment of the beam to the MMF. The 110µm MMF is slightly larger than the 105µm DCF’s inner cladding so that mode noise can be more efficiently eliminated from this coupling (see section 3.3). Light that is in the MMF is coupled to a detector with a 1mm² area.
3.3 Mode noise in DCF

Unfortunately, the inner cladding is multimode due to its larger diameter and high NA. This introduces the possibility of mode noise in the system. Mode noise is noise that develops when some of the modes are blocked or the modes are unevenly affected. The modes are basically just valid ways that light can propagate through the fiber. Often the modes can be seen as a speckle pattern in light exiting a fiber. Figure 8 shows a side by side comparison of single-mode and multi-mode output from a fiber. These modes or speckles will move and change as the fiber geometry changes, the wavelength changes, or any other index of refraction changes occur. As long as all of the light is collected from a
MMF and the total optical power is measured, there will not be a problem with mode noise. Unfortunately, optics cannot always be kept perfectly clean and coupling of light is never 100% efficient. As the speckles move on and off pieces of dust, or into and out of acceptable coupling locations, an intensity noise develops. This noise is called mode noise and has been the subject of research for some time[12]. With single mode light, coupling losses are still present, but it is not noisy because modes are not discretely being coupled and uncoupled in the system.

Figure 8 – These two spots are results of light directly from a SM and MM fiber shown on a card. The SM spot is on the left and the MM spot is on the right. Notice the MM spot has discrete regions where there is and is not light (it is speckled). As these modes move, they can be individually affected, creating mode noise in data.

Unfortunately, there was too much mode noise in the DCF and DCF splitter for small rocket plume measurements to be reliably made. Whenever the wavelength was changed, the modes would change, creating mode noise. It overwhelmed the less than one percent absorption signal received through the
one inch diameter rocket plume. If a much larger rocket plume or a much stronger absorber were to be analyzed, then this system would work much better, as the absorption signal to mode noise ratio would be much higher than one.

The fatal problem with the mode noise proved to be the spatial sweeping of the beam across the mirror. Consider the following. If the fiber could be secured (i.e. set in place with a rigid thermal insulator) then the only source of mode noise in the system would be from the wavelength changing. This type of noise would be the same each time the laser scanned through wavelength, making it easy to post-process out of the data (assuming the noise frequency is lower than the detector bandwidth).

Because the beam sweeps across the mirror though, it hits different spots on the mirror even if beamsteering is not present. This changes how the modes get reflected. In general, one sweep of the laser through the plume will not have the same noise for multiple sweeps because the position of a given bin on any two different sweeps will usually be in a slightly different place. Unfortunately, the noise has features which are similar, so averaging will not take out all of the noise. Thus, even if the fiber were completely secured, the changing modes both from beamsteering and spatial sweeping would create mode noise as the beam scanned through the plume.
3.4 Future improvements

If the DCF is to be used in a system similar to what was previously described, a new DCF should be designed specifically for this situation. Using the method described in Appendix B to find the optimum coupling efficiency, 17% of the light coupled from the SMF leaks into the cladding of the DCF. It appears that better optics are required in order to improve this. However, the NA of the core and cladding can be chosen to be very different, thus helping “throw away” much of the multimode light. Figure 9 shows that as the core’s NA becomes small and the cladding’s NA becomes large, almost all the light that enters the cladding will be pitched far from where the core light is pitched. Also, a small NA for the core will result in a tighter, smaller beam which will be able to undergo more beamsteering in the test section before being malevolently affected.

![Diagram](image)

**Figure 9 -** When the inner cladding is much larger and has a larger NA than the core, light coming from the cladding is usually lost in the system.

It is obvious that a larger diameter is required for the inner cladding, but as Figure 10 shows, so is a large NA. For most engine measurements, the rays will
travel many orders of magnitude further parallel to the propagation direction than
perpendicular to the propagation direction. Since this is true, the returning light
can be far off the optical axis, even though it is very close to parallel with the
optical axis. When passing through the lens, the off axis rays become rays with
steeper angles. Thus a large NA is required.

![Diagram](image)

**Figure 10** - Rays entering a lens with no angle will end up on the optical axis no matter
how far off the optical axis they hit the lens. The rays further from the optical axis will end
up with a steeper angle (left). Rays with a nominal angle will end up at a position off of the
optical axis (right) no matter where they hit the lens. If incidence angles are small, then
the distance of the focal point off the optical axis will also be small.

Although a large diameter and large NA cladding seems ideal because of its
ability to catch light that underwent large amounts of beamsteering, it is not
desired when coupling light from the DCF to the MMF or detector. Fast detectors
are often very small in order to minimize capacitance and maximize response
time. If the beam overfills the detector, mode noise will be noticed.
Another issue to consider is that of back-reflection. Since the DCF sends a signal back along the path from which the light came, it is important to minimize back-reflections off the end of the fiber. A larger NA of the fiber will allow more back-reflection because reflections with larger angles are still trapped in the fiber. The typical reflection off glass when light is perpendicularly incident upon it is 4%. If a signal being analyzed is less than 4%, it will be increasingly difficult to make measurements in the presence of back-reflected light. For this reason, the fiber should be polished at a steep angle. Often the 8 degree angle polish which is standard on angle polished fibers is not sufficient. Instead 11 degree or 20 degree angle polishing will be needed. The exact angle polish required can be calculated from Equation 4, which is formulated using simple geometry. It is very difficult to find companies who can provide this steep angle polish however. Since the required angle polish depends on the NA of the fiber, this a reason to keep NA of the cladding as small as possible.

$$\theta = \frac{\sin^{-1}(NA)}{n_{fiber}}$$  \hspace{1cm} (4)

3.5 Other possibilities for DCF

Although DCF in its current form is not preferable for small rocket plume measurements, it is still a technology full of promise. It has already been used to measure other water absorption spectra[9], even if not at the speeds the rocket sensor requires. DCF has also been used in imaging[13] and would be useful in mode independent applications like triggering. There are also plans to use it to
measure droplet size based on the back scattering of extremely broadband light (UV-IR range).
4. Beamsteering

4.1 Introduction

Beamsteering refers to the effects that an optically turbulent medium has upon a beam of light. The optical turbulence arises from variations in the index of refraction throughout the medium. In combustion, the index of refraction is a function of temperature, chemical makeup, and density, among other qualities. Since combustion always has turbulence, there are always gradients in the index of refraction in combustive regions. Consequently, any light entering this region will be disturbed as it traverses these gradients.

In the triptane engine, the effects of beamsteering are particularly strong when the piston is at top dead center (TDC). Figure 11 shows the loss of signal due to beamsteering in the triptane engine at TDC. In the rocket plume, the turbulence is constant in time so beamsteering continually scatters the beam. Figure 12 shows the noise in the signal due to beamsteering in the turbulent rocket plume. The data is the transmission of a continuous beam that sweeps spatially across the rocket plume. The blue trace shows the signal when the engine is off (no beamsteering), and the red trace shows when the engine is firing and, thus, beamsteering is present. The difference in width of the bulk shapes is due to differences in the speed of the scan in the two experiments and is not a result of beamsteering.
Figure 11 – This is the type of beamsteering seen in combustion engines like the triptane engine used in Kranendonk’s experiment. Kranendonk’s beamsteering data shows that there was less of a loss of signal when the 4.5mm lens was used[2].
Figure 12 – The beamsteering seen in rocket engine data is a noise added to the signal, rather than the sudden loss seen in Figure 11. The difference in widths of the overall shapes is only due to a variation of the spatial scan speed between the two runs.

In order to minimize the effects of beamsteering, different methods have been used to choose the optics before and after the beamsteering region. Choosing the correct optics will ensure that the beam is pitched in and captured in as efficient a manner as possible. One of the issues that must be considered when choosing the optimum optics is selecting a model to accurately represent the beamsteering.
There are several different ways to model the beamsteering, and thus the beam propagation itself. The literature is rich in analysis of waves traversing turbulent mediums[14, 15], but often it is not specifically concerned with coupling efficiency in applications such as rocket engine sensing. Instead of viewing light as a wave or wave front, this paper investigates other models of light. The first way discussed here, assumes that as the beam traverses the beamsteering region, its optical extent increases[2]. Furthermore, at any location into the beamsteering region, all points of light within the beam contain rays at all angles (up to the maximum angle corresponding to the beam’s divergence). The model proposed by this research describes the beam as a collection of rays which can be refracted by random eddies that the rays may encounter.

### 4.2 The Kranendonk model

In Kranendonk’s model, the beamsteering is modeled as a continuous diffusion due to a beamsteering coefficient $K$. Throughout this diffusion, it is assumed that all propagation angles within the divergence angle of the beam exist at every point along the beam. Typically, this is only true at the source, the thin lens surface, and at the beam’s focus. Figure 13 demonstrates what real light would look like. In Kranendonk’s model there would be three arrows in every position along plane 3.
Figure 13 - Rays propagate from the source in all directions (limited by the NA of the fiber) at any point on the fiber. The angular distribution is also the same at every point along the lens. When the rays are in other places, they no longer have an even distribution of angles. This is contrary to Kranendonk's assumptions.

In her paper, Kranendonk's model is used to select two optimum lenses for combating beam steering. The lenses are at the pitching source and collector, or catching element, as is shown in Figure 14. The optimum lenses allow the greatest amount of beamsteering with the greatest optical coupling of any lens pair. The system of interest is modeled with the pitching source and collector both placed at the focal points of the optimum lenses. Kranendonk's equations can then be solved for the focal lengths of the optimum lenses by minimizing
what is called the *optical extent* of the light after it propagates through the beamsteering region to the collector.

![Diagram](image)

**Figure 14** – Light from a fiber placed at the focal point of a lens traverses a beamsteering region causing the beam to diverge. The beam is then directed to a collection fiber that is at the focal point of a collection lens[2]. Using Kranendonk’s techniques, the lenses can be selected to allow the most amount of beamsteering without loss in coupling efficiency.

Optical extent is the product of the area of a beam of light, the solid angle the beam includes and the index of refraction. When the beam is in air, the index of refraction is one, so the optical extent reduces to the geometrical extent, which is also known as étendue, light-gathering power, throughput, and extent, among other names.[16] Extent is conserved in a system when there is no additional divergence added to the system[17]. Kranendonk introduces additional extent as the beam traverses the beamsteering region. Assuming that all angles exist at every point along the beamsteering region, the extent is always proportional to
the beam diameter squared times the NA of the beam squared. While this approximation is not typically physical for free space optics, the beamsteering adds to the angular diversity of the rays at each point, making this model more accurate with increased beamsteering (i.e. the single rays on the top and bottom edges of plane 3 in Figure 13 will actually have more than just the one angle, as rays get deflected by the eddies).

The major limitation of this model is that the source and collector must be at the focal point of the selected lenses. As section 4.3 will show, having the source and collector at the focal point of the lens is less efficient than having them further from the focal points. This weakness prevents this model from simulating the setups that actually work best. Luckily, the lenses that are better at the focal point are also usually the lenses that are better when distances are optimized. Therefore this model can be used to choose lenses with focal lengths close to the true optimum.

4.3 The ray tracing model

The computer ray tracing model suggested in this section tracks physical rays of light as they move through the beamsteering region. The interactions are randomly determined each time the program is run, so the results vary slightly. This model does not have discretely separated cells that are traversed by the rays; instead, a ray propagates straight until it arrives at a unique, statistically determined position. At the chosen position, the ray is randomly directed in another direction, as is shown in Figure 15. This model is of particular interest
because simulations of it can be easily run in a commercial ray-tracing software package called Zemax.

Figure 15 – This is an extreme beamsteering case modeled in Zemax. In this exaggerated example, the eddies can deflect a ray by up to 10 degrees. The beam travels through free space before going into two identical beamsteering regions back to back. The light the returns to free space.

Zemax has a mode of operation that will use Gaussian optics to model Gaussian beams; however, it has no sufficient way of modeling beamsteering in this mode. The other mode uses ray tracing, but it is less powerful for analysis. For example, with the exception of detector position, it will not optimize the optics in ray tracing mode. As a result, each iteration in the optimization process must be run by hand. Every new lens pair and every fiber position must be entered
before each sequence. Hopefully, the benefits of this analysis will lead to collaboration between Zemax and the University of Wisconsin to make an automation feature that can be included in Zemax.

For this model, there are two degrees of freedom that describe the beamsteering effects upon a ray: the distance the ray travels before being deflected, and the deflection angle. Zemax allows these two variables to be input into the program as a \textit{mean free path}, and a \textit{maximum scatter angle}. The mean free path is simply the average distance between turbulent eddies. Rays traveling through the beamsteering region a distance $x$ have an integrated probability of having been scattered given by Equation 5.

$$p(x) = 1 - e^{-\frac{x}{M}} \quad (5)$$

where $p$ is the probability that the ray is scattered, and $M$ is the mean free path. By randomly selecting a value between 0 and 1, Zemax solves for $x$ to find the distance the ray should travel before being scattered. If the distance the ray would travel before scattering is longer than the ray’s remaining path through the beamsteering region, then the ray will not scatter.

Unfortunately, there is a limited number of times a ray can scatter before Zemax ignores all subsequent interactions in the beamsteering region. For this reason, several beamsteering regions must be placed back to back (i.e. Figure 15 has
two regions back to back, then a free space region). Since the regions are still orders of magnitude larger than the mean free path, the statistics of the scattering are negligibly affected by boundary effects of the sub regions.

Once the ray’s scattering position is determined, a scatter angle is randomly chosen to be within a cone of possibilities. The cone lies along the axis of the ray’s current direction and opens with an angle equal to the maximum scatter angle set by the user as shown in Figure 16.

![Figure 16](image)

**Figure 16** - The new angle of the ray is chosen at random to be within a cone of possibilities. The maximum angle is chosen by the user.

Since two collinear rays will not be scattered at the same point nor in the same direction, this model is not completely physical. However, the statistical advantage of having many rays gives a result that reflects physical reality for rays. This model is also not physical for beams that approach the diffraction limit. When dealing with Gaussian optics, lenses and focal points should be modified to be accurately modeled by rays. The differences arise from the fact
that the Gaussian beam is composed of a wavefront. For example, for Gaussian light to be reflected back along its path off of a spherical mirror, it can be focused at the mirror’s focal point when the Rayleigh range is equal to the radius of curvature of the mirror. In example the wavefront’s curvature matches the curvature of the mirror, and retro-reflection occurs. On the other hand, for the same retro-reflective behavior, rays would need to be focused at twice the mirror’s focal point (the center of curvature) of the same spherical mirror. This Gaussian beam can be modeled by halving the focal distance of the mirror and focusing the rays where the Gaussian beam is focused. This is demonstrated in Figure 17.

![Figure 17](image.png)

Figure 17 – The focal length of the mirror in the ray traced simulation must be halved in order for the rays to exhibit the correct physics for the Gaussian beam below.

Appendix C includes a Zemax beamsteering model in which the focal length of the spherical “mirror” had to be halved because the beam was within the Rayleigh range and therefore had a different incidence angle upon the mirror than the rays would.
In the next sections, the focal or collimation points refer the points which
collimate the rays emanating from a given point, not necessarily the point that
collimates a diffraction limited Gaussian beam.

4.3.1 The source

The Zemax code does an excellent job of modeling the beamsteering region, but
what about the source of the rays? In most cases, the source is a single mode
fiber, for which the exiting light is Gaussian and has a beam waist. Fortunately,
there are few times when the beam interacts with optics while it is within the
Rayleigh length. The Rayleigh length is defined as the distance from the beam
waist of a Gaussian beam to where the beam radius is less than or equal to $\sqrt{2}$
times the beam waist, $\omega_0$ (the beam area is twice the mode area of a circular
beam). Equation 6 gives the Rayleigh length, $Z_R$. Because the beam interacts
with the lenses outside the Rayleigh length, the beam can be modeled as coming
from a source that emits rays.

$$Z_R = \frac{\pi \omega_0^2}{\lambda} \quad (6)$$

In almost every simulation discussed in this work, the surface of the source, or
fiber, is modeled using Zemax's Two Angle source. The Two Angle source
places the origin of each ray uniformly across the face of the source. The
angular distribution of the rays is uniform as well, with the maximum angle set by
the user.
For modeling perfectly collimated beams, one can use Zemax's Gaussian source. The beam waist corresponding to the $1/e^2$ value is used to describe the thickness of this beam. The Gaussian source was not used when the beams were not collimated because modeling uncollimated light with this option was not well understood. Additionally, the central limit theorem shows that the beam from a Two Angle source becomes Gaussian as more beamsteering is traversed.

It is possible to write a DLL file to simulate any source desired; however, this is a more advanced option that is more expensive, as it requires technical support, and more time consuming than a preliminary analysis requires.

### 4.3.2 Choosing the values

Now that a simulator has been developed to simulate beamsteering, the inputs of the system must be related to what actually occurs in the physical experiment. The first input to select is the mean free path.

In order to find the mean path, the turbulence in the region of interest must be characterized. To estimate turbulence in the triptane engine, the experts in numerical simulation of turbulent combustion were consulted[18]. According to them, the turbulence scale could be between 1µm and 100µm, with 100µm being the more realistic value. Upon their recommendation, all simulations used 100µm as the mean free path.
The spread of temperatures in the engine near 25 degrees before top dead center (TDC) was determined to be between 50 K and 100 K, with a center temperature between 700 K and 760 K[18], and a pressure assumed to be 40 bar. The index of refraction of air at various atmospheric temperatures was found for the wavelength of 1350nm using the National Institute of Standards and Technology’s Ciddor Equation calculator. The Ciddor Equation calculator is designed for a temperature and pressure range that is commonly seen in the atmosphere, but can be used to get first order estimates of these higher temperature conditions by extrapolation via Equation 7.

\[
\frac{P}{T} \propto N \propto (n-1) \rightarrow k \frac{P}{T} = (n-1) = 2.68E-4 \tag{7}
\]

In Equation 7, \(N\) is the number density and \(n\) is the index of refraction. Using the values from the Ciddor Equation, the constant \(k\) was found to be 0.07857. \(k\) was then used with \(P = 40\) bar to linearly estimate the index of refraction of the gasses in the chamber for \(T = 710\) K and 770 K. The difference in the index of refraction for the two temperatures was found to be approximately 5%.

Next, the eddies were assumed to be spherical with a constant temperature inside the eddy and a constant temperature outside the eddy. As shown in Figure 18, geometrical reasoning can be used to find the angle that parallel light will bend when it hits a cold eddy. (A hot eddy surrounded by cold gas gives almost the same angle of bend, but in the opposite direction.) This angle is a
function of the ray’s position from the center axis of the eddy. This position can then be mapped to an angle of incidence, $\alpha$, of the ray upon the eddy. Equation 8 shows the resulting half-angle, $\gamma$, that the deflected ray makes with its original position.

\[
\gamma = \alpha - \beta = \alpha - \sin^{-1}\left(\frac{n_1}{n_2} \sin \alpha\right) \quad (8)
\]

Figure 18 - A cold eddy surrounded by a hot environment with a light ray incident upon it. The hot eddy surrounded by a cold environment yields the same result but the ray is bent in the opposite direction.

Given a uniform distribution of rays along the side of the eddy, $\alpha$ will have a distribution following the arcsine of the distance off the eddy’s center axis. Unfortunately, Zemax only gives a uniform distribution for the refracted angle. In order to compensate for the fact that the angle is chosen uniformly between two
extremes, the maximum angle is set to be a fraction of the arcsine distribution. The distribution of the actual and modeled simulations look similar after only a few scattering events.

4.3.3 Statistical analysis of model properties

To find out if the number of rays used was sufficient for repeatable, and therefore precise simulations, the simulation was run five times with four slightly different setups. Each setup was identical except for the position of the pitching fiber with regards to the pitching lens. In each simulation, the percentage of rays successfully coupled into a fiber with diameter 22.5μm and an NA of .275 was calculated and to determine the coupling efficiency. If all of the rays were successfully coupled into the collection fiber, it is 100% efficient. The standard deviation in each setup was assumed to be the same because the same numbers of rays were used and the rays follow the same statistical deflections. Because the standard deviation is the identical for each setup, it was possible to combine the means and standard deviations of each of the four setups to find a more precise standard deviation for all setups. The standard deviation of the efficiency was then calculated using Equation 9[19] to yield a standard deviation of efficiency of 1.85% when 500 rays are used. In Equation 9 $x_{ij}$ is the efficiency in setup $i$, run $j$, $\bar{x}_i$ is the average of setup $i$, $n_i$ is the number of runs in setup $i$, and $N_{sets}$ is the number of total setups.
\[ \hat{\sigma}^2 = \frac{1}{\sum_{i=1}^{N_{\text{sets}}} n_i} \sum_{i=1}^{N_{\text{rays}}} \sum_{j=1}^{n_i} (x_{i,j} - \bar{x}_j)^2 \] (9)

### 4.3.4 Zemax output

Once a system is modeled in Zemax, a group of rays are traced one at a time through the entire system, similar to Figure 15. Eventually, all of the rays are traced and the resulting rays are recorded with angle, position, and intensity. In this model, rays are either bent or not, so the intensities of the rays are constant throughout the rays’ paths – there are no partial scatterings.

The positions and intensities of each ray are analyzed using Matlab. The Matlab scripts are attached in Appendix D. Figure 19 shows a plot of what the physical spot looks like when it hits the detector. The blue spots show where rays hit when there was no beamsteering; the red dots show where the rays hit when there was beamsteering present.
Figure 19 – A spot diagram showing where the rays hit the collector when there was no beamsteering (blue) and when there was beamsteering (red). Both plots show the optimum detector position when the pitching fiber is at the point of collimation for the pitching lens (2.972mm) and the detector is at its optimum position (2.985mm after the collection lens for the non-beamsteering case, and 3.105mm after the collection lens for the beamsteering case).

Figure 20 shows another useful plot – the distance versus angle plot. The distance is defined as how far a point is from the optical axis, or $\sqrt{x^2 + y^2}$ in the spot diagram shown in Figure 19. The angle referred to is the angle the ray makes with the optical axis. If the ray is moving away from the optical axis, it has a positive angle; conversely, if the ray is moving towards the optical axis, it has a
negative angle. The distance versus angle plot is useful for determining the angular distribution of rays at various distances from the optical axis. One practical application of this plot is for helping to choose as smallest MMF possible. The bias of the rays in either the negative or positive direction also corresponds to whether the detector is too close or too far from the focal point of the rays. When a larger NA for the MMF is not an option, and the NA of the fiber is too small for some of the rays, then those rays are disregarded from the optimization. The remaining rays show the true maximum fiber diameter required.

![Graph showing distance versus angle with optical axis at the collector at the optimum position for collecting rays. Positive angles mean the ray is moving away from the optical axis, negative angles mean the ray is moving towards the optical axis. Both no beamsteering and beamsteering are shown.](image)

Figure 20 - Distance off optical axis versus angle with optical axis at the collector at the optimum position for collecting rays. Positive angles mean the ray is moving away from the optical axis, negative angles mean the ray is moving towards the optical axis. Both
plots show the optimum detector position when the pitching fiber is at the point of
collimation for the pitching lens (2.972mm) and the detector is 2.985mm after the
collection lens for the non-beamsteering case, and 3.105mm after the collection lens for
the beamsteering case.

4.3.5 Findings of optimum source/detector positions

One of the most interesting features of Zemax modeling is the ability to analyze
an optical system when the fibers and/or detectors are not at the focal point of
their lenses. Lab experience suggests that in order to maximize efficiency, the
fibers and/or detectors should actually be placed a little farther from the lens than
the focal length. The Zemax model supports this observation. For the model of
the triptane engine used in the next section (see Figure 23), the width of the
beam that contains 86.5% of the rays (corresponding to the 1/e² value), which do
not exceed an NA of 0.275, is extrapolated around the focal point to produce the
plot shown in Figure 21. The lowest point in the plot shows where the rays are
closest together – corresponding to the optimum detector position. Because this
extrapolation takes the resulting angle and location of a ray and extrapolates it
forward and backward from the collector’s location, it is only valid until a ray
comes in contact with another optical surface (i.e. the ray changes direction).
The Matlab program that extrapolates the rays is included in Appendix D.
The optimum detector position is found by first extrapolating the rays. Since Zemax outputs the location and angles of each ray, the ray can be extrapolated along its paths to find its exact position at any distance from the lens. Then, for each discrete location, the rays that are at too steep an angle to be collected by the collector (fiber/detector) are removed. The remaining rays are then sorted according to their distance from the optical axis. Only the closest 86.5% are kept in order to exclude any outliers from the data analysis. Of the remaining rays,
the ray that is furthest from the optical axis is then the value reported along the y-axis of Figure 21

The most interesting feature of Figure 21 is the fact that as the beamsteering is introduced, the optimal location for the collector moves back. This is consistent with the beamsteering acting as a lens. As the beamsteering increases, the collimated light diverges and so is focused behind the focal point of the collecting lens.

Now that the optimum detector position can be determined, the source position should also be investigated. It is reasonable to assume that if the optimum location for the detector depends on beamsteering, so will the location of the source. When the system shown in Figure 23 is modeled, we find a location of optimal coupling efficiency by producing a plot like that in shown Figure 22. Each point in Figure 22 is produced by placing the source at the location shown on the x-axis. Using the method discussed above, the detector is then moved to its optimal position for that source location. The optimal efficiency of the system is realized when the source is at that point. The overall best efficiency of a lens pair can be read off this graph.
Figure 22 – Efficiencies of coupling light into a 22.5µm fiber with .275 NA when detector position is optimized for different pitching positions.

4.3.6 Comparison to Kranendonk’s data

Using Kranendonk’s model, the best size lenses for the setup shown in Figure 23 can be numerically solved for using EES. Since the lenses that EES directly calculates are often not lenses that are readily available, EES can be used to select the best lenses from an inputted list of available lenses. Once the best available lenses are chosen, they can be simulated with Zemax.
The comparison of two lens pairs, a 4.5mm and 4.5mm, and a 3.1mm with a 6.16mm, were calculated using Kranendonk’s equations, and then in Zemax. According to Kranendonk’s model, the efficiency of coupling light is 98% for the 4.5mm pair, and 61% for the 3.1mm and 6.16mm pair. Comparing this to the Zemax model with a catch fiber of 22.5µm and an NA of .275, we get efficiencies of over 98% and 89% respectfully. These values can be read off Figure 20 directly. In order to compare these two models, the Kranendonk model was calibrated (beamsteering coefficient \( K \) was chosen to be .00002575) to match the Zemax model’s prediction for the 4.5mm - 4.5mm pair.

It should be noted that these efficiency predictions of the two models should not line up exactly because Kranendonk’s model only allows the fibers to be at the focal point of the lenses. For instance, the beamsteering coefficient, \( K \), needs to be reduced to 0.0000415 to get 89% of the light coupled with the 3.1mm and 6.16mm pair of lenses when the fiber is at the focal point of the lens. The
important conclusion is that Kranendonk’s model and the Zemax model predict the same trends. This not only lends evidence to support the correctness of the Zemax model, but also shows that Kranendonk’s model is still the first place to start when selecting lenses. It selects lenses that are close to perfect quite easily, and then Zemax can be used to run several situations of fiber to lens distances.

4.3.7 Comparison to DCF data

Comparing the Zemax model to the DCF experiment described below is another useful comparison for the model because unlike the Kranendonk model, the fibers do not have to be at the focal points of the pitching and catching lenses. In fact, when the experiment is run, the nominal distance from the lens is not known. Instead, the distance of the fiber from the lens is manipulated during the experiment in order to maximize transmission (minimize the effects of beam steering).

The DCF experiment shown in Figure 24 was run as follows. The motoring triptane engine was used as the beamsteering test cell. At the input to the cell was the DCF and a lens on a stage. On the output of the cell was one of a variety of reflectors. Some of the reflectors tested include a retro-reflective material from 3M, a retro-reflector cube, and spherical mirrors with focal lengths equal to 50mm and 100mm.
Figure 24 – DCF, reflector and beamsteering test experiment setup. The same engine is used as in Figure 23, but the collection lens is replaced with a reflector. The DCF is both the pitching fiber and the catching fiber.

Since the 100mm focal length spherical mirror has a radius of curvature equal to 200mm, and the mirror is about 200mm away from the lens on the other side of the cylinder, it was expected that this would be a better reflector than the 50mm mirror. The 3M material introduces a large amount of diffraction, which is a major source of loss in the system. The retroreflector cube was believed to be the best before the test; however, it was found that it translates the beam slightly as it reflects. This shift causes the beam to hit the lens at a different spot, corresponding to a large incidence angle upon the fiber. The NA of the fiber is often not sufficient to capture the light at these high angles, and the returning light is lost. The beam could not be centered on the retro-reflector cube because that point did not reflect well.
The engine test results showed that in fact, the 50mm and 100mm mirror were the best choices for reflectors. As Figure 25 shows, two traces of engine data are overlaid on each other. The blue trace is data from the 100mm mirror and the red trace is data from the 50mm mirror. Notice that although the 100mm mirror alignment had initially more amplitude as predicted, it suffered a greater loss than the 50mm mirror when beamsteering was present. The 50mm mirror setup suffered only a 60% loss in signal compared to the 75% loss that the 100mm mirror underwent. Even with the advantage in signal strength that the 100mm mirror has when no beamsteering is present; the signal is stronger during beamsteering when the 50mm mirror is used. This demonstrates one of the difficulties of designing optical systems with beamsteering: different amounts of beamsteering have different optimum lens characteristics.
Figure 25 - DCF data shows three and a half cycles of the engine. The drops in signal occur at TDC. Notice that the 50mm mirror setup initially has a weaker signal than the 100mm setup, but suffers less from beam steering than the 100mm mirror system does.

To model these setups in Zemax, two beamsteering regions are created and the system is symmetrical about the center “reflector”, as seen in Figure 26. Because the retro reflectors are very difficult to model in Zemax, only the 5cm and 10cm mirror were chosen to be modeled. In Zemax, the mirrors are modeled as bi-convex lenses with the same respective focal points.
Figure 26- Top: The system with a mirror is modeled like a symmetric system with a center lens that is equivalent to the reflector. Bottom: The above model as displayed in Zemax. Stratified lines in the beamsteering sections are the smaller regions mentioned in section 4.3.

When modeled in Zemax, the same conclusion was drawn. Figure 27 shows the efficiencies of the two mirror setups at different fiber positions. During beamsteering, the 50mm mirror outperforms the 100mm mirror by about 3:2. This agreement with experimental data supports the idea that the Zemax modeling is an accurate way to simulate beamsteering in optical setups.
Figure 27 - The efficiency of coupling light from the core of the DCF though the system and catching it again in the cladding is plotted against the distance between the DCF and the lens for a system with a 100mm focal length mirror (blue) and a 50mm mirror (red).

4.3.8 Comparison to M. Richter

Now that the experimental results have verified predictions made by Zemax, they can be applied to other works in the literature which ignored beamsteering in order to demonstrate the importance of including beamsteering in one’s results. In the experiments done by Richter et. al., the authors ignore the effects of beamsteering and quote a spatial resolution of 250 µm at the center of the engine[20]. They use 266nm light and the engine cylinder has a 127mm bore.
By calculating the Rayleigh length ($Z_R$) for this color of light with Equation 6, we find that the $Z_R$ is 185 mm.

Since the light can be collimated before and after the beam waist, there is actually a 370 mm length along which the assumption that the rays are all parallel can be made for this analysis. This can be verified by calculating the divergence angle of the beam at the Rayleigh range and comparing it to the divergence angle of a collimated beam in beamsteering. Using Equation 10 we can approximate the divergence angle, $\theta$, of the collimated Gaussian beam is .004 degrees. The average angle of a collimated beam in Zemax which passes through the beamsteering region is .189 degrees. Thus, the additional divergence due to beamsteering is orders of magnitude greater than the beam’s natural divergence and the parallel ray assumption is acceptable.

$$\theta \approx \left( \frac{\sqrt{2} - 1}{2 \pi \omega_0} \right) \lambda$$

(10)

In Zemax, the parallel rays are used with the same beamsteering values used in the other simulations. The source is Gaussian, and the authors claim that the resolution of the beam is 250 µm. In Zemax, this means all the light is within 250 µm, but the rays have a Gaussian distribution within the beam. When beamsteering is taken into account, and the 1% outlying rays are removed, the beam waist at the center of the cylinder is 305 µm compared to the 243 µm Zemax gives without beamsteering (1% of the rays also removed from the 250
\[ \text{\( \mu \text{m beam}) \). This is a 25\% difference in claimed resolution, a significant difference which, again, demonstrates the importance of including beamsteering in analysis.}

### 4.3.9 Other Zemax options and improvements

There are other ways to model beamsteering using Zemax. Zernikie polynomials (wavefront analysis) can be used to show the effects of turbulence on an optical wave, but once a set of polynomials is chosen, the exact same result will occur for every calculation. There is no random turbulence, instead it acts more like a way to warp an initial wave into another corrupted wave. It is therefore usually used to represent fixed aberrations. Also, Zernikie polynomials have coefficients that must be chosen. Although the literature has information on choosing the coefficients for atmospheric turbulence, it appears that no work has been done on combustive turbulence.

Another Zemax tool that could potentially be used is the Grid Sag. The Grid Sag is a surface “over lay” which puts dimples in a surface. The heights and depths of the Grid Sag points are chosen by the user at discrete points along the grid. The surface is then constructed by interpolating the points either linearly or with a bicubic spline. This method might work if several hundred or thousands of these surfaces could be stacked back to back. The grid points and sag magnitudes can be chosen at random or made to be random with specified scattering properties[21] before being inputted in Zemax, but then the turbulent region is set and no longer random every time the simulation is run. The random selections of grid points and magnitudes would be chosen within limits to reflect the eddy size
and angular refraction which the beam would pass through in the engine. Although this method would work well for modeling beamsteering, there are prohibitive memory requirements for the thousands of Grid Sag sheets with hundreds of Grid Sag points.

Finally, the gradient index of refraction (GRIN) lens was investigated as a way to model beamsteering. For this case, there are GRIN lenses that already exist in Zemax, however none of them come close to describing random turbulence. Instead, a system of equations that would give the appearance of a random distribution of index of refractions was created. The equation with over 20 user inputs included random numbers that could be recalculated every run. The massive equation (included in the Appendix E) would then be sent to Zemax developers who would program in an interface and a DLL file so that the “beamsteering lens” could be created in Zemax. Unfortunately, because of the required symmetries in the Zemax code and the inability to find functions that could simulate random noise, this technique ended up not being able to sufficiently model beamsteering.

Although the bulk scattering used in this paper worked well, there are still some improvements to it that could be made. The first improvement could be made in the source. Currently, the built-in 2-angle Zemax source is used. It is as described in section 4.3.1 with a uniform distribution of rays chosen in a cone of possible angles. Instead of the current source, a custom source can be made. It
is possible to write a DLL file that will give a Gaussian (or any other) distribution to the rays in both location and in angle. This would obviously be preferred when more accurate solutions are required.

On Zemax's end, it would be nice if Zemax extended the automatic optimization feature to the non-sequential mode. Currently, Zemax does not include optimization for anything other than detector position in non-sequential mode. If enough rays are run, and the outliers are removed, something close to an optimized system should be able to be found. This would save much time in setting up the simulations, as currently each lens and each distance from the source to the lens must be manually inputted before a simulation can be run.

The analysis of the rays is still done in Matlab, so it may be helpful to include parts of the Matlab code in Zemax. Specifically, the plot of distance versus angle would be useful. The spot diagrams are already plotted in Zemax, and if the automated optimization is included, the optimal collector position plot (i.e. Figure 21) will not be needed.

4.4 Beamsteering conclusions

When designing optical setups, one will prefer to use the method suggested in this paper. One should choose the lenses using Kranendonk’s method, even though the fibers are always assumed to be at the focal length of the collimation lenses. As the ray model presented here and experimentation have shown, the focal point of the lens is not usually the optimum place for the fibers to be, but for
the selecting of lens pairs, it is often close enough. Once the lenses are
selected, Zemax can be used to determine the optimum locations and accurate
efficiencies of coupling for the beamsteering situation. If simple systems with the
minimum number of optical components are used, Kranendonk’s method will
save time in finding the optimum lenses; however, if the system is complex with
several beamsteering paths and optical components which are not conveniently
at focal points, an exhaustive search using Zemax will be required for finding the
optimum lens pairs.
5. Wavelength Scanning – The Laser

We obtained a high speed wavelength scanning laser from the Massachusetts Institute of Technology through the generosity of Dr. Robert Huber. The laser is capable of scanning over 50 nm in 5 µs. The wavelengths of interest can be varied, but the range of 1330 nm to 1380 nm was chosen in order to measure the waterlines in the R-branch of the \((\nu_1 + \nu_3)\) water absorption band[22]. The speed of this laser is important for making measurements faster than the optical properties change in the beamsteering region. In previous setups, the laser scans very slowly in color requiring several assumptions to be made for measurements in rocket plumes. The biggest assumptions are that the flow is steady and uniform throughout the measurement time – which has previously been over 3 seconds long [1].

5.1. How the laser works

The laser recommended for rocket engine test measurements and discussed here is a Fourier Domain Mode-Locking (FDML) laser[23] shown in Figure 28. The laser is in essence composed of a silicon optical amplifier (SOA), a fiber loop cavity, and a tunable filter. The SOA creates broadband light which traverses the fiber cavity until it comes to a fiber Fabry Perot tunable filter (FFP-TF). The filter has a 0.13 nm bandwidth. This light then undergoes gain amplification and the SOA lazes at the permitted color. The center frequency of the filter is varied at a frequency that corresponds to the round trip time of the light in the 2km cavity. The filter tunes through all of the wavelengths before the original wavelength of
light reaches the filter again. By maintaining the proper timing, the complete spectrum of the laser exists in the fiber cavity at all times (see Figure 29). One photon of light will make about three round trips through the system before leaving the cavity via a 30% outcoupler.

Figure 28 - FDML setup from MIT. The 2km spool is hidden under the breadboard. The complete schematic is shown in Figure 30, but the fiber connections are labeled here. The letters flank the devices: A – SOA, B – isolator, C – 70/30 coupler input, C_{30} – 30% output from C, C_{70} – 70% output from C, D – 90/10 coupler input, D_{10} – 10% output from D, D_{90} – 90% output from D, E – delay spool #1, F – delay spool #2, G – FFP-TF, H – polarization controllers, I – isolator, O_{10} – to 10% out on front panel, O_{90} – to 90% out on front panel.
Figure 29 - The complete spectrum the laser scans exists in the ring at once and travels around the fiber ring at the speed of light. This is a simplification of Figure 30.

The complete FDML laser schematic is shown in Figure 30. It has been used to make low resolution measurements (as compared with rocket plume measurements) in the triptane engine[22]. Problems arise when high resolution is required (as in a rocket plume).
Figure 30 - The schematic of the FDML laser shows that the laser light travels in one direction thanks to the isolators on either side of the SOA. 30% of the light is coupled out of the loop before the 70% goes to the delay fiber. The FFP-TF is driven with a 100kHz sine wave.

Because the light is traveling through a fiber, there are dispersion and polarization concerns which affect the laser's resolution. The dispersion causes the resolution of the measured spectrum to blur. These blurring effects reduce the ability to resolve narrow water lines, making it harder to pair the measured spectrum with known spectra. This in turn prohibits one from determining the state of the absorber. The overlap of color can also create intensity noise as the different wavelengths compete to laze. The intensity noise can disturb the scan so severely that the water lines can become impossible to differentiate from the noise.
5.2 The issue of dispersion

Dispersion is the spreading out of light pulses as they propagate through a medium. There are four main types of dispersion: modal, waveguide, polarization, and material. Modal dispersion only occurs in multimode fiber; it is the spreading out of the pulse due to the fact that the modes in the fiber take different amounts of time to traverse the fiber. Since the FDML uses purely single mode fiber, this type of dispersion will not affect the system. Waveguide dispersion is dispersion due to the fact that the way a waveguide guides light through the fiber is wavelength dependent. It is usually a small effect in a setup like the one discussed here and is often lumped in with material dispersion under the collective name chromatic dispersion. Polarization-mode dispersion is caused by the fact that two different polarizations travel at slightly different speeds due to the inherent birefringence of the fiber. Typically, in non-polarization maintaining fiber, the difference in speed between the two polarizations is much less than the chromatic dispersion and can be ignored. Material dispersion is when a pulse of light spreads out due to the different speeds that different wavelengths of light have in the medium[10]. The units are usually picoseconds that the pulse spreads out per nanometer of spectral width per kilometer of fiber and it is wavelength dependent. The most prevalent dispersion in this setup is material dispersion.

5.2.1 The problem

One of the finer details about light traveling through a fiber is that the speed of light is wavelength dependent. Unless free-space is used, there is always some
dispersion of the light in the cavity. This dispersion creates an overlap of colors in the fiber, which decreases the spectral resolution of the laser.

When the dispersion in the fiber is close to zero, there is no difference in the speeds of different wavelengths of light. Unfortunately, there is non-zero dispersion everywhere other than 1310nm (for SMF-28 fiber) so the timing for each color does vary. To get an idea of how much variation is acceptable, one can detune the frequency of the filter until the laser becomes too noisy to use. A slight amount of variation in speed can be acceptable, but as Figure 31 shows, there is a limit. A change of 0.1 Hz in the filter frequency can be the difference between a well behaved laser and a very noisy light source. Although the 0.1 Hz made a difference in Figure 31, further increases of up to 0.3 Hz still allow good laser behavior.
Figure 31 - When the frequency of the FFP-TF is too far from the optimal frequency, the laser behaves poorly. Here the 98,934.9 Hz driving frequency causes the laser to behave well in a small region. The resolution of the frequency driver is probably not an issue, as 98,935.0 Hz also resolves the same section of the scan. The sinusoidal humps are caused by an etalon in the low pressure cell approximately 0.1nm wide, which the laser passed through. The sharp downward spikes in the clean region are water absorption lines. In these figures, the scan is over the full 50 nm.

The width and the position of the well behaved region vary with the dispersion of the fiber. In order to improve the width of the region, Fujikura USS fiber can be substituted for the SMF-28 in the delay loop. Fujikura USS fiber has a lower dispersion slope, so there is not as much dispersion over the 50nm range. Additionally, the zero dispersion point is at 1365nm so the dispersion numbers
are closer to zero in the range of 1330-1380nm than they would be for SMF-28, which has its zero at 1310nm. Figure 32 shows the difference in laser scans with the two different delay fibers. The additional noise on the sides of the Fujikura fiber scans are believed to be polarization related.

![Graph showing laser scans for SMF-28 and Fujikura fibers with time and amplitude on the axes.](image)

Figure 32 - Although the noisy part of the Fujikura fiber scan is more noisy than the SMF-28, the Fujikura fiber has a longer region completely free of noise.

### 5.2.2 Dispersion figure of merit

In order to quantify the goodness of any given fiber with respect to dispersion, a figure of merit was devised. Figures 33 and 34 show this figure of merit for wavelength scans of 50 nm with the wavelength tuning sinusoidally in time at 100
kHz. Since the wavelength scans are sinusoidal, the laser spends more time on the edges of the scan than in the middle, thus the difference in round trip time is not as important at these places. Figures 33 and 34 reflect this fact by showing that the edges of the figure drop to 0 (perfect laser performance).

A function of merit, \( Y(\lambda) \) is constructed as follows: the time difference for a given wavelength with a dispersion of \( D(\lambda) \) is calculated by multiplying its dispersion by the length of the fiber cavity, \( L \). Next, the time, \( \tau \), that the filter spends at a given wavelength with width \( d\lambda \) is calculated taking into account the sinusoidal speed of the sweep. \( Y(\lambda) \) is then the ratio of the dispersion time to the filter time.

\[
Y(\lambda) = \frac{D(\lambda) L}{\tau(\lambda)} \quad (11)
\]

\[
\tau(\lambda) = \frac{dt}{d\lambda} = \frac{1}{2\pi Af \sqrt{1 - \left(\frac{\lambda - \lambda_c}{A}\right)^2}} \quad (12)
\]

where \( A \) is the amplitude of the wavelength scan in nanometers, \( f \) is the frequency of the wavelength scan, and \( \lambda_c \) is the center of the wavelength scan.

The function of merit for the wavelength scan that covers the R-branch is plotted in Figure 33. Dispersion is the worst at the place of maximum deviation from zero. At this location, the laser performance is most compromised. The two cases compared in Figure 33 are for SMF-28 and Fujikura USS fiber. The
Fujikura USS fiber is a single-mode fiber with a lower dispersion slope than SMF-28 and a zero dispersion point at 1365nm. The Fujikura fiber has about 5 times better performance than the SMF-28 in our wavelength range.

Figure 33 - Figure of Merit for Fujikura USS fiber and SMF-28 fiber.

Figure 34 shows comparisons of other wavelength scans for the SMF-28 and the desired wavelength scan with the Fujikura USS fiber. The optimum scan will always be centered near the wavelength with zero dispersion for a typical fiber. For SMF-28, this wavelength is 1310nm. This explains why the SMF-28 figure of merit improves as the center wavelength is lowered – it is approaching SMF-28's
zero dispersion point. The important thing to note, however, is that even at SMF-28’s optimum scan range, when compared to the Fujikura USS fiber (which is not at its optimum scan range) the Fujikura USS fiber has a better (lower) figure of merit due to its lower dispersion slope.

Figure 34 - Comparison of SMF-28 figure of merit for 50nm scan with different center wavelength.
5.3 Polarization

After substituting the USS fiber into the setup, it became apparent that the dispersion was only part of the story. Only a narrow band (approximately 5nm) could be made to look noise free with a narrow bandwidth. At the edges of this optimum range was the sudden presence of noise. This obvious border can be seen in Figure 35. The optimum region could be moved to slightly different wavelengths by adjusting the polarization control paddles, suggesting that only certain wavelengths were being conditioned properly by the polarization controller.

![Figure 35 - Typical data from the oscilloscope after the beam passes through a low pressure cell. The polarization controllers bring the left side of the data into focus, but the right side remains noisy. The sinusoidal structure is caused by an etalon in the low pressure cell with a free spectral range of ~0.1nm.](image-url)
5.3.1 The problem

The FDML laser is powered and amplified by the SOA which has a polarization dependent gain of 3dB. If the light exited the SOA and returned with exactly the same polarization it had when it left, there would not be polarization issues because the system would remain steady. Unfortunately, the polarization wanders and eventually becomes elliptical as it moves throughout the fibers in the system\cite{24}. The amount that it rotates and the extent to which it becomes elliptical is different for every wavelength. Additionally, temperature changes or movement of the fiber itself changes the stress in the fiber resulting in different effects on the polarization.

When the fibers are undisturbed, the modified polarization will remain constant and a polarization controller can be used to rotate the polarization back to an optimum orientation. The polarization controller is essentially a fiber that is wrapped and strained in such a way that the polarization of light in the fiber depends on how the user orients the fiber loops, or polarization paddles (see Figure 36). The polarization paddles act as fractional waveplates which rotate the polarization and, unfortunately are wavelength dependent. This explains why only a narrow band of the scanning region can be perfected at a time – there is only a small bandwidth of light that is being rotated by the polarization controller the correct amount.
5.3.2 Solution 1 – Polarization maintaining fiber

The first possible solution could be to use polarization maintaining (PM) fiber throughout the system. While the literature has used PM fiber in a ring laser[25], it seems it has only been done with constant wavelength lasers which have a wider bandwidths. PM fiber will take any linearly polarized light aligned with the fast or slow axis of the fiber and preserve it throughout the fiber[26]. This will work for any wavelength of light. One of the issues that arises when using PM fiber comes from misalignment of the polarization fiber connectors so that some of the polarized light that is in line with the fiber’s fast axis leaks into the slow axis. When this happens, there is a problem similar to dispersion. The two polarizations travel at different speeds so a portion of the colors end up getting to the filter at the wrong time. The time difference for the two polarizations in PM fiber can be found from Equation 13.
\[ \Delta t = \frac{L}{nc} - \frac{L}{(n + \Delta n)c} \quad (13) \]

where \( L \) is the length of the fiber, \( \Delta t \) is the difference in time for the two polarizations, \( n \) is the index of refraction, \( \Delta n \) is the birefringence, and \( c \) is the speed of light.

Using this equation, the difference in time between the light in the two different polarizations can be calculated. For a system with 2km of fiber, the difference is over 200ps. Unfortunately, the time difference is on the order of the detector speed, so competition in lazing between the two polarizations will be noticed.

The possibility of accidentally getting light in the PM fiber at the wrong orientation is not the only reason that this solution should be used reluctantly. The PM fiber itself is expensive, with quotes of over $15,000 for 2 km, plus in-line fiber polarizers would be required which cost around $1000 each.

5.3.3 Solution 2 – Depolarized light

Another solution to the polarization problem would be to completely depolarize the light once it leaves the SOA. This way, there is no polarization dependence, and no polarization control. In the extreme situation that the SOA only amplifies one polarization and absorbs or blocks another, there would be a 50% loss every time the SOA amplified the light. After experimental investigation, it was found that the laser will still laze with this loss from the depolarizer. Thus, the laser will
still function as it should with the added benefit of no intensity noise (due to polarization).

There are active and passive depolarizers; technically, neither of them depolarizes light. Instead, they scramble the light using different methods. The active depolarizers actually have an element inside that moves around randomly so that the light is not polarized when integrated over time. These devices are much too slow to be of any use in an application with a laser as fast as the FDML. Passive depolarizers scramble the polarization independent of time, and therefore are a good choice for this type of system.

There are two types of passive depolarizers predominantly used in optics. The first depolarizer is the Lyot; the second is the wedge. The Lyot depolarizer counts on the laser having a significantly broad wavelength. One of the benefits of the Lyot depolarizer is that it is often in the form of a fiber. This makes setup simple and reliable. As light moves through the fiber, it is rotated several times with a high dependence on wavelength. In other words, different colors are rotated different amounts. The same effect that the Lyot depolarizer depends on to function is the effect that leads to problems in the polarization controller.

The literature has used a Lyot depolarizer in a ring laser before[27]. Unfortunately, the Lyot depolarizer only works well for broader band light.
Monochromatic or narrow band light does not get depolarized by the Lyot depolarizer because all the light gets modified by the same amount.

The second polarizer is a birefringent crystal wedge, usually quartz. This depolarizer works based on spatial differences. The beam is expanded so that it fills a sufficient portion of the wedge. As the polarized light passes through the wedge, it rotates at a constant rate. Since different rays of light go through different amounts of the wedge based on where they hit the wedge, various rays’ polarizations will be rotated by different amounts. One problem with this depolarizer is that there is a fast axis and a slow axis for the light. In order for the depolarizer to work best, the polarized light should enter at a 45% angle to the two axes. If the polarized light is parallel to either axis, then no rotation will occur. The light has to have some component along each axis in order for there to be a change in orientation. When the polarization direction wanders before the wedge input, intensity noise is introduced because the depolarizer’s response varies with input orientation. By using a short fiber or a PM fiber from the SOA to the depolarization wedge, this problem should be able to be avoided.

Another problem with the wedge polarizer is that it must be used in free space. This issue actually has two parts. First, the beam must be made large enough to fill a significant part of the wedge. If too small an area of the wedge is illuminated, the scrambling will be poor at best. Large collimation packages are available, but coupling efficiencies between them are low.
Second and most importantly, the different polarizations diverge in the wedge, and in space after they leave the wedge. This fundamental effect in birefringent materials is shown in Figure 37.

![Figure 37 - The different polarizations will refract differently in birefringent material. This problem is amplified by the wedged shape of the crystal which causes even similar polarizations to be no longer parallel.](image)

This diverging beam effect can be demonstrated in the lab with a strongly polarized light source, as in Figure 38. As the light is depolarized, the different polarizations spread away from each other. In fact, even rays of the same polarization spread away from each other. Far from the source, it becomes obvious that there are distinct beams coming from the depolarizer. Since the light has to be focused back into single mode fiber (which has a diameter of ~9µm) the scrambled polarizations can not all be coupled into the fiber at once. Thus the problem of polarized light remains.
Figure 38 - On the left is a spot from a polarized laser. On the right is the same beam but a depolarizer was placed in the beam's path. The spots are on the order of a few millimeters wide and are approximately 1 m from the source and depolarizer.

There is a third way to deal with polarization issues using single mode fiber[28]. One can use a cascaded 2 x 2 SMF coupler and form a loop with one input and one output port. As the light traverses the loop, it becomes linearly and circularly rotated before rejoining the light in the other branch of the coupler. In order for this to work, the resonant behavior of the loop must be negligible (i.e. creating a Mach-Zehnder device would produce unwanted intensity fluctuations). This non-resonant requirement can only be satisfied when the length of the fiber loop is much greater than the coherence length of the laser.

Since the FDML has a fairly long coherence length (~1.5m), the fiber loop would have to be at least 3m long. This would be too long of a delay for the FDML laser (10ns).
5.4 FDML Laser Conclusions

Although a solution to the polarization problem was not found, there are some avenues which would be worth investigating. If more time and more spools of fiber could be acquired, one solution might be to use several (~10) delay loops in parallel – similar to Shen’s cascaded coupler idea[28]. This would allow different polarizations to arrive at the SOA at once. Another possible solution is to introduce scattering in the fiber. Forward scattered light is unpolarized[29] so if there were enough scattering events, the light might become depolarized enough for the FDML to behave well. An exhaustive list of possibilities is provided in Table 1.

If money was easier to find than time, the best solution would most likely be to use PM fiber with PM optical components. The 2km segment should be placed between polarizers in order to help block out unwanted pulses from polarization misalignment.
Table 1 - Exhaustive list of polarization solutions along with main difficulty.

<table>
<thead>
<tr>
<th>Possible Solutions</th>
<th>Difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarize Light before or after SOA</td>
<td>Polarization wanders in fiber, producing intensity noise similar to having no polarization control at all.</td>
</tr>
<tr>
<td>Free space delay</td>
<td>Various wavelengths are rotated differently in fiber sections. This means that some wavelengths will be in the right polarization while others will not.</td>
</tr>
<tr>
<td>Polarization insensitive OSA</td>
<td>Would work if available. All wavelengths and polarizations would be amplified the same amount; thus, there would be no intensity noise.</td>
</tr>
<tr>
<td>Wavelength independent polarization control</td>
<td>Fiber sections would still affect various wavelengths differently. Even if all wavelengths were rotated the same amount, the various wavelengths would be affected differently in the rest of the system.</td>
</tr>
<tr>
<td>Active depolarization</td>
<td>Random polarization changes are slow compared to wavelength change. Even the fastest depolarizers are effectively frozen in comparison to the speed with which the wavelength changes.</td>
</tr>
<tr>
<td>Lyot depolarizer</td>
<td>Laser bandwidth is too narrow - all wavelengths rotate the same amount. The Lyot depolarizer is designed for broad band light because the rotation is a function of wavelength.</td>
</tr>
<tr>
<td>Depolarization wedge</td>
<td>The alignment is wavelength sensitive. In addition to wavelength sensitivity, spatial differences in wedge thickness introduce divergence, which is obstructive to SM - SM coupling.</td>
</tr>
<tr>
<td>Splitter with polarization controls</td>
<td>Lengths must be exact within one wavelength of light. Beating and Interference can occur in the light.</td>
</tr>
<tr>
<td>Fiber loop cascade</td>
<td>Wavelength change rate is too high compared to loop trip time. The loops need to be larger than the coherence length, and the transit time of the loops would disturb the timing of the delay by overlapping different wavelengths of light.</td>
</tr>
<tr>
<td>Forward scattering</td>
<td>High losses make this method prohibitive.</td>
</tr>
<tr>
<td>Polarization maintaining fiber</td>
<td>In the absence of a polarization insensitive OSA, this is the best choice. All components must be PM and all connections must be carefully aligned so that one type of polarization does not cross over into the other one.</td>
</tr>
</tbody>
</table>
6. Conclusions

This research investigated several technologies associated with rocket engine sensing. The three most important contributions were in the development and understanding of the dual-clad fiber used for transmission of the signal, the influence of beamsteering on beam propagation and on optimizing the collection of light, and ways to improve the performance of a new type of laser, the Fourier Domain Mode-Locking laser. It was determined that the DCF would not be an acceptable component for small rocket engine sensors because the mode noise proved too difficult manage.

A model for beamsteering has been developed which accurately models optical setups in the presence of homogeneous optical turbulence. Design decisions regarding lens selection for minimizing losses due to beamsteering can be made using this model. Agreement with both experiment and the literature has been demonstrated, as well as the importance of taking beamsteering into account when reporting spatial resolutions of beams in a turbulent region.

Finally, intensity noise in the FDML laser was successfully identified and several possible solutions to alleviate the problems were discussed. Chromatic dispersion in the fiber-ring cavity was identified as a main source of noise. A dispersion figure of merit was developed to evaluate and characterize potential fibers, and a through analysis was investigated. Polarization was identified as the other main source of noise in the laser’s output. While some solutions like
the Lyot and wedge depolarizers were shown to be ineffective, other solutions like the PM fiber show promise.

The research in these three areas have advanced the development of a rocket sensing system by demonstrating both the ideas that work, and by identifying the solutions that do not.
7. References


[18] Jhavar, R., and Narayanaswamy, K., 2006,


8. Appendix

A. Spatial Scanning – The steering mechanism

In order to do tomography, we need to have information at various locations in the rocket plume. In this section we address the problem of moving the laser to various locations. Several designs were considered. The first was a stationary system with no moving parts. The laser was split several times and pulses were sequentially shot through the plume at different locations. The second was a vibrating mirror which deflected the laser beam through the plume. The third was a rotating polygon which deflected the laser beam, and the fourth was an oscillating lens which steered the beam through the plume.

The first design would be a good design as far as repeatability and fatigue are concerned; however, it would be very difficult to get enough bins in the plume. Since the fibers would have to be in the same plane, and they would all have to have their own lenses. Thus it would be difficult to make. Also, once it is designed and built, there is no easy way to change the bin resolution.
In the second design, a collimated beam is reflected off of a mirror which vibrates back and forth. The mirror is at the focus of a lens which redirects the light so that the beam is perpendicular to the planar mirror on the other side of the plume. As the mirror vibrates back and forth, the beam scans across the rocket plume.

In the third design, the beam is reflected off a rotating polygon to a spherical mirror. The improvement of this design over the previous is that there is one less optical component. Unfortunately, after further investigation, it was found that the point at which the beam reflects off the polygon moves as the polygon rotates. This is bad because it complicates the tomography, and lessens the advantage that the spherical mirror gives.

In the fourth and final design, there is no mirror at all. Instead, the lens itself moves back and forth. The fiber is at the focal point of the vibrating lens which directs the beam across the plume and spherical mirror. This is the design that was ultimately selected.
We selected a 2.95mm lens for the vibrating lens. This was based of Kranendonk’s model of beamsteering. It was also checked against beam diameter, lens diameter, and the requirement of the lens translation in order to achieve the necessary sweep angle.

The mirror was selected to have the smallest f/#. The f/# is the ratio of the focal length to lens diameter. It corresponds to the largest sweep angle that the mirror will reflect back from the center of curvature of the mirror. The result was a spherical mirror with a 50 mm diameter and a 50 mm focal length (f/1). If a smaller f/# was available, that would be preferred, but this is the best that was available.
B. **Double Clad Fiber coupling efficiency**

When coupling light from the single mode fiber to the primary core of the DCF, there will almost definitely be some light which misses the primary core, but that is still coupled into the inner cladding. This light will propagate through the fiber and could introduce mode noise into the measurement section. The efficiency we’re concerned with is the ratio of light coupled unintentionally into the inner cladding to the light actually coupled into the single mode core of the DCF. The best way to measure this is as follows.

First couple the light into the DCF without worrying if the light is in the core or the cladding. Next find the center of the fiber by misaligning the setup until the top and bottom edges of the fiber are found. You will know when you are at the edge by a drop in the signal strength. By using the same signal level for the top and bottom, you will ensure that the midpoint of the edges will be in the center. For example, if your photo-detector reads 1.0 V when you are coupled in the DCF, you might define the top and bottom edge as the position that reads only 0.5 V. Use the same technique for the left and right edges. The alignment should be close to center now.

In order to optimize the single mode to DCF core alignment, the following setup shown in Figure 1 is recommended. Collimate the beam from the DCF. If the most of the light is in the core, you should be able to see a bright circle inside of a dimmer, larger circle (see Figure 2). The small inner circle is light from the
core, and the outer circle is light from the cladding. By letting the beam diverge over a meter or two, the cladding light should diverge significantly from the core. A lens can then be placed in front of a detector in order to focus all the light (both cladding and core) into the detector. Be sure that the lens is large enough to capture the full extent of the cladding's divergence; the goal is to capture all the light.

Figure 39 - DCF light is collimated and allowed to diverge over a distance. A beam block stops light from the core from continuing to the detector. A lens then focuses the light from the unblocked cladding light onto the detector.

A beam block the width of the core's circle can be placed in front of the core's circle in order to measure only the cladding signal. If the setup shown in Figure 2 is used, there will be a little bit of cladding blocked as well. Also, the beam block is most likely not the exact size and shape of the core circle. It is better to make it a little larger so that it blocks
more of the cladding rather than letting more of the core through. This is because optimization depends on minimizing the power in the cladding. If light is moved from the cladding to the core, but core light is still allowed to be detected, there will be no way of knowing if the alignment is improving. If the cladding is blocked, and we assume that light fills the cladding uniformly, light that is coupled into the core will reduce the remaining cladding signal.

An iris could then be placed in the beams path in order to block the light from the cladding but not the core light. If this is done, there would still be some light from the cladding that would pass though the iris. The further away the iris is placed, the less light from the cladding would be allowed to pass through. The detector would then give you the amount of light that is in the core.

As an alternative, however, it is usually better if the complete signal is recorded and the cladding signal subtracted from it to give the core signal. Not only is this an easier setup, but the cladding signal is so much smaller than the core signal that measuring the total signal usually does not effect the precision of the measurements.
Figure 40 – The core light is blocked by a rod on the left side of the top picture. The cladding light continues to a lens on the right that focuses the light onto the detector. The picture on the bottom left shows the collimated light before the core is blocked out. The picture on the bottom right shows the resulting light that is allowed to go to the detector after being blocked.

Once the cladding and core levels are recorded, an efficiency ratio can be determined. It is important to note that because of the accidental obstruction of cladding light when blocking out the core light, the calculated efficiency will be slightly higher than the actual.

Special thanks to Keith Rein for the photographs.
C. Adjusting a lens for ray trace modeling of Gaussian beam

(An excerpt from "Optical System for Tomographic Measurements in Rocket Plumes" by Schmidt, Caswell, and Sanders)

After the initial design, the setup was modeled in a program called Zemax in order to make sure that beam steering would not be a problem. Zemax is in part, a ray tracing program that calculates the effects of every optical event upon several hundreds or thousands of rays. A beamsteering region was produced in Zemax using a bulk scattering region the size of the test region. The Zemax setup is shown in Figure 1. Instead of modeling the reflection off of the spherical mirror, everything was kept unidirectional. The mirror was replaced with an equal focal length lens, and the return path of the light was remodeled again, complete with another beamsteering region.

![Figure 1 - Zemax setup with rays.](image)

The spherical mirror is within the Rayleigh range of the beam waist but Zemax only models the light as rays. Figure 2 describes the problem pictorially. In order to work
around this, the spherical mirror’s focal point had to be changed to reflect the physics that actually occurs at that surface. Since the curvature of the wave front at the mirror’s surface matches that of the mirror, it will reflect the beam directly back along its path. To simulate this with rays, we need to change the focal point of the “lens” (corresponding to the mirror) so that it is half the distance to the focus of the beam.

![Diagram of spherical mirror and focal points](image)

**Figure 42 - The same mirror that refocuses a Gaussian beam to its focal point will not focus classical rays to the same point.**

After the simulation was run in Zemax, the ray data is analyzed using Matlab. Figures 3 and 4 show the distribution of position verses angle for each ray. In the figures, each point represents one ray. It is evident from these figures that the beamsteering will not be a problem in this experiment, as the detector has a 1mm surface and accepts angles much greater than 3 degrees.
Figure 43 - Distance off the optical axis verses angle of rays with no beamsteering.
Figure 44 - Distance off the optical axis verses angle of rays with beamsteering.
D. Matlab Codes

1. Distance_angle_Zemax_reader.m

clear all
clc
namez = '12_30_05 mirror B1d.dat'
Z0 = 0;%2.99944; %Used only for Plot A axis & when using fstar
f1 = 3.1;
f2 = 6.16;
fstar = 0;%2.97211;% detector position investigated - change below to use fstar
%focal point calculated in sequential mode - used when comparing to Kranendonk
% 350230 = 4.5mm, f* = 2.97211mm
% 350330 = 3.1mm, f* = 1.788695mm
% 350170 = 6.16mm, f* = 3.178275mm
lenses = ['f_1 = ','num2str(f1),'mm, f_2 = ','num2str(f2),'mm'];

fid = fopen(namez);
[A,count] = fread(fid,'float32');
j=0;

for i = 53:7:count-6
    j = j+1;
    x(j) = A(i);
    y(j) = A(i+1);
    z(j) = A(i+2);
    l(j) = A(i+3);
    m(j) = A(i+4);
    n(j) = A(i+5);
    Inten(j) = A(i+6);
end

for i = 1:j
    dist(i) = sqrt( x(i)^2 + y(i)^2 );
end

theta=atan2(-y,x);
phi=atan2(m,l);
angle = n.*sign(sin(cos(theta+phi)));
angleD = 180/pi*acos(angle);

for i=1:length(n)
    if angleD(i)>90
        angleD(i) = angleD(i)-180;
    end
end

%%%%%%% Detector Placement %%%%%%%%
Zrange = [Z0-.05:.001:Z0+.05];
newdist = extrap(dist,angleD,Z0,Zrange);
[newang,angsign] = extrapang(dist,angleD,Z0,Zrange);
[distmax,kept,notkept] = det_opt(newdist,angleD,.225,.865);
distatpoint = det_opt(dist,angleD,.275,.865); % where Zemax's det plane is

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plot A - ray diameter @ z %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % Only can be used when outside beamsteering region
% % (When rays are straight and can be extrapolated)
%
% figure
% hold on
% plot(Zrange,distmax*1000,'r-')
% xlabel('distance from lens [mm]')
% ylabel('86.5% of rays in this diameter [mum]')
% title(['Finding Optimum Detector Location - ', namez])
% text(Z0-.1,(.7*max(distmax)+.3*min(distmax))*1000,lenses)

locbestZ = find(distmax==min(abs(distmax)));
bestZ = Zrange(locbestZ);
bestdist = extrap(dist,angleD,Z0,fstar); % bestZ; %
% Use fstar when need detector at a certain place, bestZ when optimizing
% % det position - when using fstar, don't forget to plug in Z0 at top
%
% hold on
% plot (bestZ,min(abs(distmax*1000)),'o')
%
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Efficiency %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% [E,rayskept] = effic(bestdist,angleD,22.5E-3,.225);
E
z(1);

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plot B - Dist vs Ang %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% for i = [485 605]
% figure
% hold on
% plot(abs(newdist(i,:))*1000,newang(i,:),'r.'
% xlabel('distance [mum]')
% ylabel('angle [deg]')
% axis('square')mina [max(dist)] 0 2])
% end
% title(namez)
% text(5,0,'4mm into beamsteering region')
%
% figure
plot(dist*1000,angleD,'b.')

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plot C - Spot %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% figure
% hold on
% plot(x*1000,y*1000,'b.' )
% xlabel('x [mum]')
% ylabel('y [mum]')
% axis('equal')[-15 .15 -.15 .15])
% title(namez)
%
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plot D - Ray extrapolation %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% figure
% plot(Zrange,abs(newdist(:,rayskept)))

2. Extrap.m

function newdist = extrap(dist,angleD,z0,range)
  %
  % use extrap(dist,angleD,z0,range)
  %
  % dist and angleD are from distance_angle_zemax_reader
  % maxNA is the maximum NA the catching fiber can obtain
  % z0 is the distance from the lens dist and angleD are defined at
  % range is the set of z values (measured from the lens) at which you want
  % the efficiency calculated

  k = 1;
  for z = range
    newdist(k,:) = dist + (z-z0)*tan(pi*angleD/180);
    k = k+1;
  end

3. ExtrapAng.m

function [newang,angsign] = extrapang(dist,angleD,z0,range)
  %
  % use extrapang(dist,angleD,z0,range)
  %
  % dist and angleD are from distance_angle_zemax_reader
  % maxNA is the maximum NA the catching fiber can obtain
  % z0 is the distance from the lens dist and angleD are defined at
  % range is the set of z values (measured from the lens) at which you want
  % the efficiency calculated
  % angsign = ones(length(range),length(angleD));
  % newang = angsign;
  k = 1;
  for z = range
    angsign(k,:) = sign(dist + (z-z0)*tan(pi*angleD/180));
    size(angsing)
    size(angsing(k,:))
    size(angleD)
    newang(k,:) = angleD.*angsign(k,:);
    k = k+1;
  end

4. Det_opt.m

function [distmax,keptrays2,notkept] = det_opt(dist,angleD,maxNA,perc)

[M,N] = size(dist);
keptrays2 = NaN; %*ones(1,N);
notkept = NaN; %*ones(1,N);
j=1;
jj=1;
for i = 1:N
if angleD(i) < 180/pi*asin(maxNA)
    kepstrays2(j) = i;
    j = j+1;
else
    notkept(jj) = i;
    jj=jj+1;
end
end

% kepstrays2;
% dist(:,kepstrays2);
dist = abs(dist);

for k = 1:M
    [sortedrays,order] = sort(dist(k,kepstrays2));
    cutoff = floor(perc*length(kepstrays2));%.865% is 1/e^2 value (Gaussian Power)
    distmax(k) = sortedrays(cutoff);
end

if nargout == 1
    kepstrays2 = []; % kepstrays2 = kepstrays2;
    notkept = [];
end

5. Effic.m

function [effic,totalpointskept] = effic(dist,angleD,d,NA)
  % Use: effic(dist,angleD,62.5E-3,.275)
  %
  % Inputs:
  % dist is vector of distances from distance_angle_zemax_reader
  % angleD is vector of angles from distance_angle_zemax_reader
  % d is cutoff diameter of fiber in [mm]
  % NA is cutoff NA for fiber
  %
  % Outputs:
  % effic is the percent of rays that will be accepted by the fiber

  j=1;
  for i=1:length(dist)
      if dist(i) <= d/2
          distkept(j) = dist(i);
          pointskept(j) = i;
          j=j+1;
      end
  end
  j=1;
  totalpointskept = NaN;
  for i=pointskept
      if abs(angleD(i)) <= 180/pi*asin(NA)
          anglekept(j) = angleD(i);
          totalpointskept(j) = i;
          j=j+1;
      end
  end
effic = \frac{\text{length(totalpointskept)}}{\text{length(dist)}} \times 100;
E. GRIN Lens equation and User Inputs

The most general form of the equation is:

\[
n(z, y) = (M(z, y) + 1) \left( \frac{1}{N_G} \sum_{i=1}^{N_G} G_i(z, y) + \frac{1}{N_S} \sum_{i=1}^{N_S} S_i(z, y) \right) + L(z, y) + O
\]

where

\[
G_i(z, y) = A_i \exp \left( -\frac{(z - \mu_{Gz,i})^2}{\sigma_{Gz,i}^2} - \frac{(y - \mu_{Gy,i})^2}{\sigma_{Gy,i}^2} \right)
\]

\[
S_i(z, y) = \begin{cases} 
\frac{(z - c_{zi})^2}{a_i^2} + \frac{(y - c_{yi})^2}{b_i^2} & \geq 1 \quad 0 \\
\frac{(z - c_{zi})^2}{a_i^2} + \frac{(y - c_{yi})^2}{b_i^2} & < 1 \quad B_i 
\end{cases}
\]

\[
M(z, y) = h_M \sum_{i=1}^{N_M} \left[ \text{rand}_{M1,i} \sin(k_{Mz,i} z + \text{rand}_{M2,i}) + \text{rand}_{M3,i} \sin(k_{My,i} y + \text{rand}_{M4,i}) \right]
\]

\[
L(z, y) = h_L \sum_{i=1}^{N_L} \left[ \text{rand}_{L1,i} \sin(k_{Lz,i} z + \text{rand}_{L2,i}) + \text{rand}_{L2,i} \sin(k_{Ly,i} y + \text{rand}_{L4,i}) \right]
\]

\[
k_{Mz,i} = i(\text{rand}_{k1,i} + 1) \frac{2\pi k_{z,\text{min}} - k_{z,\text{max}}}{N_M - 1} + k_{z,\text{min}} \quad k_{My,i} = i(\text{rand}_{k2,i} + 1) \frac{2\pi k_{y,\text{min}} - k_{y,\text{max}}}{N_M - 1} + k_{y,\text{min}}
\]

\[
k_{Lz,i} = i(\text{rand}_{k3,i} + 1) \frac{2\pi k_{z,\text{min}} - k_{z,\text{max}}}{N_L - 1} + k_{z,\text{min}} \quad k_{Ly,i} = i(\text{rand}_{k4,i} + 1) \frac{2\pi k_{y,\text{min}} - k_{y,\text{max}}}{N_L - 1} + k_{y,\text{min}}
\]

\[
k_{z,\text{min}} = \frac{2\pi}{T} \quad k_{y,\text{min}} = \frac{2\pi}{D}
\]

and, \( z = \) the optical axis defined in ZEMAX

\( y = \) the vertical axis defined in ZEMAX

\( n = \) index of refraction

\( \text{rand}_{M1,i} = \) \( i \)-th random number (between 0 and 1) generated for M1 term

(similar for other rand numbers)
User Defined Parameters in Lens Data Editor of ZEMAX

\( N_G \) = number of Gaussian curves included (initial value = 3)

\( N_S \) = number of “top hat” curves included (initial value = 1)

\( O \) = off set (user defined parameter in Lens Data Editor of ZEMAX; initial value = 0)

\( A_i \) = height of \( i^{th} \) Gaussian curve (initial value = 0.4)

\( \sigma_{Gz,i} \) = standard deviation of \( i^{th} \) Gaussian curve in z-direction (initial value = 1)

\( \sigma_{Gy,i} \) = standard deviation of \( i^{th} \) Gaussian curve in y-direction (initial value = 1)

\( \mu_{Gz,i} \) = center of \( i^{th} \) Gaussian curve in z-direction (initial value = 0)

\( \mu_{Gy,i} \) = center of \( i^{th} \) Gaussian curve in y-direction (initial value = 0)

\( B_i \) = amplitude of \( i^{th} \) “top hat” curve (initial value = 1)

\( a_i \) = \( i^{th} \) z-semi-axis (initial value = 600)

\( b_i \) = \( i^{th} \) y-semi-axis (initial value = 600)

\( c_{zi} \) = \( i^{th} \) offset of “top hat” center in z-direction (initial value = 0)

\( c_{yi} \) = \( i^{th} \) offset of “top hat” center in y-direction (initial value = 0)

\( N_M \) = number of Fourier series terms (initial value = 20)

\( h_M \) = height (top to bottom) of multiplicative noise (initial value = 0.001)

\( N_L \) = number of Fourier series terms (initial value = 20)

\( h_L \) = height (top to bottom) of additive noise (initial value = 0.001)

\( k_{max} \) = maximum noise width (initial value = 10 mm\(^{-1}\))

\( T \) = thickness of GRIN surface in the z direction (initial value = 600 mm)

\( D \) = semi-diameter of GRIN surface in the y direction (initial value = 600 mm)