# Finite Element Modeling of Fluid Flows and Solid Deformations

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## 1. Introduction

In the Finite Element Method (FEM) the physical system to be analyzed is broken up into simple discrete elements. Often times the number of elements required to accurately represent the physical system is quite large. By representing the larger physical system as the sum of smaller and simpler elements the solution is likewise simplified to the sum of smaller and simpler analyses. Figure 1, provides a simple introductory example of the finite element method by discretizing a circle to find $\pi$ [1].

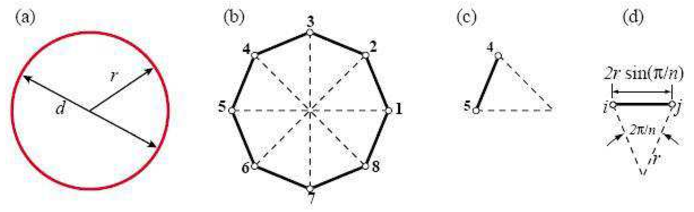


FIGURE 1 The "find $\pi$" problem treated with FEM concepts: (a) continuum object, (b) a discrete approximation by inscribed regular polygons, (c) disconnected element, (d) generic element.

## 2. The Code - Elmer

Elmer

The program selected to solve problems via the Finite Element Method was Elmer. Elmer is an open source multiphysical simulation software developed by CSC, the Finnish IT center for science [3]. Elmer supplies a complete package capable of mesh generation, simulation, and post processing. As a multiphysics package, Elmer is capable of modeling many more physical situations than we were interested in solving [4]. The area that was most vital to this research was Elmers fluid mechanics capabilities.

## 3. The Problems we Analyzed

1. Incompressible flow through an expanding pipe. Similar to problem 1, here we modelled the flow of an incompressible fluid through a region with a linear expansion.

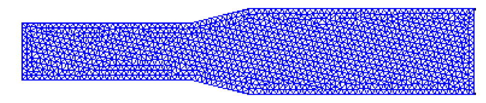

**Figure 2: The mesh defining an expanding pipe**

2. Incompressible flow over a step. In this problem we modelled the flow of an incompressible fluid through a region with a sudden change in width.

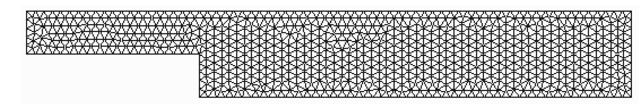

**Figure 3: The mesh defining a step in the tube**

3. Deflection of an elastic beam. This problem involved finding the displacement of an elastic beam which is subject to a uniform load.

**Figure 4: The mesh defining an elastic beam**

## 4. Fluid Flow Through an Expanding Pipe

To analyze fluid flow through the expanding pipe we used the Hagen-Poiseuille equation. The Hagen-Poiseuille equation models the velocity of laminar flow through a tube as a function of the distance from the center of the tube [2].

$$v = -\frac{1}{4\eta}\frac{\Delta P}{\Delta x}(R^2 - r^2)$$

**Hagen-Poiseuille Constants:**
$\eta = Viscosity$
$\Delta P = Pressuredropoverthewholetube$
$\Delta x = Lengthofthetube$
$R = Radiusofthetube$

We found that the results of the flow simulation were indeed Hagen-Poiseuille like.

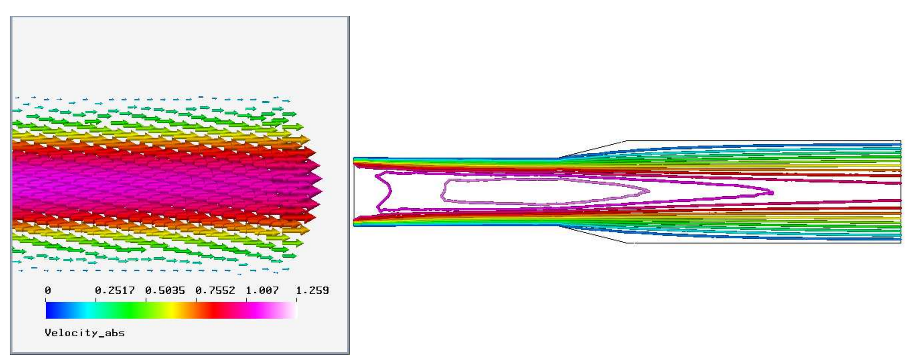

**Figure 5: Hagen-Poiseuille behavior presents itself in the results of the expanding pipe simulation**

## 5. Incompressible Flow Over a Step

When analyzing incompressible fluid flow over an expanding pipe the primary parameter we wanted to concentrate on was pressure. With the pressure from the system we could verify the systems adherence to the Bernoulli continuity equation.

$$p_1V_1 = p_2V_2$$

**Bernoulli Continuity Equation Variables:**
$p_1 = Pressurealongisocontour1$
$V_1 = Velocityalongisocontour1$
$p_2 = Pressurealongisocontour1$
$V_2 = Velocityalongisocontour2$

An important qualitative realization of the Bernoulli continuity equation is that a drop in pressure means velocity will increase. This realization manifests itself quite nicely in this simulation.

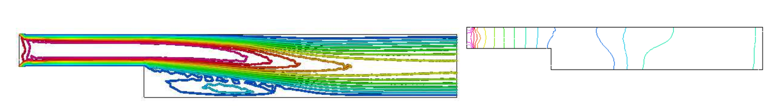

**Figure 6: Pressure isocontours along the tube** Most of the pressure isocontours conform quite nicely to what should be expected from the Bernoulli equation.

## 6. Bending an Elastic Beam

To analyze the deflection of an elastic beam under a uniform load we used the deflection equation that describes a beam with one fixed end, under uniform load. This is a standard deflection equation derived expressly for the situation this simulation models[5].

$$\delta(x) = -\frac{Fx^2}{24EI}(x^2 + 6L^2 + 4Lx)$$

**Constants:**
$F = Force$
$E = Young'smodulus$
$I = Momentofinertia$
$L = Lengthofthebeam$

As in our prior simulations, this one provided nice output as well. We once again found good agreement to typical results for such simulations.

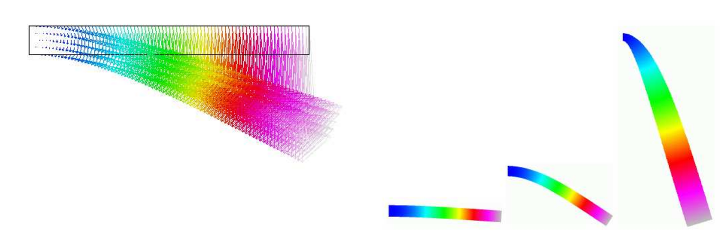

**Figure 7: Vectors indicating deformation of a beam, as well as multiple deformations of a loaded beam**

## References

[1] Felippa, Carlos, "Introduction to Finite Element Methods (ASEN 5007)," Department of Aerospace Engineering Science, University of Colorado at Boulder, http://www.colorado.edu/engineering/CAS/courses.d/IFEM.d/IFEM.Ch01.d/IFEM.Ch01.pdf.

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I would first like to thank my research mentor, Dr. Paul Thomas,
as well as Dr. Phil Ihinger.
Secondly, I would like to thank the Office of Research and
Sponsored Programs for funding.