Phase Shift Characteristics of Oscillating Flow in Pulse Tube Regenerators

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ABSTRACT

Experiments are performed to measure the dynamic velocities and pressures of regenerators under different conditions to better understand the regenerator phase relation. A method is developed to investigate the phase characteristic and resistance coefficient of oscillating regenerators. The main oscillating phase shifts are shown and discussed.

INTRODUCTION

The most distinct characteristics of high frequency oscillating regenerators are phase relations between parameters. The phase angle could reflect the cooling performance, so to predict phase shift in oscillating flow is crucially important for the optimum design of pulse tube cryocoolers. Most previous researches focus on the oscillating friction factor, while several researches pay attention on phase shift. For example, F. Kuriyama et al.[1] set up a numerical model for pulse tube cryocoolers (PTC) and analyze the phase by phasor diagrams, Hofman[2] measured and analyzed the volume flow rates at the hot end of the regenerator and the hot end of the pulse tube, Ju et al.[3] measured the phase shift of pressure and velocity at both ends of the regenerator at 50 Hz, and Sungry et al.[4] used a new parameter to depict the phase shift between the instantaneous pressure drop and the velocity at the inlet of the regenerator.

This paper centers on the phase angle relations in a PTC.

EXPERIMENTAL SYSTEM

Figure 1 shows the experimental apparatus[5] used for dynamic measuring. The apparatus resembles a real cryocooler but operates at room temperature. It could measure the dynamic pressure and velocity at both ends of the tested regenerators. A linear compressor (swept volume: 2 cc) is used to generate oscillating pressure (30-60 Hz). The experiment is operated at an average pressure of 2 MPa.

There is a stabilizing section between the compressor and the regenerator to make the flow more stable and several pieces of wire screens are arranged before and after the sensors for the

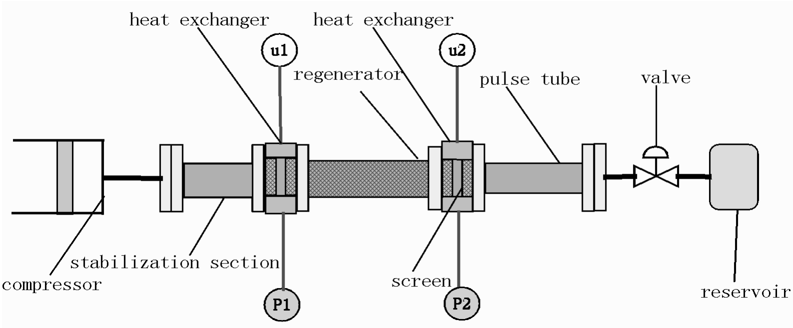

Figure 1. Experiment apparatus for testing the regenerators

uniformity of velocity. A reservoir is connected to the pulse tube by an adjustable needle orifice valve.

The lengths of regenerators are 40 mm, 50 mm, 70 mm and 80 mm, respectively. They are packed with stainless-steel plainly woven wire screens mesh no. 400.

A fine hot wire (DANTEC, Model 90N10) anemometer and a piezoelectric pressure transducer (KISTLER, Type 601A) is arranged at each end of the regenerator to measure the dynamic pressure and the instantaneous oscillating velocity through the regenerator, respectively, as shown in Figure 1.

In order to get the system characteristics, various conditions such as different opening of orifice (20, 30, 40, 50, 60), different input power (4 W, 9 W, 16 W) and different frequency (30 Hz, 40 Hz, 50 Hz, 60 Hz, et al. are changed for testing. The input power is limited by 16 W in this experiment because the measured velocity curve becomes unstable at higher input power.

In this paper, instantaneous pressure and velocity at the inlet of a regenerator, or other elements, are denoted by P_1 and U_1, and those at the outlet denoted by P_2 and U_2, respectively. The phase shift is denoted by $\Phi(U_1,U_2)$, $\Phi(U_1,P_2)$, $\Phi(U_1,P_1)$.

Figure 2 shows a typical group of measured velocity and pressure waves at the inlet (U_1,P_1) and outlet (U_2,P_2) of a regenerator. The waves clearly show the different amplitude and the phase shift. The phase angle of each parameter is processed by a Fast Fourier Transform (FFT), the same as other work.[1]

One should notice that, since the wave of velocity is not exactly sinusoidal and not very smooth, the phase result processed by FFT is not very accurate. So the error of the phase up to several degrees is possible.

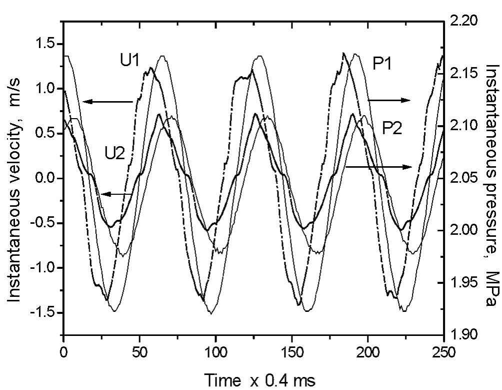

Figure 2. Typical experimental curves

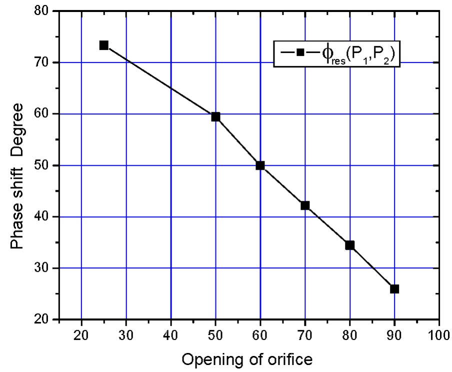

Figure 3. Phase shift of pressures at both sides of a resistance element.

EXPERIMENTAL RESULTS AND ANALYSIS

Phase Shift of Pure Resistance Element

For a resistance element plus the reservoir with a small volume, like the orifice and the reservoir structure in PTCs, the velocity phase shift between the orifice two ends is nearly zero, while the phase of the pressure waves, Φ_{res} (P_1,P_2), varies a lot and gets larger with the resistance increasing, as shown in Figure 3. Small opening of an orifice valve, delegating large resistance coefficient, corresponds to a large phase shift of Φ_{res} (P_1,P_2).

It is easy to comprehend that the mass flow rate must be the same at both ends of a non-volume element for the conservation of mass. When the resistance element delays the mass flow rate in oscillating flow, the phase shift between the corresponding pressure exists and offers a pressure gradient for the mass flow to overcome the resistance. The phase shift of pressures at ends of a resistance element is mainly caused by the resistance of the element.

Phase Shift of Volume

It is also observed from experiments that, for a capacity element with little resistance, like a pulse-tube, Φ_{cap} (P_1, P_2) is nearly zero, while Φ_{cap} (U_1, U_2) gets larger with the volume increasing, which can be computed by a simple model.

Figure 4 shows the simple model for analysis. The pressure is nearly equal in the whole pulse tube. The reservoir is large enough to assure the pressure P_0 constant when the system works.
The pressure is:

$$P_{in} = P_{1cap} = P_{2cap} = P_0 + |P|\sin(\omega t) \tag{1}$$

here $|P|$ is the amplitude of pressure wave. Choose the right direction as positive. With small pressure amplitude in the experiment, the instantaneous mass flow rate can be approximately described as:

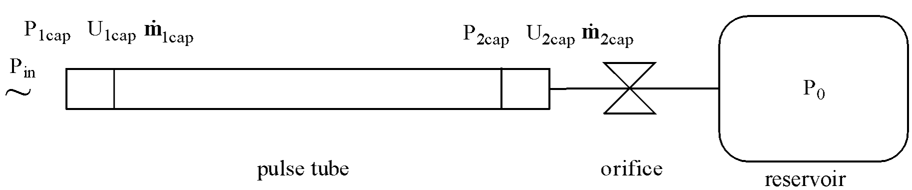

Figure 4. Computed model for phase shift of a capacity element

$$\dot{m}_{2cap} = C_o(P_{2cap} - P_0) = C_o(|P|\sin(\omega t)) \tag{2}$$

where C is the coefficient defined, corresponding to the opening of orifice.
The mass conservation equation for the pulse tube is

$$\frac{dm_{cap}}{dt} = \dot{m}_{1cap} - \dot{m}_{2cap} \tag{3}$$

The expression for the mass in the tube is:

$$m_{cap} = \frac{P_{in}V_{cap}}{RT} \tag{4}$$

Then, from equation (2), (3), and (4)

$$\begin{aligned}\dot{m}_{1cap} &= \omega\cdot\frac{|P|V_{cap}}{RT}\cos(\omega t) + C_o|P|\sin(\omega t)\\ &= \sqrt{(\omega\cdot\frac{|P|V_{cap}}{RT})^2 + (C_o|P|)^2}\cdot\sin(\omega t+\theta)\end{aligned} \tag{5}$$

$$tg\theta = \frac{\omega\cdot\frac{|P|V_{cap}}{RT}}{C_o|P|} = \frac{\omega V_{cap}}{C_o RT} \tag{6}$$

where $\theta = \Phi_{cap}(m_1,P_1)$ is the phase shift between $\dot{m}_{1cap}$ and P_{1cap}•$\Phi_{cap}(m_1,P_1)$ is the most important phase angle at cold end tip in Radebaugh's enthalpy flow theory[6] that decides the cooling performance of the cryocooler. Since $|P|$ has the small amplitude, density could be neglected and the phase in $\dot{m}$ is almost same in U. The resistance of the hot end, $C_{o,}$ is directly related to the phase angle. The phase $\Phi_{cap}(U_1, P_1) = \theta$ will be 90^0 when C_o=0 and $\Phi_{cap}(U_1,P_1)=\theta$ will be 0 when C_o is infinite. Compared with the experimental results, as shown in Figure 5, the simple model is reliable. The resistance coefficient is gained from experiments as its definition. The phase shift between the velocities at the ends of a capacity element is mainly be caused by the capacity of the element.

Phase Shift of Regenerators

The regenerator is a complex element that contains both resistance and capacity elements. General experimental results demonstrate that it can be studied by $\Phi_{cap}(P_1,P_2)$ and $\Phi_{reg}(U_1,U_2)$ for resistance and capacity characteristics, respectively, under different conditions. The phase shift $\Phi_{reg}(U_2,P_2)$ at the cold end of the regenerator has no relationship with the regenerator, but is affected by the situation behind. Some graphs representing some of the many experiments are shown in Figure 6~9. In each graph, only one parameter is changed.

Figure 6 shows the situation when the orifice valve is opened up. $\Phi_{reg}(U_2,P_2)$ diminishes evidently, while $\Phi_{reg}(P_1,P_2)$ and $\Phi_{reg}(U_1,U_2)$ does not change much. A different resistance at the end of

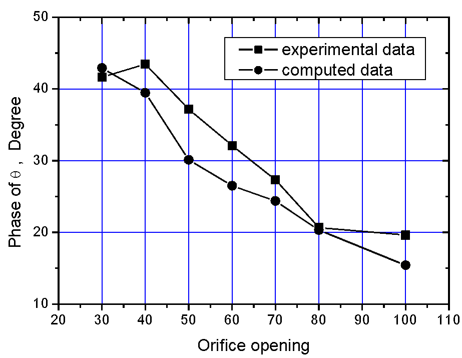

Figure 5. Computed data comparing to experimental data

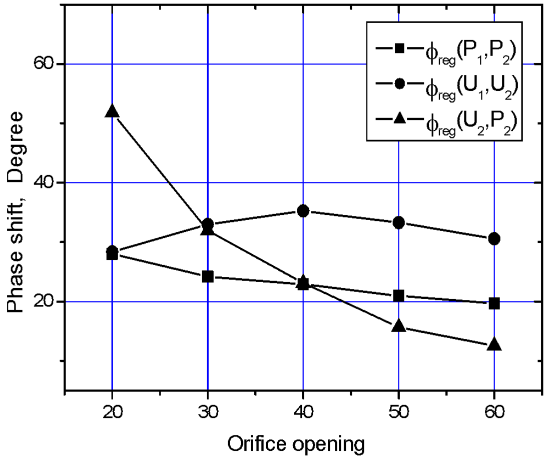

Figure 6. Phase shift depending on orifice opening

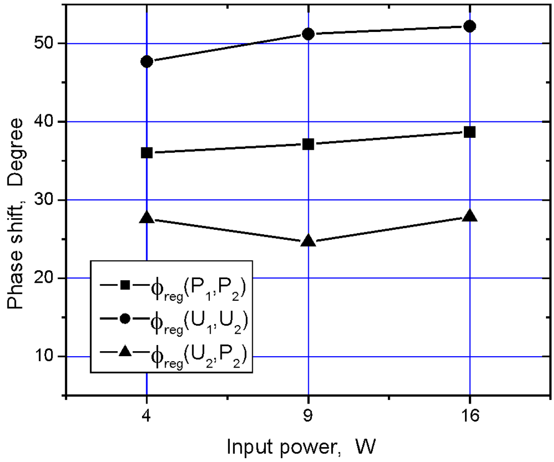

Figure 7. Phase shift depending on input power

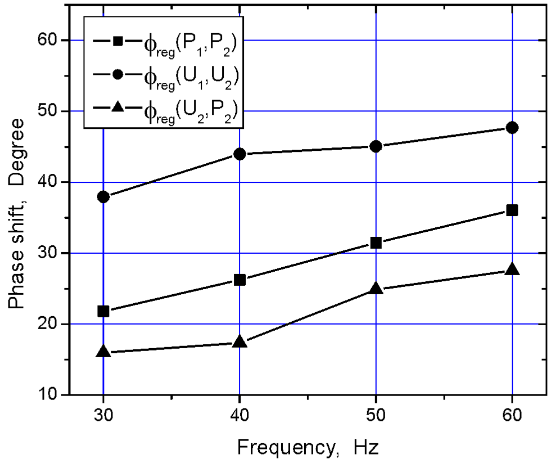

Figure 8. Phase shift depending on frequency

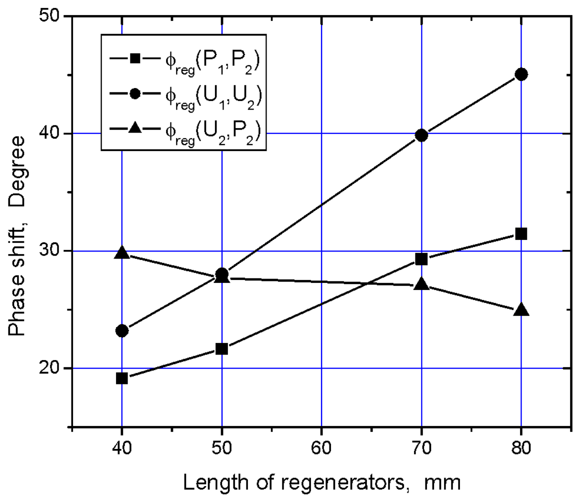

Figure 9. Phase shift depending on length of regenerators

pulse tube changes $\Phi_{reg}(U_2,P_2)$, but the same regenerator decides the velocity phase shift and the pressure phase shift. Analogously, Figure 7 presents three almost constant phase shift angles under different input powers for the same regenerator and the same orifice opening. In Figure 8, all of the three phase shift angles increase with frequency. The phase shift $\Phi_{reg}(U_2,P_2)$ increases because ω in equation (6) increases with the frequency, and the same as $\Phi_{reg}(U_1,U_2)$. In Figure 9, with the increase of the regenerators' length, both $\Phi_{reg}(P_1,P_2)$ and $\Phi_{reg}(U_1,U_2)$ increase apparently, because of increasing resistance and capacity, while $\Phi_{reg}(U_2,P_2)$ does not vary much at the same situation at the end.

Pressure Drop and Resistance Coefficient

Some former works[4,7] focus on the phase shift angle between the velocity at the inlet of a regenerator U_{1reg} and the instantaneous pressure drop ΔP_{reg} (define $\Delta P_{reg}=P_{1reg}-P_{2reg}$), which is also sinusoidal. However, experimental results demonstrate that, under various conditions, ΔP_{reg} almost has the same phase with the linear-average velocity U_{reg} ($U_{reg}=(U_{1reg}+U_{2reg})/2$), while it has a phase shift from U_{1reg}. Figure 10 and Figure 11 demonstrate the difference at different frequency and different regenerators.

For a regenerator with a high frequency oscillating flow, because there exists a phase shift $\Phi_{reg}(U_1,U_2)$ between velocities at both ends, which can be up to 50 degree, the oscillating resistance coefficient

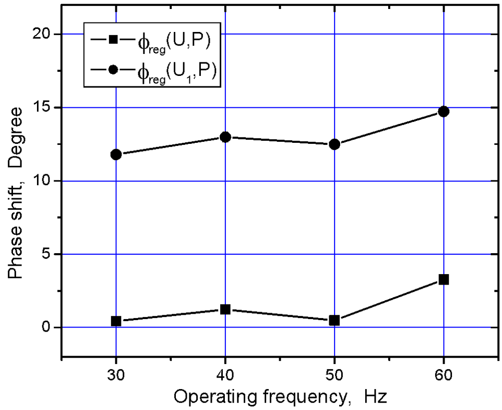

Figure 10. Phase shift at different frequency

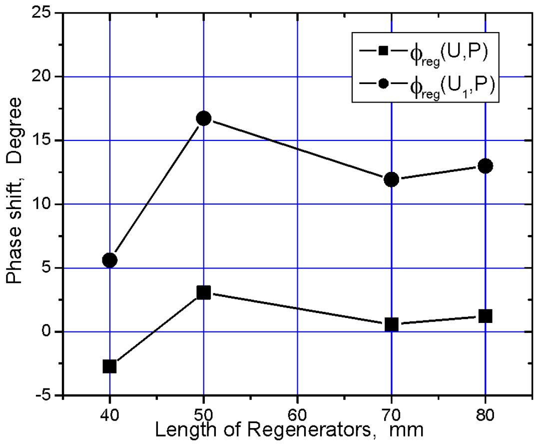

Figure 11. Phase shift of different regenerators

shouldn't be calculated by U_1, only. The velocity of gas at each tiny section inside the regenerator is different at any instantaneous time, corresponding to a different tiny pressure drop. The total oscillating pressure drop should be the integral result of each part, different from steady flow. If the resistance coefficient is calculated by U_{1reg} and ΔP_{reg}, it will vary with time because of the phase shift and it will be a different value under different experimental conditions. The amplitude of U_{2reg} is around half or even less than that of U_{1reg}, so it's not correct to use U_{1reg} to delegate the velocity of a regenerator. Though the entrance effect and the nonlinear effect must exist to a certain extent, using U_{reg} and ΔP_{reg} to investigate the resistance of a regenerator is commonly available.

CONCLUSION

Phase shifts of pressures and velocities at both ends of an oscillating flow regenerator are separately determined by the resistance and the capacity character of the regenerator. To investigate the oscillating resistance coefficient of regenerators, the instantaneous velocities at both ends should be considered to determine the pressure drop.

ACKNOWLEDGMENTS

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