

FLUX DIFFUSION LOSSES IN STABILIZED CONDUCTORS*

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Abstract - The "ac" losses in normal metal sheaths around superconductor composites are calculated. Three cases are considered: (1) the composite region is assumed to have a finite constant resistance, (2) flux lines are assumed to move from the normal metal into the superconductor at constant velocity, and (3) complete equations of motion and continuity of flux are used in both the normal and superconductor region. Case (1) is incorrect since it predicts a decrease in losses for an increase in rate of change of applied magnetic field, in contradiction to case (3). Case (3) is, of course, complete and accurate but complex enough to require numerical solutions. Case (2) however is only in error by a few percent for charging rates in the order of one tesla/h. Case (3) can be applied for any charging rate as long as the superconductor filaments are transposed, not just twisted.

INTRODUCTION

Superconducting magnets which are continuously charged and discharged, such as those used for energy storage in utility systems and divertor and transformer coils used in fusion reactors are subject to the full range of "ac" losses. It is the purpose of this paper to discuss losses in the normal metal external to the composite region during charge and discharge.

Energy losses in multifilament composite conductors have been the subject of many investigations [1-3]. In addition, in a stabilized conductor, there are considerable losses associated with flux motion through the normal metal sheath around the composite segment. The composite conductor affects the magnitude of these losses because of the boundary conditions it imposes at the interface between the normal metal and the composite portion.

The normal metal losses are calculated for a slab of normal metal in perfect contact with a slab of superconductor. Three cases are considered. In the first case, the superconductor is assumed to have a finite resistance. In the second case, the flux lines are assumed to move from the normal metal to the superconductor with a constant velocity. In the last case, the diffusion equation in the normal metal and the equations of continuity and motion of flux lines in the superconductor are solved numerically.

Examples are given applying to the calculation of losses in large energy storage solenoids for use by electric utility systems.

FINITE CONSTANT RESISTANCE SUPERCONDUCTOR

The problem is solved for a long solenoid. The windings can be taken as two slabs of normal metal and superconductors as shown in Fig. 1. The field is assumed to be changing linearly with time on the inner

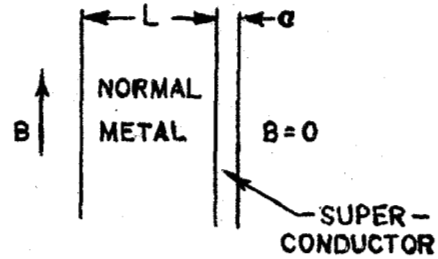


Fig. 1. Two slab model of a single layer solenoid.

side and zero on the outer sides of the slabs. The diffusion equations are

$$\frac{\partial^2 B_1}{\partial x^2} = \frac{1}{\alpha_1} \frac{\partial B_1}{\partial t} \tag{1}$$

and

$$\frac{\partial^2 B_2}{\partial x^2} = \frac{1}{\alpha_2} \frac{\partial B_2}{\partial t} \tag{2}$$

where

B_1 and B_2 are the fields in the normal metal and the superconductor respectively.

α_1 and α_2 are the diffusivities in the normal metal and superconductor respectively.

The boundary conditions are

$$B_1(0, t) = B_a(t), \tag{3}$$

$$B_2(L, t) = 0 \tag{4}$$

and

$$\rho_1 \frac{\partial B_1}{\partial x} = \rho_2 \frac{\partial B_2}{\partial x} \tag{5}$$

where

ρ_1 and ρ_2 are the resistivities of normal metal and superconductor respectively, L is the normal slab thickness, and B_a is the field at the surface $x=0$.

Boundary condition, Eq. (5), results from requiring the same electric field on both sides of the interface.

This problem was solved using Duhamel's theorem. First, a solution was found for a unit step in applied field. Second, since the applied field is a continuously changing field, the integral part of Duhamel's theorem was used to give the solution for $J_{1,2}(x, t)$ and $B_{1,2}(x, t)$. The solutions, which are cumbersome and lengthy, have been used to calculate energy losses.

The energy loss per m^2 in the normal slab is presented in Figs. 2 and 3 for a linear field change from 2.5 to 5T in 10 h or 1h, which are typical values for utility system storage usage. In this case, the superconductor thickness is 0.1 cm. The resistivity ratio between the slabs is varied from 10 to 10^4 and the normal slab thickness is varied from 4 to 20 cm. In Figs. 2 and 3 the loss for 10 h charging times is greater than for 1 h. Note that these results are unrealistic since faster charging should produce larger losses. The problem arises due to the model: a constant resistance superconductor implies a fractional current in the normal metal at all times.

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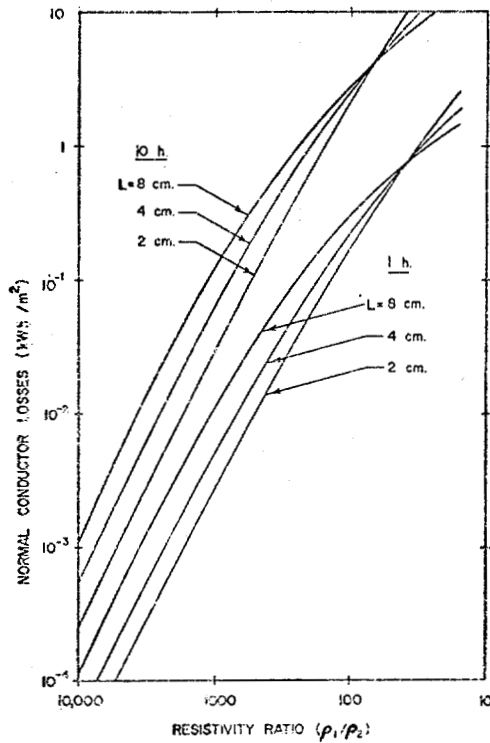


Fig. 2. Losses for case (1) vs. resistivity ratio.

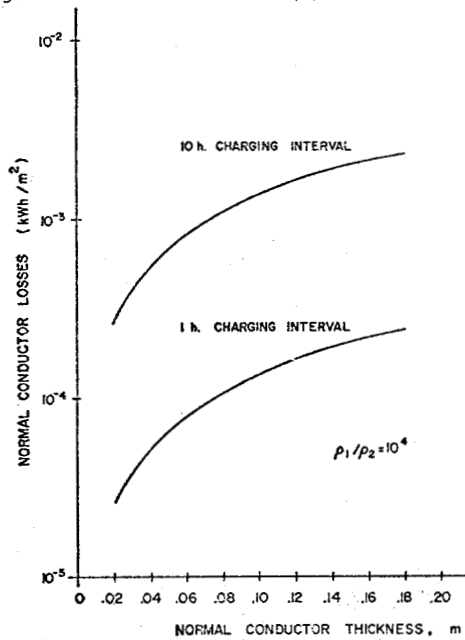


Fig. 3. Losses for case (1) vs. normal metal thickness.

CONSTANT FLUX VELOCITY ACROSS THE INTERFACE

In this section it is assumed that the flux lines move into the superconductor with a constant velocity. The equation of motion and the continuity equation of the flux lines introduced by Irie [4] are used to calculate an approximate value of this velocity. The following assumptions are made to calculate the velocity.

1. In the unsteady state, e.g., when charging, the current density is assumed to be higher than the critical current density. The Lorentz force will therefore exceed the pinning force and the flux lines move.
2. The current density in the superconductor is constant even with changing field (Bean model).

For example, transposed multifilaments become a Bean model slab.

If the magnetic field applied parallel to the surface is increasing, then the continuity equation and the equation of motion in the superconductor slab are:

$$\frac{1}{\mu} \frac{\partial B^2}{\partial x} - \alpha B - \beta v = 0 \quad \text{Equation of motion} \quad (6)$$

$$\frac{\partial B}{\partial t} - \frac{\partial}{\partial x} (vB) = 0 \quad \text{Continuity equation when charging} \quad (7)$$

For constant J , the velocity of the flux lines will be given by $v = 2J/\beta - \alpha/\beta$

where α is the pinning strength, and β is the viscosity coefficient of flux lines. Substituting v in Eq. (7), one has

$$J^2 + \alpha J/2 - (\beta/2\mu) \frac{\partial B}{\partial t} = 0.$$

Because of the assumption of constant J , this means that $\partial B/\partial t = \text{constant}$ everywhere, except where $B = 0$, inside the sample. This can be approximately valid if $\partial B/\partial t$ is constant at the superconductive surface, which is true for rates of one tesla/h. Eq. (1) gives

$$J = (J_C + \sqrt{J_C^2 + \frac{2\beta}{\mu} \frac{\partial B}{\partial t}}) / 2, \text{ and} \quad (8)$$

$$v = \frac{2}{\beta} (J - J_C) \approx \frac{2}{\mu J_C} \left(\frac{\partial B}{\partial t} \right)_s$$

where the subscript s refers to the value of $\partial B/\partial t$ at the interface between the normal metal and the superconductor.

The calculation of losses in the normal slab is possible after evaluating the velocity of flux lines into the superconductor. For a step function at the free surface of the normal metal, the differential equation and the boundary conditions are

$$\frac{\partial^2 B^*}{\partial x^{*2}} = \frac{\partial B^*}{\partial \theta} \quad \text{and} \quad \frac{\partial B^*}{\partial x^*} = h B^*$$

where B^* is the step function solution
 x^* is dimensionless and equal to x/L
 θ is dimensionless and equal to $\alpha x/L^2$
 h is similar to the heat transfer coefficient and is equal to $(Lv\mu)/\rho$.

Using the same Duhamel's method, solutions for B and J were found. From these, energy losses in the normal slab for the 2.5 to 5 T field change were calculated for various thicknesses, see Fig. 4. Charging times of 1 h and 10 h and $\rho = 10^{-10} \Omega \cdot m$ were used. In calculating the velocity to calculate h , Eq. (8), v is related to \dot{B}_a at the free surface of the normal metal as follows:

$$v \approx \frac{2}{\mu J_C} \left(\frac{dB}{dt} \right)_s \approx \frac{2 \dot{B}_a}{\mu J_C (1+h)}, \text{ and}$$

$$h \approx \frac{1}{2} \left\{ -1 + \sqrt{1 + \frac{8L \dot{B}_a}{\rho J_C}} \right\}.$$

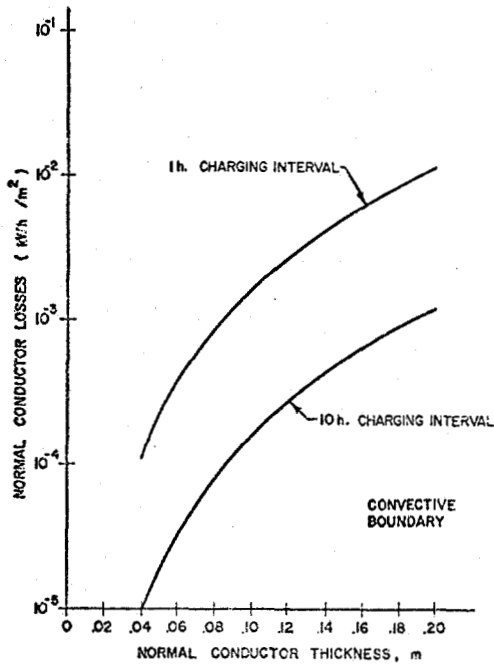


Fig. 4. Losses for constant velocity - convective boundary vs. thickness (case 2).

NORMAL SUPERCONDUCTOR SLABS

The diffusion of magnetic flux and current through the normal metal into the superconductor region is governed by the diffusion equation in the normal metal. In the superconductor the equations of motion and continuity of flux, Eqs. (6) and (7), govern the diffusion of flux. The solution of these equations for the two slab system yields the field and current distributions in both slabs from which losses can be calculated. Because of the non-linearity of the flux equations in the superconductor, an iterative solution is found using the integral approximation technique. Combining Eqs. (6) and (7), one gets

$$\frac{1}{\mu} \frac{\partial^2 B^2}{\partial x^2} - \alpha \frac{\partial B}{\partial x} = \beta \frac{\partial B}{\partial t}$$

The above equation integrated once reduces to

$$\frac{1}{\mu} \left[\frac{\partial B^2}{\partial x} \right]_L^{L+\delta(t)} - \alpha \left[B \right]_L^{L+\delta} = \beta \frac{d}{dt} \int_L^{L+\delta(t)} B dx \quad (9)$$

where $\delta(t)$ is the thickness of the superconductor penetrated by the field and is a function of time. If the field is assumed linear, Eq. (9) reduces to

$$\frac{d}{dt} B_s \delta = \frac{4 B_s^2}{\beta \delta} - \frac{\alpha B_s}{\beta} \quad (10)$$

where B_s is the field at the interface. The above equation with the diffusion equation in the normal metal can be solved numerically as shown in Appendix A.

From Eq. (9) and for steady state α is given by $\alpha = 2 J_C$. The critical density, J_C , is taken equal to $2 \times 10^9 \text{ A/m}^2$ for Nb-Ti. β is related to the normal resistance of Nb-Ti as follows [5]:

$$\beta = B_{C2}(0) / \rho_n$$

where $B_{C2}(0)$ is the critical field and ρ_n is the normal resistance.

It may be noticed that Eq. (2) for β is valid if the temperature is small compared to the critical temperature, i.e., for small values of T/T_C . For larger values of T/T_C , β may be an order of magnitude larger. As seen later, the value of β does not affect the results significantly. The normal resistance of Nb-44%Ti is $3 \times 10^{-7} \Omega \text{ m}$ [6] and $B_{C2}(0)$ is 12 tesla.

Figure 5 is a plot of losses vs. L , the normal metal thickness, for one hour and 10 hour charging respectively. It may be noticed that the losses calculated using the equation of motion of flux lines are slightly different from the losses calculated assuming constant flux velocity at the interface.

For the small values of β and small dB/dt used here it is seen in Fig. 5 that losses are almost independent of β ; this was also seen in the previous constant velocity-convective solution. Figure 6 is a plot of $\delta(t)$ which is found to be almost linear.

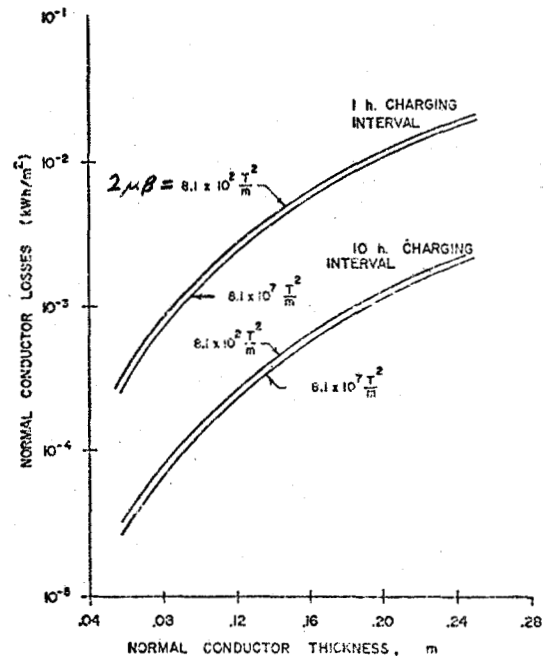


Fig. 5. Losses for normal and superconductor slabs vs. thickness (case 3).

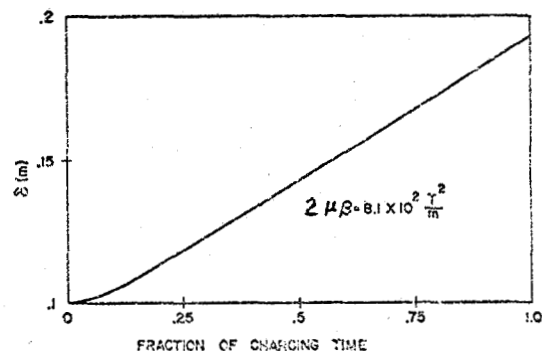


Fig. 6. Superconducting thickness vs. time for case 3.

CONCLUSIONS

The charging losses in the normal metal sheath around a superconductor composite have been found for an idealized single layer solenoid. Such solenoids are typical of those planned for energy storage usage. Two models have been employed to represent the solenoid turns: model (1) is two normal slabs in which the "superconductor" slab has much lower resistivity than the normal slab, and model (2) is two slabs of which the first is normal and the second slab a superconductor. Transposed filaments share the current in space so that a solid superconductor slab is a reasonable equivalent. The following has been found:

1. Model (1), the resistive two-slab model, yields an incorrect representation of the losses. In this case, some current must always flow in normal metal sheath which invalidates the loss calculations for the longer time periods.

2. The most accurate solutions to the problem involve the use of the complete flux equations of motion with model (2) slabs. In this case, pinning forces were taken equal to αB , following Irie [4]. The formulation of the problem allows for other pinning force dependencies.

3. The second solution to model (2) is a simple approximate solution which is accurate for small rates of change of field. In this case, the assumption is that the velocity of flux at the interface is constant, which is true for slow processes in which dB/dt is constant.

4. As an example, energy loss per m^2 was calculated numerically for a single layer solenoid. The superconductor filament bundle location, L , was varied between 4 cm and 24 cm from the inner solenoid surface, see Fig. 5. The losses at 24 cm are about 100 times larger than at 4 cm. In a 10,000 MWh storage solenoid, the loss per daily cycle is 27 MWh if $L = 12$ cm (our original design) and only 1 MWh if $L = 4$ cm (losses are given as room temperature inputs to the refrigerators). Therefore care must be exercised in locating filamentary bundles in high purity metal conductors.

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APPENDIX A - NORMAL SUPERCONDUCTOR SLABS

The field and the current distribution inside normal metal is found by using the finite difference technique. If the field distribution at time t is known, the field can be calculated everywhere in the normal metal except at the interface. The field at the interface can be found by solving Eq. (10) and by satisfying the requirement that the electric field is the same on both sides of the interface. It is assumed that between t_0 and $(t_0 + \Delta t)$ the penetration thickness $\delta(t)$ can be written as

$$\delta(t) = \delta(t_0) + a(t-t_0) \quad t_0 \leq t \leq t_0 + \Delta t$$

and Eq. (10) reduces to

$$\delta(t) \frac{dz}{d\delta} + \left(\frac{2\alpha}{a\beta} + 2 \right) z - \frac{4}{a\beta} z^2 = 0$$

where $z = B_s/\delta(t)$, and the solution of the equation

$$\text{gives } B_s(t) = \delta(t) \frac{(C_1/C_2) K (\delta(t)/\delta(t_0))^{C_1}}{1 - K (\delta(t)/\delta(t_0))^{C_1}} \quad (\text{A-1})$$

where $C_1 = -(2\alpha/a\beta + 2)$, $C_2 = 4/a\beta$, and

$$K = \frac{B_s(t_0)/\delta(t_0)}{B_s(t_0)/\delta(t_0) + C_1/C_2}$$

Because of the requirement of equal electric fields one has

$$\frac{\rho}{\mu} \left(\frac{\partial B}{\partial x} \right) = - \frac{d}{dt} \frac{B_s \delta}{2}$$

↑ normal metal

$$\text{i.e. } \frac{\rho}{\mu} \left(\frac{\partial B}{\partial x} \right) = \frac{\alpha B_s(t)}{\beta} - \frac{2 B_s^2(t)}{\beta \delta(t)} \quad (\text{A-2})$$

↑ normal metal

In Eq. (A-2) $\frac{\partial B}{\partial x}$ in the normal metal can be written as a function of B_s . It is possible to solve Eqs. (A-1) and (A-2) simultaneously to find both $B_s(t)$ and $\delta(t)$. These two values are considered $B_s(t_0)$ and $\delta(t_0)$ for the next iteration and the solution can be found accordingly.