

CONSIDERATIONS OF A LARGE FORCE BALANCED MAGNETIC ENERGY STORAGE SYSTEM

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ABSTRACT

We show in this paper that any generalized toroid-dipole force balancing system will require a minimum unidirectionally stressed structure equal to the minimum amount required by the virial theorem.^{1,2,3} The analytical proof is general and independent of configuration, current distribution, and the fraction of energy stored in the poloidal and toroidal fields.

INTRODUCTION

Concepts for superconductive energy coils involving force balancing systems have been recently suggested.^{4,5} In these concepts the forces on the first windings oppose the natural forces of a second set of windings. In that way the second coil might be considered as a partial replacement for the structure ordinarily needed to confine the first coil and vice versa. In one example the inner coil is a toroid whose outward forced turns are partially restrained by the inward forces from an externally wound poloidal coil which is essentially wound on the surface of the toroid, see Fig. 1. Although no detailed design has been presented of such two coil systems, it has been claimed that structure saving would result. We show in this paper that any generalized toroid-dipole force balancing system will still require a minimum unidirectionally stressed confining structure whose mass is $\frac{\rho}{\sigma}E$, where ρ is the density of structure, σ is the average structural stress and E is the magnetic energy stored in the two coil system. Therefore, there is no structure saved by this balancing technique since $\frac{\rho}{\sigma}E$ is the minimum tensile structure required for any magnet.

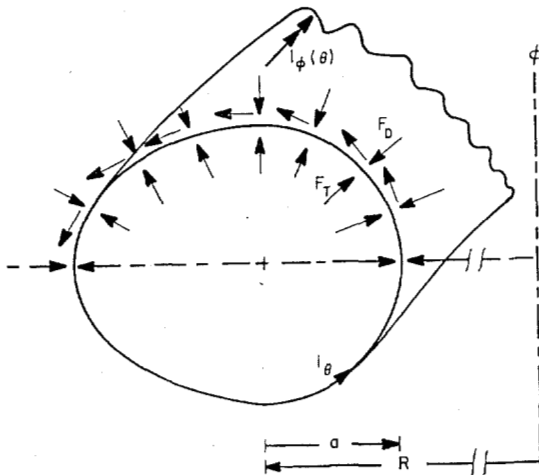


Fig. 1 Magnetic force distribution in a force compensated coil of arbitrary cross-section. F_T is the normal force due to I_0 the toroidal surface current. The dipole surface current $I_\phi(\theta)$ causes normal and tangential forces F_D .

THE VIRIAL THEOREM

The virial theorem of Clausius² as described by Levy¹ for electromagnetic systems states that for simple unidirectionally stressed structure of average stress $|\pm\sigma|$ and density ρ the total structural mass is

$$M \geq \frac{\rho E}{\sigma} \quad (1)$$

This statement alone can be very misleading, as noted by Levy, since any structure in compression must be equaled by a corresponding structure under tension. Thus, if a mass M_t is in tension and a mass M_c is in compression, one can write

$$M_t - M_c \geq \frac{\rho E}{\sigma} \quad \text{or} \quad M \geq 2 M_c + \frac{\rho E}{\sigma} \quad (2)$$

where the total structural mass $M = M_t + M_c$. Obviously, M_c (compression) should be zero if possible.

Based on the Virial Theorem it is impossible for the force balanced system of toroid-dipole windings to reduce structural mass below $\frac{\rho E}{\sigma}$. That limit is true for any coil or system of coils. However, the persistence of the force-balanced concept and the surprising lack of general acceptance of the Virial Theorem has led us to provide a direct proof which does not rely on the Virial Theorem.

STRUCTURE REQUIREMENT

For a system that consists of a toroidal configuration, a poloidal configuration (dipole), or both, the energy stored, E , is

$$E = \frac{1}{2} LI^2 \quad (3)$$

and

$$L = \mu_0 R f(\beta) \quad (4)$$

where R is the major radius, β is an aspect ratio $= a/R$, a is a characteristic length, I is the current, and f is a function of β that contains information related to the shape of the configuration, the surface current distribution and the number of turns in both the toroidal and poloidal field configurations. For example, in the case of a low aspect ratio ($\beta < 5$) circular cross-section force balanced magnet system with a uniform dipole surface current flowing perpendicular to a uniform toroidal surface current the inductance relations give:

$$f(\beta) = n_T^2 \{1 - (1 - \beta^2)^{1/2}\} + n_D^2 [\ln \frac{8}{\beta} - 2] \quad (5)$$

where n_T and n_D are the number of turns in the toroidal and poloidal windings respectively. The characteristic length a is the minor radius.

A. Net Radial Force

In the case of a toroidal winding, forces F_T are pointing outward and normal to the surface as shown in Fig. 1. The total vector sum of these forces on the

toroid is in the negative radial direction R which requires a central compressive structure. In the case of a dipole, forces are pointing inward normal to the surface and tangential as sketched in Fig. 1. The vector sum of these forces on the dipole is in the positive radial direction R opposite to net force on the toroid. In case of a force balanced system, the structure required to contain the net radial force F_R may be under tension or compression depending on which force is larger, the poloidal or the toroidal. The tension or compression bearing mass

$$M_R \text{ is } M_R = 2\pi R \rho |T|/\sigma \quad (6)$$

where T , the tension or compression in the structure wall, is

$$T = -\frac{1}{4\pi} \frac{\partial L}{\partial R} I^2 \quad (7)$$

Consequently from equations (4), (6), and (7)

$$M_R = |Q_R| \frac{\rho E}{\sigma} \quad (8)$$

where Q_R is

$$Q_R = -\left(1 - \frac{\beta}{f} \frac{\partial f}{\partial \beta}\right) \quad (9)$$

A negative Q is for structure under tension while a positive Q is for structure under compression.

B. Circumferential Forces

In the case of a torus, the forces point outward normal to the surface, thus requiring structure under tension. In the case of a dipole with a shielding surface current distribution⁶ that excludes the field inside the winding, forces point inward normal to the surface, thus requiring structure under compression. There are no tangential forces for a perfect shielding surface current distribution on a dipole surface. For other surface current distributions there are tangential forces which can be contained by the structure M_R .

In a force balanced system, the structure required to contain the circumferential forces may be under tension or compression depending on which force is larger, the toroidal or the poloidal.

In order to calculate the amount of structure required to contain these forces, we will use a virtual work model, Fig. 2, which allows the circumferential structure to expand by ΔS . The energy stored change ΔE is

$$\Delta E = - \int T \delta S, \quad (10)$$

where T is the tension or compression in the structure wall and $\delta S = \Delta r d\theta$. In order to conserve the configuration during the virtual expansion, Δr is

$$\Delta r = \frac{\Delta a}{a} r \quad (11)$$

Substituting Eq. (11) in (10) and taking the limit $\Delta r \rightarrow 0$

$$\int T r d\theta = a \frac{\partial E}{\partial a} \quad (12)$$

The tension or compression bearing mass is

$$M_a = \frac{\rho}{\sigma} \int |T| r d\theta = \left| \frac{\rho}{\sigma} a \frac{\partial E}{\partial a} \right| \quad (13)$$

Consequently from equations (13) and (4)

$$M_a = |Q_a| \frac{\rho E}{\sigma} \quad (14)$$

where

$$Q_a = -\frac{\beta}{f} \frac{\partial f}{\partial \beta} \quad (15)$$

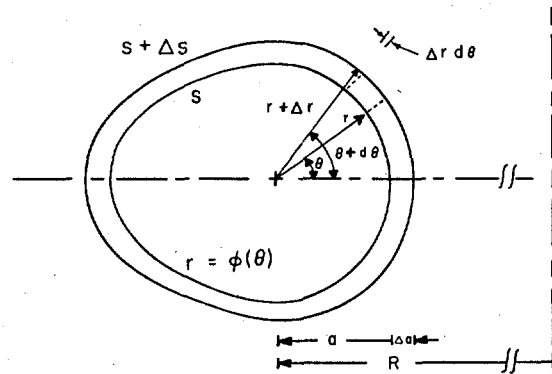


Fig. 2 A virtual work model for calculating the structure required to contain net magnetic forces normal to the magnet surface S .

The structure required to contain circumferential forces is given by equation (14) only if all the circumferential forces are pointing outward or inward. If circumferential forces are positive in some regions and negative in other regions then the structure requirement may exceed M_a in eq. (14).

From equations (9) and (15) it is impossible to cancel Q_R and Q_a simultaneously. The total structural mass required is

$$M = \frac{\rho E}{\sigma} \{ |Q_R| + |Q_a| \} \geq \frac{\rho E}{\sigma} \quad (16)$$

If $0 \leq \frac{\beta}{f} \frac{\partial f}{\partial \beta} < 1$, then both M_R and M_a are under tension and the sum of the magnitude of Q_R and Q_a is always equal to unity. For values of $\frac{\beta}{f} \frac{\partial f}{\partial \beta}$ outside this range one structure is in tension while the other is in compression. The sum of the magnitude of Q_R and Q_a is

$$|Q_R| + |Q_a| = 1 + 2 \left| \frac{\beta}{f} \frac{\partial f}{\partial \beta} \right|, \text{ for } \frac{\beta}{f} \frac{\partial f}{\partial \beta} \leq 0 \quad (17)$$

$$= 2 \frac{\beta}{f} \frac{\partial f}{\partial \beta} - 1, \text{ for } \frac{\beta}{f} \frac{\partial f}{\partial \beta} \geq 1 \quad (18)$$

Thus $|Q_R| + |Q_a|$ is always greater than one.

The condition $\frac{\beta}{f} \frac{\partial f}{\partial \beta} = 1$ exists when the net radial forces of the dipole and the torus cancel each other. The condition $\frac{\beta}{f} \frac{\partial f}{\partial \beta} = 0$ can be satisfied if the forces

Table I. Analytical Formulas Of f , Q_R , and Q_a For Simple Toroidal, Poloidal, and Solenoidal Configurations

	TOROID "Circular Cross-Section"	DIPOLE Uniform Surface Current "Circular Cross-Section"	SHORT SOLENOID $\beta < 0.3$ $\beta = \text{Height/Diameter}$
f	$1 - (1 - \beta^2)^{1/2}$	$(1 + \frac{\beta^2}{4}) \ln \frac{8}{\beta} - 2$ $\sim \ln \frac{8}{\beta} - 2$	$\sim \ln \frac{4}{\beta} - \frac{1}{2}$
Q_R	$(1 - \beta^2)^{-1/2}$	$\sim \frac{\ln \frac{8}{\beta} - 1}{\ln \frac{8}{\beta} - 2}$	$\sim \frac{\ln \frac{4}{\beta} + \frac{1}{2}}{\ln \frac{4}{\beta} - \frac{1}{2}}$
Q_a	$-(1 + (1 - \beta^2)^{-1/2})$	$\sim \frac{1}{\ln \frac{8}{\beta} - 2}$	$\sim \frac{1}{\ln \frac{4}{\beta} - \frac{1}{2}}$

Table II. Numerical Calculations of Structure Factors Q_R and Q_a as a Function of β for Toroidal and Poloidal Configurations with Circular Cross-Section.

β	TOROID		DIPOLE Uniform Surface Current		DIPOLE Shielding Surface Current	
	Q_R	Q_a	Q_R	Q_a	Q_R	Q_a
0.1	1.005	-2.005	-1.41	0.41	-1.44	0.44
0.2	1.02	-2.02	-1.54	0.54	-1.67	0.67
0.3	1.05	-2.05	-1.64	0.64	-1.94	0.94
0.4	1.09	-2.09	-1.72	0.72	-2.3	1.30
0.5	1.16	-2.16	-1.76	0.76	-2.79	1.79

normal to the surface of the dipole and the torus cancel each other everywhere on the surface. The last condition requires special current distribution on the surface of the dipole. Such current distribution will produce forces tangential to the dipole surface which will require structure in tension $M_R = \rho E / \sigma$.

It can be shown that the condition $0 \leq \frac{\beta}{f} \frac{\partial f}{\partial \beta} \leq 1$ is equivalent to:

$$\frac{|Q_a^T|}{|Q_a^D|} \geq \frac{E_D}{E_T} \geq \frac{|Q_R^T|}{|Q_R^D|} \quad (19)$$

where D and T correspond to dipole and toroid. The energies stored in the toroidal field and the poloidal field are E_D and E_T .

Tables I and II are lists of expressions and numerical values of f , Q_a , and Q_R for simple toroids and dipoles. It should be noticed from equations (17) and (18) and from Tables I and II that equation (2) derived by Levy is always satisfied. It also may be concluded that for any configuration, there will be no shielding dipole surface current that will give forces opposite and equal to forces resulting from toroidal current flowing on the same surface.

Accordingly, for magnetic energy E confined by a structural material, the mass of structure can not be less than that indicated by Eq. (16), which agrees exactly with the Virial Theorem. There is no apparent benefit in reduced structure by going to a complicated force balanced double magnet system of a toroid and a dipole.

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