Technical Notes and Correspondence

Existence and Computation of Infinite Horizon Model Predictive Control With Active Steady-State Input Constraints

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***Abstract*—This note addresses the existence and implementation of the infinite-horizon controller for the case of active steady-state input constraints. This case is important because, in many practical applications, controllers are required to operate at the boundary of the feasible region (for instance, in order to maximize global economic objectives). For this case, the usual finite horizon parameterizations with terminal cost cannot be applied, and optimal solutions are not generally available. We propose here an iterative algorithm that generates two finite-horizon approximations to the true infinite-horizon problem, where the solution to one of the approximations yields an upper bound on the true optimum, while the other approximation yields a lower bound. We show convergence of both bounding approximations to the optimal solution, as the horizon length in the approximations is increased. We outline a procedure, based on this result, to provide a solution to the infinite-horizon problem that is exact to within any user-specified tolerance. Finally, we present an example that includes a comparison between optimal and suboptimal controllers.**

***Index Terms*—Model predictive control (MPC), optimal control, steady-state constraints.**

I. Introduction

Model predictive control (MPC) is a technique in which a process model is used to forecast future process behavior, and the sequence of future control inputs is computed as the solution to an open-loop optimization problem. The first element of the optimal input sequence is used as the process input, and the remaining elements of the input sequence are discarded. The optimization procedure is repeated at each sampling time. Feedback from measurements is considered by correcting the model prediction, based on the error between the measurement and prediction. Many methods are available for this correction. Several recent reviews [1]–[3] summarize the theoretical formulations and industrial implementations of MPC.

In this note, we consider the infinite horizon formulation of model predictive control (IHMPC) and address the case of constraints that are active at steady state. Process constraints arise both from physical limitations (for example, a valve can be at maximum fully open and at minimum totally closed) and from safety and performance specifications. Most papers on constrained infinite horizon MPC rely on the assumption that the origin is in the interior of the feasible region [4]–[6].

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It is frequently the case, however, that in order to maximize performance objectives, the MPC controller operates at the boundary of the feasible region with respect to both input and output constraints. Moreover, when a nonzero disturbance enters the process, it is often the case that one or more manipulated inputs ride at their corresponding saturation values during a period of steady-state operation. These cases give rise to problem formulations in which the origin lies on the boundary of the feasible region. This situation was treated in [7], where a suboptimal solution for this problem is given. The main contribution of this note is to provide an algorithm for finding upper and lower bounds on the optimum of the constrained infinite horizon optimization problem, and a procedure for identifying the optimal solution to any given level of accuracy.

Several authors have addressed the solution of convex optimization problems on an infinite-dimensional space via finite-dimensional approximations; some, like us, specialize their analysis to quadratic programs from control. A lower bound approximation is derived in [8] in a general setting. Convergence of finite approximation schemes for problems related to control are discussed in [9] and [10], but these works make key assumptions on the properties of the functions and solutions that are not satisfied in our case, and the second presents a stopping rule that is difficult to implement.

II. Formulation of the Problem

In this note, we consider linear, time-invariant, discrete systems described by

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + B_d d_k \\ d_{k+1} &= d_k \\ y_k &= Cx_k + C_d d_k \end{aligned} \tag{1}$$

where

$x \in \mathbb{R}^n$	state;
$u \in \mathbb{R}^m$	input;
$y \in \mathbb{R}^p$	output;
$d \in \mathbb{R}^p$	integrated disturbance added for offset-free purpose;
A, B, B_d, C, and C_d	matrices of appropriate dimensions.

It is assumed that the pair (A, B) is stabilizable, the pair (C, A) is detectable, and (B_d, C_d) are such that the augmented system (1) is detectable. We assume that, at each sampling time k, a state estimator designed for (1) (e.g., Kalman filter) provides estimate of the state $\hat{x}_{k|k}$ and of the disturbance $\hat{d}_{k|k}$.

Given the current disturbance estimate $\hat{d}_{k|k}$, we solve the following target calculation problem [7]:

$$\min_{x_{s,k},\, u_{s,k},\, y_{s,k},\, \eta} \frac{1}{2}\left\{\eta^T Q_s \eta + (u_{s,k} - \overline{u})^T R_s (u_{s,k} - \overline{u})\right\} + q_s^T \eta \tag{2}$$

subject to

$$\begin{aligned} x_{s,k} &= Ax_{s,k} + Bu_{s,k} + B_d \hat{d}_{k|k} \\ y_{s,k} &= Cx_{s,k} + C_d \hat{d}_{k|k} \\ \overline{y} - \eta \le y_{s,k} &\le \overline{y} + \eta, \qquad \eta \ge 0 \\ u_{\min} \le u_{s,k} &\le u_{\max}, \qquad y_{\min} \le y_{s,k} \le y_{\max} \end{aligned}$$

in which Q_s and R_s are positive–definite matrices and $\overline{y}$ and $\overline{u}$ are the setpoints for the output and input, respectively. The target optimization handles the case in which the setpoints $\overline{y}$ and $\overline{u}$ may come from a plant-wide optimization using a model and disturbance different from those in (2). The target problem provides a steady state for the model in (2) close to the setpoint, which serves as the origin for the deviation variables used subsequently in (4). We assume that the strict inequalities $y_{\min} < y_{\max}$, $u_{\min} < u_{\max}$ are satisfied. For appropriate choices of q_s, the linear penalty $q_s^T \eta$ guarantees that the output slack variable η is zero whenever it is possible to use this value without violating feasibility; that is, whenever the target value $\overline{y}$ is a feasible choice for $y_{s,k}$.

Next, we pose the following infinite horizon optimization problem to compute the control action:

$$\min_{\{x_{k+j}, u_{k+j}, y_{k+j}\}_{j=0}^{\infty}} \frac{1}{2}\sum_{j=0}^{\infty}(y_{k+j}-y_{s,k})^T Q (y_{k+j}-y_{s,k}) + (u_{k+j}-u_{s,k})^T R (u_{k+j}-u_{s,k}) \quad \text{(3a)}$$

subject to

$$\hat{x}_{k+j+1|k} = A\hat{x}_{k+j|k} + Bu_{k+j} + B_d\hat{d}_{k|k}, \quad j = 0, 1, 2, \ldots \quad \text{(3b)}$$

$$y_{k+j} = C\hat{x}_{k+j|k} + C_d\hat{d}_{k|k}, \quad j = 0, 1, 2, \ldots \quad \text{(3c)}$$

$$u_{\min} \le u_{k+j} \le u_{\max}, \quad j = 0, 1, 2 \ldots \quad \text{(3d)}$$

$$y_{\min} \le y_{k+j} \le y_{\max}, \quad j = 0, 1, 2, \ldots. \quad \text{(3e)}$$

We assume that Q and R are positive definite matrices of appropriate dimension. This optimization problem can be rewritten in a more convenient way by considering the following new deviation variables:

$$w_j = \hat{x}_{k+j|k} - x_{s,k} \quad v_j = u_{k+j} - u_{s,k} \quad Q \leftarrow C^T Q C$$

$$D = \begin{bmatrix} I \\ -I \end{bmatrix} \quad d = \begin{bmatrix} u_{\max} - u_{s,k} \\ -u_{\min} + u_{s,k} \end{bmatrix}$$

$$E = \begin{bmatrix} C \\ -C \end{bmatrix} \quad e = \begin{bmatrix} y_{\max} - y_{s,k} \\ -y_{\min} + y_{s,k} \end{bmatrix}.$$

Notice that from (2), we have that $d \ge 0$ and $e \ge 0$. Thus, (3) becomes

$$\mathcal{O}(N)\colon \min_{\{w_j, v_j\}_{j=0}^{\infty}} \frac{1}{2}\sum_{j=0}^{\infty} w_j^T Q w_j + v_j^T R v_j \quad \text{(4a)}$$

subject to

$$w_0 = w^{\text{init}} \quad w_{j+1} = Aw_j + Bv_j, \qquad j = 0, 1, 2, \ldots \quad \text{(4b)}$$

$$Dv_j \le d, \qquad j = 0, 1, 2, \ldots \quad \text{(4c)}$$

$$Ew_j \le e, \qquad j = 0, 1, 2, \ldots. \quad \text{(4d)}$$

Problem (4) may be infeasible due to the presence of input and state constraints. For example, a disturbance may enter the plant and cause the current state w^{init} to leave the set of admissible initial conditions. The problem of transient infeasibility for MPC caused by inconsistent state constraints has been addressed in several ways [11]–[13]. Any of these state constraint softening approaches can be incorporated into the methodology proposed here. Therefore, we can assume that for the given w^{init}, a sequence $\{v_j, w_j\}_{j=0}^{\infty}$ exists that is feasible with respect to constraints (4b), (4c), and (4d), and gives a finite value of the objective function in (4). This assumption is commonly referred to as *constrained stabilizability*. It is important to note that the pair $(Q^{1/2}, A)$ is detectable, because (C, A) is detectable and the "original" Q in (3) is positive definite. Because $(Q^{1/2}, A)$ is detectable, unstable modes cannot evolve without affecting the objective function. This condition and the fact that R is positive definite imply that for a feasible sequence $\{(w_j, v_j)\}_{j=0}^{\infty}$ in (4), we have $w_j, v_j \to 0$ as $j \to \infty$ [6]. In fact, positive definiteness of R implies that the aggregated input vectors $(v_0, v_1, v_2, \ldots)$ belongs to the space ℓ^2, which is the set of infinite-dimensional real vectors $(z_1, z_2, z_3, \ldots)$ for which $\sum_{j=1}^{\infty} z_j^2 < \infty$.

When all components of d and e are strictly positive, the origin is in the interior of the feasible region, and algorithms for solving (4) are available [5], [6]. If, on the other hand, state constraints (4d) are active at steady state (i.e., some components of e are zero), then arbitrarily small constant disturbances can render the hard constrained problem infeasible, which means that there is no feasible sequence that brings the system to the origin without persistently violating the active state constraints. This condition should register a process exception and possibly shut down the process. We assume, therefore, that state constraints are not active at steady state; that is, $e > 0$. Finally, we consider the case in which the input constraints *are* active at steady state; that is, some components of d are zero (equivalently, some components of $u_{s,k}$ equal their lower or upper bound). This case is often encountered in practice; for example, if the setpoint $\overline{y}$ is unreachable, the target problem produces $u_{s,k}$ at a bound. We consider this case in the remainder of the note.

III. Optimal Solution of the Infinite Horizon Problem

In this section, we present a method for finding the solution of problem (4) to arbitrary accuracy. Our approach is to construct two finite-horizon problems that approximate (4) and for which solutions can be calculated. One of these approximations has an optimal objective value that is an upper bound for the optimal objective of (4), while the other approximation yields a lower bound. We show that the two bounds approach each other as the horizon length N approaches infinity, and use the difference between the bounds to estimate the difference between the solution of the approximating problems and the solution of (4). An added benefit of our analysis is that it proves existence of the optimal solution to problem (4) whenever a feasible sequence exists.

A. Upper Bound on the Optimal Solution

An upper bound on the optimal objective Φ^* of (4) can be computed by using the method proposed in [7]. In this approach, a suboptimal solution to (4) is found by restricting the evolution of the input and state trajectories to the null space of the active steady-state constraints, after the finite horizon $N > 0$. This solution is found by minimizing the following infinite horizon objective function:

$$\mathcal{U}(N)\colon \min_{\{w_j, v_j\}_{j=0}^{\infty}} \frac{1}{2}\sum_{j=0}^{\infty} w_j^T Q w_j + v_j^T R v_j \quad \text{(5a)}$$

subject to: (4b), (4c), (4d), and (5b)

$$\overline{D}v_j = 0, \qquad j = N, N+1, \ldots \quad \text{(5c)}$$

in which $\overline{D}$ denotes the row submatrix of D corresponding to input inequality constraints active at steady state, that is the rows of D for which the corresponding elements of d are zero. Let $\overline{\Phi}_N$ be the optimal objective value for $\mathcal{U}(N)$ in (5). Since (5) has more constraints than (4), its feasible region is no larger, so we have

$$\Phi^* \le \overline{\Phi}_N. \quad \text{(6)}$$

We can reformulate the infinite horizon problem (5) as a finite-horizon problem (for sufficiently large N) that can be solved by practical means as follows:

$$\min_{\{w_j\}_{j=0}^{N}, \{v_j\}_{j=0}^{N-1}} \frac{1}{2} \sum_{j=0}^{N-1} \left\{ w_j^T Q w_j + v_j^T R v_j \right\} + \frac{1}{2} w_N^T \overline{\Pi} w_N \quad \text{(7a)}$$

subject to

$$w_0 = w^{\text{init}} \quad w_{j+1} = Aw_j + Bv_j, \quad j = 0, 1, 2, \cdots, N-1 \quad \text{(7b)}$$

$$Dv_j \le d, \qquad j = 0, 1, 2, \ldots N-1 \quad \text{(7c)}$$

$$Ew_j \le e, \qquad j = 0, 1, 2, \ldots N \quad \text{(7d)}$$

in which the cost-to-go matrix $\overline{\Pi}$ is associated with the unconstrained control law

$$v_j = \overline{K} w_j. \quad \text{(8)}$$

The offline computation of $\overline{K}$ is described in [7]; we outline the procedure here. Let $\mathcal{N}_{\overline{D}}$ be an orthonormal basis for the null space of $\overline{D}$, so that vectors v_j that satisfy (5c) have the form $\mathcal{N}_{\overline{D}} p_j$, $j = N, N+1, \ldots$ for arbitrary p_j. We then solve the optimal unconstrained LQR problem for the system with characteristic matrices $(A, B\mathcal{N}_{\overline{D}})$ and with $(Q, \mathcal{N}_{\overline{D}}^T R \mathcal{N}_{\overline{D}})$ as state and input penalty, respectively, to obtain optimal gain and cost-to-go matrices $K_{\overline{D}}$ and $\overline{\Pi}$, respectively. It follows that $\overline{K} = \mathcal{N}_{\overline{D}} K_{\overline{D}}$. In order for such a linear control law to exist, the pair $(A, B\mathcal{N}_{\overline{D}})$ must be stabilizable. When this condition is not satisfied, we require the controller to zero, at the end of the finite horizon N, the unstable modes that are not controllable in this subspace.

We have the following existence result.

Theorem 1: If the optimization problem (4) is feasible, there exists a finite integer N' such that (5) is feasible for any $N \ge N'$.

Proof: See [14, App. A.1]. ■

Note that the solution obtained from (7) and (8) yields a valid solution for (5) [and, hence, a valid upper bound for (4)] only if the terminal state w_N from (7) belongs to the output admissible set of the system with characteristic matrices $(A, B\mathcal{N}_{\overline{D}})$ and with the inactive constraints only [7]. That is, we need to ensure that the sequence $\{v_j, w_j\}_{j=N}^{\infty}$ generated by (4b) together with (8) satisfies the inactive constraints from (4c), (4d) at all subsequent stages $j = N, N+1, \ldots$.

B. Lower Bound on the Optimal Solution

A lower bound on the optimal objective Φ^* of (4) can be found by minimizing the following infinite horizon objective function:

$$\mathcal{L}(N): \min_{\{w_j, v_j\}_{j=0}^{\infty}} \frac{1}{2} \sum_{j=0}^{\infty} w_j^T Q w_j + v_j^T R v_j \quad \text{(9a)}$$

subject to: (4b), (7c), (7d). (9b)

Notice that constraints (7c) and (7d) are enforced over a finite horizon N only. Therefore, if the optimal problem (4) [equivalently, the upper bounding problem (5)] is feasible, we have that (9) also is feasible.

Let $\underline{\Phi}_N$ be the optimal objective value for $\mathcal{L}(N)$. Since (9) has fewer constraints than (4), we have

$$\underline{\Phi}_N \le \Phi^* \qquad \forall N > 0. \quad \text{(10)}$$

The infinite horizon problem (9) can be solved by using a finite parameterization, since after stage N the optimal control law is $v_j = Kw_j$ in which K is the well-known unconstrained LQR gain computed from the corresponding Riccati equation. Thus, we solve the following problem:

$$\min_{\{w_j\}_{j=0}^{N}, \{v_j\}_{j=0}^{N-1}} \frac{1}{2} \sum_{j=0}^{N-1} \left\{ w_j^T Q w_j + v_j^T R v_j \right\} + \frac{1}{2} w_N^T \Pi w_N \quad \text{(11a)}$$

subject to: (7b), (7c), (7d) (11b)

in which Π is the steady-state solution of the Riccati equation. Note that since the constraints (7c) and (7d) are imposed only over a finite horizon, the solution of (11) may violate input and state constraints (4c) and (4d) at some stages $j > N$.

C. Convergence of the Optimal Sequences

The results of this section are proved in [14, App. A.2]. Our analysis relies on the fact that all aggregated (infinite-dimensional) input vectors $(v_0, v_1, v_2, \ldots)$ of interest lie in the space ℓ_2, which is a separable Hilbert space when equipped with the obvious inner product. In particular, in the notation of the results below, we have that z^*, $\overline{z}_N$, and $\underline{z}_N$ all belong to ℓ^2.

Theorem 2: Let z^* and $\overline{z}_N$ be the optimal infinite-dimensional input sequences solution to the optimal problem (4) and to the upper bounding problem (5), respectively. We have

$$\lim_{N\to\infty} \overline{z}_N = z^* \qquad \overline{\Phi}_N \downarrow \Phi^* \quad \text{(12)}$$

where the last limit indicates that $\{\overline{\Phi}_N\}$ is nonincreasing and converges to Φ^*.

Theorem 3: Let z^* and $\underline{z}_N$ be the optimal infinite-dimensional input sequences solution to the optimal problem (4) and to the lower bounding problem (9), respectively

$$\lim_{N\to\infty} \underline{z}_N = z^* \qquad \underline{\Phi}_N \uparrow \Phi^* \quad \text{(13)}$$

where the last limit indicates that $\{\underline{\Phi}_N\}$ is nondecreasing and converges to Φ^*.

Theorem 4: Let v_0^* and $\overline{v}_{N,0}$ be the first input component of the solution to the optimal problem (4) and to the upper bounding problem (5), respectively. There exists a positive scalar α such that for any N the following inequality holds:

$$\|\overline{v}_{N,0} - v_0^*\| \le \|\overline{z}_N - z^*\| \le \left[\frac{2}{\alpha} \left(\overline{\Phi}_N - \underline{\Phi}_N \right) \right]^{1/2}. \quad \text{(14)}$$

The scalar α depends on the problem data and in particular on the matrix R, as we show at the end of the next section.

IV. Implementation Issues

The results of the previous section suggest an iterative approach to determining an approximation to the v_0^* component of the solution of the infinite horizon problem (4). In this approach, we solve a series of quadratic programs for the upper and the lower bound problems (5) and (9). If the difference between the optimal objective values for these problems does not satisfy a chosen stopping criterion, the horizon is increased; otherwise the first input $\overline{v}_{N,0}$ of the computed sequence of the upper bound problem (5) is accepted as a good approximation to $v_0 **$, and is injected into the plant.

As stopping criterion we use a relative difference between the upper and the lower bound solution

$$\frac{\overline{\Phi}_N - \underline{\Phi}_N}{1 + \underline{\Phi}_N} \leq \rho \tag{15}$$

where ρ is a small positive number.

At each sampling time we apply the following algorithm.

Algorithm 1 Start with a positive horizon $N > 0$.

1) Solve (7). If the problem is infeasible, go to 5). Otherwise, let $\overline{\Phi}_N$ be the optimal value of its objective function.
2) If the final state w_N does not belong to the output admissible set for constraints inactive at steady state [7], go to 5).
3) Solve (11). Let $\underline{\Phi}_N$ be the optimal value of its objective function.
4) Check (15). If satisfied, go to 6).
5) Increase the horizon N and go to 1).
6) Set v_0 equal to the first solution component of (7).

The proposed algorithm always terminates because from (13) and (12) we have that

$$\lim_{N \to \infty} \overline{\Phi}_N - \underline{\Phi}_N = 0 \tag{16}$$

which implies that for any $\rho > 0$ there exists a N' such that for $N > N'$ the stopping criterion (15) is satisfied.

In (14), the monotonicity constant α appears in the denominator, so the bound is tight when α is large. It is straightforward to show that $\alpha \geq \lambda_{\min}(R) > 0$, in which we used the fact that R is positive definite.

V. Case Study

As example we consider the heavy oil fractionator of the Shell Control Problem [15]. A linear model of the system is

$$G(s) = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} & \frac{6.90e^{-15s}}{40s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.20}{19s+1} \end{bmatrix}. \tag{17}$$

The following input and output constraints are considered:

$$-0.5 \leq u_i \leq 0.5, \qquad i = 1, 2, 3$$
$$-0.5 \leq y_1 \leq 0.5, \qquad -0.5 \leq y_3.$$

The controller tuning matrices are $Q = R = I$. We consider a setpoint: $\overline{y} = [0.3 \;\; 0.3 \;\; -0.3]^T$, which is not reachable due to input constraints. The target calculations return (with $Q_s = R_s = I$, $q_s^T = 10^6 \cdot \mathbf{1}$) the following feasible targets for input and output:

$$u_{s,k} = \begin{bmatrix} 0.5 \\ -0.1026 \\ -0.2702 \end{bmatrix} \quad y_{s,k} = \begin{bmatrix} 0.2540 \\ 0.2436 \\ -0.2086 \end{bmatrix}, \quad k = 0, 1, 2, \ldots. \tag{18}$$

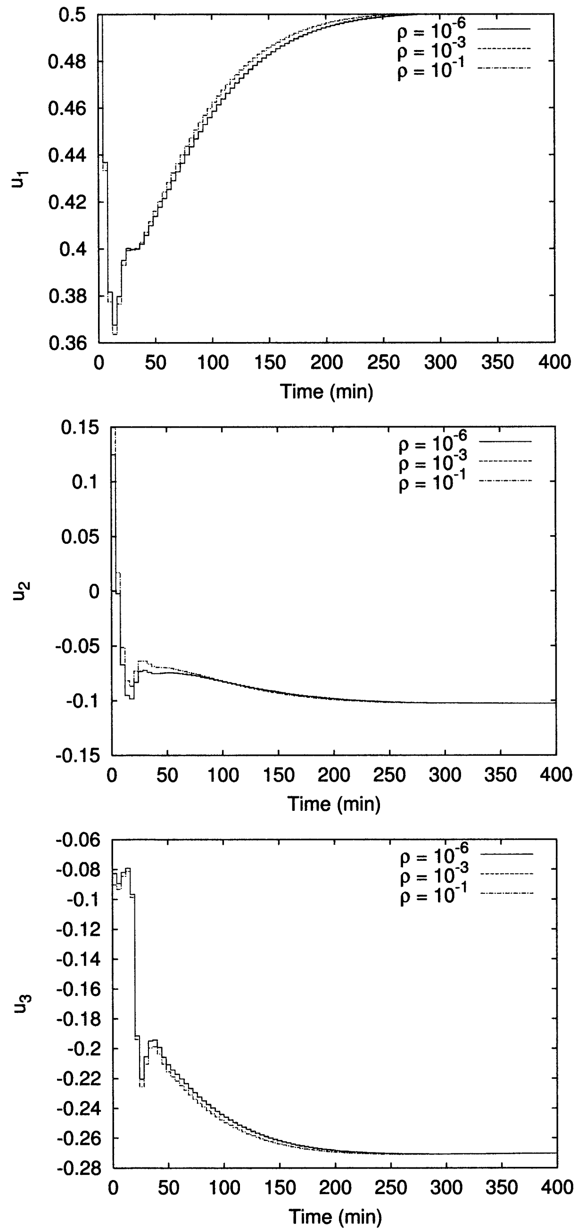

Fig. 1. Closed-loop inputs.

Because the setpoint is unreachable, the first component of $u_{s,k}$ equals its upper bound, and the origin lies on the boundary of the feasible region.

In Figs. 1 and 2, we report inputs and outputs for three optimal regulators obtained with different relative tolerance ($\rho = 10^{-6}$, 10^{-3}, 10^{-1}, respectively). The regulators with relative tolerance of 10^{-6} and 10^{-3} show essentially the same closed-loop response. Also, the regulator with relative tolerance of 10^{-1} guarantees a performance not too far from optimal. Clearly, the use of a larger tolerance has a direct impact on the computation time because the horizon length N required to satisfy the stopping criterion decreases. We show in Fig. 3 the horizon required by each controller to meet the specified convergence tolerance, at each point in the simulation. It is interesting to notice that the required horizon is larger at the beginning of the horizon because the system state is far away from its steady state, and then it decreases.

For this particular example the sampling time is 4 min [15], while the average computation time required for the controller with $\rho = 10^{-6}$

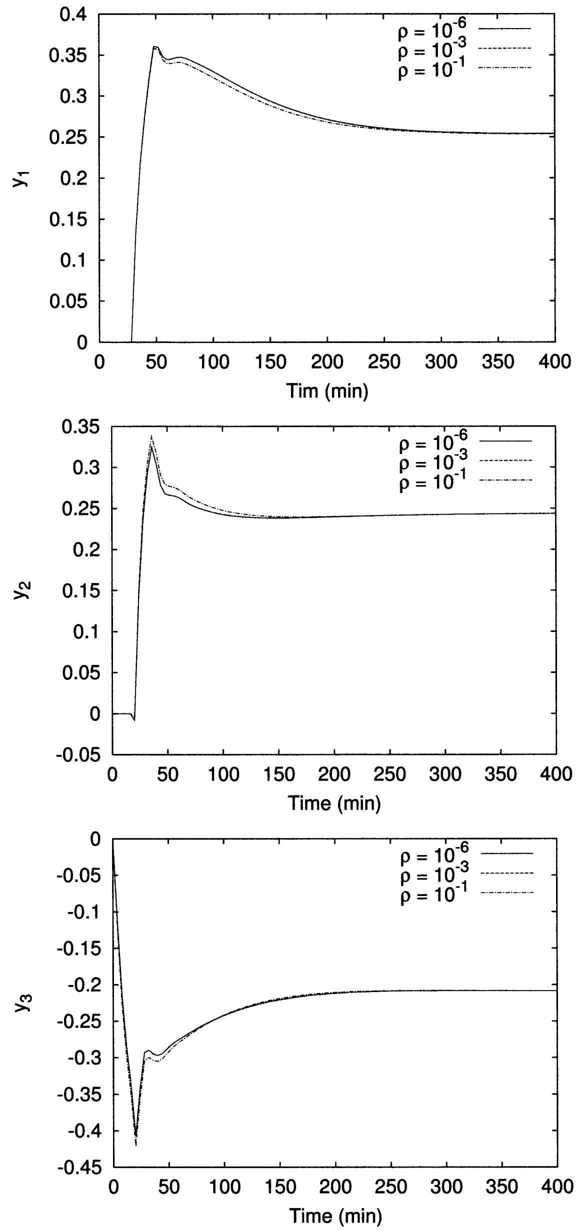

Fig. 2. Closed-loop outputs.

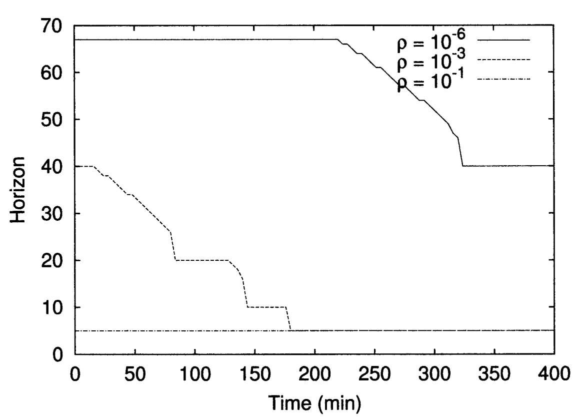

Fig. 3. Closed-loop required horizon.

is about 4 s on a 1.2-GHz Athlon computer and using a dense Hessian approach, which scales cubically with the horizon length. In fact, interior-point structured solvers for the MPC problem that scale linearly with the horizon are available and can be directly applied to this algorithm.

VI. Conclusion

In this note, the existence and the implementation of the infinite horizon controller for the case of active steady-state input constraints has been discussed. This case is important because, in practical applications, controllers are often required to operate at the boundary of the feasible region. Previously, only suboptimal solutions were available for this case, based on finite horizon formulations with terminal equality constraints or infinite horizon formulations with appropriate suboptimal finite parameterization. We presented here an iterative algorithm that determines the optimal solution of this problem within a user specified tolerance. Availability of a near-optimal solution makes the proposed controller simple to understand and tune, and improves its performance. Finally, when the computation time is a limiting factor we can still apply this algorithm with a larger tolerance (shorter horizon) and obtain a bound on the difference between the optimal and the computed suboptimal input and cost.

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