

POWER SPECTRUM OF A SIGMA-DELTA MODULATOR WITH HEXAGONAL VECTOR QUANTIZATION AND CONSTANT INPUT

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ABSTRACT

A new application of sigma-delta modulation has emerged in soft switching three phase power electronic converters. For this application, a conventional sigma-delta modulator with scalar signals and binary quantizer is generalized to a sigma-delta modulator with vector signals and a truncated hexagonal lattice quantizer. This vector sigma-delta modulator has an interesting output spectrum which is of importance in assessing performance and interpreting results. This paper analytically derives the output spectrum for the case of a generic constant input using ergodic theory and Fourier series on the hexagon.

1. INTRODUCTION

Three phase soft switching converters are of current interest in power electronics. In particular, the resonant link converter uses zero voltage switching to limit switching losses and attain relatively high switching frequencies [1]. Conventional modulation techniques such as PWM are not applicable to resonant link converters because of the discrete timing of the switchings. However, the sigma-delta ($\Sigma\Delta$) architecture is well suited to modulation of the resonant link converter. Indeed switching converters can be viewed as analog to digital converters in which an analog reference is coded into a low resolution set of discrete switching states [2]. The relatively high switching frequency of resonant link converters corresponds to a high oversampling ratio with typical values of 20–50. Experimental and simulation results for first and second order $\Sigma\Delta$ hexagonal modulators in a resonant dc link system switching at 100kHz with a bandwidth of 2kHz are presented in [2].

To apply the conventional $\Sigma\Delta$ architecture with binary quantization and a single feedback loop to three phase converters requires some generalization. First, the output voltages (line-to-neutral) of a three phase converter with balanced load are limited to a set of seven output vectors (dots

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in Fig. 2) which form a truncated hexagonal lattice. If we assume a nearest neighbor partition as in the binary quantizer case, the appropriate generalization is the truncated hexagonal lattice quantizer which is well known in communications [3]. Second, all modulator signals are augmented from scalar quantities to vectors and the scalar integrator is replaced by a vector integrator. This generalized modulator is inherently two dimensional and cannot be simply analyzed as the Cartesian product of two scalar modulators.

The output spectrum of a scalar $\Sigma\Delta$ modulator is complicated and it is useful in interpreting results to analytically derive the output spectrum. (The approximate white noise model of quantization error is useful in design but can be misleading for the single-loop case [4].) Powerful techniques from ergodic theory have been deployed by Gray [5, 4], Delchamps [6, 7], and He et al. [8] to derive exact formulas for the spectra of scalar modulators for various inputs. In this paper, we build on their methods to derive the spectrum of a vector $\Sigma\Delta$ modulator with a hexagonal quantizer and a generic constant input.

2. HEXAGONAL LATTICES

It is convenient to define the hexagonal lattice in the plane $\mathcal{P} = \{(x, y, z)^t \in \mathbb{R}^3 \mid x+y+z = 0\}$. Define $W : \mathbb{R}^2 \rightarrow \mathcal{P}$ and $V : \mathcal{P} \rightarrow \mathbb{R}^2$ to be

$$W = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & -1 \end{bmatrix}, \quad V = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix} \quad (1)$$

VW is the identity on \mathbb{R}^2 and WV is the identity on \mathcal{P} .

The hexagonal lattice is $\Lambda = \bigcup\{Wk \mid k \in \mathbb{Z}^2\}$. The lattice dual [3] to Λ is

$$\Lambda^* = \{p \in \mathcal{P}^* \mid p\lambda \in \mathbb{Z} \text{ for all } \lambda \in \Lambda\} = \bigcup\{qV \mid q \in \mathbb{Z}^{*2}\}$$

Vectors in \mathcal{P} or Λ are written as column vectors and dual vectors in \mathcal{P}^* or Λ^* are written as row vectors. For example, $n_a \in \mathcal{P}$ in Fig. 2 is the column vector $\frac{1}{3}(2, -1, -1)^t$. The columns of W generate Λ and the rows of V generate Λ^* .

The Voronoi cells (points closest to a given lattice point) of Λ are hexagons of side $\sqrt{2/3}$. Define the set H to be the

interior of the Voronoi cell containing 0, together with a specific choice of 3 non-opposite hexagon sides and 2 opposite hexagon vertices. (These choices ensure that lattice translates of H tile the plane with no overlapping points.) H is the dark central hexagonal region of Fig. 2.

3. DISCRETE-TIME MODEL

A discrete time hexagonal $\Sigma\Delta$ modulator is shown in Fig. 1; the signals β , u_n , q_n , and e_n are vectors in the plane $\mathcal{P} \subset \mathbb{R}^3$. The nearest neighbor quantizer q is shown in Fig. 2. The input u to q is a point in the plane \mathcal{P} and the output $q(u)$ is the nearest to u of the 7 truncated hexagonal lattice points $\{0, \pm n_a^\perp, \pm n_b^\perp, \pm n_c^\perp\}$ in Fig. 2.

From Fig. 1, one can write the following difference equations in discrete time $n = 0, 1, 2, \dots$:

$$u_{n+1} = \beta + u_n - q(u_n) \quad (2)$$

where β is the constant input, u_n is the modulator state, and $q_n = q(u_n)$ is the quantizer output at time n .

The quantizer error sequence is

$$e_n = e(u_n) = u_n - q(u_n) \quad (3)$$

and the integrator state u_n and quantizer output q_n can be expressed in terms of e_n :

$$u_{n+1} = e_n + \beta \quad (4)$$

$$q_{n+1} = e_n - e_{n+1} + \beta \quad (5)$$

Rewriting (2) in terms of e_n yields the nonlinear equations:

$$\begin{aligned} e_0 &= e(u_0) \\ e_{n+1} &= e(e_n + \beta), \quad n = 0, 1, 2, \dots \end{aligned} \quad (6)$$

4. SOLUTION OF THE DIFFERENCE EQUATION

Our analysis requires the modulator state u_n , $n = 0, 1, 2, \dots$ to be contained in the no overload region R of the quantizer. R is the shaded region of Fig. 2 consisting of the 7 hexagons closest to zero. S is the star shaped shaded region of Fig. 2. The following sufficient condition for no overload can be shown by induction: If $\beta \in S$ and $e_0 \in H$, then $e_n \in H$ and $u_n \in R$ for all $n = 0, 1, 2, \dots$. We assume $\beta \in S$ and $e_0 \in H$ and hence no overload for the rest of the paper.

A function f on \mathcal{P} is *hexagonally periodic* if $f(x+\lambda) = f(x)$ for all $\lambda \in \Lambda$. Define $\square : \mathcal{P} \rightarrow H$ as the identity on H and extend the definition of \square to \mathcal{P} by making \square hexagonally periodic. Let $x, y \in \mathcal{P}$. Then

$$\square(x+y) = \square(\square(x) + y) \quad (7)$$

Since the function $e(u) = u - q(u)$ coincides with \square on the no overload region R , the no overload assumption implies that the difference equation (6) can be written as

$$e_{n+1} = \square(e_n + \beta) \quad (8)$$

Property (7) can be used to check that the solution to (8) is

$$e_n = \square(e_0 + n\beta) \quad (9)$$

5. FOURIER AND ERGODIC RESULTS

We state results about Fourier analysis and ergodic shifts. Let $f, g : \mathcal{P} \rightarrow \mathbb{C}$ be hexagonally periodic and Lebesgue square integrable on H . Then $f(x) = \sum_{p \in \Lambda^*} \hat{f}_p e^{i2\pi p x}$ where the equality is interpreted in the \mathcal{L}^2 sense and the Fourier coefficients are $\hat{f}_p = \frac{1}{|H|} \int_H f(s) e^{-i2\pi p s} ds$. $|H| = \sqrt{3}$ is the area of H . Parseval's formula is

$$\frac{1}{|H|} \int_H f(s) g^*(s) ds = \sum_{p \in \Lambda^*} \hat{f}_p \hat{g}_p^* \quad (10)$$

These Fourier results can be obtained either as sketched in appendix A or as a particular case of harmonic analysis on compact Abelian groups [9].

Identify points of \mathcal{P} differing by vectors in Λ to define $\mathcal{H} = \mathcal{P}/\Lambda = \{x + \Lambda \mid x \in \mathcal{P}\}$. \mathcal{H} is a compact Abelian group. A function $f : H \rightarrow \mathbb{C}$ lifts to a function $\tilde{f} : \mathcal{H} \rightarrow \mathbb{C}$ if $\tilde{f}(x + \Lambda) = f(x)$ for all x . Lemma 1 gives conditions on the input β for the dynamics (9) to induce a uniquely ergodic shift on \mathcal{H} so that time averages of a function f can be evaluated as an integral over H :

Lemma 1. *Let $\beta \in \mathcal{P}$ be such that the only $p \in \Lambda^*$ with $p\beta \in \mathbb{Z}$ is $p = 0$. Let $f : H \rightarrow \mathbb{R}$ have a continuous lifting $\tilde{f} : \mathcal{H} \rightarrow \mathbb{R}$. Then for all $e_0 \in \mathcal{P}$,*

$$\lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} f(\square(e_0 + n\beta)) = \frac{1}{|H|} \int_H f(s) ds \quad (11)$$

Corollary 1. *The result of Lemma 1 extends to functions $f : H \rightarrow \mathbb{R}$ for which there exist sequences of functions $\tilde{f}_k, \underline{f}_k, k = 1, 2, \dots$ with continuous liftings such that $\underline{f}_k(x) \leq f(x) \leq \tilde{f}_k(x)$ for all x and $|\underline{f}_k - \tilde{f}_k|_\infty \rightarrow 0$ as $k \rightarrow \infty$.*

Lemma 1 can be obtained from standard results on the torus [10] as indicated in appendix A.

6. SPECTRAL ANALYSIS

We make the generic assumption that the input β satisfies the condition of Lemma 1. (If, exceptionally, β does not satisfy the condition of Lemma 1 then either e_n is periodic or e_n is aperiodic and constrained to a submanifold of H .)

The mean of the noise process is, using (9),

$$\bar{e} = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} e_n = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} \square(e_0 + n\beta)$$

Applying Lemma 1 to each component of the identity map on H yields

$$\bar{e} = \frac{1}{|H|} \int_H s ds = 0$$

so that \bar{e} is well defined and independent of e_0 .

The autocorrelation matrix of the noise process is

$$R_e(k) = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} e_n \otimes e_{n+k}$$

where \otimes denotes outer product. Using (9) and (7) gives

$$R_e(k) = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} \overline{e_0 + n\beta} \otimes \overline{e_0 + n\beta + k\beta}$$

Let $f: H \rightarrow \mathcal{P} \times \mathcal{P}$ be defined by $f(s) = s \otimes \overline{s + k\beta}$. Each component of f is continuous except on several line segments and satisfies the conditions of Corollary 1. Hence

$$R_e(k) = \frac{1}{|H|} \int_H \overline{s} \otimes \overline{s + k\beta} ds \quad (12)$$

Partition $\Lambda^* = \Lambda^t \cup (n_a^t + \Lambda^t) \cup (-n_a^t + \Lambda^t)$. Using coordinates $p = (p_a, p_b, p_c)$, define σ and Π on Λ^* by

$$\sigma(p) = \begin{cases} 0 & \text{if } p \in \Lambda^t, p_a p_b p_c \neq 0 \text{ or } p = 0, \\ 2\pi/\sqrt{3} & \text{if } p \in \Lambda^t, p_a p_b p_c = 0 \text{ and } p \neq 0, \\ 1 & \text{if } p \in n_a^t + \Lambda^t, \\ -1 & \text{if } p \in -n_a^t + \Lambda^t. \end{cases}$$

$\Pi(p)$ = product of nonzero elements of $\{p_a, p_b, p_c\}$, $p \neq 0$ and $\Pi(0) = 1$. Let c_p be the Fourier coefficients of $\overline{\square}$:

$$c_p = \frac{1}{|H|} \int_H \overline{s} e^{-i2\pi p s} ds \quad (13)$$

Appendix B computes c_p as

$$c_p = \frac{-i \sigma(p) p^t}{4\sqrt{3}\pi^2 \Pi(p)} \quad (14)$$

The Fourier coefficients of $f(x) = \overline{x + k\beta}$ are $c_p e^{i2\pi p \beta k}$. Each entry of the outer product in (12) can be regarded as an inner product of functions over H and applying Parseval's formula (10) to each entry gives

$$R_e(k) = \sum_{p \in \Lambda^*} c_p^* \otimes c_p e^{i2\pi p \beta k} \quad (15)$$

The Bohr-Fourier series (15) implies that the quantization error spectral density matrix S_e is purely discrete having amplitudes $c_p^* \otimes c_p$ at frequencies $\langle p\beta \rangle$ for $p \in \Lambda^*$:

$$S_e(\omega) = \sum_{p \in \Lambda^*} c_p^* \otimes c_p \delta(\omega - \langle p\beta \rangle) \quad (16)$$

where $\langle p\beta \rangle$ denotes the fractional part of $p\beta$. The quantizer output q_n is obtained by differencing e_n and adding β according to (5). Hence the output spectral density matrix $S_q(\omega) = \beta \otimes \beta \delta(\omega) + 4 \sin^2(\pi\omega) S_e(\omega)$ and, finally,

$$S_q(\omega) = \beta \otimes \beta \delta(\omega) + \sum_{p \in \Lambda^*} \frac{\sin^2(\pi\omega) \sigma^2(p)}{12\pi^4 \Pi^2(p)} p \otimes p \delta(\omega - \langle p\beta \rangle)$$

7. APPENDIX A

We transform standard Fourier and ergodic results from the square $[0, 1]^2$ to H . Relate coordinates x' on $[0, 1]^2$ to coordinates x on H by $x' = \langle Vx \rangle$ and $x = \langle Wx' \rangle$. Relate coordinates p on Λ^* to coordinates p' on \mathbb{Z}^{*2} by $p' = pW$. Suppose $f' \in \mathcal{L}^2([0, 1]^2)$ and let $f = f' \circ \langle \cdot \rangle \circ V \in \mathcal{L}^2(H)$. Then $f'(x') = f'(\langle Vx \rangle) = f(x)$, $dx = |H| dx'$ and $\langle p'x' \rangle = \langle pW \langle Vx \rangle \rangle = \langle px \rangle$. Write $(f, g) = \frac{1}{|H|} \int_H f(x) g^*(x) dx$ and $(f', g') = \int_{[0, 1]^2} f'(x') g'^*(x') dx'$. Then $(f, g) = (f', g')$ and $\hat{f}_p = (f, e^{-i2\pi p x}) = (f', e^{-i2\pi p' x'}) = \hat{f}'_{p'}$. Moreover, $f(\langle x+y \rangle) = f'(\langle V \langle x+y \rangle \rangle) = f'(\langle V(x+y) \rangle) = f'(\langle x' + y' \rangle)$. Hence the following results can be transformed to the results of section 5:

Fourier analysis on $[0, 1]^2$ [11]: $\mathcal{L}^2([0, 1]^2)$ is a Hilbert space with inner product (f', g') . $\{e^{i2\pi p' x'} \mid p' \in \mathbb{Z}^{*2}\}$ is a complete orthonormal basis. $f'(x') = \sum_{p' \in \mathbb{Z}^{*2}} \hat{f}'_{p'} e^{i2\pi p' x'}$ where the equality is interpreted in the \mathcal{L}^2 sense and $\hat{f}'_{p'} = (f', e^{-i2\pi p' x'})$. The Parseval formula $(f', g') = \sum_{p' \in \mathbb{Z}^{*2}} \hat{f}'_{p'} \hat{g}'_{p'}^*$ holds.

Mané [10] (thm. II 3.2, prop. II 2.7, thm. I 9.2) implies: Let $\beta' \in \mathbb{R}^2$ be such that the only $p' \in \mathbb{Z}^{*2}$ with $p' \beta' \in \mathbb{Z}$ is $p' = 0$. Let $\tilde{f}' : \mathbb{R}^2/\mathbb{Z}^2 \rightarrow \mathbb{R}$ be continuous. Let π be the canonical projection $\mathbb{R}^2 \rightarrow \mathbb{R}^2/\mathbb{Z}^2$. Then translation by $\pi\beta'$ is a uniquely ergodic shift on $\mathbb{R}^2/\mathbb{Z}^2$ and, for all $e'_0 \in \mathbb{R}^2$,

$$\lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} \tilde{f}'(\pi(e'_0 + n\beta')) = \int_{\mathbb{R}^2/\mathbb{Z}^2} \tilde{f}'(s') ds'$$

(Note that $\pi(e'_0 + n\beta') = \pi(e'_0 + n\beta')$.) Suppose that $f' : [0, 1]^2 \rightarrow \mathbb{R}$ lifts to \tilde{f}' so that $f' = \tilde{f}' \circ \pi$. Then

$$\lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=0}^{L-1} f'(\langle e'_0 + n\beta' \rangle) = \int_{[0, 1]^2} f'(s') ds'$$

8. APPENDIX B: Compute Fourier coefficients c_p

It is convenient to define vectors n and n^\perp (see Fig. 2):

$$[n_a \mid n_b \mid n_c] = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$[n_a^\perp \mid n_b^\perp \mid n_c^\perp] = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Also define $n_{-s} = -n_s$ and $n_{-s}^\perp = -n_s^\perp$ for $s = a, b, c$.

Equation (13) implies $c_0 = 0$. Now we compute c_p for $p \neq 0$. Define $a : \mathcal{P}^* \rightarrow \mathbb{R}$ by $a(p) = -\int_{\mathcal{H}} e^{-i2\pi p x} dx$. Then

$$c_p = \frac{1}{i2\pi|\mathcal{H}|} (D_p a(p))^t \quad (17)$$

and the problem becomes finding an expression for $a(p)$:

$$\begin{aligned} a(p) &= \frac{p}{i2\pi p p^t} \int_{\mathcal{H}} (D_x e^{-i2\pi p x})^t dx \\ &= \frac{p}{i2\pi|p|^2|n^\perp|} \oint_{\partial\mathcal{H}} e^{-i2\pi p t} n_t^\perp dt \\ &= \frac{|n|}{i2\pi|p|^2|n^\perp|} \sum_{s \in \pm\{a,b,c\}} p n_s^\perp \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-i2\pi p \sigma_s(t)} dt \end{aligned}$$

where $\sigma_s(t) = \frac{1}{2} n_s^\perp + n_s t$, $t \in [-1/2, 1/2]$ parameterizes the hexagon edges. Then letting $p_s = p n_s$ and $p_s^\perp = p n_s^\perp$ gives

$$\begin{aligned} a(p) &= \frac{-|n|}{\pi|n^\perp||p|^2} \sum_{s \in \{a,b,c\}} p_s^\perp \int_{-\frac{1}{2}}^{\frac{1}{2}} \sin(2\pi p \sigma_s(t)) dt \\ &= \frac{-1}{\pi^2 \sqrt{3} |p|^2} \sum_{s \in \{a,b,c\}} \frac{p_s^\perp}{p_s} \sin(\pi p_s) \sin(\pi p_s^\perp) \quad (18) \end{aligned}$$

Substituting (18) in (17), differentiating, and evaluating at $p \in \Lambda^*$ (then $p_s^\perp \in \mathbb{Z}$ and $\sin(\pi p_s^\perp) = 0$) yields

$$c_p = \frac{i}{6\pi^2 |p|^2} \sum_{s \in \{a,b,c\}} \frac{p_s^\perp}{p_s} \sin(\pi p_s) \cos(\pi p_s^\perp) n_s^\perp \quad (19)$$

In the case $p \in \Lambda^* - \{0\}$, $p_s \in \mathbb{Z}$ and $\sin(\pi p_s)/(\pi p_s) = 0$ and $c_p = 0$ except when $p_s = 0$. When $p_s = 0$, $p = k n_s^\perp$, $k \in \mathbb{Z}$, $p_s^\perp = 2k$ and (14) follows from (19).

To simplify (19) in the case $p \in \Lambda^* - \Lambda^t$, consider $f_s : \Lambda^* \rightarrow \mathbb{R}$ defined by

$$f_s(p) = 2 \sin(\pi p_s) \cos(\pi p_s^\perp) = \sin(\pi p n_s^+) + \sin(\pi p n_s^-)$$

where $n_s^\pm = n_s \pm n_s^\perp$. Notice that $n_s^\pm \in 2\Lambda^*$. Let $\lambda \in \Lambda$. Then

$$f_s(p + \lambda^t) = \sin(\pi(p + \lambda^t) n_s^+) + \sin(\pi(p + \lambda^t) n_s^-) = f_s(p)$$

since $n_s^\pm \in 2\Lambda^*$ implies that $\lambda^t n_s^\pm$ is an even integer. This periodicity of f_s implies that f_s is constant on each of $n_a^t + \Lambda^t$ and $-n_a^t + \Lambda^t$. The respective constants can be directly calculated: $f_s(\pm n_a^t) = \pm\sqrt{3}$ for $s = a, b, c$. Then $f_s(p) = \sigma(p)\sqrt{3}$ and

$$c_p = \frac{i \sigma(p)}{4\sqrt{3}\pi^2 |p|^2} \sum_{s \in \{a,b,c\}} \frac{p_s^\perp}{p_s} n_s^\perp \quad ; \quad p \in \Lambda^* - \Lambda^t.$$

Now (14) follows by algebra and $p_a + p_b + p_c = 0$.

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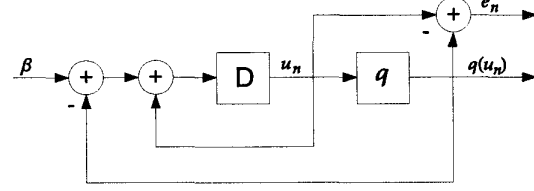


Figure 1: Discrete-time hexagonal $\Sigma\Delta$ modulator. D : unit delay. q : hexagonal quantizer.

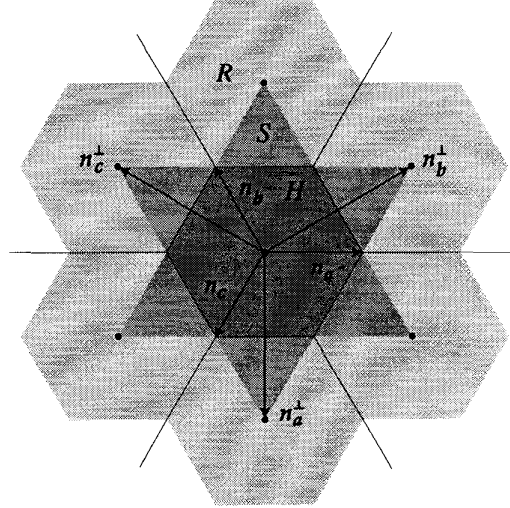


Figure 2: Hexagonal quantizer q and regions H , S , and R .

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