Decoupled Control of a Four-Leg Inverter via a New $4 \times 4$ Transformation Matrix

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Abstract—Four-Leg (3-phase 4-wire) inverters are developed to power unbalanced/nonlinear three-phase loads. A unique $4 \times 4$ decoupling transformation matrix is used that enables direct transformation between the four degree-of-freedom (DOF) leg-modulation space of the inverter and its corresponding 3-DOF output-voltage space. This is analogous to the well-known $3 \times 3$ “abc-qd” transformation developed for the three-leg inverter. Details of this new $4 \times 4$ “Quad” transform are provided, along with a depiction of the voltage-vectors produced. Advanced synchronous-frame control techniques are applied with this 4-to-3 “abcnc-qdnc” transform to create a UPS-style inverter with sinewave output. Experimental results for an 8.6 kVA prototype inverter are presented.

Index Terms—Current control, inverters, transforms, uninterruptible power systems, voltage control.

I. INTRODUCTION

FOR SOURCING power to unbalanced and/or nonlinear three-phase loads four-leg inverters have been developed, where the fourth leg connects to the load neutral [15]. The topology is commonly a “full-bridge” VSI with LC filters suitable for producing sinusoidal output voltages [2], [4]. Previous analyses of the four-leg topology have utilized the well-known $3 \times 3$ “abc-qd” transformation [7] matrix in modeling the operation of the four-leg inverter [8], [10], [11], [14]. While this “abc-qd” transformation is widely used for the modeling and control of three-leg inverters, it does not adequately address the extra degree-of-freedom (DOF) the four-leg inverter provides [1], [12].

A unique $4 \times 4$ decoupling transformation matrix is used for the four-leg inverter that enables direct transformation between the 4-DOF leg-modulation space of the inverter and its corresponding 3-DOF output-voltage space [1]. With this new $4 \times 4$, or Quad-Transformation matrix the legs of a four-leg inverter can be deterministically modulated to produce arbitrary phase voltages, regardless of loading. Advanced synchronous-frame control techniques are applied with this 4-to-3 Quad-Transform to create a UPS-style inverter with sinusoidal output voltage. Experimental results from an 8.6 kVA prototype inverter are provided.

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Fig. 2. Four-leg inverter primary voltage vectors: [a b c n].

Fig. 2. Note that viewed from “above” the [a b c] vectors in Fig. 2 form the familiar triad of a three-phase system in the qd-plane [2], [7]. The [a b c n] vectors of Fig. 2 comprise the upper part of the new “abcn-quad” or “Quad” transform matrix [1]

$$\mathbf{T}_{qd\alpha\beta} = \frac{2}{3} \begin{bmatrix} \sqrt{3} & \sqrt{3} & \sqrt{3} & \sqrt{3} \\ 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \sqrt{3} & -\sqrt{3} & 0 \\ 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} \end{bmatrix}$$

The $\mathbf{T}_{qd\alpha\beta}$ matrix provides an amplitude-invariant decoupling transformation from “abcn” phase-quantities to an orthogonal qd-space, as given in

$$\begin{bmatrix} f_q \\ f_d \\ f_o \\ f_z \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \sqrt{3} & -\sqrt{3} & 0 \\ 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \\ f_n \end{bmatrix}$$

where $f$ represents the phase quantity of interest (voltage, current, etc.) The $f_z$ term again represents a place-holder term such that a square (and hence, invertible) transform is formed. Note that the familiar “abc-quad” definitions used for the three-leg inverter are preserved [1], [7].

The four-leg inverter has sixteen ($2^4$) switching-states/voltage-vectors, which project into qd-space as shown in Fig. 3. Note that the fourteen nonzero vectors in Fig. 3 fill out a rough “sphere” in qd-space; this is analogous to the “circle” formed by the six nonzero voltage-vectors of the three-leg inverter in the qd-plane [1]. The inverse of $\mathbf{T}_{qd\alpha\beta}$ is found as

$$\begin{bmatrix} f_q \\ f_d \\ f_o \\ f_z \end{bmatrix} = \mathbf{T}_{qd\alpha\beta}^{-1} \begin{bmatrix} f_q \\ f_d \\ f_o \\ f_z \end{bmatrix}$$

As the four-leg inverter produces only three independent output voltages, subsets of the Quad-Transform are formed as

$$\begin{bmatrix} f_q \\ f_d \\ f_o \end{bmatrix} = \mathbf{T}_{qd\alpha\beta} \begin{bmatrix} f_q \\ f_d \\ f_o \end{bmatrix}$$

where

$$\mathbf{T}_{qd\alpha\beta} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \sqrt{3} & -\sqrt{3} & 0 \\ 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} \end{bmatrix}$$
Fig. 4. Equivalent single-phase models of four-leg inverter in \(q\delta d\)-space.

\[
\begin{bmatrix}
\alpha
\beta
\gamma
\end{bmatrix} = T_{q\delta d} \begin{bmatrix}
\alpha
\beta
\gamma
\end{bmatrix}
\]

where the general vector quantities \(T_{q\delta d} = [\alpha \beta \gamma] \) and \(T_{q\delta d} = [\alpha \beta \gamma] \) have been defined. Combining (1) and (5), the \(q\delta d\) phase voltages of the inverter may be found as functions of the modulation indices

\[
\begin{bmatrix}
f_{\alpha}
f_{\beta}
f_{\gamma}
\end{bmatrix} = T_{q\delta d} \begin{bmatrix}
f_{\delta}
f_{\beta}
f_{\gamma}
\end{bmatrix}
\]

Equation (7) shows that \(\alpha = \beta = \gamma\); thus, the four-leg inverter can now be represented as three decoupled single-phase systems, as depicted in Fig. 4. Previous representations of the four-leg inverter system using the ordinary \(3 \times 3\) \(q\delta\)-transform could only depict decoupled systems after phase current regulators were applied: the fundamental phase voltage coupling was still present. Note that the zero sequence output voltage is explicitly controlled by the inverter modulation. In Fig. 4 the \(q\delta d\) circuit components equal those of the actual three-phase filter: \(L_{\alpha} = L_{\beta} = L_{\gamma} = L_{f}; R_{\alpha} = R_{\beta} = R_{\gamma} = R_{f}; C_{\alpha} = C_{\beta} = C_{\gamma} = C_{f}\).

III. SYNCHRONOUS-FRAME CONTROLLER

The 3-D “\(q\delta d\)” output space of the inverter is represented in Fig. 5 as three orthogonal axes (“right-hand-rule” convention [9]) where an arbitrary vector, \(f_{q\delta d} = [f_{\delta} f_{\beta} f_{\gamma}]\), is shown.

\[
\begin{bmatrix}
f_{\delta}
f_{\beta}
f_{\gamma}
\end{bmatrix} = T_{q\delta d} \begin{bmatrix}
f_{\delta}
f_{\beta}
f_{\gamma}
\end{bmatrix}
\]

A vector representing balanced positive-sequence 3-phase voltage rotates counter-clockwise in the \(q\delta\)-plane at \(\omega_{e}t\) [rads], with \(\omega_{e}\) the fundamental frequency. A second \(q\delta d\)-frame is constructed by rotating around the \(\alpha\)-axis by angle \(\theta\) as depicted in Fig. 6. Note in Fig. 6 that the \(f_{\alpha}^{e}\) and \(f_{\beta}^{e}\) axes point out of the page. The original frame of reference is the stationary-frame (superscript \(s\)), and the rotated frame is the synchronous-frame (superscript \(e\)). Note also in Fig. 6 that the \(f_{\alpha}^{e}\) and \(f_{\beta}^{e}\) axes are aligned. Thus, synchronous-frame zero-sequence quantities are identically equal to their stationary-frame counterparts: \(f_{\alpha}^{e} = f_{\alpha}^{s}\).

A vector in the stationary-frame is transformed to synchronous-frame coordinates by a 3-D rotation matrix [9] representing a rotation about the \(\alpha\)-axis by an angle \(\theta\)

\[
\begin{bmatrix}
f_{\delta}^{e}
f_{\beta}^{e}
f_{\gamma}^{e}
\end{bmatrix} = R_{3}(\theta) \begin{bmatrix}
f_{\delta}^{s}
f_{\beta}^{s}
f_{\gamma}^{s}
\end{bmatrix}
\]

where the general vector quantities \(T_{s\delta d}^{e} = [f_{\delta}^{e} f_{\beta}^{e} f_{\gamma}^{e}]\) have been defined.
Combining (5) and (8) the synchronous-frame $qdo$ quantities, $\mathbf{f}_{qdo}$, are found from the phase quantities, $\mathbf{f}_{abcn}
abla\nabla$.

$$
\begin{bmatrix}
    f_q \\
    f_d \\
    f_o \\
    f_n
\end{bmatrix} = R_d(\theta) T_{qdo} \begin{bmatrix}
    f_a \\
    f_b \\
    f_c \\
    f_n
\end{bmatrix}
$$

where the shorthand notation $C(\theta) \equiv \cos(\theta)$ and $S(\theta) \equiv \sin(\theta)$ is used. The inverse of $T_{qdo}(\theta)$ is found as

$$
\begin{bmatrix}
    f_a \\
    f_b \\
    f_c \\
    f_n
\end{bmatrix} = T_{qdo}^{-1}(\theta) \begin{bmatrix}
    f_q \\
    f_d \\
    f_o \\
    f_n
\end{bmatrix}
$$

where an arbitrary sinusoid has been commanded to the $d$-axis. For balanced three-phase voltage:

$$
V_{qdo} = \begin{bmatrix}
    v_q^e \\
    v_d^e \\
    v_o^e
\end{bmatrix}
$$

where $\theta^e$ is an arbitrary phase angle for the reference vectors with respect to the synchronous frame (reference Fig. 7).
inverter voltage commands, \([v_{\text{ref}q} \ v_{\text{ref}d}]^T\), and thus they are not usually implemented (see parameters in Table I). The \([v_{\text{ref}q} \omega_C \ v_{\text{ref}d} \omega_C]^T\) terms represent the fundamental current through the \(q\delta\) filter capacitors, which are decoupled by the \(q\delta\) capacitor current command: \([v_{\text{ref}q}^T \ (\omega_C \ v_{\text{ref}d})]^T\).

The equivalent, decoupled system in the synchronous-frame is now depicted in Fig. 10.

As seen in Fig. 10, the equivalent controller is actually three single-phase controllers operating independently. The reference commands for all three axes are DC quantities and, thus, the PI-based controllers will regulate steady-state errors to zero. Since the equivalent controllers depicted in Fig. 10 operate independently, the closed-loop (CL) four-leg inverter can be represented as three sinusoidal voltage sources in the \(q\delta\)-frame. These are depicted in Fig. 11.

The \(Z_q\), \(Z_d\) and \(Z_0\) terms in Fig. 11 denote the equivalent CL output impedances of the respective phases. These impedances are determined by the filter parameters and controller gains, and define the dynamic-stiffness of the inverter [2], [3], [13]. Note in Fig. 11 that the four-leg inverter produces a regulated zero-sequence voltage to the load, and as such will conduct neutral currents associated with unbalanced and/or nonlinear loads.

### IV. Experimental Results

The inverter and controller parameters for the four-leg inverter are listed in Table I. A general-purpose Pentium™-based digital controller was used to implement the control [2].

#### A. Unbalanced Load

Results from an unbalanced load test, \([R_a \ R_b \ R_c] = [25.2 \ 5.2 \ 4.8] \Omega\), are shown in Fig. 13 for the four-leg inverter. For comparison, Fig. 12 depicts the performance of a three-leg inverter (with an equivalent controller) sourcing the same load. As seen in Fig. 12 (top), the three-leg inverter can readily regulate the \(q\delta\)-voltages of the load (equivalent to the line-to-line voltages), but it cannot compensate for the zero-sequence voltage, \(v_{\text{seq}}\), created by the unbalanced load. This is an inherent limitation of three-leg inverter systems. Plotted in \(q\delta\)-space, the unbalance produces a trajectory “tilted” out of the \(q\delta\)-plane as seen in Fig. 12 (bot). Conversely, the four-leg inverter in Fig. 13 (top) regulates the zero-sequence voltage to zero, and accurately tracks the \(q\delta\)-reference in the \(q\delta\)-plane; Fig. 13 (bot). (Note: the high-frequency oscillations seen are due to the limits of the general purpose controller.)
As projected, the Quad-Transform enables the four-leg inverter system to maintain three balanced line-to-neutral voltages in the presence of unbalanced loads.

**B. Mixed Loads**

To emulate the loads typically found in an office building, where single-phase computer power supply type loads are prevalent, the system was tested under mixed load conditions

Phase-A: FW Diode Rectifier, \( C_{dc} = 1000 \mu F \), \( R_{dc} = 26 \Omega \)

Phase-B: \( R_0 = 5.1 \Omega \)

Phase-C: \( R_c = 5.1 \Omega \)

where all loads are line-to-neutral. Fig. 14 depicts the phase-a current; Fig. 15 depicts the \( qdo \)-phase voltages. In Fig. 15 there can now be seen some distortion of the \( qdo \)-phase voltages due to the nonlinear nature of the phase-a current. Even in the presence of such a harsh nonlinear load, with current spikes of \( \pm 50 \ A_{\text{pk}} \), the phase-a output voltage regulation is maintained with a THD of only 5.3%. Fig. 16 depicts the \( qdo \)-phase voltages, and their reference, in \( qdo \)-space.

In Fig. 16, it is seen that \( V_{qdo} \) closely tracks the reference circle in \( qdo \)-space. As depicted in Figs. 14–16, the Quad-Transform...
enables the four-leg inverter system to maintain three balanced line-to-neutral voltages in the presence of nonlinear and/or unbalanced loads. Output impedance at 60 [Hz] is fundamentally zero, and is found to be less than 0.4 [Ω] below 500 [Hz]. In all test cases voltage regulation is to within 1%.

V. CONCLUSIONS

This paper presents a four-leg (3-phase 4-wire) sinewave inverter and controller, where the new $4 \times 4$ decoupling Quad-Transformation matrix is used. Details on the controller implementation are provided, along with plots of experimental results. Points to note as follows.

1) Four-leg inverters are uniquely able to source power to unbalanced and/or nonlinear three-phase loads.

2) The familiar $3 \times 3$ “$abc$-$dq$” transform does not adequately address the extra DOF of the four-leg inverter.

3) The new $4 \times 4$ Quad-Transform enables a direct transformation between the 4-DOF leg-voltage space of the four-leg inverter and its 3-DOF output-voltage space.

4) With the Quad-Transform, the four-leg inverter can be modeled as three decoupled single-phase systems down to the inverter phase-voltage level.

5) A synchronous-frame controller is developed that deterministically regulates all three components of the output-voltage space: $v_d$, $v_q$, and $v_0$.

As is seen in the experimental results, the four-leg sinewave inverter system is able to accurately regulate balanced, positive-sequence three-phase voltage in the presence of unbalanced and/or nonlinear loads.

REFERENCES


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