SYNCHRONOUS MACHINE PARAMETERS FROM SUDDEN-SHORT TESTS BY BACK-SOLVING

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ABSTRACT

Calculation of synchronous machine parameters from sudden short circuit measurements has been subject to unnecessary assumptions and approximations. These assumptions are due in part to the unavailability of measurements, and in part to the cumbersome nature of the calculations needed to remove these approximations. This paper describes the nature of these approximations and a procedure to remove them. It also illustrates how a new philosophy for problem formulation and solution using backsolving programs can simplify and clarify the solution process. Finally, the paper introduces a new way of calculating eigenvalues based on a generalization of the eigenvalue problem. **KEYWORDS:** Synchronous Machines, Short Circuit Tests, Machine Parameters, Symbolic Computation.

INTRODUCTION

The calculation of synchronous machine parameters from sudden short circuit measurements has been subject to one unnecessary assumption and two approximations for mathematical convenience.

- The first approximation involves neglecting the coupling between transient and subtransient periods. Considering that the time constants associated with the transient and subtransient periods are independent of each other, simple formulas for the calculation of machine inductances are possible. Considering that these time constants are not independent but represent eigenvalues of the natural response of the machine under sudden short circuit conditions, coupling between these time constants can be accounted for. Derivation of these formulas requires an analytic solution of a two by two eigenvalue problem. This work was done by Dommel et al. [1,2,3] as an extension to Canay [4].
- The second approximation involves neglecting armature resistance. If the armature resistance is considered, coupling occurs between the direct and quadrature parameters of the machine. It is still possible to view the time constants as eigenvalues, but all four eigenvalues are coupled. An analytic solution is nearly impossible with conventional tools.
- The assumption is the result of the measurements normally available from tests not being sufficient to determine all the machine inductances. A new test proposed by Canay resolves this problem.

This paper describes a new approach to formulation of the problem that relies on the use of SOLVER-Q, a symbolically assisted numeric equation solving program [8,9]

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to calculate the machine parameters. With this program, no approximations are needed. An exact solution to the problem is obtained directly from its formulation.

Results of calculations for typical machine parameters indicate that the usual approximations are quite good for typical ranges of values of machine parameters. With this new tool no approximations are necessary. This improves the accuracy of machine models for typical machines and allows the extension of the theory to unusual design machines. Furthermore, because the solution is obtained directly from the defining formulas of the machine without further manipulations, additional physical insight into the meaning of all machine parameters can be gained without the need to become deeply involved in unnecessary mathematical manipulations.

NOTATION

We do not describe nuances of units and scaling associated with machine parameters. We use an accepted formulation of the machine equations using a more or less standard notation. A slightly different notation is required by the back-solver. Table 1 illustrates the notation for main variables. Table 2 illustrates the notation for subscripts. Mutual inductances have a single subscript whenever this is sufficient to avoid confusion.

Table 1: Notation for variables.

Variable	In Equation	In SOLVER-Q
Flux Linkage	λ	l
Current	i	i
Voltage	v	v
Self Inductance	L	x
Mutual	M	xm
Resistance	R	r
Eigenvalues	μ	μ
Time Constants	T	t

Table 2: Notation for subscripts.

Subscript	In Equations	SOLVER-Q
Radian Frequency	ω	w
Direct Axis Quantities	d	d
Direct Axis Armature Reaction	d	d
Field Winding	f	f
Direct Axis Damper Winding	D	dp
Quadrature Axis Quantities	q	q
Quad. Axis Armature Reaction	q	q
Quad. Axis Damper Winding	Q	qp
Quad. Axis 2nd Winding	g	g
Open Circuit	o	o
Leakage	ℓ	l

BACKGROUND

The fundamental normalized equations used to describe the transient behavior of the machine are those for the *ideal synchronous machine* as defined in [13, page 700]:

$$\begin{bmatrix} \lambda_d \\ \lambda_f \\ \lambda_D \end{bmatrix} = \begin{bmatrix} L_{dd} & M_d & M_d \\ M_d & L_{ff} & M_d \\ M_d & M_d & L_{DD} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_D \end{bmatrix} \quad (1a)$$

$$-\begin{bmatrix} \frac{d\lambda_f}{dt} \\ \frac{d\lambda_D}{dt} \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_D \end{bmatrix} \begin{bmatrix} i_f \\ i_D \end{bmatrix} + \begin{bmatrix} v_f \\ 0 \end{bmatrix} \quad (1b)$$

$$\begin{bmatrix} \lambda_q \\ \lambda_g \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{qq} & M_q & M_q \\ M_q & L_{gg} & M_q \\ M_q & M_q & L_{QQ} \end{bmatrix} \begin{bmatrix} i_q \\ i_g \\ i_Q \end{bmatrix} \quad (1c)$$

$$-\begin{bmatrix} \frac{d\lambda_q}{dt} \\ \frac{d\lambda_Q}{dt} \end{bmatrix} = \begin{bmatrix} R_g & 0 \\ 0 & R_Q \end{bmatrix} \begin{bmatrix} i_g \\ i_Q \end{bmatrix} \quad (1d)$$

These equations ignore armature resistance. As a result, the direct and quadrature axis of the machine are decoupled and can be dealt with separately. Since the calculation procedures are the same for both axis, we describe them only for one axis.

The parameters of the machine can be determined from sudden short circuit tests prescribed by standards [5,6]. The measurements in Table 3 are available. Some of the normalized machine parameters above can be obtained directly from the test parameters in Table 3:

Table 3: Measurements Available.

Measurement	Symbol
Armature resistance	R_a
Armature leakage reactance	X_{ℓ}
Zero sequence reactance	X_0
Synchronous reactances	X_d, X_q
Transient reactances	X'_d, X'_q
Subtransient reactances	X''_d, X''_q
Transient short cct time const.	T'_d, T'_q
Subtransient short cct time const.	T''_d, T''_q

$$\begin{aligned} L_{dd} &= \frac{X_d}{\omega} & M_d &= \frac{X_d - X_{\ell}}{\omega} \\ L_{qq} &= \frac{X_q}{\omega} & M_q &= \frac{X_q - X_{\ell}}{\omega} \end{aligned} \quad (2)$$

In order to find the remaining direct axis parameters, first define the open circuit time constants. The traditional (approximate) formulas relating the open circuit and short circuit time constants are [7]:

$$T'_{do} = \frac{X_d}{X'_d} T'_d \quad T''_{do} = \frac{X_d}{X''_d} T''_d \quad (3)$$

The traditional formulas for calculation of the remaining internal parameters assume that the transient and subtransient time constants are sufficiently different that the flux decays do not interact. These formulas are:

$$\frac{1}{X_{ff}} = \frac{1}{X'_d - X_{\ell}} - \frac{1}{X_{Md}} \quad (4a)$$

$$\frac{1}{X_{DD}} = \frac{1}{X'_d - X_{\ell}} - \frac{1}{X''_d - X_{\ell}} \quad (4b)$$

$$R_f = \frac{X_{ff} + X_{Md}}{\omega T'_{do}} \quad (4c)$$

$$R_D = \frac{X_{DD} X_{Md} + X_{DD} X_{ff} + X_{ff} X_{Md}}{\omega T''_{do} (X_{ff} + X_{Md})} \quad (4d)$$

Similar equations apply to the quadrature axis.

NEW IDEAS FOR EIGENVALUE COMPUTATIONS

A back solving (or declarative style) program is a program capable of solving consistent sets of algebraic equations [12]. The philosophy of this paper is to establish a consistent set of equations, no matter how obtained, and to solve them as they are without further alteration. This paper uses SOLVER-Q [8,9], a declarative style equation handler that solves sets of nonlinear equations of arbitrary form. Thus, rather than obtaining explicit expressions for the

machine parameters in terms of available measurements, we are satisfied with a set of consistent *inverse* expressions for measurements in terms of machine parameters. We then rely on the back solving ability of SOLVER-Q to perform the numerical computation. The desired parameters are uniquely but implicitly defined.

The relationship between parameters and measurements requires the solution of an eigenvalue problem. It is of interest to see how to express an eigenvalue problem for direct solution by back solving methods, so we digress slightly from our main topic of interest.

Eigenvalues by Back Solving Methods

Consider first the classical eigenvalue and eigenvector problem. The problem arises in the context of the solution of dynamic equations of the form:

$$\frac{dy}{dt} = A y + u \quad (5)$$

where y is a state vector and u is a forcing function.

The natural modes of response of this system are determined by the eigenvalues of this system. The eigenvalues and eigenvectors of this system are values of the scalar μ and the vector y that satisfy:

$$A y = \mu y \quad (6)$$

The problem is to find a nonzero value for y plus a value for μ . If A is a matrix of dimension n , it is known that in general there will be up to n distinct eigenvalues and eigenvectors. Considering that the coupling between modes has been entirely ignored in most calculations to date, we do not incur a serious error if we limit this discussion to the case of distinct real eigenvalues.

In the definition above, there are $n+1$ variables (the n components of y and μ), but only n nonlinear equations. An additional equation is required to insure that the eigenvectors are not zero. There are many ways of establishing this condition, but the simplest two are to either set the norm of the eigenvector y to unity, or to arbitrarily designate a single element of the eigenvector y as one. Thus, the additional equations are either:

$$\sum_{i=1}^n y_i^2 = 1 \quad \text{or} \quad y_1 = 1 \quad (7)$$

This latter approach can sometimes lead to problems, but is more expedient. Thus, the philosophy of reducing eigenvalue problems to ordinary nonlinear algebraic equation problems results in the need to solve:

$$\begin{aligned} a_{11} y_1 + a_{12} y_2 + \dots &= \mu y_1; \\ a_{21} y_2 + a_{22} y_2 + \dots &= \mu y_2; \\ &\vdots \\ y_1 &= 1; \end{aligned} \quad (8)$$

Other solution techniques to solve the eigenvalue problem above are described in references such as [10]. Here we need a more general view of what is an eigenvalue problem. Consider the differential equation:

$$B \frac{dy}{dt} = A y + u \quad (9)$$

The eigenvalues and eigenvectors are the solution of the following set of equations:

$$A y = \mu B y \quad (10)$$

B must be nonsingular. In theory, this problem is reducible to the former problem as follows:

$$J y = \mu y \quad \text{where:} \quad J = B^{-1} A \quad (11)$$

There are, however, both computational and physical reasoning advantages for leaving the equations as originally stated in (10). Sparsity, symmetry and insight are lost. The eigenvalue problem can be formulated directly from (10) as the solution of the following set of nonlinear equations:

$$\begin{aligned} a_{11} y_1 + a_{12} y_2 + \dots &= \mu b_{11} y_1 + \mu b_{12} y_2 + \dots; \\ a_{21} y_2 + a_{22} y_2 + \dots &= \mu b_{21} y_1 + \mu b_{22} y_2 + \dots; \\ &\dots \end{aligned} \quad (12)$$

$$y_1 = 1;$$

The next step in the generalization of the eigenvalue problem is to recognize that a set of linear equality constraints among dynamic variables can be considered in much the same way. In order for this to be possible in general, the differential equation must depend on additional variables y that are linearly related to the state variable x according to the following formulation:

$$K \frac{dy}{dt} = A y + B x + u \quad (13a)$$

$$C y + D x = v \quad (13b)$$

If D and K are nonsingular, this problem can also be reduced to a standard eigenvalue problem as follows:

$$J y = \mu y \quad \text{where: } J = K^{-1}(A - BD^{-1}C) x \quad (14)$$

The next step in the generalization of the problem is to recognize that not all variables may appear in the original differential equations. There may be additional variables introduced as part of the formulation process. We refer to the vector of these variables as z . Consider the following dynamic problem:

$$K \frac{dy}{dt} = A y + B x + u \quad (15)$$

$$C y + D x + E z = v$$

$$F y + G x + H z = 0$$

where both u and v are forcing functions and A, B, C, D, E, F, G and K are matrices. E, K and $(G - HE^{-1}D)$ are nonsingular. The state variables y now participate not only in the differential equations, but are subject to additional equality constraints. Additional variables z are introduced, one per additional equality constraint. The eigenvalue problem associated with this system can be expressed as:

$$A y + B x = \mu K y \quad (16)$$

$$C y + D x + E z = 0$$

$$F y + G x + H z = 0$$

This problem is reducible to an ordinary eigenvalue problem as follows:

$$J y = \mu y \quad (17a)$$

$$\text{where: } J = K^{-1}[A + B(G - HE^{-1}D)^{-1}(HE^{-1}C - F)] \quad (17b)$$

Often it is easier to solve the eigenvalue problem using the complete set of equations in (16) than the reduced set in (17). The solution of (16) is more direct and simpler due to the loss of symmetry and sparsity in J .

In the problem of machine parameter determination from terminal measurements, we face a question opposite to the eigenvalue problem: we are given the eigenvalues, and we wish to find the values of the parameters that are necessary to produce the observed behavior. In the equations above, it is as if we were given the values of the μ 's and were asked to obtain the values of some of the matrix coefficients. For every eigenvalue known, one coefficient can be obtained.

Eigenvectors and Initial Conditions

Solving the eigenvalue and eigenvector problems simplifies the solution of the complete problem including constant forcing terms u and v . Assume that all n

eigenvalues μ_i and eigenvectors y^i have been found and that are all distinct. Assume further that initial conditions for the state variables y^0 are available. The complete time domain response of this system can be determined from:

$$y(t) = \alpha_1 y^1 e^{\mu_1 t} + \alpha_2 y^2 e^{\mu_2 t} + \dots + y^p(t) \quad (18a)$$

$$z(t) = \alpha_1 z^1 e^{\mu_1 t} + \alpha_2 z^2 e^{\mu_2 t} + \dots + z^p(t) \quad (18b)$$

where $y^p(t)$ and $z^p(t)$ are the forced response of the system, determined from:

$$A y^p + B x^p + u = 0 \quad (19)$$

$$C y^p + D x^p + E z^p = v$$

$$F y^p + G x^p + H z^p = 0$$

The values of α_1, α_2 , are calculated from:

$$y(0) = \alpha_1 y^1 + \alpha_2 y^2 + \dots + y^p(0) \quad (20)$$

FIRST STEP: DECOUPLED D AND Q AXIS

Removing the approximations of the classic methods requires eliminating the assumption of independence between the transient and subtransient periods. We first neglect armature resistance, decoupling the d and q axis. This part of the paper parallels the work of Canay and Dommel, although the method used to solve the problem is quite different and sets the stage for the following sections.

In the problem of machine parameter determination, the objective is to obtain the values of some of the equation parameters. Using the back solving philosophy, both the formulation and the final form for the problem are the same.

The equations that determine the dynamic behavior of the direct axis of the machine are:

$$\lambda_d = L_{dd} i_d + M_d i_f + M_d i_D \quad (21)$$

$$\lambda_f = M_d i_d + L_{ff} i_f + M_d i_D$$

$$\lambda_D = M_d i_d + M_d i_f + L_{DD} i_D$$

$$-\frac{d\phi_f}{dt} = R_f i_f + v_f$$

$$-\frac{d\phi_D}{dt} = R_D i_D$$

These equations fit the form of the generalized eigenvalue problem with linear equality constraints. Two alternatives for solution exist. The first is to obtain an explicit systematic procedure for the exact calculation of eigenvalues and eigenvectors as done by Canay. For convenience, this procedure assumes the use of unsaturated open circuit time constants for the machine obtained from:

$$T_{do} + T_{do}'' = \frac{X_d}{X_d'} T_d' + \left(1 - \frac{X_d}{X_d'} + \frac{X_d}{X_d''}\right) T_d'' \quad (22a)$$

$$T_{do}' T_{do}'' = T_d' T_d'' \frac{X_d}{X_d'} \quad (22b)$$

The remainder of the Canay procedure is outlined in Appendix A. The Canay procedure determines the unknown machine parameters R_f, R_D, L_{ff} and L_{DD} . The procedure is simple and direct but has some shortcomings:

- The original formulas require considerable modification, obscuring insight.
- Explicit formulas can only be obtained for systems of low order, 2 in Canay's case. Analytic expressions for high order systems are impossible.
- Slight variations in available information can cause havoc. For example, if R_f happens to be known instead of X_d' .

The back solving procedure requires that we formulate the problem as above, in a consistent unambiguous manner,

and then use a back solver to obtain results. This approach can accommodate to any manner of measurements and specifications with ease. The formulation for the solution of the machine problem considering that $R_a=0$ consists of several kinds of equations assembled as one set. Each equation kind is described separately, but in the end they are all solved simultaneously.

a) Eigenvalue/eigenvector equations:

These equations describe the natural response modes of the system. The eigenvalue equations for the machine consist of two sets of 5 equations, one for each eigenvalue. The set of equations that define in implicit form the eigenvalues is:

$$\begin{aligned} -\mu_1 \lambda_{f1} &= R_f I_{f1} \\ -\mu_1 \lambda_{D1} &= R_D I_{D1} \\ 0 &= L_{dd} I_{d1} + M_d I_{f1} + M_d I_{D1} \\ \lambda_{d1} &= M_d I_{d1} + L_{ff} I_{f1} + M_d I_{D1} \\ \lambda_{D1} &= M_d I_{d1} + M_d I_{f1} + L_{DD} I_{D1} \end{aligned} \quad (23)$$

To obtain the first set, let $i=1$. To obtain the second set, let $i=2$. The forcing functions v_f and λ_d are removed from the natural response determination. They can be found by solving a set of steady state equations.

b) Equations for μ_1 and μ_2 from measurements:

Our problem is a special case of the generalized eigenvalue problem, where the vector Z is now the scalar i_d . The solution to this problem has the form:

$$i_d(t) = i_d^0(t) + \alpha_1 i_{d1} e^{\mu_1 t} + \alpha_2 i_{d2} e^{\mu_2 t} \quad (24)$$

Figure 1: The determination of time constants and reactances from measurements.

According to standards, the short circuit current is defined as:

$$i_d = \frac{E_q}{X_d} + E_q \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + E_q \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''} \quad (25)$$

From these equations we conclude that the values of the eigenvalues μ and the scaling coefficients α are known in terms of some of the (still unknown) parameters. Thus, we establish some additional equations:

$$\begin{aligned} \mu_1 &= -\frac{1}{T_d'} & \mu_2 &= -\frac{1}{T_d''} \\ \alpha_1 i_{d1} &= E_q \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) & \alpha_2 i_{d2} &= E_q \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) \end{aligned} \quad (26)$$

c) Initial condition and steady state equations:

α_1 , α_2 and E_d are calculated based on initial and final conditions. The initial condition equations are determined from the state variables λ_f and λ_D before the sudden short circuit. If the machine is operating under open circuit nominal conditions prior to the fault, the following equations can be established:

$$\begin{aligned} \lambda_{d0-} &= \frac{1}{\omega} \\ R_f I_{f0-} + v_f &= 0 \\ \lambda_{D0-} &= M_d I_{f0-} \end{aligned} \quad (27)$$

$$\lambda_{r0} = L_{ff} I_{f0-}$$

$$\lambda_{D0} = M_d I_{f0-}$$

The equations for final values (the short circuit at steady state) are:

$$\begin{aligned} R_f I_{fp} + v_f &= 0 \\ 0 &= L_{dd} I_{dp} + M_d I_{fp} \\ \lambda_{fp} &= M_d I_{dp} + L_{ff} I_{fp} \\ \lambda_{Dp} &= M_d (I_{dp} + I_{fp}) \end{aligned} \quad (28)$$

Once we know these values, α_1 , α_2 and E_q can be calculated from the following equations:

$$\begin{aligned} \alpha_1 \lambda_{f1} + \alpha_2 \lambda_{f2} &= \lambda_{f0} - \lambda_{fp} \\ \alpha_1 \lambda_{D1} + \alpha_2 \lambda_{D2} &= \lambda_{D0} - \lambda_{Dp} \\ E_q &= -\omega \lambda_{d0-} \end{aligned} \quad (29)$$

Assembling equations (23-29) is sufficient to solve the problem provided a reasonable initial guess is available. There is no need for an explicit analytic solution in the traditional sense. Appendix B describes the basis for this assertion. Reasonable initial guesses can be obtained by solving the approximate problem using traditional formulas.

SECOND STEP: THE COUPLED EIGENVALUE PROBLEM

If the armature resistance is not ignored, the problem of calculating the machine parameters from an eigenvalue formulation appears to become far more involved. However, the philosophy is identical to the one above, and in principle as well as practice, the problem is not much harder than before.

The eigenvalue/eigenvector equations to be solved include now the linkage fluxes for the q and the d axis; other equations neglect this interaction. This assumption is required because the classical definition of i_d for the sudden short circuit test, since this current would depend on four time constants, not two. This does not agree with the data extracted from measured short circuit currents as illustrated in Figure 1. The numerical example shown below supports the validity of this slight approximation.

The equations to be solved are four sets of eigenvalue definition equations, one for each eigenvalue. Two eigenvalues correspond to the d axis, two to the q axis.

$$\begin{aligned} -\mu_1 \lambda_{d1} &= R_a I_{d1} + \omega \lambda_{q1} & -\mu_1 \lambda_{q1} &= R_a I_{q1} - \omega \lambda_{d1} \\ -\mu_1 \lambda_{f1} &= R_f I_{f1} & -\mu_1 \lambda_{g1} &= R_g I_{g1} \\ -\mu_1 \lambda_{D1} &= R_D I_{D1} & -\mu_1 \lambda_{Q1} &= R_Q I_{Q1} \\ \lambda_{d1} &= L_{dd} I_{d1} + M_d I_{f1} + M_d I_{D1} & \lambda_{q1} &= L_{qq} I_{q1} + M_q I_{g1} + M_q I_{Q1} \\ \lambda_{f1} &= M_d I_{d1} + L_{ff} I_{f1} + M_d I_{D1} & \lambda_{g1} &= L_{qg} I_{q1} + M_q I_{g1} + M_q I_{Q1} \\ \lambda_{D1} &= M_d I_{d1} + M_d I_{f1} + L_{DD} I_{D1} & \lambda_{Q1} &= M_q I_{q1} + M_q I_{g1} + L_{QQ} I_{Q1} \end{aligned} \quad (30)$$

Four sets of these equations are obtained by letting i be 1, 2, 3 and 4, since there are four time constants for the d and q axis:

$$\mu_1 = -\frac{1}{T_d'} \quad \mu_2 = -\frac{1}{T_d''} \quad \mu_3 = -\frac{1}{T_q'} \quad \mu_4 = -\frac{1}{T_q''} \quad (31)$$

We now establish general initial condition equations. We do not require an open circuit initial condition during the sudden short circuit test.

$$\begin{aligned} \alpha_1 i_{d1} &= E_q \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) & \alpha_2 i_{d2} &= E_q \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) \\ \beta_1 i_{q3} &= -E_d \left(\frac{1}{X_q'} - \frac{1}{X_q} \right) & \beta_2 i_{q4} &= -E_d \left(\frac{1}{X_q''} - \frac{1}{X_q'} \right) \\ E_d &= +\omega \lambda_{q0-} & E_q &= -\omega \lambda_{d0-} \end{aligned} \quad (32)$$

The initial conditions also give expressions in terms of the values of λ_{r0} , λ_{D0} , λ_{g0} , and λ_{Q0} . Short circuit steady state equations (considering $i_{fp} = i_{f0-}$), allow us to calculate

λ_f^D , λ_D^D , λ_g^D , and λ_Q^D . Both sets of flux linkage are used to determine another set of expressions for α_1 , α_2 , β_1 , and β_2 , as follows:

$$\begin{aligned}\alpha_1 \lambda_{f1} + \alpha_2 \lambda_{f2} &= \lambda_{f0} - \lambda_{fp} \\ \alpha_1 \lambda_{D1} + \alpha_2 \lambda_{D2} &= \lambda_{D0} - \lambda_{Dp} \\ \beta_1 \lambda_{g3} + \beta_2 \lambda_{g4} &= \lambda_{g0} - \lambda_{gp} \\ \beta_1 \lambda_{Q3} + \beta_2 \lambda_{Q4} &= \lambda_{Q0} - \lambda_{Qp}\end{aligned}\quad (33)$$

The complete set of equations (30-33) gives enough conditions to calculate the eight parameters (R_f , R_D , L_{ff} , and L_{DD} for the direct axis and R_g , R_q , L_{gg} , and L_{QQ} for the quadrature axis).

FINAL STEP: THE CANAY INDUCTANCE

The final step in removing the shortcomings of traditional machine parameter calculation methods requires the definition and measurement of a new machine parameter, the Canay inductance. The Canay inductance is of consequence only for the direct axis.

One of the assumptions in (1) is that the mutual inductances between all windings on one axis are numerically equal. The implication of this assumption is that the instantaneous values of some of the winding currents may be in error, but their effect as seen from the armature is correct. When the winding is a damper winding this makes sense, as the winding in question is only fictitious. However, when the winding in question is the field winding, we may be interested in its actual currents. In this case the actual value of the mutual inductance between armature and field in the direct axis is of interest. However, there are simply not enough measurements provided in the traditional tests to permit the calculation of this inductance. Thus, it is necessary to resort to an additional measurement. This was proposed by Canay. We describe the calculation of machine inductances when the Canay inductance is available.

While Canay and others have insisted on the need for an equivalent circuit representation of the magnetic circuit, we argue that introducing the electric circuit equivalent of the magnetic circuit obscures and limits the problem. We deal directly with the magnetic circuit and its equations, without reference to an equivalent circuit.

The equations for the direct axis of the machine, before referring quantities to the armature, are:

$$\begin{bmatrix} \lambda_d \\ \lambda_f \\ \lambda_D \end{bmatrix} = \begin{bmatrix} L_{dd} & M_{df} & M_{dD} \\ M_{df} & L_{ff} & M_{fD} \\ M_{dD} & M_{fD} & L_{DD} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_D \end{bmatrix}\quad (34a)$$

$$-\begin{bmatrix} \frac{d\lambda_f}{dt} \\ \frac{d\lambda_D}{dt} \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_D \end{bmatrix} \begin{bmatrix} i_f \\ i_D \end{bmatrix} + \begin{bmatrix} v_f \\ 0 \end{bmatrix}\quad (34b)$$

The leakage reactance X_ℓ obtained from measurements represents leakage flux between armature and rotor windings, the damper winding in particular. If we know the mmf i_d on the armature winding, the difference between the fluxes in the armature and the damper winding is:

$$\lambda_d - \lambda_D = X_\ell i_d\quad (35a)$$

From equation (34), this quantity is also equal to:

$$\lambda_d - \lambda_D = (L_{dd} - M_{dD}) i_d\quad (35b)$$

Therefore the leakage flux allows us to establish:

$$M_{dD} = L_{dd} - X_\ell\quad (35c)$$

Assume that we also know the flux that links the field winding but fails to link the armature and the damper windings of the machine. Call this the Canay flux, and its

reactance the Canay reactance X_c (Also known as the field leakage reactance X_{f1}). According to this definition:

$$\lambda_f - \lambda_D = X_c i_f\quad (36a)$$

$$\lambda_f - \lambda_D = (L_f - M_{fD}) i_f\quad (36b)$$

$$M_{fD} = L_f - X_c\quad (36c)$$

Knowledge of these leakage fluxes from measurements allows us to find two additional parameters. We observe that there are three unknown mutual couplings. One additional condition is obtained as follows. Define:

$$\lambda_f' = k_f \lambda_f i_f' = \frac{1}{k_f} i_f\quad (37a)$$

$$\lambda_D' = k_D \lambda_D i_D' = \frac{1}{k_D} i_D\quad (37b)$$

Substitution into equation (34) yields:

$$\begin{bmatrix} \lambda_d \\ \lambda_f' \\ \lambda_D' \end{bmatrix} = \begin{bmatrix} L_{dd} & k_f M_{df} & k_D M_{dD} \\ k_f M_{df} & k_f^2 L_{ff} & k_f k_D M_{fD} \\ k_D M_{dD} & k_f k_D M_{fD} & k_D^2 L_{DD} \end{bmatrix} \begin{bmatrix} i_d \\ i_f' \\ i_D' \end{bmatrix}\quad (38a)$$

$$-\begin{bmatrix} \frac{d\lambda_f'}{dt} \\ \frac{d\lambda_D'}{dt} \end{bmatrix} = \begin{bmatrix} k_f^2 R_f & 0 \\ 0 & k_D^2 R_D \end{bmatrix} \begin{bmatrix} i_f' \\ i_D' \end{bmatrix} + \begin{bmatrix} k_f v_f \\ 0 \end{bmatrix}\quad (38b)$$

The primes can be dropped. The values of k_f and k_D are chosen so that the equations above are reduced to the form in (1). In order for this to be the case we choose:

$$k_f = \frac{M_{dD}}{M_{fD}} \quad k_D = \frac{M_{df}}{M_{fD}}\quad (39)$$

Because we do not care about the actual values of damper winding flux and current, we can arbitrarily select $k_D = 1$, which implies $M_{fD} = M_{dD}$. This provides the needed added "measurement." If the Canay leakage inductance is *not* known, then we make an additional (albeit incorrect) assumption. We assume that $k_f = 1$.

The method for measuring the Canay reactance is left to the original paper. For the interested reader, the suggested procedure is to obtain a trace of the field current amplitude during the sudden short circuit test. The ratio of the initial amplitude of the field current oscillations to the initial field current indirectly defines the Canay reactance. Exact computation of this reactance is not trivial, and provides another opportunity for use of the back solving methodology advocated in this paper.

NUMERICAL EXAMPLES

We illustrate the new method with a numerical example from [11] and variations. In most cases there is little difference between traditional methods of calculation and the new method. It is only as the values of the armature resistance increase that significant difference can be observed.

We first reproduce the values of the desired parameters calculated using both Canay's explicit method as outlined in Appendix A and two variations of the method presented in this paper: we consider first both the decoupled d/q formulation (which implies $R_a = 0$) and then the complete formulation, but with the armature resistance R_a set equal to zero. Table 4 illustrates the data used and Table 5 shows the results. We obtain the same results with both the Canay approach and either of the SOLVER-Q approaches.

From the complete eigenvalue formulation we can study the effect of R_a on any parameter. Figures 2-5 show how changes in R_a affect the calculated parameters. Notice

that both the field resistance and inductance are relatively insensitive to changes in R_a , while other parameters are quite sensitive to R_a beyond a certain range. A slight dependency of R_g and X_g on the sudden short initial conditions was observed when R_a was not zero, but is not shown.

Table 4: Data for example from [7].

	d axis	q axis
X_d, X_q	1.79	1.71
X'_d, X'_q	0.169	0.228
X''_d, X''_q	0.135	0.200
T_{d0}, T_{q0}	4.3 s	0.85 s
T'_{d0}, T'_{q0}	0.032 s	0.05 s

Table 5: Calculated Parameters neglecting R_a . Both methods give the same results.

	Canay	SOLVER-Q
T'_d (s)	0.40001568	0.40001662
T''_d (s)	0.02594312	0.02594313
R_f (p.u.)	0.00140679	0.00140679
X_{fr} (p.u.)	1.72178915	1.72178910
R_D (p.u.)	0.00406988	0.00406988
X_{DD} (p.u.)	1.66545811	1.66545811
T'_q (s)	0.10734264	0.10734264
T''_q (s)	0.04630742	0.04630742
R_g (p.u.)	0.01414519	0.01414519
X_{gg} (p.u.)	1.90927836	1.90927848
R_Q (p.u.)	0.00819421	0.00819421
X_{QQ} (p.u.)	1.67419869	1.67419868

Figure 2: R_f and R_{dp} as a function of R_a . The variation is slight. This assumes that R_f was calculated, not measured.

Figure 3: Resistances R_g and R_{qp} as a function of R_a

Table 6 shows that the modal coupling between the d and q axis due the resistance R_a is small. Even though the armature resistance results in interaction that affects the values of the *own axis* time constants themselves, the decay terms in either axis with the *opposite axis* time constants are small in amplitude.

Figure 4: X_g as a function of R_a

Figure 5: Other reactances as a function of R_a

Table 6: Selected eigenvector values for λ_f calculated for the eigenvalues T'_d and T'_q . A value of 0 for λ_{fz} denotes no interaction between axis.

R_a (p.u.)	$\lambda_{f1}(T'_d)$	$\lambda_{fz}(T'_q)$
0	1.0	0
0.00001	1.0	0.000014
0.0001	1.0	0.000142
0.001	1.0	0.001418
0.01	1.0	0.014274
0.05	1.0	0.068049
0.1	1.0	0.112482

CONCLUSIONS

The generalization of the eigenvalue problem along with the use of a back solver permits the removal of approximations during machine parameter determination. Approximations are unnecessary and the formulation is much cleaner than in previous efforts. Procedures that neglect R_a in the calculation process (such as Canay's) introduce errors when time constants are re-calculated based on the parameters obtained (about 4% for T'_q with $R_a=0.04$ pu). The present procedure introduces no such errors. Even for large values of R_a the time constants are correctly computed, since the coupling between axis is considered.

ACKNOWLEDGEMENT

The authors wish to thank the Electric Power Research Institute. The ideas upon which this work is based were developed as part of our work on instructional materials for EMTP use. In the process of attempting to describe how the EMTP performs the calculation of machine parameters we got to thinking about how it ought to be done in this era where sufficient computing power renders many approximations unnecessary. Also, special thanks are due to the reviewers of this paper for many valuable suggestions.

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APPENDIX A: THE CANAY PROCEDURE

The Canay procedure to calculate the parameters R_f , R_D , L_{ff} and L_{DD} of the synchronous machine is as follows:

- a) Obtain the "f-branch" and "D-branch" time constants (some fictitious branches):

$$T_1 = \frac{L_f}{R_f} \quad T_2 = \frac{L_D}{R_D}$$

$$L_f = L_{ff} - M_d \quad L_D = L_{DD} - M_d$$

These values are obtained as the solution of:

$$T_1 + T_2 = (T_{d0}' + T_{d0}'') \frac{M_d - L_d}{M_d} + (T_d' + T_d'') \frac{L_d}{M_d}$$

$$T_1 T_2 = T_{d0}' T_{d0}'' \frac{L_{drD}}{M_d}$$

Where L_{drD} is found from the following equation:

$$L_{drD} = M_d - L_d + L_d''$$

- b) Using the values of T_1 and T_2 calculated in (a), the inductance L_{dr} can be obtained from:

$$L_{dr} = \frac{M_d (T_1 + T_2)}{T_{d0}' + T_{d0}'' - \left(1 + \frac{M_d}{L_{drD}}\right) T_2}$$

- c) Based on these parallel inductances (L_{drD} and L_{dr}), L_f and L_D can be calculated:

$$L_f = \frac{L_{dr} M_d}{M_d - L_{dr}} \quad L_D = \frac{L_{drD} L_{dr}}{L_{dr} - L_{drD}}$$

- d) Knowing the values of L_f and L_D , the desired machine parameters can be obtained from:

$$R_f = \frac{L_f}{T_1} \quad R_D = \frac{L_D}{T_2}$$

$$L_{ff} = L_f + M_d \quad L_{DD} = L_D + M_d$$

```
{ Canay Equations for the calculation of the Parameters
for the d axis of the Synchronous Machine Data }
w = 2*pi*60 ;
xds = 0.135 ; xdt = 0.169 ; xd = 1.790 ;
xl = 0.1300 ; tsdo = 0.032 ; ttdd = 4.300 ;

{Relation between Open Circuit and Short Circuit Time
Constants}
a = xd/xdt ;
b = ttdd + tsdo ;
c = ttdd * tsdo * xds * (1/xd - 1/xdt + 1/xds) ;
ttdd = (b + SQRT(b^2 - 4*a*c))/(2*a) ;
tsd = ttdd * tsdo * xds/(xd * ttdd) ;
{ Mutual Impedance }
xmd = xd - xl ;
{ Canay Procedure }
{a.- Calculate Ldp in parallel with Lf and Md: }
xdpf = xmd - xd + xds ;
{a.- Calculate T1 and T2: }
a = (ttdd + tsdo)*(xmd - xd)/xmd + (ttdd +
tsd)*xd/xmd ;
p = ttdd * tsdo * xdpf / xmd ;
td1 = s/2 + SQRT(s^2 - 4*p)/2 ;
td2 = s/2 - SQRT(s^2 - 4*p)/2 ;
{b.- Calculate Lf in parallel with Md: }
xfp = xmd*(td1 - td2)/(ttdd + tsdo - (1 +
xmd/xdpf)*td2) ;
{c.- Calculate Lf: }
xf = xmd + xfp * xmd /(xmd - xfp) ;
{d.- Calculate Ldp: }
xdp = xmd + xdpf * xfp /(xfp - xdpf) ;
{e.- Calculate the resistances Rf and Rdp: }
rf*td1 = (xf - xmd)/w ;
rdp*td2 = (xdp - xmd)/w ;
```

Figure B-1: SOLVER-Q file for the Canay procedure.
Refer to Tables 1 and 2 for notation.

```
{ Eigenvalue Equations }
{Transient}
ttdd = 0.40001662 ;
lfl = ttdd*rf*ifl ;
ldp1 = ttdd*rdp*idp1 ;
xd * id1 + xmd * ifl + xmd * idp1 = w*ld1 ;
xmd * id1 + xf * ifl + xmd * idp1 = w*lf1 ;
xmd * id1 + xmd * ifl + xdp * idp1 = w*ldp1 ;
{Subtransient}
tsd = 0.02594313 ;
lf2 = tsd*rf*if2 ;
ldp2 = tsd*rdp*idp2 ;
xd * id2 + xmd * if2 + xmd * idp2 = w*ld2 ;
xmd * id2 + xf * if2 + xmd * idp2 = w*lf2 ;
xmd * id2 + xmd * if2 + xdp * idp2 = w*ldp2 ;
{ Short Circuit condition ld = 0 (forcing function) }
ld1 = 0 ; ld2 = 0 ;
{ Force the solution of the Eigenvalue problem }
lf1 = 1 ; ldp2 = 1 ;
{ Initial fluxes for delta = 0 => Vd = 0 and Vq = 1 }
l = xmd * ifom ;
w*lf0 = xf * ifom ;
w*ldp0 = xmd * ifom ;
{ Steady State Fluxes after the Short Circuit }
ifss = ifom ;
0 = xd * idss + xmd * ifss ;
w*lfss = xmd * idss + xf * ifss ;
w*ldpss = xmd * (idss + ifss) ;
{ Scaling the Eigenvectors }
lfo - lfss = alpha1*lfl + alpha2*lf2 ;
ldp0 - ldpss = alpha1*ldp1 + alpha2*ldp2 ;
{ Transient and Subtransient elements of id }
alpha1 * id1 = ed * (1/xdt - 1/xd) ;
alpha2 * id2 = ed * (1/xds - 1/xdt) ;
ed = -1. ;
{ Mutual Inductance Md }
xmd = xd - xl ;
{ Data }
w = 2*pi*60 ; xd = 1.79 ; xl = 0.13 ;
xdt = 0.169 ; xds = 0.135 ;
```

Figure B-2: SOLVER-Q file for the decoupled problem.
Equations shown are for the d axis.

```

{ Eigenvalue Equations }
{ T'd }
ttd = 0.40001662;
ld1 = ttd*(ra*id1 + 2*pi*f*1q1);
lfl1 = ttd*rf*ifl1;
ldp1 = ttd*rdp*idp1;
lql1 = ttd*(ra*iq1 - 2*pi*f*1d1);
lqp1 = ttd*rg*iq1;
xd * id1 + xmd * ifl1 + xmd * idp1 = w*ld1;
xmd * id1 + xf * ifl1 + xmd * idp1 = w*lf1;
xmd * id1 + xmd * ifl1 + xdp * idp1 = w*ldp1;
xq * iq1 + xmq * iq1 + xmq * iqp1 = w*1q1;
xmq * iq1 + xq * iq1 + xmq * iqp1 = w*1q1;
xmq * iq1 + xmq * iq1 + xqp * iqp1 = w*1qp1;

{ T''d }
tsd = 0.02594313;
ld2 = tsd*(ra*id2 + 2*pi*f*1q2);
lfl2 = tsd*rf*ifl2;
ldp2 = tsd*rdp*idp2;
lq2 = tsd*(ra*iq2 - 2*pi*f*1d2);
lg2 = tsd*rg*iq2;
lqp2 = tsd*rg*iqp2;
xd * id2 + xmd * ifl2 + xmd * idp2 = w*ld2;
xmd * id2 + xf * ifl2 + xmd * idp2 = w*lf2;
xmd * id2 + xmd * ifl2 + xdp * idp2 = w*ldp2;
xq * iq2 + xmq * iq2 + xmq * iqp2 = w*1q2;
xmq * iq2 + xq * iq2 + xmq * iqp2 = w*1q2;
xmq * iq2 + xmq * iq2 + xqp * iqp2 = w*1qp2;

{ T'q }
ttq = 0.107342639;
ld3 = ttq*(ra*id3 + 2*pi*f*1q3);
lfl3 = ttq*rf*ifl3;
ldp3 = ttq*rdp*idp3;
lq3 = ttq*(ra*iq3 - 2*pi*f*1d3);
lqp3 = ttq*rg*iq3;
lqp3 = ttq*rg*iqp3;
xd * id3 + xmd * ifl3 + xmd * idp3 = w*ld3;
xmd * id3 + xf * ifl3 + xmd * idp3 = w*lf3;
xmd * id3 + xmd * ifl3 + xdp * idp3 = w*ldp3;
xq * iq3 + xmq * iq3 + xmq * iqp3 = w*1q3;
xmq * iq3 + xq * iq3 + xmq * iqp3 = w*1q3;
xmq * iq3 + xmq * iq3 + xqp * iqp3 = w*1qp3;

{ T''q }
tsq = 0.046307422;
ld4 = tsq*(ra*id4 + w*1q4);
lfl4 = tsq*rf*ifl4;
ldp4 = tsq*rdp*idp4;
lq4 = tsq*(ra*iq4 - w*1d4);
lg4 = tsq*rg*iq4;
lqp4 = tsq*rg*iqp4;
xd * id4 + xmd * ifl4 + xmd * idp4 = w*ld4;
xmd * id4 + xf * ifl4 + xmd * idp4 = w*lf4;
xmd * id4 + xmd * ifl4 + xdp * idp4 = w*ldp4;
xq * iq4 + xmq * iq4 + xmq * iqp4 = w*1q4;
xmq * iq4 + xq * iq4 + xmq * iqp4 = w*1q4;
xmq * iq4 + xmq * iq4 + xqp * iqp4 = w*1qp4;

{ Force the nontrivial solution of the Eigenvalue problem }
ifl1 = 1; ldp2 = 1; lq3 = 1; lqp4 = 1;

{ Initial Cond.:  $\delta=45^\circ \Rightarrow V_{do}=\sin(45^\circ)$  and  $V_{qo}=\cos(45^\circ)$  }
SIN( $\pi/4$ ) = -ra * idom - w*1qo;
w*ldo = xd * idom + xmd * ifom;
w*lfo = xmd * idom + xf * ifom;
w*ldpo = xmd * idom + xmd * ifom;
COS( $\pi/4$ ) = -ra * iqom + w*ldo;
w*lqo = xq * iqom;
w*lgo = xmq * iqom;
w*ldpo = xmq * iqom;
1 = SIN( $\pi/4$ ) * idom + COS( $\pi/4$ ) * iqom;

{ Steady State Linkage Fluxes after the Short Circuit }
ifss = ifom;
0 = ra * idss + w*1qss;
w*ldss = xd * idss + xmd * ifss;
w*lfss = xmd * idss + xf * ifss;
w*ldpss = xmd * (idss + ifss);
0 = ra * iqss - w*ldss;
w*1qss = xq * iqss;
w*lgss = xmq * iqss;
w*ldpss = xmq * iqss;

{ Scaling the Eigenvectors }
lfo = ifss =  $\alpha_1$ *lfl1 +  $\alpha_2$ *lfl2;
ldpo = ldpss =  $\alpha_1$ *ldp1 +  $\alpha_2$ *ldp2;
lgo = lqss =  $\beta_1$ *lq3 +  $\beta_2$ *lq4;
ldpo = lqpss =  $\beta_1$ *lqp3 +  $\beta_2$ *lqp4;

{ Voltages in the d and q axis }
eq = -w*ldo; ed = w*1qo;

{ Transient and Subtransient elements of id and iq }
 $\alpha_1$  * id1 = eq * (1/xdt - 1/xd);
 $\alpha_2$  * id2 = eq * (1/xds - 1/xdt);
 $\beta_1$  * iq3 = -ed * (1/xqt - 1/xq);
 $\beta_2$  * iq4 = -ed * (1/xqs - 1/xqt);

{ Mutual inductances Md and Mq }
xmd = xd - x1; xmq = xq - x1;

{ Data }
ra {ra} = 0.000 + 0.001;
w = 2*pi*60; x1 = 0.13;
xd = 1.79; xdt = 0.169; xds = 0.135;
xq = 1.71; xqt = 0.228; xqs = 0.2;

```

Figure B-3: SOLVER-Q file for the complete eigenvalue problem (considering R_A). Initial guesses not shown.

APPENDIX B: SOLVER-Q SETUP

The present work has been rendered practical only because of the recent introduction of a powerful symbolically assisted numeric equation solving package capable of manipulating large sets of coupled nonlinear equations with ease. This package, named SOLVER-Q, allows the direct transition from a well posed problem formulation to its numerical solution. This program is designed to operate in an IBM PC environment, where it can handle up to 1000 simultaneous nonlinear equations.

Here we present the complete setup in SOLVER-Q format for the example used in this paper. Notice that because of the limited font set on the IBM computer and because of SOLVER-Q's inability to distinguish lowercase from uppercase in variable names, some of the symbols have been changed. Specifically, we use 1 instead of λ for flux and the subscript dp instead of D to denote direct axis damper winding quantities.

SOLVER-Q can solve the problem starting for any consistent set of equations. It converges better if a reasonable initial guess is given. A reasonable initial guess for all the variables is the approximate solution obtained from the traditional formulas.

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Discussion

H.W. Dommel (The University of British Columbia, Vancouver, Canada) and I.I. Dommel (Power Systems Consultants, Vancouver, Canada): The authors present a novel and useful approach for the conversion of synchronous machine parameters, which takes the stator resistance R_a accurately into account. For typical data, the improved accuracy over Canay's data conversion may be insignificant, but an accurate solution is always preferable to remove doubts about the magnitude of approximation errors. The authors' method is therefore a valuable advance in the data conversion problem.

We have used the system of 6 differential equations for a sudden three-phase short-circuit as the authors did, not for data conversion, but to check how well the resistances and inductances from various data conversion procedures reproduce the transient and subtransient reactances and time constants. While carrying out this work, we discovered that our data conversion of [1] was slightly inaccurate.

By diagonalizing the 6 differential equations with the eigenvalue/eigenvector approach, it is possible to write down the solution explicitly in the form of Eq.(25). The time constants are the negative reciprocals of the eigenvalues, and the coefficients $(1/X'_d - 1/X_d)$ and $(1/X''_d - 1/X'_d)$ are obtained from the elements of the eigenvector matrix and its inverse. Since the currents cannot jump instantaneously, Eq.(25) has an additional oscillating term

$$i_d\text{-oscillating} = \frac{E_q}{X''_d} e^{-\alpha t} \cos \omega t + \frac{E_q}{X_{sine}} e^{-\alpha t} \sin \omega t, \quad (I)$$

which makes $i_d = 0$ at $t = 0$. The amplitude of the sine-term is so small that it can be ignored (equivalent to a reactance X_{sine} which is 40 times larger than X''_d in the paper's example with $R_a = 0$).

This oscillating term is equivalent to the dc offset in phase quantities, which decays with a time constant $T_d = 1/\alpha$, with α nearly equal to

$$\alpha = -\frac{\omega R_a}{2} \left(\frac{1}{X''_d} + \frac{1}{X''_q} \right). \quad (II)$$

In the exact solution, $\alpha \pm j\omega$ is an extra pair of conjugate complex eigenvalues, in addition to the 4 eigenvalues used in the paper.

For $R_a = 0$, the oscillating term can be found independently from the two equations

$$\frac{d\lambda_d}{dt} = -\omega\lambda_q \quad \text{and} \quad \frac{d\lambda_q}{dt} = \omega\lambda_d, \quad (III)$$

which has the solution

$$\lambda_d = E_q \cos \omega t \quad \text{and} \quad \lambda_q = E_q \sin \omega t. \quad (IV)$$

Fig. A. Direct axis circuit with sinusoidal voltage source $E_q \sin \omega t$.

In the direct axis, this appears as a known forcing function in Fig. A, which produces the steady-state oscillating term

$$i_d\text{-oscillating} = \frac{E_q}{\sqrt{R^2 + X^2}} \cos(\omega t + \theta) \quad (V)$$

where $R + jX$ is the complex impedance seen by the voltage source in Fig. A, and

$$\theta = \tan^{-1}(R/X). \quad (VI)$$

This impedance can either be calculated from

$$R + jX = j\omega L_1 + \frac{1}{A + B + C}, \quad (VII)$$

where $A = 1/j\omega M_d$,
 $B = 1/(R_f + j\omega L_f)$,
 $C = 1/(R_D + j\omega L_D)$,

or from

$$R + jX = j\omega L_d \frac{(1 + j\omega T'_d)(1 + j\omega T''_d)}{(1 + j\omega T''_{d0})(1 + j\omega T''_{d0})}. \quad (VIII)$$

Eq.(V) can be split into a cosine and sine term,

$$i_d\text{-oscillating} = \frac{E_q}{\sqrt{R^2 + X^2}} \cos \theta \cos \omega t + \frac{E_q}{\sqrt{R^2 + X^2}} \sin \theta \sin \omega t \quad (IX)$$

which, when compared with Eq.(I), shows that the subtransient reactance obtained from the exact solution is

$$X''_d = \frac{\sqrt{R^2 + X^2}}{\cos \theta}, \quad (Xa)$$

or with Eq.(VIII),

$$X''_d = \frac{X_d}{(1 + j\omega T''_{d0})(1 + j\omega T''_d)} = \frac{X_d}{(1 + j\omega T'_d)(1 + j\omega T''_{d0})}. \quad (Xb)$$

Using the parameters of Table 5 of the paper for $R_a = 0$ produces an X''_d from Eq.(X) which is 0.212% higher than the specified value of 0.135 p.u. Canay's data conversion with 4 time constants T'_d , T''_d , T'_{d0} , T''_{d0} is

exact for $R_d = 0$, but the conversion of X'_d , X''_d to a pair of time constants, in cases where only one pair is known, is slightly inaccurate by neglecting R_f and R_p in Fig. A. From a practical standpoint, the inaccuracy of 0.212% in X''_d is of course negligible.

For the exact conversion from X'_d and X''_d to a pair of time constants for $R_d = 0$, Eq.(22b) should be replaced by Eq.(Xb), and Eq.(22a) should be replaced by

$$\left[1 + \frac{1}{(\omega T'_d)^2}\right] \cdot \begin{pmatrix} 1 & 1 \\ L'_d & L_d \end{pmatrix} = \frac{1}{L_d} \cdot \frac{(T'_d - T''_d)(T'_d - T''_d)}{T'_d(T'_d - T''_d)} \quad (XI)$$

The reciprocal of the factor

$$\left[1 + \frac{1}{(\omega T'_d)^2}\right]$$

results from the integration of the forcing function

$$\int_0^\infty e^{-(t-u)/T'_d} \cdot d\lambda_d \cdot du$$

in our Eq.(VI.27) of [1]. In [1] we assumed that this factor is exactly 1.0, which leads to Eq. (22a) if X_d/X''_d is replaced by $T'_d T''_d / (T'_d T''_d)$. In the example, the factor is 1.000044 and setting it to 1.0 produces practically identical results. It appears that a third definition of X''_d can be obtained from our Eq.(VI.20) of [1] if the third term is equated to

$$\left[1 + \frac{1}{(\omega T''_d)^2}\right] \cdot \begin{pmatrix} 1 & 1 \\ L''_d & L'_d \end{pmatrix}$$

in an analogous way.

In our Eq.(VI.30) of [1] for the definition of L_{dfp} , which the authors show in Appendix A just before paragraph b), L''_d must be replaced by $L_d T'_d T''_d / (T'_d T''_d)$ for the more accurate conversion proposed here.

Canay's original data conversion is still useful. The errors which it produces in the resistances and inductances are usually negligible from a practical standpoint. Furthermore, it produces initial guesses for the authors' method. Canay's data conversion can also serve as an indicator of inconsistencies in the measured data if a negative argument is encountered in the square root calculation. In such a case, more approximate data conversions are usually applied.

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T. J. Hammons (Glasgow University, Glasgow G12 8QQ, Scotland, U.K.): The authors have provided useful information on the calculation of Synchronous Machine subtransient and transient parameters from sudden short-circuit test by back-solving. They have highlighted unnecessary assumptions and approximations that have been made in the past due in part to the unavailability of measurements and to the cumbersome nature of the calculations needed to remove these approximations. They have described a procedure to remove approximations, and have illustrated how a new philosophy for problem formulation and solution using back-solving programs can simplify and clarify the solution process. In addition, they

have introduced a new way of calculating eigenvalues giving decay of the components based on a generalization of the eigenvalue problem.

Investigations performed by this Discussor and reported in Reference [A] indicate that detailed generator models which simulate direct- and quadrature axis sub-transient quantities are necessary for realistic assessments of transient electrical phenomena in large synchronous generators. Significant difference in rotor swing is observed when detailed (2d, 3q) or reduced (1d, 2q) and (1d, 1q) damper models are used to model the machine's damper circuits. In addition, shaft torque response following a supply system disturbance is noticeably different when detailed or reduced damper models are employed. Damper circuit modelling also has quite a noticeable influence on electrical damping of torsional vibrations in turbine-generator shafts. This is particularly true at low torsional frequencies in addition to that frequency which relates to rotor swing [B]. The influence of sub-transient quantities [(2d, eq) model] should not be neglected in response torque investigations and in predicting rotor swing.

It should be emphasized that rotor circuit losses which give rise to electrical damping torques are not only in the field and damper windings, but especially in steam turbine generators, mainly in the solid parts of the rotor. It is necessary to consider the losses in the solid iron parts for a realistic and precise computation of electrical damping of rotor swing and torsional vibrations in the machine shaft. This effect is more important for the quadrature-axis due to the absence of the field winding. The effect of solid iron rotor parts is more dominant, for the same reason, in machines without an explicit damper winding. By increasing the number of rotor circuits to three on each axis, i.e., by incorporating the sub-transient quantities, the effect of the solid iron parts at subsynchronous frequencies can be considered in a realistic way.

In the case of perturbations in generator rotor speed resulting from rotor swing, and torsional vibration of the shaft when the generator operates on light load, i.e., at a small load angle, electrical damping is effected by currents which flow in the quadrature-axis damper circuits of the synchronous machine. Conversely, at high load angles, direct-axis rotor circuits become the effective mechanism for electrical damping of rotor swing and torsional vibrations in the machine shaft.

It should be noted that detailed governor and avr simulation where gain and phase shift is faithfully modelled at predominant frequencies is also necessary for realistic assessments of rotor swing (and damping of torsional vibrations in turbine-generator shafts). Steam viscous damping acting on turbine rotors should also be simulated in calculating eigenvalues and overall damping of vibrations in turbine-generator shafts. Steam viscous damping acting at each turbine state is assumed to be proportional to respective turbine load. Turbine-generator response following severe disturbances on the system supply is analysed in References [A], [B] and [C].

Figure A illustrates the effect damper circuit modelling (and the fault clearing process) has on rotor swing following a 185ms L-L-L non-ground HV system short-circuit with clearance. The responses relate to a 500MW, 3600 rpm machine. Automatic voltage regulator (avr) and governor action has been ignored.

Curves (i)-(iii) depict that rotor acceleration is less following fault incidence when the detailed (2d, 3q) damper model is employed. Peak rotor angle to effective infinite busbar corresponding to the (2d, 3q), (1d, 2q) and (1d, 1q) damper models for fault clearance at fault current zeros where supply system infeed is simulated is 92.6°, 99.0° and 97.0° respectively. Peak rotor angle corresponding to the detailed (2d, 3q) damper representation following simultaneous fault clearance is 91.1°. Peak rotor angle is significantly less when the detailed representation is employed on account of improved low-frequency electrical damping of rotor swing. Peak rotor

Table A. Time Constants Determined from Frequency Response Loci for 500MW, 3600 r.p.m. Machine

Model 1:	$x'_d=1.77$	$x''_d=0.311$	$x'''_d=0.274$	$x''''_d=0.221$
	$x_c=0.0825$	$T'_d=0.816$	$T''_d=0.069$	$T'''_d=0.0031$
	$x_q=1.65$	$x''_q=0.767$	$x'''_q=0.424$	$x''''_q=0.240$
		$T'_q=0.226$	$T''_q=0.0314$	$T'''_q=0.0024$
Model 2:	$x'_d=0.301$	$x''_d=0.256$	$x'_q=0.62$	$x''_q=0.322$
	$T'_d=0.77$	$T''_d=0.022$	$T'_q=0.140$	$T''_q=0.011$
Model 3:	$x'_d=0.301$	$x''_d=0.256$	$x'_q=0.322$	
	$T'_d=0.77$	$T''_d=0.022$	$T'_q=0.011$

Figure A. Effect of Damper Modelling on Rotor Angle following L-L-L non-ground HV System Disturbance, 500MW, 3600 r.p.m. Machine

angle is marginally lower following simultaneous interruption on account of reduced effective duration of the fault.

For the machine model, three, two and one rotor circuit fits to standstill frequency response (SFR) test data (or equivalent data computed by finite-element analysis) as depicted by the equivalent circuits of Figure 2 of Reference [A] were made. The fits were then corrected as appropriate for the low flux level of the SFR tests through a comparison of the open-circuit test airgap line gradient with that obtained from measurement of the stator-rotor transfer function at very low frequency. It should be noted that all three models yield nearly the same short-circuit behaviour (the deviations are negligible). This is on account of the sub-subtransient time constant of the d-axis (0.0031s) being very small, so that the main portion of

$$V_q[1/x_d - 1/x_d'] \exp(-t/T_d')$$

is not visible in the oscillographic record of the phase current following a sudden short-circuit. Time constants and reactances for the three models are given in Table A. Determination of Synchronous Machine parameters from characteristic quantities applicable to sub-subtransient data is discussed in Reference [D].

It would seem from the above that detailed generator models which simulate direct- and quadrature-axis sub-subtransient phenomena are essential for predicting generator transient behaviour following a severe supply-system event, such as rotor swing and shaft torsional response. It would appear that this phenomena has not been represented in the analyses the authors have made. Could the authors' method be extended to include determination of synchronous machine sub-subtransient parameters, in addition to subtransient and transient parameters, from sudden short-circuit tests by back-solving? While generalization of the eigenvalue problem along with the use of a back solver permits the removal of approximations for a (1d, 2q) subtransient model during machine parameter determination, such as neglect of R_a , sub-subtransient components may decay too fast for the procedure to be adapted to give realistic sub-subtransient constants for the machine. The authors' views on this Discussor's observations would enhance usefulness of the presented results.

References

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FERNANDO L. ALVARADO AND CLAUDIO CAÑIZARES

The authors wish to thank the discussors for their interest in our paper. We respond to Professor Dommel's comment first. Professor Dommel discusses a slight inaccuracy in the calculation of the parameters. The effect of this inaccuracy is that it is not possible to exactly reproduce the "measured" data with the calculated parameters. The inaccuracy occurs whether the "back-solving" procedure advocated in the paper or any other calculation procedure is used. The reason for this error is attributed to the effect of the field and damper winding resistance on the sudden-short circuit currents. We agree with his comment, and wish to take a slightly different viewpoint more consistent with our back-solving approach.

The problem is due in part to the incomplete nature of the definition of sudden short currents used in the standard tests. The standard definition (IEEE standard 115-1983) of the sudden-short circuit current for the d axis that we reproduce here is shown in equation (25) in the paper:

$$i_d = \frac{E_q}{X_d} + E_q \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + E_q \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''}$$

This definition neglects the armature resistance R_a . It also neglects the sinusoidal oscillation of the field current and considers only its average effect. To take into account the complete solution of the sudden-short circuit current for this specific case, another term should be added. This additional term would have to be the oscillating term that comes from the phasor solution of the machine equations due to the flux λ_d . In addition to equations (1) in the paper, the complete differential equations describing machine behavior include additional differential equations for λ_d and λ_q as follows:

$$\frac{d\lambda_d}{dt} = -v_d - R_a i_d - \omega \lambda_q \quad \frac{d\lambda_q}{dt} = -v_q - R_a i_q + \omega \lambda_d$$

Under short circuit conditions both v_d and v_q are zero. If we neglect R_a these equations reduce to:

$$\frac{d\lambda_q}{dt} = -\omega \lambda_q \quad \frac{d\lambda_d}{dt} = \omega \lambda_d \quad (a)$$

If we solve these equations for no-load initial conditions then:

$$\lambda_d = -\frac{E_q}{\omega} \cos(\omega t) \quad \lambda_q = -\frac{E_q}{\omega} \sin(\omega t) \quad (b)$$

The average value for both these terms is zero. If the armature resistance had been considered, these fluxes would eventually decay to zero amplitude as well. It is

possible to solve these armature equations separately from the remaining rotor circuit equations. We now consider the effect of these armature fluxes on the currents:

$$\begin{aligned}\frac{d\lambda_f}{dt} &= -R_f i_f - v_f \\ \frac{d\lambda_d}{dt} &= -R_D i_d \\ \omega \lambda_d &= X_d i_d + X_{Md} i_f + X_{Md} i_D \\ \omega \lambda_f &= X_{Md} i_d + X_{ff} i_f + X_{Md} i_D \\ \omega \lambda_D &= X_{Md} i_d + X_{Md} i_f + X_{DD} i_D\end{aligned}\quad (c)$$

Because we have already determined v_d , we can treat this problem as having two forcing functions, namely: v_f (DC) and λ_d (AC). We then use superposition to find a steady state solution for i_d . The DC term can be calculated by setting the derivatives (d/dt) and the cosine forcing function (λ_d) equal to zero, obtaining:

$$i_{dDC} = \frac{E_q}{X_d} \quad (d)$$

which agrees with the definition in the standards. For the AC term, the DC forcing function (v_f) has to be set equal to zero and the derivative term (d/dt) has to be replaced by $j\omega$. This produces a set of phasor equations. In this case the phasor form of the forcing function is:

$$\tilde{\lambda}_d = -\frac{E_q}{\omega} \quad (e)$$

The analytic solution to this set of phasor equations is:

$$\begin{aligned}\tilde{i}_d &= -\frac{E_q}{X_d} \left\{ 1 + \frac{j \frac{X_{Md}^2}{X_d} [R_f + R_D + j(X_{ff} + X_{DD} - 2X_{Md})]}{\left(R_f + jX_{ff} - j \frac{X_{Md}^2}{X_d}\right) \left(R_D + jX_{DD} - j \frac{X_{Md}^2}{X_d}\right) + \left(X_{Md} - \frac{X_{Md}^2}{X_d}\right)^2} \right\} \\ \tilde{i}_d &= -\frac{E_q}{Z_{eq}} e^{j(\omega t + \phi)}\end{aligned}\quad (f)$$

In the time domain this solution produces the oscillating term:

$$i_{dosc1} = -\frac{E_q}{Z_{eq}} \cos(\omega t + \phi) \quad (g)$$

If this term is added to the standard definition for i_d , the equation is not entirely consistent. In particular, the initial current is not zero. What it has been usually done to avoid this problem is to define the oscillating term as:

$$i_{dosc2} = -\frac{E_q}{X_d} \cos(\omega t) \quad (h)$$

In this case X_d is defined as the equivalent of the parallel reactances in the star circuit model of the machine, which is what we get if R_f and R_D are neglected in equation (f).

For the IEEE benchmark model of the machine used in the paper, the difference between the two possible magnitudes of the oscillating term can be calculated using the SOLVER-Q complex setup shown below. This produces a calculated subtransient impedance (X_d) 0.2118% off the specified value. This agrees with Dr. Dommel's calculation.

```
{ Machine equations }
i1*w*lfss = -rf*ifss;
i1*w*ldpss = -rdp*idpss;
xd * idss + xmd* ifss + xmd * idpss = eq;
xmd* idss + xf * ifss + xmd * idpss = w*lfss;
xmd* idss + xmd* ifss + xdp * idpss = w*ldpss;
```

The small discrepancy between these two values obtained for the oscillating part of the sudden-short circuit current does not appear in any other of the terms that form part of i_d . For these terms, the eigenvalue analysis of the decoupled problem gives the same values as the ones obtained from the definition in the standards. This suggests that equations (22a) and (22b), which are based on the definition of X_d as the parallel of the reactances in the star circuit, are in slight error. These equations are also used by the Canay procedure. H. Dommel outlines a procedure to eliminate this approximation. We consider an alternative. The error can be avoided by using the open circuit time constants directly, together with X_d' and X_d'' . To do this the following equations have to be added to the analysis:

$$\begin{aligned}T_{do}' + T_{do}'' &= \frac{L_{ff}}{R_f} + \frac{L_{DD}}{R_D} \\ T_{do}' T_{do}'' &= \frac{L_{ff} L_{DD} - M_d^2}{R_f R_D}\end{aligned}\quad (i)$$

These equations come directly from the eigenvalue analysis of the open circuit machine equations, where R_a and the approximations on the transient and subtransient reactances do not play any role (i_d and i_q are zero).

A second cause of slight error comes from the assumption that the sinusoidal terms of the field and damper winding fluxes do not influence the determination of the initial conditions, also a consequence of neglecting the resistances R_f and R_D in the definition of X_d . This error can be suppressed by adding to our problem the set of equations shown above in the SOLVER-Q setup for the AC steady-state analysis ($R_a = 0$). The equations used to determine the scaling factors for the eigenvectors have to be replaced by equations (j) that include these oscillating terms at $t=0$.

$$\begin{aligned}\lambda_{f0} - \lambda_{fDC} &= \alpha_1 \lambda_{f1} + \alpha_2 \lambda_{f2} + \text{Re}(\tilde{\lambda}_{fAC}) \\ \lambda_{D0} - \lambda_{DDC} &= \alpha_1 \lambda_{D1} + \alpha_2 \lambda_{D2} + \text{Re}(\tilde{\lambda}_{DAC})\end{aligned}\quad (j)$$

These implicit equations can be used without any further manipulation directly into a SOLVER-Q setup. The solution to this problem for the d axis gives slightly different results from the ones shown in the paper for the two winding machine model:

$$\begin{aligned}R_f &= 0.00140679 & X_f &= 1.72178915 \\ R_D &= 0.00406988 & X_D &= 1.66545811\end{aligned}$$

Not mentioned by either discussor or in the paper is perhaps a greater potential source of error. The standard definition neglects the possible existence of quadrature axis terms in the direct axis sudden-short current. However, the presence of R_a may require the consideration of a more complete form for this current if sudden-short circuit tests are to be used:

$$i_d = \beta_1 + \beta_2 e^{-t/T_d'} + \beta_3 e^{-t/T_d''} + \beta_4 e^{-t/T_q'} + \beta_5 e^{-t/T_q''} + \beta_6 e^{-t/T_a} \cos(\omega t + \phi)$$

As a result, we believe that a more accurate way of calculating the circuit parameters can be achieved by using the results of the open circuit tests, where the coupling between axis does not exist.

Professor Hammons starts his discussion with an excellent overview of the contributions of our paper. We could not have stated them better ourselves. Then he proceeds to discuss the merits of considering a sub-transient winding (probably due to eddy current effects) on machine transients. Introducing a third winding in the simulation of the synchronous machine yields a model closer to reality, specially for the analysis of torques and rotor swings. He then asks whether the proposed procedure would be capable of producing

reasonable circuit parameters for such model. We show in this closure that the answer is yes. We created a SOLVER-Q setup that included the additional windings for the d-axis. The transient, subtransient and sub-subtransient parameters were those given by Hammons. Just a few changes had to be made to the original file shown in the paper to include into the simulation the third winding. We calculated initial guesses using the Canay procedure. *Good initial guesses proved essential for good convergence. These were obtained from the two winding model.*

The results obtained (for $R_a = 0$) are as follows:

$R_f = 0.001127819$	$X_f = 1.955267642$
$R_D = 0.045878795$	$X_D = 2.821476343$
$R_{DD} = 0.474653206$	$X_{DD} = 2.184718014$
$R_g = 0.015743180$	$X_g = 2.828957507$
$R_Q = 0.064774170$	$X_Q = 2.259965955$
$R_{QQ} = 0.391118670$	$X_{QQ} = 1.855319504$

We regard these values as reasonable, based on the set of measurements available to us and on the values calculated

with Canay's equations from the set of data for the two winding model, which are shown below.

$R_f = 0.0011408$	$X_f = 1.9398053$
$R_D = 0.1070040$	$X_D = 2.5155586$
$R_g = 0.0174220$	$X_g = 2.4075808$
$R_Q = 0.1198928$	$X_Q = 1.9935677$

The inclusion of R_a into the determination of the parameters should not be a problem. In this case the d and q axes would be considered together, and with good initial guesses obtained from the decoupled analysis, the solution would be straight forward.

We conclude by stating that the even with a tool such as SOLVER-Q, we encountered convergence difficulties. Very good initial guesses (obtained from classic methods) were required in some cases. We look forward to improved versions of this type of software that will have even more robust global convergence characteristics.

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