Magnetohydrodynamic simulations of direct current helicity injection for current drive in tokamaks

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Magnetohydrodynamic (MHD) computations of direct current (DC) helicity injection for tokamak-like configurations show current drive with no “loop voltage” in a resistive, pressureless plasma. Self-consistently induced, hollow current profiles are unstable to resistive modes that partially relax the profile through the MHD dynamo mechanism. The resulting current profiles remain quite hollow, however, and tokamaks are not generated. The current driven by the fluctuations leads to closed contours of average poloidal flux, but the 1% fluctuation level is large enough to produce a region of stochastic magnetic field. A limited Lundquist number (S) scan from 2.5×10^3 to 2×10^4 indicates that both the fluctuation level and current profile relaxation increase with S . A simple quasilinear power scaling is consistent with these results at low S and suggests that at larger S , the fluctuation level decreases. © 1996 American Institute of Physics. [S1070-664X(96)00202-5]

I. INTRODUCTION

An innovative approach to the problem of steady-state current drive for tokamaks is direct current (DC) helicity injection.¹ The basic concept is to intercept a fraction of the magnetic flux with electrodes and place a potential along the open field lines. Current is driven directly along the intercepted lines, and a desirable current distribution results from magnetic fluctuations, which serve to reduce current density gradients. Indeed, the Taylor hypothesis² is often used as a justification for expectations of complete relaxation, where the J/B ratio is constant throughout the plasma. In this paper, we present results of three-dimensional (3-D) nonlinear resistive magnetohydrodynamic (MHD) simulations for this concept. To our knowledge, these are the first published solutions of basic equations that test current drive via DC helicity injection for tokamak configurations. (Simulations of DC injection in reversed-field pinches for the purpose of profile control and stabilization have been reported.³)

Experimentally, DC helicity injection has been explored on the Current Drive Experiment (CDX),^{4,5} the Continuous Current Tokamak (CCT),⁵ and the Helicity Injected Tokamak (HIT).⁶ The CDX (and CCT) configuration uses a toroidally local cathode that is mounted near the surface of the chamber. The cathode emits an electron beam in one toroidal direction, and a vertical magnetic field deflects the beam toward the center of the chamber. Probe measurements indicate that the current is peaked within a poloidal cross section of a 3 cm minor radius.⁴ The discharge is adjacent to the cathode and does not occupy the entire 10 cm minor radius chamber. In HIT, a magnetized coaxial plasma gun is used as a source of plasma and for toroidally symmetric helicity injection. Profile information—reconstructed from surface probe data by an equilibrium code—indicates substantial relaxation.⁶ The discharge fills the chamber and generates closed poloidal flux contours.

The goal of our study is to examine three important issues for using DC helicity injection as an exclusive means to sustain the tokamak. The issues are (1) current drive without

“loop voltage,” (2) the extent of relaxation, and (3) the level of magnetic fluctuation. The first issue is the motivation for the concept—a current drive scheme where steady-state operation is possible. The second and third issues reflect how well the magnetic fields confine plasma. If there is little relaxation, then the magnetic fields provide little insulation between the plasma and the electrode surfaces. If there is substantial relaxation, but the magnetic fluctuation level is large enough to prevent the formation of closed flux surfaces, energy and particle confinement will be poor relative to conventional tokamaks.

We investigate these issues from first principles by applying MHD computation to configurations that have the same magnetic field topology as HIT. The simulations have no applied “loop voltage,” and electrostatic helicity injection is generated through boundary conditions on the electric and magnetic fields. When the injection is weak, the vacuum magnetic fields guide the current uniformly through the chamber. The resulting current profile is flat, but the field lines are open. As the injection is increased, the current profile becomes increasingly hollow. Resistive current-gradient-driven modes become unstable, but their growth is limited by both quasilinear and nonlinear saturation mechanisms. The quasilinear saturation stems from a MHD dynamo, where the fluctuation-induced electric field enhances interior current. This relaxes the current profile and produces closed poloidal flux contours, amplifying the initial open poloidal flux. The simulations show 20%–50% flux amplification when the applied potential is large. In these cases, the presaturated current profile is extremely hollow, and the relaxation is slight. In addition, magnetic field-line puncture plots show that the 1% fluctuation level is sufficiently large to make the field stochastic in the region of relaxation. Therefore, the closed contours do not represent closed flux surfaces.

The topics addressed in this paper are not unrelated to the current profile relaxation considered by Weening and Boozer,⁷ but the current drive mechanisms and the mathematical approaches are different. In both cases, resistive MHD instabilities lead to the relaxation. However, we con-

sider externally driven current from an electrostatic potential, while Weening and Boozer focus on bootstrap currents driven by a pressure gradient in the interior of the plasma. Also, the MHD code used for this work models the fluctuations in detail, including nonlinear interactions and feedback on the mean fields—in contrast to the quasilinear “current viscosity” approach used by Weening and Boozer.

This paper is organized as follows: A brief description of the code is presented in Sec. II, along with a discussion of the geometry and the boundary conditions used to simulate the injection. General results illustrating the current drive and relaxation are presented in Sec. III, and S -scaling information is presented in Sec. IV. We conclude with a discussion of the results and their implications in Sec. V.

II. NUMERICAL MODEL

The DC helicity injection is modeled with the zero-beta version of the DEBS MHD code⁸ that solves initial value problems in the geometry of a straight periodic cylinder. This code has been used extensively for reversed-field pinch simulations, but various versions have also been applied to astrophysics and solar physics phenomena. The coupled partial differential equations in MKS units are

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} = \mathbf{V} \times \mathbf{B} - \eta \mathbf{J}, \quad (1)$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} = -\rho \mathbf{V} \cdot \nabla \mathbf{V} + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}, \quad (2)$$

where $\mathbf{B} = \nabla \times \mathbf{A}$. The equations allow compressible flow, but the density (ρ) is not evolved and is uniform throughout the domain. The resistivity (η) and viscosity (ν) are also uniform in these simulations. The geometry makes Fourier representation appropriate in both the axial (toroidal) and azimuthal (poloidal) directions. Simulations for the nonlinear S scaling in Sec. IV A have $|n| \leq 5$ axial modes and $0 \leq m \leq 10$ azimuthal modes resolved. This covers the dominant resonant modes and the important nonlinearly coupled modes. The comparisons in Sec. III are obtained with a reduced resolution of $|n| \leq 1$ and $0 \leq m \leq 5$ or $0 \leq m \leq 10$. This reduction has a moderate impact and does not change the salient features of the solutions—details are given in Sec. IV A. The number of cells needed in the radial direction depends on S ; the maximum used here is 509 for the $S = 2 \times 10^4$ case.

The boundary conditions at the outer wall of the cylinder ($r = a$) are used to produce the helicity injection. Through the vector potential, $B_r(a)$ is specified to produce a vertical field, and $E_\theta(a)$ is specified to apply the electrostatic potential. The simple arrangement used for most of this study is illustrated in Fig. 1 and is topologically similar to the HIT experiment. The fields that penetrate the plasma surface can be considered vacuum fields. They effect the helicity injection, as the rate of change in helicity is

$$\frac{dK}{dt} = -2 \int \mathbf{E} \cdot \mathbf{B} \, d\mathbf{x} + 2 \int \mathbf{E}_v \cdot \mathbf{B}_v \, d\mathbf{x}, \quad (3)$$

where $K = \int \mathbf{A} \cdot \mathbf{B} \, d\mathbf{x} - \int \mathbf{A}_v \cdot \mathbf{B}_v \, d\mathbf{x}$ is a relative helicity, and the v -subscripted fields are the vacuum fields.⁹ The integrals

FIG. 1. Schematic of the applied fields for DC injection (poloidal slice illustrated). Only the region within the periodic cylinder contains plasma and is in the problem domain. The orientation of $\theta = 0$ is shown for the I_{inj} definition.

extend only over the plasma volume, and the use of the vacuum fields maintains gauge independence when $\mathbf{B}(a) \cdot \mathbf{n} \neq 0$, provided $\mathbf{B}_v(a) \cdot \mathbf{n} = \mathbf{B}(a) \cdot \mathbf{n}$ and $d[\mathbf{B}_v(a) \cdot \mathbf{n}]/dt = d[\mathbf{B}(a) \cdot \mathbf{n}]/dt$. For DC injection, the second term on the right side of Eq. (3) reduces to $-2 \int \phi \mathbf{B} \cdot \mathbf{n} \, dS$, with ϕ being the applied electrostatic potential on the surface of the plasma.

The velocity at the boundary is specified to be the local $\mathbf{E} \times \mathbf{B}$ drift based on the applied electrostatic field and the local magnetic field. This avoids generating a poloidal surface current in response to the potential, and is a reasonable approximation of experimental conditions. In HIT, for example, gas is ionized in the plasma gun and accelerated by a $\mathbf{J} \times \mathbf{B}$ force, so that the “confinement” region, which is downstream of the gun, is subject to a flow of plasma.⁶ The problem domain in the simulations represents a “confinement” region, and a gun region or surface layer is not modeled. In the limit of $\mathbf{B}_v = 0$, applying the potential produces a drift across the chamber without inducing current.

III. RESULTS

We classify our simulations by the level of applied electric field. In the weak-drive limit, the parallel current profile is flat, and there are no MHD modes and no flux surfaces. This limit is relatively uninteresting, but it demonstrates the sustainment of axial current without “loop voltage.” When the applied electric field is large, the induced current distorts the poloidal field, which in turn distorts the current path. This produces hollow current profiles that are unstable to resistive MHD modes.

The simulations are initialized with a large uniform axial magnetic field, a vertical magnetic field, and small random perturbations in the vector potential. The normal component

FIG. 2. Weak-drive steady-state results of (a) parallel current ($\lambda = \mu_0 a \langle \mathbf{J} \cdot \langle \mathbf{B} \rangle / \langle B \rangle^2$) and (b) axially symmetric poloidal flux contours from an $S=5000$, $R/a=6$ simulation. The magnitude of $\mathbf{E}_v$ is $0.8B_z(0)a/\tau_r$.

of magnetic field at the surface determines $\mathbf{B}_v$ and is fixed for all time. The electrostatic potential is initially zero, and its level is increased over the first 0.1 or $0.2\tau_r$, where $\tau_r = \mu_0 a^2 / \eta$ is the resistive diffusion time. After this ramp phase, the potential is held constant to sustain the injected current against resistive loss. This current is driven by the component of $\mathbf{E}_v$ that is parallel to the magnetic field, which also injects helicity. Current enters the domain on the top of the cylinder in Fig. 1 and leaves when it reaches the bottom. The current density vectors are primarily axial, but there is no “loop voltage” because the axial component of electric field from $\mathbf{V} \times \mathbf{B}$ cancels ηJ_z . If the applied potential is reduced, the current decays.

For weak-drive cases, the magnetic field remains essentially unaltered from vacuum conditions, and current simply follows the field lines. This amounts to the generation of a magnetically guided, diffuse electron beam. Results from a weakly driven simulation show a “parallel current” profile, $\lambda = \mu_0 a \langle \mathbf{J} \cdot \langle \mathbf{B} \rangle / \langle B \rangle^2$, that is rather flat [Fig. 2(a)] [$\langle \rangle$ denotes the ($m=0$, $n=0$) Fourier mode or the θ - z mean]. In addition, the axially symmetric poloidal flux contours are only slightly distorted from their vacuum state [Fig. 2(b)]. In the limit of a vanishing electrostatic field, the ratio of axial current to injected current is determined by the magnitude of the vertical magnetic field: $I_z/I_{inj} = aB_z/4RB_v$, where

FIG. 3. Temporal evolution of the axially symmetric poloidal flux contours from a strongly driven $S=5000$, $R/a=6$ simulation. The four plots show the (a) initial conditions prior to the application of electrostatic potential, (b) the point of half potential, (c) full potential with growing instabilities, and (d) the final relaxed state with saturated fluctuations.

$$I_z = \int_0^{2\pi} \int_0^a J_z r \, dr \, d\theta,$$

$$I_{inj} = - \int_0^\pi \int_0^{2\pi R} J_r \, dz \, a \, d\theta,$$

and the orientation of $\theta=0$ is as shown in Fig. 1. For the simulated conditions, where the cylinder aspect ratio (R/a) is 6 and $B_v=0.0125$ (normalized to the initial B_z), the ratio is 3.33 in the limit, while the simulation result is 3.23.

When the current drive is increased, the self-induced poloidal magnetic field becomes larger than the vertical field. This excludes injected current from the center, and unstable, hollow current profiles develop. In the following sections, we discuss the relaxation and flux amplification (Sec. III A), the vertical magnetic field and applied voltage magnitudes (Sec. III B), and some special cases that indicate the significance of other parameters (Sec. III C).

A. Relaxation and flux amplification

When the applied electrostatic potential is ramped to a large level, the poloidal flux is forced toward the surface of the cylinder. This is clearly shown in frames (a)–(c) of Fig. 3, a time sequence of axially symmetric poloidal flux contours. The distortion of the poloidal flux is accompanied by a distortion of the current path, and the λ profile becomes quite hollow when the full potential is applied. The solid λ trace in Fig. 4 shows the unrelaxed state corresponding to the flux plot of Fig. 3(c). Resistive MHD modes are unstable at this point but have not grown to an appreciable level. The modes subsequently saturate and relax the λ profile to the dashed trace in Fig. 4. This relaxation produces the closed flux con-

FIG. 4. Parallel current profiles before and after the saturation of the fluctuations. This comparison shows a modest level of relaxation.

tours in Fig. 3(d). The change in the λ profile is small, but the amplification of poloidal flux is more than 30%. In this simulation, a true steady state is reached approximately $0.5\tau_r$ after the initial saturation.

The MHD modes induce a net Poynting flux and relax the current profile via the dynamo electric field, $\mathbf{E}_f = -(\mathbf{v} \times \mathbf{b})$. [Small case fields are modes other than the (0,0).] The $\mathbf{E}_f \cdot \langle \mathbf{J} \rangle$ distribution from the resonant $n=1$ modes [Fig. 5(a)] is positive in the exterior and negative in the interior. This represents a MHD dynamo that is driving interior current at the expense of exterior current. It is comparable to the MHD dynamo in the reversed-field pinch,¹⁰ but the $\mathbf{E}_f \cdot \langle \mathbf{J} \rangle$ distribution is flipped. Here, the electromagnetic energy transport from the resonant modes is inward. Figure 5(b) shows a comparison of the $\langle \mathbf{e} \times \mathbf{b} \rangle_r$ Poynting flux from the resonant $n=1$ modes and that from the applied $n=0$ fields. The poloidal field distortion excludes the $n=0$ power flux from the interior, and the resonant modes provide more penetration for a small amount of the total power. The Poynting flux from a given resonant mode is not confined to its tearing layer; however, it is radially localized. Since the λ gradient is near the wall, there is little drive for the fluctuations in the center of the cylinder. On the axis itself, only $m=1$ modes can have nonzero poloidal components of $\mathbf{v}$ and $\mathbf{b}$ to drive $\langle J_z \rangle$. None of the cases examined here have been driven so hard that an $m=1$ mode is resonant anywhere except near the wall.

Relaxation unfortunately comes with a price—the magnetic fluctuation level. The volume average of the fluctuation level is on the order of 1% of the axial field. The spectrum is broad and peaked at the $n=1$ mode that is resonant nearest the wall (see Figs. 6 and 7). This level of fluctuation is large enough to make the field lines stochastic in the region where the dynamo drives current. The comparison of poloidal flux contours with a field-line puncture plot, shown in Fig. 8, demonstrates that the flux surfaces formed by relaxation are not truly closed.

FIG. 5. Relaxed-state distributions of (a) dynamo power density from $n=1$ modes and (b) radial Poynting fluxes induced by the applied fields ($n=0$ modes) and the resonant fluctuations ($n=1$ modes) overlaid with different y-axis scales.

B. Applied field variations

We have varied the vertical field strength and the applied voltage to find better performance, i.e., less hollow current profiles with substantial flux amplification. These tests were

FIG. 6. Magnetic energy spectra for the $n \neq 0$ modes from a fully resolved $S=2 \times 10^4$, $R/a=6$ simulation. There are temporal fluctuations, so spectra are averaged over $0.2\tau_r$, and standard deviations are roughly 50% for all modes. The $m > 0$, $n < 0$ modes are denoted by the conjugate indices.

FIG. 7. Safety factor ($q = r\langle B_z \rangle / R\langle B_\theta \rangle$) profile from the $S=2 \times 10^4$, $R/a=6$ simulation. The unusual shape is generated self-consistently by the DC helicity injection, and resonance surface locations are indicated for the largest modes. (This trace is actually the inverse of the time average of $R\langle B_\theta \rangle / r\langle B_z \rangle$. Fluctuations occasionally force $\langle B_\theta \rangle$ to zero near the axis.)

conducted with simulations having $S=5000$, $R/a=6$, and only $|n| \leq 1$ modes resolved. The results are listed as cases A–F in Table I, and they indicate the following trends: First, I_z predictably increases with either E_v or with B_v . Second, both the fluctuation level ($|n| \neq 0$ modes only to exclude the applied perturbations) and the poloidal flux amplification increase with E_v . Third, I_z/I_{inj} decreases with increasing E_v or

B_v —the current path is more poloidal when either of the applied fields is increased, an effect not fully compensated by the relaxation. (The ratios are also much less than what is produced in the weak-drive limit.) While the flux amplification provides a measure of the dynamo-driven current, the I_z/I_{inj} ratio depends on both the applied fields and the relaxation provided by the dynamo. Thus, the I_z/I_{inj} ratio provides a measure of relaxation only among cases where the applied fields are the same.

Although the flux amplification increases with E_v (and, to some extent, with B_v), increasing the applied voltage does not enhance the flattening of the λ profile. The profiles for cases C and D are compared in Fig. 9. The parallel current in the relaxed region is larger for case D, but the spike of current at the wall is much larger, so the entire profile is actually more hollow. Also, the relaxed region is not pushed farther in toward the axis. Increasing B_v makes the λ profile more hollow as well (see Fig. 10), but this does extend the relaxed region toward the axis. The difference lies in the effect of the applied fields on the resonance surfaces of the unstable modes. Increasing either E_v or B_v tends to decrease the safety factor ($q = r\langle B_z \rangle / R\langle B_\theta \rangle$) at the wall. However, increasing E_v alone forces the injected current path closer to the wall, so the resonance surfaces of the largest modes do not move inward. In contrast, increasing B_v tends to move the current path and the resonance surfaces away from the wall.

These results show that it is not possible to adjust the externally controllable parameters to generate current profiles that are closer to tokamak profiles, while retaining substantial flux amplification. The current profile must be quite hollow to generate significant dynamo activity with the large axial field. Further increasing the applied fields produces more MHD dynamo, but the resulting current profile is more hollow, not less.

The distinction between changes in E_v and B_v leads to a more general conclusion—that the final state cannot depend on the helicity injection rate alone. The rate of helicity injection is $2\int \mathbf{E}_v \cdot \mathbf{B}_v d\mathbf{x}$, and it is directly proportional to the product of E_v and B_v for the cases with vertical fields. Cases D and F in Table I, for example, have the same helicity injection rate, but the resulting current profiles in Figs. 9 and 10 are not the same.

C. Special cases

Additional simulations have been performed to assess the significance of parameters other than the magnitudes of the applied fields. In case G of Table I, the viscosity has been reduced by a factor of 4 from the value used for all other simulations—the standard value is $\nu = a^2 \rho / \tau_r$. In all other respects, case G is the same as case E, and it is clear that the viscosity is small enough that it has little impact on the results. Case H is the only exception to the flat resistivity prescription. The normalized profile is $[1 + (r/a)^2]^2$, and the effect is substantial. The large resistivity at the wall inhibits the formation of an unstable current profile, so the dynamo is not observed at the voltage level applied.

The distribution of the vacuum poloidal field over the surface of the cylinder has also been considered. Case I is the

FIG. 8. Comparison of (a) axially symmetric flux contours and (b) a field-line puncture plot for the $S=2 \times 10^4$, $R/a=6$ simulation.

TABLE I. Helicity injection simulation results with $S=5000$ and $|n|\leq 1$ modes resolved. The base parameters are $\nu=a^2\rho/\tau_r$, uniform resistivity, vertical applied fields, $R/a=6$, and constant axial flux; exceptions are noted for cases G–L. Here B_v is normalized by the initial B_z , E_v is normalized by $B_z(0)a/\tau_r$, and $S_k=2\int \mathbf{E}_v \cdot \mathbf{B}_v d\mathbf{x}$ is the normalized helicity injection rate.

Case	E_v	B_v	S_k	I_{inj}	I_z	I_z/I_{inj}	Flux amp.	rms (b) all $n/n\neq 0$
A	20	0.008 33	39.5	0.80	0.52	0.65	<1%	1.9%/0.23%
B	25	0.008 33	49.3	1.25	0.64	0.51	<1%	2.1%/0.25%
C	20	0.0125	59.2	1.49	0.78	0.52	32%	2.6%/1.3%
D	30	0.0125	88.8	3.49	1.15	0.33	51%	3.0%/1.4%
E	15	0.018 75	66.6	1.89	0.86	0.46	22%	3.3%/1.4%
F	20	0.018 75	88.8	3.36	1.16	0.35	36%	3.6%/1.6%
G ^a	15	0.018 75	66.6	1.87	0.87	0.47	21%	3.3%/1.4%
H ^b	30	0.0125	88.8	0.48	0.43	0.89	$\leq 1\%$	2.1%/2.6 $\times 10^{-5}\%$
I ^c	30	0.0125 ^c	73.8	1.77	0.96	0.54	<1%	2.4%/0.6%
J ^d	20	0.0125	19.7	0.58	0.77	1.33	<1%	2.8%/0.6%
K ^e	20	0.0125	98.7	2.49	0.78	0.31	30%	2.5%/0.9%
L ^f	20	0.0125	59.2	1.45	0.77	0.53	32%	2.6%/1.3%

^aViscosity is reduced by 4.

^bResistivity is a function of radius, $\eta(r)=(1+r^2)^2$.

^cHere $B_v(a)$ is from an axial line current located at $r=1.3a$. The resulting poloidal flux through the cylinder is equivalent that in case C.

^d $R/a=2$.

^e $R/a=10$.

^fAxial flux is allowed to change.

exception to the vacuum magnetic field configuration that is illustrated in Fig. 1. For this case the surface distribution of $\mathbf{B}_v \cdot \mathbf{n}$ is that from a line of current, parallel to the cylinder axis and located $0.3a$ outside the cylinder wall in the plane of zero potential. The total poloidal flux passing through the cylinder is equivalent to the $B_v=0.0125$ cases with the vertical field configuration. The E_v distribution is unchanged. This produces a current profile that is less hollow than case D. The dynamo is relatively weak, but it exhibits the same kind of behavior as it does in the other cases.

Another geometric parameter that affects the relaxation is the aspect ratio. For a given λ profile, R/a affects the magnitude of the safety factor at all radii, so it determines which modes are resonant. Case J, and another similar simulation with $S=10^4$, have $R/a=2$. The dominant $n=1$ modes have poloidal mode numbers that are roughly three times larger than in the comparable case C, so they tend to be more stable. In addition, nonlinear coupling between the ($m=1$, $n=0$) source of power (the vertical applied fields) and the low m , $n=1$ modes is inhibited, because modes with $m<4$ are not resonant. In the simulations with $R/a=6$, the low m ,

FIG. 9. Parallel current profiles from simulations with different applied potentials. The case with larger potential generates more interior current through fluctuation activity, but the directly driven current at the wall is much larger.

FIG. 10. Parallel current profiles from simulations with different applied vertical fields. Increasing the vertical field enhances the penetration of the fluctuation-driven current.

$n=1$ modes are enhanced by this nonlinear coupling, and thereby generate more dynamo current.¹¹ Case K has $R/a=10$ to determine if the trend continues for aspect ratios larger than the usual $R/a=6$. The (1,1) mode is resonant in this case, but the poloidal flux amplification is virtually the same as in case C. (We usually use $R/a=6$, so that fewer modes, relative to smaller R/a cases, need to be resolved to accurately capture the important quasilinear and nonlinear interactions.)

A final special case considers the effect of allowing the axial flux in the cylinder to change at a rate determined by an external inductance. In case L, $\langle E_\theta \rangle$ at the wall, which is proportional to a “gap” voltage for an imaginary conducting shell that is cut in the axial direction, is allowed to be non-zero. Its value varies with the rate of change of poloidal shell current, $2\pi R \langle B_z(a) \rangle / \mu_0$. Otherwise, the conditions are again the same as in case C. There results a small amount of paramagnetism (0.6%) from a pinching effect with little impact on the relaxation.

These special cases show that while geometric and resistivity-related effects are important, the general behavior remains consistent with the basic configuration. We therefore concentrate on the basic configuration and expect that the applied field trends in Sec. III B and the S scaling in Sec. IV may be generalized to other configurations.

IV. LUNDQUIST NUMBER SCALING

We have investigated the resistivity dependence of the fluctuation level and relaxation by performing computations over a limited range of S ($2.5 \times 10^3 - 2 \times 10^4$). This range is close to contemporary experiments (e.g., in HIT, $S \sim 10^5$). However, it is far from reactor conditions, so scaling information is relevant. Computations at larger S are possible, but temporal and spatial resolution requirements are prohibitive at present. Numerical results from the full nonlinear system are presented in Sec. IV A. The interpretation of these results is facilitated by quasilinear simulations (Sec. IV B) and a heuristic scaling (Sec. IV C).

A. Nonlinear computations

For the nonlinear S scan, we employ the parameters of case C in Table I. The normalized electric field [$E_v \tau_r / a B(0)$] is held constant, and the normalized viscosity [$\nu \tau_r / a^2 \rho$] is fixed at unity. To produce accurate fluctuation levels, full modal resolution ($0 \leq m \leq 10$, $|n| \leq 5$) is used for each simulation. The results show that the current profile flattening and the magnetic fluctuation level increase with S over this range (see Table II). The change in profile flattening is evident from the I_z/I_{inj} ratios—the applied fields are the same for all cases, and it is illustrated by the comparison of λ profiles in Fig. 11. The spike of current at the wall is reduced and the bump from relaxation is increased as S is increased.

These simulations also show the significance of the $|n| > 1$ modes. The flux amplification is smaller in case N than it is in case C of Table I, where these modes are not resolved. For the fully resolved cases, some fraction of the power sustains a nonlinear cascade to $n > 1$ modes. These modes generate dynamo current drive, like the $n=1$ modes,

TABLE II. Lundquist number scaling from simulations with $|n| \leq 5$ modes resolved. All cases use the usual conditions of vertical applied fields, $E_v=20$, $B_v=0.0125$, and $R/a=6$. Results are averaged over $0.2\tau_r$ after saturation.

Case	S	I_{inj}	I_z	I_z/I_{inj}	Flux amp.	rms (b) all $n/n \neq 0$
M	2500	1.86	0.77	0.42	<1%	2.81%/0.16%
N	5000	1.47	0.76	0.51	22%	2.71%/1.23%
O	10 000	1.37	0.79	0.58	33%	2.50%/1.33%
P	20 000	1.26	0.79	0.63	42%	2.39%/1.39%

but they tend to produce a larger Ohmic dissipation rate. In addition, the cases with $|n| > 1$ modes do not settle into a true steady state. The resonant modes fluctuate on a time scale between the Alfvén time and the diffusion time, and quasi-steady conditions are sustained after relaxation. The information in Table II is therefore averaged over a period that is long in comparison with the temporal fluctuations.

B. Quasilinear computations

A set of computations without nonlinear interactions has been completed to examine the resistivity dependence resulting from the quasilinear (self-interaction) terms alone. In these cases, the modes interact with the mean field but not with each other, and the advective term in Eq. (2) is eliminated. The simulations have S ranging from 5×10^3 to 2×10^4 , and other parameters are the same as those used for the nonlinear S scan. The results in Table III show that the trends of increasing fluctuation level and current profile flattening are similar to those produced by the full nonlinear simulations. However, there are only three significant Fourier modes: the mean field, the (1,0) applied vertical field, and one resonant $n=1$ mode. The final state is free of temporal fluctuations.

In the absence of nonlinear coupling, the resistivity dependence may be interpreted through a simple power balance

FIG. 11. Parallel current profiles from fully resolved simulations with different S . Each profile is a time average over approximately $0.2\tau_r$. This comparison shows that relaxation increases with S .

TABLE III. Results of the quasilinear cases. The parameters are the same as those used for the S scan, but only $0 \leq m \leq 5$ and $|n| \leq 1$ modes are resolved.

Case	S	I_{inj}	I_z	I_z/I_{inj}	Flux amp.	rms (b) all $n/n \neq 0$
R	5000	1.64	0.77	0.47	$\leq 1\%$	3.21%/0.66%
T	10 000	1.58	0.78	0.49	1.2%	3.06%/0.94%
U	20 000	1.40	0.77	0.55	9.1%	3.03%/1.55%

for the $n=1$ mode. The radial Poynting flux from this mode vanishes at the wall, so Poynting's theorem gives $\int \mathcal{R}e(\mathbf{e}_{n=1} \cdot \mathbf{j}_{n=1}^*) d\mathbf{x} = 0$ in steady state. We expand the quasilinear electric field into $\mathbf{e}_{n=1} \equiv -\mathbf{v}_{n=1} \times \langle \mathbf{B} \rangle + \eta \mathbf{j}_{n=1}$, assuming that the mean flow is small. Then, a quasilinear force balance, $\mathbf{j}_{n=1}^* \times \langle \mathbf{B} \rangle \equiv -\langle \mathbf{J} \rangle \times \mathbf{b}_{n=1}^*$, is applied to produce

$$\int_{r_s}^a \mathbf{E}_f \cdot \langle \mathbf{J} \rangle r dr \equiv \int_{r_s-\epsilon}^{r_s+\epsilon} \eta j^2 r dr - \int_0^{r_s} \mathbf{E}_f \cdot \langle \mathbf{J} \rangle r dr, \quad (4)$$

where $\mathbf{E}_f = \mathcal{R}e(-\mathbf{v}_{n=1} \times \mathbf{b}_{n=1}^*)$. Ohmic dissipation is significant within the resistive layer ($r_s - \epsilon < r < r_s + \epsilon$), and the resonance surface ($r = r_s$) is used to divide the dynamo into two parts. For $r > r_s$, $\mathbf{E}_f$ absorbs power from the mean current which is sustained by the applied $n=0$ fields, so the left-hand side represents power input for the mode. For $r < r_s$, the second term on the right-hand side represents power transfer to the current in the interior. This scenario is similar to the situation shown in Fig. 5(a), where several modes contribute to the $\mathbf{E}_f \cdot \langle \mathbf{J} \rangle$ distribution. We observe that as the resistivity is decreased in the simulations (by increasing S), the Ohmic loss becomes a smaller fraction of the input power. The mode saturates at a larger level, with a larger dynamo current drive, so the relaxation is enhanced.

A comparison of the nonlinear simulations in Table II with the quasilinear simulations in Table III shows that nonlinear effects are important. The nonlinear simulations produce much more poloidal flux amplification, though the fluctuation levels are not necessarily larger. Nonetheless, the quasilinear terms provide the resonant modes with most of their input power, and the current profile flattening results from quasilinear terms, so we surmise that the quasilinear S scaling is an important part of the full nonlinear S scaling.

C. Heuristic scaling

Equation (4) may be converted into a heuristic scaling for the quasilinear fluctuation level. This serves two purposes. First, it illustrates the different resistivity dependencies of the terms in Eq. (4), which lead to the scaling of the fluctuation level and dynamo-driven current. Second, it may be used to extrapolate results beyond the range of S that has been simulated.

Upon saturation, the radial profiles of the perturbed velocity and magnetic field remain close to their linear forms. We therefore assume that changes in the radial profiles are not significant in the integrals in Eq. (4). In the same spirit, J_o and J_i represent the mean axial current density outside and inside r_s , respectively. The former is essentially fixed by the applied fields, and the latter is sustained by the dynamo electric field, $\eta J_i \sim v b$. We relate v and b through Faraday's

law in the ideal regions, $v \sim \gamma b$, using a linear growth rate that scales as a fractional power of resistivity, $\gamma \sim \eta^\nu$. (For example, $\nu=3/5$ for tearing modes.¹²) The dynamo electric field is then proportional to $\eta^\nu b^2$. The Ohmic loss for the mode is proportional to $\epsilon(\eta j^2)$, where $j \sim b/\epsilon$, and we use a resistive skin depth as the width of the tearing layer, $\epsilon \sim (\eta/\gamma)^{1/2}$. Incorporating these simplifications and scalings in Eq. (4) produces

$$C \eta^\nu b^2 = D \eta^{(\nu+1)/2} b^2 + \eta^{2\nu-1} b^4, \quad (5)$$

where C and D are positive constants.

The simplified power balance, Eq. (5), can be rearranged into a relation for the fluctuation level as a function of S ,

$$b^2 \sim C S^{-(1-\nu)} - D S^{-3(1-\nu)/2}, \quad (6)$$

where the domain of S , which is proportional to η^{-1} , has a lower bound so that $b^2 > 0$. At low S , the fluctuation level increases with S , consistent with the simulation results. At high S , Eq. (6) suggests that the contribution of the Ohmic term becomes negligible and that the fluctuation level decreases with increasing S . Since J_i is proportional to $b^2 S^{1-\nu}$, the relaxation of the λ profile is a monotonic increasing function of S . This is also consistent with the simulations in their limited range.

V. SUMMARY AND CONCLUSIONS

The MHD simulations reported here demonstrate that DC helicity injection can drive axial (toroidal) current in a resistive plasma with no net axial electric field. When the poloidal magnetic field induced by the driven current is larger than the applied vertical field, the electrostatic potential sustains an unstable current profile. This produces resistive MHD modes that partially relax the current profile by the well-understood MHD dynamo mechanism. This relaxation generates closed contours of poloidal flux. There are also nonlinear effects. The most important is the coupling between the applied vertical fields and the resonant fluctuations, which enhances the relaxation in large aspect ratio cases.

Complete relaxation to a constant J/B state does not occur in this driven system. The current density profiles remain quite hollow and do not resemble tokamak profiles. In addition, the relatively large fluctuation level of 1% yields a stochastic field-line puncture plot, which would lead to significant energy transport in an actual experiment. The current profile becomes less hollow when S is increased, but this is accompanied by an increasing fluctuation level. A heuristic quasilinear power balance suggests that the fluctuation level decreases with S in the limit of large S , but this range has not been observed in the simulations.

The simulation results agree with the HIT experiment in that robust flux amplification and hollow current profiles occur, but there is not agreement on the extent of either phenomenon. In CDX a peaked current channel arises near the cathode. The toroidally localized, thermionic cathode arrangement in this experiment emits a beam of electrons that may have kinetic properties that are outside the scope of MHD.

In terms of the three important issues for using DC helicity injection as a sole means of sustaining tokamaks, the simulation results are discouraging. While the electrostatic current drive has been reproduced, the extent of relaxation is weak, and it is accompanied by a significant fluctuation level. The relaxation improves as S is increased, and optimization of the geometry or including toroidal effects may help. However, it is unlikely that an order of magnitude improvement in dynamo power—which would be necessary for significant relaxation—can be realized. Furthermore, any improvement in the dynamo current drive will come with magnetic fluctuation levels that are even larger than those in the simulations. We therefore do not expect that DC helicity injection will be successful in sustaining tokamaks with good confinement properties.

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¹¹The importance of nonlinear coupling suggests that poloidal coupling from toroidal effects may also play an important role, especially in low aspect ratio cases. Such effects are beyond the scope of this study.

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