Parametric Variations in Dynamic Models of Induction Machine Clusters

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Abstract — This paper presents a probabilistic approach to the characterization of dynamical models of induction machine clusters. Our method derives bounds on eigenvalue variations for linearized models expressed in terms of stochastic norms. In our examples of modeling of power system loads this characterization tends to be less conservative than alternative deterministic approaches. We consider examples of induction machines with different ratings (classes), and allow for wide variations of electrical and mechanical parameters. We describe a stochastic norm approach to: 1) efficiently describe the dynamical model variations for a cluster of similar machines without having to perform repeated eigenvalue calculations, e.g. in a wind farm application, and 2) suggest the order of the reduced model in power system load modeling where the tightness of the bounds of eigenvalue variations is used for guidance in decisions regarding the number of different classes that would efficiently represent a given composite load.

Key Words — load modeling, probabilistic methods, induction machines, parametric variations.

I. INTRODUCTION

The goal of this paper is to examine the effects of parameter variations on linearized models of induction machines. This sort of dynamical characterization is useful in modeling, operation and control of large clusters of induction machines often encountered in energy systems as power sources or loads (e.g. wind farms, industrial loads). The interval matrix methodology from control system literature [1] is of limited use at this point as analytical results are missing for systems of realistic order and the number of Monte Carlo simulations needed for reliable conclusions is quite heuristic. Even if available, such deterministic results tend to be conservative (with the possi-

ble implications of underutilization of available resources), as they include parameter variations that are unlikely (or even impossible) in actual operation. The method presented here is based on a statistical description of expected parameter variations; the user has an explicit handle on the “degree of certainty” that is required for dynamical assessments such as stability and model variability.

This work is also intended to provide information that will be useful for aggregate load modeling in power systems. Specifically, it will help identify the required order of the dynamic model by identifying distinct clusters of eigenvalues associated with load dynamics. The aggregate modeling of power system load components is necessary since it is not feasible to include a dynamic model for every device connected to the system. Despite numerous efforts reported in the engineering literature, accurate load modeling remains challenging. To account for dynamics associated with energy storage capabilities, models of individual load components are used to represent all of the loads of a certain type. For example, a single induction motor model, or small number of motors, may be used to represent the entire motor portion of the load. It is unreasonable to expect that such an aggregate will exactly capture the load dynamics. It is hoped instead that the model will reflect some basic, physical property associated with motor loads.

It has been suggested in the literature that one, two, or more individual induction motor models may be necessary to accurately capture the dynamics of the entire motor load (see [2] and its references). In [3] it is noted that the “large” and “small” motors used in that study exhibit significantly different behavior, implying that different models need to be used for each. This paper discusses the use of stochastic norms to determine when a single or small number of individual component models may be used to represent an aggregate for an entire class of loads. Specifically, we examine aggregate induction motor models. Analogous analyses are feasible for other types of (linearized) dynamic loads.

In a previous paper we noted that an aggregate model for many similar induction motors connected in parallel was possible if the eigenvalues of the all the individual motors appeared in identifiable clusters [4]. In that paper and in [5] we introduced preliminary results using the stochastic norm as a tool for load modeling, but we have predominately addressed variations in mechanical loading, and developed an

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expected value dynamic system to represent the machines. The main contribution of this paper is an optimization procedure to determine the least conservative weighting matrices for stochastic norm-based calculations. We examine linearized models for many induction motors (large and small) connected in a parallel cluster. A straightforward approach to eigenvalue clustering is to directly calculate eigenvalues for nominal "small" and "large" motors with random variations in machine parameters, loading, and voltage, and then visually inspect the results to determine if any identifiable clusters appear. This procedure easily becomes computationally intensive and quite subjective, especially if more classes of machine sizes are to be included. Instead, we propose a systematic approach in which nominal eigenvalues associated with the (nominal) models are computed first, and stochastic norm theory is applied to estimate bounds on the variations from the nominal eigenvalues based on a statistical description of parameter, loading, and voltage variations. The minimum order of the system aggregate models is determined by: 1) the number of (perceived) distinct regions in the complex plane, and 2) by the tightness of the stochastic norm bound in the case of overlapping regions. As a procedure, one would increase the order of the aggregate model until the tightness of the stochastic norm does not improve significantly (in a statistical sense). This will yield the order of the aggregate model. Philosophically, this approach is analogous to model order determination techniques used in the system identification literature [6] except here the stochastic norm is used as the measure.

In Section II we present a standard induction machine model, mostly for the purpose of introducing the notation; in Section III we summarize our simulations and in Section IV we provide background on the stochastic norm. In Section V we formulate and solve an optimization problem that results in non-conservative weighting matrices for the modeling and analysis purposes in the proposed framework.

II. INDUCTION MACHINE MODEL

We consider induction machine clusters containing machines of several (two in the example) classes (power ratings) connected in parallel. Each machine has different mechanical load, and electrical and mechanical parameters that vary within bounds for the respective class. Nominal machine data used in our study are from [7], and parameter variations described in Tables 1 and 2 correspond to extremal values for each class listed in [7]. One of the machine classes corresponds to nominal rating of 11 kVA, while the other has rating of 877 kVA. These two classes outline the machine rating span often encountered in power system loads and in alternative energy sources (e.g. wind farms). Both sets of parameters are based on a 100 kVA power base.

We write a $D - Q$ model for each induction machine in a

cluster as

$$2H \frac{ds}{dt} = T_m - E'_D I_D - E'_Q I_Q \quad (1)$$

$$T_o \frac{dE'_D}{dt} = -E'_D + s \frac{X_r}{R_r} E'_Q - \frac{X_m^2}{X_r} I_Q \quad (2)$$

$$T_o \frac{dE'_Q}{dt} = -E'_Q - s \frac{X_r}{R_r} E'_D + \frac{X_m^2}{X_r} I_D \quad (3)$$

$$0 = V_D - E'_D - R_s I_D + X' I_Q \quad (4)$$

$$0 = V_Q - E'_Q - R_s I_Q - X' I_D \quad (5)$$

We can solve the algebraic constraints for the current to achieve a third-order system of nonlinear differential equations with the state vector x being (for example) the slip, s , and E'_D and E'_Q . Linearization of the equations around an operating point gives a linear model in standard state-space form:

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} \quad (6)$$

where $\tilde{x} = [\tilde{s} \ \tilde{E}'_D \ \tilde{E}'_Q]^T$ and $\tilde{u} = [\tilde{T}_m \ \tilde{V}]^T$ are increments of state variables and inputs, respectively. We are interested in an efficient characterization of changes in A (denoted as ΔA) as machine parameters, mechanical load, T_m , and supply voltage, V , vary.

III. MONTE CARLO SIMULATIONS OF PARAMETRIC VARIATIONS

In this section we consider the case of simultaneous, independent variations of several machine parameters and operating conditions. We apply a nominal 1 p.u. voltage to all machines (equivalent to the assumption that the machines are connected through ideal transformers to a common bus). We assume (for simplicity) that the (independent) parameter deviations are governed by normal probability density functions with means, standard deviations and allowed limits given in Tables 1 and 2. The method presented here is equally applicable to other probability densities. Whenever a randomly generated parameter value in our Monte Carlo simulations falls outside the range in the respective table, we assign it to the nearest value in the table (such events are quite infrequent, as the assumed probability densities are concentrated in the given parameter limits).

Here $X_r = X_s = X_m + X_{rs}$, $X' = X_s - X_m^2/X_r$ and $T_o = X_r/(120\pi R_r)$ (with $120 \mapsto 100$ for a 50 Hz system). For the class of 11 kVA motors $X_m = 8.9$. The nominal A matrix (denoted with A_s) is then

$$A_s = \begin{bmatrix} 0 & 0.72 & 8.71 \\ -26.26 & -27.28 & 5.30 \\ -325.27 & -5.30 & -27.28 \end{bmatrix} \quad (7)$$

with eigenvalues $[-13.76 \pm j51.89 \quad -27.02]$. Parameter variations for the class of large 877 kVA machines are given in Table 2 (with $X_m = 0.46$).

Table 1: Parameter Variations of the 11 kVA Induction Machine Model (100 kVA power base)

Param.	E[·]	$\sigma(\cdot)$	Limits
R_r	0.085	0.01	0.064–0.120
R_s	0.13	0.02	0.10–0.21
X_{rs}	0.60	0.05	0.40–0.77
H	0.048	0.01	0.018–0.110
V	1.00	0.01	0.95–1.05
T_m	0.07	0.02	0.005–0.15

Table 2: Parameter Variations of the 877 kVA Induction Machine Model (100 kVA power base)

Param.	E[·]	$\sigma(\cdot)$	Limits
R_r	0.0016	0.0003	0.0010–0.0048
R_s	0.0017	0.0002	0.0010–0.0027
X_{rs}	0.015	0.001	0.013–0.017
H	2.6	0.2	2–3
V	1.00	0.01	0.95–1.05
T_m	5	2	0.5–14

The nominal $A = A_l$ matrix is then

$$A_l = \begin{bmatrix} 0 & 0.33 & 6.42 \\ -36.01 & -20.37 & 4.34 \\ -354.19 & -4.34 & -20.37 \end{bmatrix} \quad (8)$$

with eigenvalues $[-10.37 \pm j46.87 \quad -19.98]$. Please note the large numerical differences in entries (1,3) and (1,2) of the A matrices – these will affect our attempt to produce a single machine equivalent for a cluster containing both types of machines. In Fig. 1 we show eigenvalues of the state matrix for the 11 kVA class, and in Fig. 2 we do the same for the 877 kVA class. We observe considerable overlap which may be surprising given the large difference in nominal ratings, but agrees with the entries of A matrices being comparable in a common power base.

We turn our attention next to the statistical properties of the *eigenvalues* corresponding to each class of machines. While these are very difficult to address analytically, we use the results of our simulations and in Figs. 3 we present the histogram of the real part of the eigenvalue from the conjugate pair in the second quadrant (denoted as λ_1) for the 877 kVA machine case (similar histograms are obtained for imaginary parts and for the 11 kVA class). These histograms are well approximated by a normal probability density, and the standard deviations of eigenvalues (moduli, real and imaginary parts) for both classes are given in Table 3. Our goal is to derive similar bounds on eigenvalue variations **without** having to perform eigenvalue calculations as a part of each

Figure 1: Eigenvalues for the 11 kVA example, 1000 simulations

Table 3: Eigenvalue Variations of the Induction Machine Models

Class	$\sigma(\lambda_1)$	$\sigma(\lambda_2)$	$\sigma(\lambda_3)$
11 kVA, Moduli	7.51	7.51	3.82
11 kVA, Real pt.	1.97	1.97	3.82
11 kVA, Imag. pt.	7.25	7.25	0
877 kVA, Moduli	3.26	3.26	4.15
877 kVA, Real pt.	2.16	2.16	4.15
877 kVA, Imag. pt.	2.44	2.44	0

simulation. We review the necessary analytical tools in the next section.

IV. BACKGROUND ON THE STOCHASTIC NORM

In this section we first review the stochastic perturbation theory as developed by Stewart [8]. In this framework the perturbed quantity is approximated by a first-order perturbation expansion, and the perturbation is described in statistical terms. The appealing property of these statistics is that they tend to be more realistic (less conservative) than the worst-case perturbation bounds obtained in terms of (deterministic) norms.

Suppose that the entries of a matrix A are random variables. We measure the size of this random matrix by the **stochastic** norm defined by

$$\|A\|_S^2 = \mathcal{E}(\|A\|_F^2) \quad (9)$$

where $\mathcal{E}$ is the expectation operator, and $\|\cdot\|_F$ is the Frobe-

Figure 2: Eigenvalues for the 877 kVA example, 1000 simulations

nus norm, i.e. $\|A\|_F^2 = \text{tr}(A^H A)$, where tr denotes the trace (sum of diagonal components), and A^H denotes the Hermitian (complex conjugate transpose) of A . It turns out that the stochastic norm satisfies a number of useful mathematical expressions that are common to other norms (see [8]).

We first need to establish the relationship between the stochastic norm and the actual error. In the most general case, this question is answered using the Chebyshev inequality, which asserts that it is improbable that a function of a random matrix will be much larger than the stochastic norm of that function (the function being an eigenvalue in our case). These bounds are very weak - if we have more knowledge about the statistical mechanism generating the perturbation matrix (e.g. independent, normally distributed entries), then we can establish substantially stronger results.

We will assume that the uncertain matrix A can be decomposed into the known fixed part (mean) $\hat{A}$ and perturbation ΔA :

$$A = \hat{A} + \Delta A.$$

Certain conditions have to be imposed on the perturbation matrix in order to yield a tractable analysis. We assume that the perturbation can be written in the form

$$\Delta A = S_c E S_r^H \quad (10)$$

where S_c and S_r are positive semidefinite scaling matrices for the columns and rows, respectively, and the elements of E are uncorrelated with mean zero and variance 1. For such perturbations,

$$\|\Delta A\|_F^2 = \|S_r\|_F^2 \cdot \|S_c\|_F^2. \quad (11)$$

While the assumed form of ΔA is driven by requirements for analytical tractability, we show later in our examples that

Figure 3: Real parts of λ_1 for the 877 kVA example, 1000 simulations

an optimization problem can be formulated that produces very efficient scaling matrices for the modeling of clusters of induction machines.

Let us interpret the stochastic norm in the case when ΔA is a 3×3 matrix, $S_c = S_r = I$, and entries of E are independent, normal random variables with unit variance. Then $\|\Delta A\|_F^2$ is a sum of 9 independent random variables y_k , each of which has mean 1, variance 2, and χ_1^2 probability distribution. Thus $\|\Delta A\|_F = 3$, and let us calculate the probability that $\|\Delta A\|_F > 1.5\|\Delta A\|_F$. The probability turns out to be 0.0164 (using tables for χ_9^2 from [9]). Thus the stochastic norm in this case characterizes the matrix very well, as there is only a 1.6 % probability of the Frobenius norm of an actual perturbation exceeding the stochastic norm by more than 50 %.

Next we consider the sensitivity of an (non-repeated) eigenvalue of A (denoted in this section with λ_1) to the perturbations ΔA described in (10). We relegate details of the sensitivity result to the appendix; y_1 is a vector related to the left eigenvector corresponding to the eigenvalue λ_1 (the exact definition is given in the Appendix, Equation (26)), and let v_1 be the corresponding unit right eigenvector. The eigenvalue sensitivity result in [8] then establishes that

$$\|\Delta \lambda_1\|_S = \|S_c y_1\|_2 \|S_r v_1\|_2 \quad (12)$$

when the norms on the right hand side are standard Euclidean vector norms. For comparison we recall the standard (deterministic) eigenvalue sensitivity result (see e.g. [10]) for a known, fixed perturbation $\Delta \hat{A}$:

$$\Delta \lambda_1 = \frac{w_1^H \Delta \hat{A} v_1}{w_1^H v_1} \quad (13)$$

where w_1 is the left eigenvector associated with the eigenvalue λ_1 . We note a similarity between the expression (13) and (12), especially if the eigenvectors are normalized such that $w_1^H v_1 = 1$.

A critical issue in applications of stochastic norms is the choice of the positive semidefinite matrices S_c and S_r . Let $S_r^2 = S_r^T S_r$ and $S_c^2 = S_c^T S_c$. It is shown in [8] that if we denote the columns of ΔA with a_k, a_l , then their covariance equals

$$\text{Cov}(a_k, a_l) = (S_r^2)_{k,l} S_c^2 \quad (14)$$

where $(\cdot)_{k,l}$ denotes the k, l -th entry of the matrix. In our examples, $\Delta A, S_c$ and S_r are 3×3 , and $\text{Cov}(A)$ is 9×9 , with the diagonal terms being variances of corresponding entries in ΔA . The previous expression can be shortened using the tensor (Kronecker) matrix product notation as

$$\text{Cov}(A) = S_r^2 \otimes S_c^2. \quad (15)$$

V. CHARACTERIZATION OF EIGENVALUE VARIATIONS

V.1. Complete Information on Cluster Composition

We consider first the case in which we know in advance the classes comprising a machine cluster (their nominal parameters and covariance matrices). Our procedure for generating S_c and S_r that will approximate the matrix variations due to parameter changes is based on computer simulations. These simulations involve the generation of (a limited number of) A matrices and the calculation of the (sampled) covariance matrix for the entries. We then perform an optimization procedure to determine the (positive semidefinite) scalings S_c and S_r that best approximate the given (sampled) covariance matrix. The scalings in turn yield bounds for eigenvalue variations. The information contained in sampled covariance matrix can come from simulations (as in our example), from repeated dynamic model identification studies [6], or from knowledge of the variability of the underlying physical processes (e.g. in industrial loads, or in wind farms).

Let $\text{Cov}(\hat{A})$ denote the sampled (experimentally obtained) covariance matrix. Then we pick matrices S_c and S_r as to achieve

$$\min \|\text{Cov}(\hat{A}) - S_r^2 \otimes S_c^2\|_2 \quad (16)$$

subject to the definiteness constraints

$$S_c \geq 0 \quad (17)$$

$$S_r \geq 0. \quad (18)$$

The choice of norm in (16) is very important – if the induced two-norm is replaced by the Frobenius norm, then the characterization of the eigenvalues becomes inferior. This is to be expected as the two-norm describes the largest amplification that a matrix can give to a unit vector, while the Frobenius

norm has no such directional information. We solve this non-linear optimization problem using a Newton-Raphson procedure with penalty factors for constraint violations.

We illustrate this procedure on the class of large machines (877 kVA) – our optimization yields

$$S_c = \begin{bmatrix} 1.0366 & -0.0010 & -0.0466 \\ -0.0010 & 4.5578 & -0.9289 \\ -0.0466 & -0.9289 & 1.7099 \end{bmatrix} \quad (19)$$

and

$$S_r = \begin{bmatrix} 3.1791 & -0.0329 & -0.4105 \\ -0.0329 & 1.2102 & -0.0471 \\ -0.4105 & -0.0471 & 0.2780 \end{bmatrix} \quad (20)$$

These choices of S_c and S_r yield $\|\Delta\lambda_{1,l}\|_S = 3.17$, which is very close to the sampled standard deviation value of 3.26. Furthermore, if one perturbs A_l with the chosen S_c and S_r , and performs eigenvalue analysis, then the resulting eigenvalue patterns are very similar to those obtained by varying machine parameters.

When the same optimization is performed for a cluster consisting of small (11 kVA) machines, the results parallel the ones just presented: the optimal S_c and S_r yield $\|\Delta\lambda_{1,s}\|_S = 6.89$, which again is close to the sampled standard deviation value of 7.51.

In Fig.4 we present results for a cluster consisting of machines of both classes (recall that this cluster composition information is assumed to be available), and we plot the circles with radii $2 \times \|\Delta\lambda_1\|_S$ for each class, centered appropriately. It appears that in this case an equivalent model consisting

Figure 4: Eigenvalues and bounds for both classes, 1000 simulations each

of a single machine (with $\|\Delta\lambda_1\|_S$ equaling that of the small machine cluster $\|\Delta\lambda_{1,s}\|_S$) would indeed be satisfactory.

V.1. Case 2: Incomplete Information on Cluster Composition

In our analysis of a cluster containing both types of machines we have been using the knowledge of the existence of two distinct classes, together with their nominal parameter values. It is interesting to investigate what happens if the cluster structure information is not available. We next consider the case when a data set (sample matrices) is available without any reference to the two machine classes that have generated it. Thus, we calculate the covariance matrix from a single simulation set consisting of 1000 cases of each class. The result is quite different from those corresponding to the individual clusters of large or small machines. The primary reasons can be traced back to the large differences in entries (1,3) and (1,2) of the A matrices for the two classes. The calculated $\|\Delta\lambda_1\|_S$ equals 13.36, which is substantially larger than the sampled (experimental) values for the either class. The cluster composition information thus appears to be crucial for obtaining non-conservative bounds on eigenvalue variations. Conversely, given this lack of tightness in the estimate, one is led to question the premise that the data has been generated by machines belonging to a single class, and further model refinements are clearly necessary (say by using two machine classes).

VI. CONCLUSIONS

In this paper we presented a statistical characterization of induction machine models under multiple parametric uncertainties. We suggested the use of the stochastic norm to characterize model variations under parametric variations. In cases when numerous sources of uncertainty are present, the stochastic norm approach could be an efficient way to obtain a useful, non-conservative bound. While we need statistics for the changes in A to calculate the stochastic norm, we do not have to perform numerically expensive eigenanalysis for each deviation in order to estimate the eigenvalue changes.

In our specific load modeling example we conclude that the described method could aid the power system analyst in deciding on the number of distinct machine classes. If the knowledge of the composition of the cluster is available, then one could construct an aggregate model for each machine class with the corresponding stochastic norm bounds. In our example, due to the overlap, a single machine would suffice. On the other hand, if the knowledge of the composition of a multi-class cluster is not available and one wrongly assumes a single class, then the resulting equivalent model does not yield a tight bound in the terms of a stochastic norm, suggesting the need for further data collection and model refinements.

APPENDIX

Following [8] we write the (uncertain) matrix A as

$$AV = V \text{ diag} \lambda_i \quad (21)$$

where V is the matrix of right eigenvectors (normalized to unit length in the standard, Euclidean vector norm), with columns v_1, v_2, v_3 . A unitary matrix G is formed next

$$G = [v_1 \ Y_2] \quad (22)$$

such that $G^H G = I$, and Y_2 is a 3×2 matrix obtained by orthogonalizing v_2, v_3 (say via Gram-Schmidt process). Then

$$G^H A G = \begin{bmatrix} \lambda_1 & A_{12} \\ 0 & A_{22} \end{bmatrix} \quad (23)$$

Next we form the matrix T

$$T = \lambda_1 I - A_{22} \quad (24)$$

and evaluate the matrix Q satisfying

$$Q = -A_{12} T^{-1} \quad (25)$$

Let us define y_1 as

$$y_1 = v_1 + Y_2 Q^H \quad (26)$$

It can be checked that (for v_1 of unit length and orthogonal to Y_2)

$$y_1^H A v_1 = \lambda_1. \quad (27)$$

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