

MIMO Capacity Scaling and Saturation in Correlated Environments

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Abstract—The capacity of MIMO systems under ideal (i.i.d.) channel conditions has been shown to increase linearly with the number of antennas. Capacity scaling is observed in a correlated channel environment as well but the constant of scaling is shown in [7] to be less than that of the i.i.d. scaling factor. Using a channel representation motivated by physical scattering considerations, the virtual channel representation, we show this effect does occur provided the number of effective scattering paths keeps asymptotically increasing with the increase in the number of antennas. Otherwise we show that saturation of capacity occurs. Using random banded matrix theory results, we also show that the effect of correlation is to reduce the effective received SNR according to the degree of correlation. This also yields a closed form expression for the asymptotic capacity of correlated MIMO channels.

I. INTRODUCTION

The capacity of MIMO systems has been shown to increase linearly with the number of antennas in an ideal i.i.d. channel [1], [2]. However an i.i.d. channel assumption is far from being true in realistic situations. Realistic channel realizations are usually modeled by a channel matrix whose entries are correlated Gaussian random variables. Chuah et al. [7] have shown that capacity scaling is still observed in correlated channels, but the scaling parameter is less than that observed in i.i.d. channels. However the correlation coefficient in [7] is on the order of 0.2 at an antenna separation of 0.5λ (due to the product correlation they assume) which is still not very significant.

Raleigh and Cioffi [3] have shown that in a high SNR situation the slope of the capacity vs. SNR curve is limited by the number of multipaths as well as the antennas. In this light, expecting a scaling in capacity inherently assumes a corresponding increase in the richness of the scattering environment. Moustakas et al. [4] point out that the capacity scaling factor increases with a decrease in correlation. But to date we have not seen a work on the connection between capacity per antenna and the number of multipaths for a correlated channel.

Adopting a channel representation that is based on a “beam-space interpretation”- the *virtual channel representation* [6] - we model a correlated environment as a virtual channel matrix with non-vanishing sub-matrices, each of which is modeled as a banded matrix with i.i.d. entries. The virtual channel matrix is obtained via a 2D-Fourier transformation of the actual correlated channel matrix. Such structure for the virtual channel matrix is motivated by physical scattering considerations [6]. Using this model, we show that capacity scaling¹ is observed provided the *bandwidth*² of the channel scales with the number

of antennas³. We observe two different regimes where capacity behavior differs drastically - one in which capacity scales (if bandwidth increases with the number of antennas) and the other where it saturates (if bandwidth remains the same even as the number of antennas is increased). Via the virtual channel representation, this behavior can in turn be related to the scattering geometry, configuration and distribution as a function of the number of antennas.

The physical interpretation of the above result is as follows. Capacity scales with the number of antennas if and only if increasing the number of antennas simultaneously brings in new scattering paths between the transmitter and receiver. If an increase in the number of antennas is not accompanied by a corresponding increase in the number of scattering paths, the capacity reaches an asymptotic limit as one can expect. Increase in effective number of scattering paths with increasing number of antennas is possible in diffuse scattering environments – the increased spatial resolution due to larger array aperture allows us to zoom finer and finer into the continuum of scatterer space [6]. On the other hand, for a finite number of scattering paths, this would not hold and we would expect saturation in capacity.

Using results from random banded matrix theory, we show that the capacity scaling parameter depends on the ratio of the bandwidth of the channel matrix to the number of antennas: k/N . Furthermore, we also show that the effect of a banded channel is to decrease the received SNR in proportion to the ratio k/N relative to an i.i.d. channel. This could be interpreted as a correlated scattering environment behaving just like an i.i.d. channel asymptotically, but with the received SNR scaled by the correlation parameter k/N .

The next section discusses the MIMO channel model used in this paper. Section III discusses some relevant results from Random Matrix Theory and Random Banded Matrix Theory. In Section IV, we analyze the capacity of a correlated channel using the virtual channel representation under two different observable regimes – capacity saturation and capacity scaling. Monte-Carlo simulations of a MIMO system are done in Section V to illustrate our results and conclusions are drawn in Section VI.

II. CHANNEL MODEL

Consider a multi-antenna system with N transmit and M receive antennas. The M -dimensional received signal \mathbf{y} and the N -dimensional transmitted signal \mathbf{x} are related by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

where \mathbf{n} is the noise and \mathbf{H} is the channel matrix coupling the transmit and the receive antennas. The statistics of \mathbf{H} depend

³In this paper, we assume that the number of transmitters is equal to the number of receivers, but extension to a non-equal configuration of transmitters and receivers is straightforward.

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¹We actually consider the average mutual information under an equal power distribution strategy at the transmitter and use the term capacity and average mutual information interchangeably.

²Bandwidth of a matrix is defined as the number of significant non-zero diagonals, e.g. a diagonal matrix has bandwidth 1 and a full $N \times N$ matrix has bandwidth N .

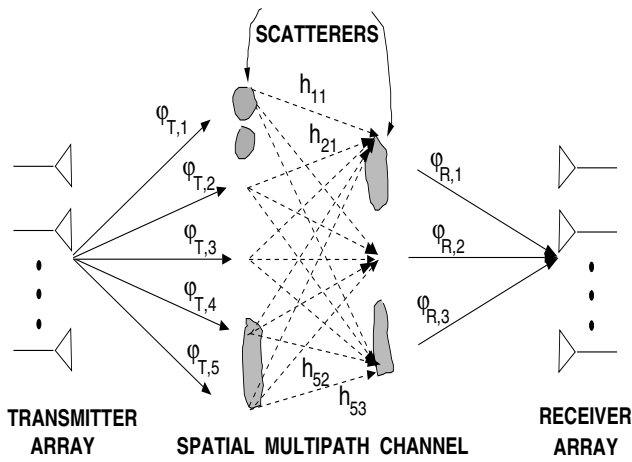


Fig. 1. A schematic illustrating the *virtual* representation of a scattering environment that corresponds to beamforming in the direction of fixed, virtual angles at the transmitter and the receiver.

on the antenna geometry, physical scattering environment, frequency of operation etc. Ideal channel modeling assumes that the entries of \mathbf{H} are i.i.d. Gaussian random variables [1], [2]. This makes the mathematical treatment tractable but realistic situations rarely have a rich scattering environment. Parametric *physical* models, on the other hand explicitly model signal copies from different directions [5].

We focus on one-dimensional uniform linear arrays (ULAs) of antennas at the transmitter and receiver and consider far-field scattering characteristics. The physical model [5] depends on the spatial angles as seen by the transmitters and the receivers in a non-linear fashion and thus makes mathematical analysis of information-theoretic issues like capacity difficult.

The recently introduced virtual channel representation [6] is a novel approach that connects the ideal statistical modeling and parametric physical modeling schemes. It imposes structure on the channel matrix \mathbf{H} by capturing essential characteristics of the physical scattering environment in a linear fashion.

The virtual representation exploits the finite dimensionality of the signal space⁴ to develop a *linear* channel representation that uses spatial beams in *fixed, virtual* directions. The virtual channel representation, illustrated in Fig. 1, can be expressed as

$$\mathbf{H} = \mathbf{A}_R \mathbf{H}_V \mathbf{A}_T^H \quad (2)$$

where fixed virtual angles result in \mathbf{A}_R and \mathbf{A}_T (matrices formed of array steering and response vectors)[6]. The $M \times N$ matrix \mathbf{H}_V is the virtual channel matrix. Uniform sampling of the principal period $\theta \in [-0.5, 0.5]$ is a natural choice for virtual spatial angles making \mathbf{A}_R and \mathbf{A}_T unitary. The resultant \mathbf{H}_V is then unitarily equivalent to \mathbf{H} and captures all channel information. In fact, \mathbf{H}_V is the two-dimensional DFT of \mathbf{H} .

Realistic scattering environments can be modeled as a superposition of clusters with limited angular spreads. The virtual matrix \mathbf{H}_V offers an intuitive interpretation as different clusters correspond to different non-vanishing sub-matrices of \mathbf{H}_V . Furthermore the non-zero entries of the virtual matrix are approximately uncorrelated [6]. The uncorrelatedness gets better as we increase the number of antennas.

⁴Due to finite number of antenna elements and finite array aperture.

The number of non-vanishing uncorrelated elements of the virtual matrix control the channel correlation [6]. On one extreme is a “diagonal” virtual channel with non-vanishing elements only on the diagonal. Physically, this corresponds to a line of scatterers between the transmitter and receiver. The diagonal channel exhibits significant correlation. On the other extreme is a fully populated virtual matrix corresponding to a rich scattering environment – this yields an i.i.d. channel. Varying levels of correlation between the two extremes can be captured by a banded virtual matrix with varying numbers of non-vanishing diagonals.

Since the entries of the virtual matrix \mathbf{H}_V are uncorrelated we model it as a banded matrix with i.i.d. Gaussian entries whose real and imaginary components are $N(0, 1/2)$ ⁵. We could also control channel correlation by varying the number and size of sub-clusters and the number of diagonals in each sub-cluster. This is discussed further later.

III. RESULTS FROM RANDOM MATRIX THEORY

In this section, we present some relevant results from Random Matrix Theory (RMT) and Random Banded Matrix Theory (RBMT). Random matrices play an important role in modeling Hamiltonians in quantum mechanics over a wide-range of classically chaotic and integrable (regular) systems [12]. The analogy between quantum systems and MIMO channels is too striking to ignore. Idealized chaotic systems are like fully i.i.d. channels and regular systems are like a channel with sparse scattering whereas realistic systems interpolate between the two extremes just as in MIMO channels.

The most important class of random matrices is the Gaussian Unitary Ensemble (GUE) [9]. The GUE of order N is defined as the set of random Hermitian matrices with i.i.d. complex entries from a $N(0, 1)$ distribution whereas the real and imaginary parts of entries along the non-diagonals are from $N(0, 1/2)$ [9]. In our analysis of MIMO channels, the GUE corresponds to the fully i.i.d. channel.

A Random Banded Matrix ensemble (RBM) is defined as the set of random Hermitian matrices with entries that are non-zero only up to a certain bandwidth $k \leq N$. The entries along the main diagonal are from a real $N(0, 1)$ distribution whereas the non-zero non-diagonal entries have real and imaginary parts chosen from a $N(0, 1/2)$ distribution. In MIMO parlance, banded matrices correspond to banded virtual channel matrices. RBMs are however much more complicated than a full bandwidth matrix because of the lack of rotational symmetry which simplifies much of RMT.

The asymptotic analysis of the GUE (and hence an i.i.d. channel) is simplified because each unordered eigenvalue shows an identical statistical behavior. RMT predicts that the unordered eigenvalues of the GUE are highly correlated and thus we could assume the same behavior for the unordered eigenvalues of the GUE in an asymptotic analysis. A similar behavior is seen with a RBM provided the bandwidth of the RBM scales with N [11]. A simple illustration of the above fact is that a diagonal matrix (which does not scale) is clearly seen to have uncorrelated eigenvalues (the entries themselves)

⁵In actuality the entries are uncorrelated but not necessarily identically distributed.

whereas a fully i.i.d. matrix (bandwidth scales with size) has correlated eigenvalues which have a Wishart distribution.

The Wigner's semi-circle Law states that the scaled eigenvalues of the GUE follow a semi-circle density function with radius 2 [9]. Let \mathbf{A} be a GUE of order N and let

$$\rho_N(x) = \frac{1}{N} \sum_{i=1}^N \delta(x - \frac{\lambda_i(\mathbf{A})}{\sqrt{N}}), \quad (3)$$

be defined as the Density of States (DOS) of the GUE where $\lambda_i(\mathbf{A})$ is the i th unordered eigenvalue of \mathbf{A} . Then

$$\lim_{N \rightarrow \infty} \rho_N(x) = \frac{\sqrt{4-x^2}}{2\pi} \quad -2 \leq x \leq 2. \quad (4)$$

RBMT results point out that the asymptotic behavior of the DOS of a RBM is essentially the same as that of the GUE if its bandwidth keeps increasing with its size according to some power law [8]. The only difference is in the way the eigenvalues of an RBM are normalized. If $k = N^\beta$ for some $0 < \beta \leq 1$ and

$$\rho'_N(x) = \frac{1}{N} \sum_{i=1}^N \delta(x - \frac{\lambda_i(\mathbf{A})}{c\sqrt{k}}), \quad (5)$$

is the Density of States (DOS) of the RBM ensemble \mathbf{A} and $\lambda_i(\mathbf{A})$ is the i th eigenvalue of \mathbf{A} , then

$$\lim_{N \rightarrow \infty} \rho'_N(x) = \frac{\sqrt{4-x^2}}{2\pi} \quad -2 \leq x \leq 2 \quad (6)$$

where c is a scaling constant depending on the way the RBM is modeled⁶ and is on the order of $\sqrt[4]{2\pi}$. We note the scaling with $c\sqrt{k} = cN^{\beta/2}$ in (5) as opposed to scaling by \sqrt{N} in (3). This case of the RBM corresponds to a MIMO channel where virtual scattering paths increase as we increase the number of antennas, but not enough to result in an i.i.d. channel.

A RBM with a constant bandwidth corresponds to a MIMO channel in which as we increase the number of antennas, the virtual scattering paths do not increase proportionately. In this case if the size of the matrix is taken to infinity, then its eigenvalue equation can be cast in a transfer-matrix form. Using results from the theory of disordered systems, we find that the eigenvalues of such a matrix are localized, i.e. the eigenvalues of a finite RBM are with high probability much smaller than the square-root of the size of the matrix⁷. Specifically, if \mathbf{A} is a finite RBM and

$$\rho''_N(x) = \frac{1}{N} \sum_{i=1}^N \delta(x - \frac{\lambda_i(\mathbf{A})}{\sqrt{N}}), \quad (7)$$

$$\text{then } \lim_{N \rightarrow \infty} \rho''_N(x) = g(x) \quad (8)$$

where the density function $g(x)$ is a delta-like function with infinitesimal support.

IV. CAPACITY ANALYSIS OF CORRELATED MIMO CHANNELS

The average mutual information of a narrowband N transmitter, N receiver MIMO system under equal power transmission

⁶Various other modeling schemes for a RBMT do exist [8].

⁷Size of a $N \times N$ matrix is defined as N .

can be written as [1]:

$$C = \mathbf{E} \left[\log_2 \det \left(I + \frac{\rho \mathbf{H}_V \mathbf{H}_V^H}{N} \right) \right] bps/Hz \quad (9)$$

where $\mathbf{E}[\cdot]$ is the expectation over the statistics of \mathbf{H}_V . As we have noted in Section III, for a finite banded RBM the DOS converges to a delta function and for a scaling bandwidth RBM, to an appropriately normalized semi-circle density function. The asymptotic capacity analysis under equal power distribution requires the way the eigenvalues of $\mathbf{H}_V \mathbf{H}_V^H$ behave under different banded constraints on \mathbf{H}_V ⁸.

For the special case when the matrix has bandwidth N , a very general result from RMT [10] says that for a matrix of the form $\mathbf{H}_V \mathbf{H}_V^H$ where \mathbf{H}_V is a random complex matrix of i.i.d. Gaussian entries and size N ,

$$\lim_{N \rightarrow \infty} f_{\lambda_{\frac{\mathbf{H}_V \mathbf{H}_V^H}{N}}}(x) = \tilde{f}_{H_V}(x) = \frac{1}{2\pi} \sqrt{\frac{4-x}{x}} \quad (10)$$

$0 \leq x \leq 4$.

The striking coincidence between the Wigner's semi-circle law and the Silverstein's density function (viz. the Silverstein's density function is obtained from the Wigner's semi-circle law by a quadratic transformation of random variable) forces one to conjecture that a similar behavior should be observed in the case of a RBM. It is indeed the case and the proof of this claim is in the Appendix.

In the case when the bandwidth scales (increase in scatterer population is proportionate to increase in antenna size), the density function of the random variable $\frac{\lambda_{\mathbf{H}_V \mathbf{H}_V^H}}{N}$ can be given based on $\frac{\lambda_{\mathbf{H}_V \mathbf{H}_V^H}}{ck}$ (where c is an appropriately chosen constant), viz.

$$\frac{\lambda_{\mathbf{H}_V \mathbf{H}_V^H}}{N} = \frac{ck}{N} \left(\frac{\lambda_{\mathbf{H}_V \mathbf{H}_V^H}}{ck} \right). \quad (11)$$

Let $a = \frac{ck}{N}$ be a constant independent of N (This will happen if k/N is a constant in $(0, 1]$). Then the density function is seen to be

$$\lim_{N \rightarrow \infty} f_{\lambda_{\frac{\mathbf{H}_V \mathbf{H}_V^H}{N}}}(x) = \tilde{f}_{H_V}(x) = \frac{1}{2\pi a} \sqrt{\frac{4a-x}{x}} \quad (12)$$

$0 \leq x \leq 4a$. The above equation is a more generalized form of the asymptotic eigenvalue distribution for an RBM with bandwidth scaling analogous to (10). The similar form of the two equations suggests that for a RBM with bandwidth scaling, normalized eigenvalue behavior is similar to that of an i.i.d. matrix.

The capacity of a channel modeled by a banded virtual matrix can then be written for large N (see footnote 8) as

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{C}{N} &\approx \int_0^{4a} \log_2(1 + \rho x) \tilde{f}_{H_V}(x) dx \\ &= \frac{1}{2\pi} \int_0^4 \log_2(1 + \rho a x) \sqrt{\frac{4-x}{x}} dx = \lim_{N \rightarrow \infty} \frac{C_{iid}(\rho a)}{N} \end{aligned} \quad (13)$$

⁸According to the Random Matrix Theory conjecture, as $N \rightarrow \infty$ the distribution of a particular unordered eigenvalue over the realizations of \mathbf{H}_V converges to the empirical distribution of the eigenvalues for a single realization of \mathbf{H}_V . The same result has been conjectured for a RBM with bandwidth scaling.

where ρ is the received SNR and $C_{iid}(\rho)$ is the capacity of an i.i.d. channel at that SNR and

$$\lim_{N \rightarrow \infty} \frac{C_{iid}(\rho)}{N} = \log_2 \left[\frac{1 + 2\rho + \sqrt{1 + 4\rho}}{2} \right] + (\log_2 e) \frac{2\rho + 1}{2\rho} \left[\frac{\sqrt{4\rho + 1}}{2\rho + 1} - 1 \right].$$

The expression for $\frac{C_{iid}(\rho)}{N}$ is computed using Silverstein's empirical distribution function as is given in (10) ([10], [13]).

Thus, we see that the asymptotic capacity supported by a k -banded channel is the same as that of an i.i.d. channel except for a scaling of SNR. One way of looking at this result is that the k -banded correlated channel presents a received SNR that is the SNR of an i.i.d. channel scaled by the parameter of correlation (k/N). Thus, using the virtual representation we are able to characterize the growth of capacity with number of antennas in correlated channels.

The case of a finite banded matrix follows exactly as we proceeded above except that the eigenvalues of a finite RBM are uncorrelated. Since the empirical distribution of the eigenvalues of a finite RBM tend to a delta-like function as $N \rightarrow \infty$, the ratio $C/N \rightarrow 0$. This is expected as the number of scattering paths remains the same as the number of antennas increases.

We extend the above results for a single scattering cluster to multiple clusters. To model a highly correlated environment, consider a virtual matrix which is a superposition of sub-matrices each of which by itself is modeled as a k -banded matrix. If \mathbf{H}_V is modeled as a block matrix with square sub-matrices having distinct support in the transmitter and the receiver virtual angles, then we could write a closed form expression for capacity in terms of the statistics of these sub-matrices⁹. If \mathbf{H}_V is modeled as a L block diagonal matrix with each block a square N_i ($\sum_{i=1}^L N_i = N$) and modeled as a k_i banded matrix

$$\mathbf{H}_V = \text{diag} [\mathbf{H}_{V_1} \dots \mathbf{H}_{V_L}] \quad (14)$$

then the capacity can be written for large N as

$$C = \sum_{i=1}^L E_{\mathbf{H}_{V_i}} \left[\log_2 \det \left(I_{N_i} + \frac{\rho \mathbf{H}_{V_i} \mathbf{H}_{V_i}^H}{N} \right) \right]. \quad (15)$$

This can then be written as

$$C = \sum_{i=1}^L \sum_{j=1}^{N_i} E_{\lambda_j} \left[\log_2 \left(1 + \frac{\rho}{N} \lambda_{j, \mathbf{H}_{V_i} \mathbf{H}_{V_i}^H} \right) \right]. \quad (16)$$

The marginal unordered eigenvalue distributions of $\mathbf{H}_{V_i} \mathbf{H}_{V_i}^H$ are the same as long as its bandwidth scales with N_i . Only those clusters whose bandwidths scale with number of antennas contribute to the asymptotic limit on capacity. The remaining clusters reach a saturation and thus contribute nothing to capacity scaling. Physically, if more scattering paths are resolved with increasing number of antennas (zooming) [6], capacity will scale. Otherwise it will saturate. Thus if we denote

⁹If however the assumption of distinct supports in the transmitter and receiver is not valid, we could still use these results to obtain bounds for the capacity scaling parameter using the results on eigenvalue inequalities of sums of Hermitian matrices. Here one should note that cross terms would vanish due to the law of large numbers arising from the asymptotic analysis.

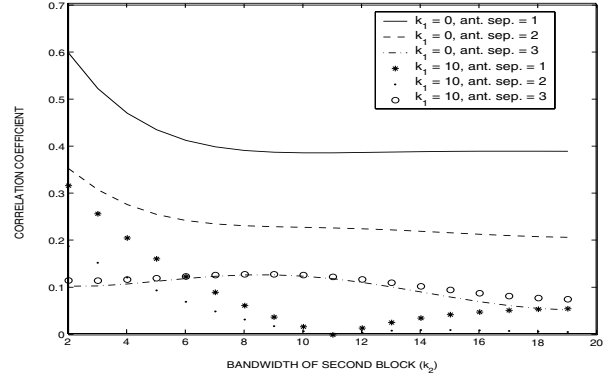


Fig. 2. A 57×57 channel matrix is modeled as a block matrix with three blocks of size 19, each of which has bandwidth k_1 , k_2 and k_3 with $k_3 = 10$ and k_1 and k_2 varied as in the figure.

by S the index of the set of sub-matrices of \mathbf{H}_V whose bandwidth scales with N , (16) becomes

$$\frac{C}{N} \approx \sum_{i \in S} \frac{N_i}{N} \left[\frac{1}{2\pi} \int_0^4 \log_2(1 + \rho a_i x) \sqrt{\frac{4-x}{x}} dx \right] \quad (17)$$

$$\lim_{N \rightarrow \infty} \frac{C}{N} = \sum_{i \in S} \frac{N_i}{N} C_{iid}(\rho \frac{c_i k_i}{N})$$

where k_i is the bandwidth of i th cluster in S^{10} .

V. NUMERICAL RESULTS

A correlated channel was simulated using \mathbf{H}_V with a block matrix structure depicting scattering clusters. A 57×57 channel matrix with three sub-matrices of size 19 each and with varying bandwidths was assumed. The correlation between different antennas (ant. sep. = 1 in the figure means correlations between antennas T_1 and T_2 say) is plotted in Fig. 2 as a function of the bandwidth of the matrix. The bandwidth of the third block k_3 is held constant at 10 and k_1 and k_2 are varied. It is noticeable that under the block model we can model a correlated channel with varying correlation by varying the bandwidth. Particularly significant is the higher correlation that can be modeled relative to the widely used product model for correlation of MIMO channels.

The average mutual information of a fading channel (using the virtual model) under equal power allocation at the transmitter was evaluated in Section IV. We also saw that capacity scaling and saturation are two different regimes of the same model. Fig. 3a is an illustration of the semi-circle law in a GUE. We consider a 200×200 random matrix and plot the empirical distribution of eigenvalues scaled by the (square-root of the) size of the matrix in Fig. 3a.

The empirical distribution of eigenvalues of an RBM ($N = 200$, $k = 15$) scaled by the bandwidth and the size, respectively, are plotted in Fig. 4a and b, respectively. If this ratio of k/N is maintained as N is increased to infinity the semi-circle density will be observed. Otherwise the density will shrink down to a delta function.

¹⁰Assuming the ratio $c_i k_i / N$ is a constant independent of N for each sub-matrix of \mathbf{H}_V .

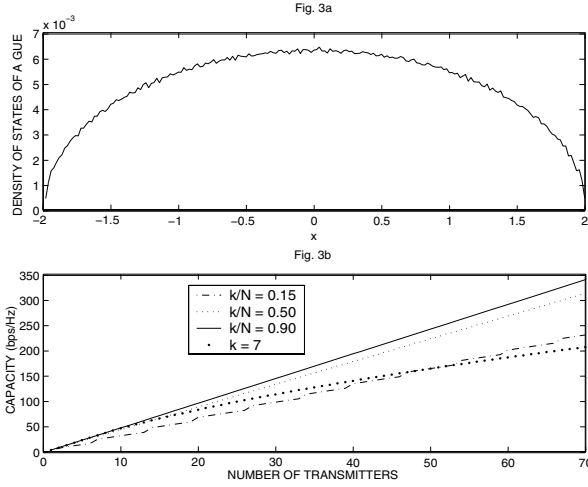


Fig. 3. The empirical distribution of eigenvalues for a GUE is plotted in *a* and *b* is a plot of capacity of a banded channel as a function of number of antennas for different values of bandwidth ratios k/N .

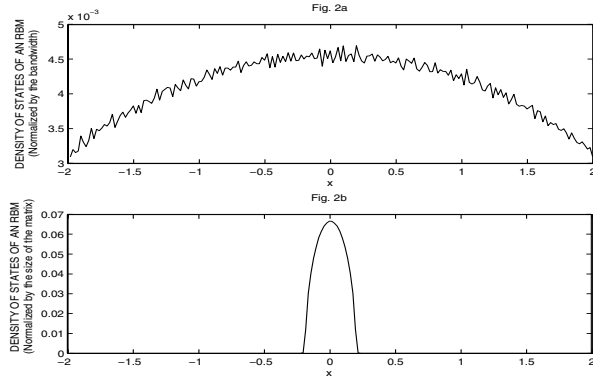


Fig. 4. RBM ensembles of size $N = 200$ and $k = 15$ were generated to plot the empirical distribution of the eigenvalues with bandwidth normalization in *a* and size normalization in *b*. This figure illustrates the semi-circle law in (a) and delta-like distribution in (b).

We simulated the channel under the virtual model and Fig. 3b is a plot of how the two different capacity regimes are easily observable with a single cluster virtual channel matrix as bandwidth of the matrix is changed from a constant to a parameter proportional to the size of the matrix. The parameter k/N is changed from 0.15 (low scatterer concentration) to 0.9 (high scatterer concentration). The plots were generated by averaging over 10,000 independent channel realizations.

VI. CONCLUSIONS

We have used results from Random Banded Matrix Theory for assessing capacity of a correlated MIMO channel in the limit of large number of antennas. The virtual channel representation allows us to use RBMT for capacity analysis of MIMO channels and we see that scaling and saturation are two sides of the same coin. The capacity scales provided that as we increase the number of antennas a proportionate number of scattering paths are resolvable. If fixed scattering paths are resolvable then the capacity reaches an asymptotic limit. The ability to resolve new scattering paths is modeled by the ratio k/N of

the banded virtual matrix and the capacity scaling parameter is seen to be proportional to this ratio. This result is in agreement with what is reported in [7] except that [7] does not identify a region where capacity does not scale. In an asymptotic sense, a correlated channel is like an i.i.d. channel except for the received SNR reduced by the factor k/N . This is formalized by a closed form expression for the asymptotic capacity of correlated MIMO channels that we obtain.

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APPENDIX

A. Proof of Claim in Section IV

The workhorse of our proof is the theorem by Grenander and Silverstein [14] which was extended to the complex case in the Appendix of [15]. This theorem states that as long as each row and column of \mathbf{H}_V have D i.i.d. $N(0, 1)$ complex entries and D scales with N , $\frac{\lambda_{\mathbf{H}_V \mathbf{H}_V^H}}{D}$ converges to the Silverstein's density function. The proof then proceeds using the fact that the norm of the differences in empirical distribution functions of two matrices $\mathbf{A}\mathbf{A}^H$ and $\mathbf{B}\mathbf{B}^H$ is bounded by the ratio of the rank of $\mathbf{A} - \mathbf{B}$ and their size [14]. Using one of these matrices as a D -connected channel matrix [15] and the other as a banded \mathbf{H}_V with $D = ck$, we are done for $k = aN^\beta$, $\beta \in (0, 1)$ where a and c are constants. We are done for $\beta = 1$ by the left continuity of the norm operator for a fixed N and a .

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