



The Impact of Cooperative Guided Reflection on Student Learning: The Case of Optimization Problem Solving in Calculus I

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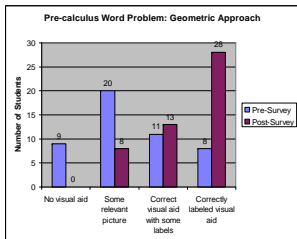
What is CGR?

Cooperative Guided Reflection (CGR) is an assignment that synthesizes mathematical goals with the pedagogical goals of writing to learn and cooperative learning.

- ❖ Guided Reflection—Students use a list of prompting questions to reflect on their correct solution of a mathematics problem.
- ❖ Cooperation—Students work outside of class in assigned groups to complete the CGR assignment.
- ❖ Writing—Students write about their reflections, their mathematics processes, and their collaboration.

This study examines the use of CGR to improve student learning on the topic: optimization problem solving in Calculus I.

CGR as an Aid in Knowledge Transfer



Students transferred knowledge of problem solving in calculus optimization to problem solving in an algebra setting. Student writing indicates that CGR is instrumental in this knowledge transfer.

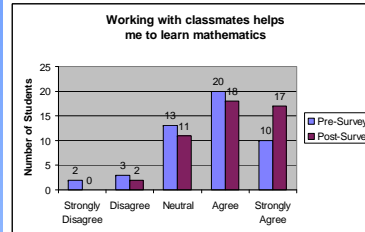
“This problem taught me a lot about labeling the picture to make it easier to work out. When I labeled it the easy way, finding the derivative was much easier.”

“You have to get the $(x-12)$ and the $(y-8)$ before you can even start the problem.”

“Since I received help on number 16, I didn’t need help on this one, being as the concepts were the same.”

- ❖ Five rubrics used to analyze student solutions: verbal, geometric, algebraic, numerical, overall solution
- ❖ 69% of students improved geometric/visual thinking
- ❖ 65% of students improved algebraic/symbolic thinking
- ❖ 71% of students improved their overall score

Value of Cooperative Work in Mathematics



33% of students raised the level they place on the value of cooperative work in mathematics

Students use CGR to reflect on the value of working with classmates to learn mathematics.

“We solved this by going over every step and talking through it.”

“It was fun because we worked very well together to get the answer.”

“Only Max in our group had this one done and I relied very much on him to help me get through this one.”

“This problem was difficult for me, but with the help of my group, I got it figured out.”

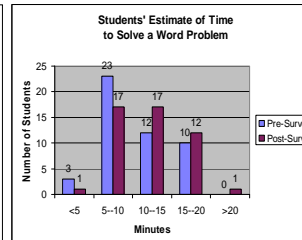
Estimate of Time to Solve a Mathematics Word Problem

Through CGR students reflect on how much time they need to solve mathematics word problems, even though time is not addressed directly in the assignment.

“It definitely takes time to get your head around the information.”

“We learned that we need to take our time when we looked back and found some simple multiplication mistakes.”

“I have come to believe that of the problems I do, there are no hard problems, just long problems.”



35% of students increased their time estimate.

Exam Score Comparison with non-CGR students

	CGR students – two sections of Calculus I instructed by the investigator (N=26 and N=25)	Non-CGR students – one section of Calculus I with a different instructor (N=16)
Common Hour Exam Test Item	mean score: 9.6/15 points*	mean score: 8.9/15 points**
Common Final Exam Test Item	mean score: 9.2/15 points*	mean score: 9.0/15 points **
Students attending to minor issues: domain of function and test of critical number	38%	0%

*Not a statistically significant difference ($p=.29$)

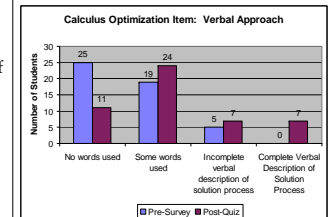
**Not a statistically significant difference ($p=.18$)

Verbal, Graphical, Algebraic and Numerical Approaches to Calculus

Student writing demonstrates that CGR aids student growth in all four approaches to calculus.

“This problem involved a lot of numerical thinking, and as with most of the problems, verbal thinking as a group.”

“Richard explained to us that we needed to then take those numbers and place them into the equations of the sides. We did a lot of our thinking verbally for this problem.”



- ❖ Five rubrics used to analyze student solutions: verbal, geometric, algebraic, numerical, overall solution
- ❖ Verbal rubric: 53% of students used words more and more appropriately on the post-quiz than on the pre-survey
- ❖ Geometric rubric: 90% of students improved their used of a visual aid
- ❖ Symbolic rubric: 98% of students made improvements in their algebraic thinking
- ❖ Numerical rubric: 41% of students improved their level of numerical approach
- ❖ Overall solution: 96% improved their score

Conclusions

- ❖ Through CGR student are getting a deeper understanding of what it means to think algebraically, graphically, numerically and verbally. They are separating these ideas; they are attaching the words to the ideas; and they are growing in their understanding of the ideas.
- ❖ CGR is an aid in transferring mathematical knowledge.
- ❖ CGR is an aid in maturing mathematical attitudes and beliefs especially: the applicability of calculus, estimate of time to solve a problem and the value of cooperation in learning mathematics.
- ❖ CGR is an aid in developing students’ resources, heuristics and control.
- ❖ Student exam performance is not diminished by the CGR assignment. Requiring students to focus on a supplementary task and attendant ideas did not impair exam performance.